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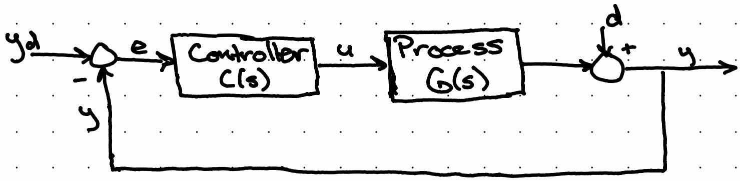
4/21/21
CH E 152B HW3

Exercise 6 MIMO Transfer Functions

Consider a multivariable process with process transfer function matrix $G(s)$ and controller transfer function matrix $C(s)$.



a) Give the four closed-loop transfer fn's between the following vector-valued signals: from setpoint y_d and output disturbance d to measured output y and manipulated input u.

b) Are any of these the sensitivity function $S(s)$? Which one(s) and what does it measure about the closed-loop system?

c) Are any of these the complementary sensitivity function $T(s)$? Which one(s)?

d) What is the relationship between $S(s)$ and $T(s)$?

Solution:

a) First, we will find the transfer fn's between y and y_d and d:

$$y = d + GC(y_d - y)$$

$$\hookrightarrow (I + GC)y = d + GCy_d$$

$$\hookrightarrow y = (I + GC)^{-1} d + (I + GC)^{-1} GC\, y_d$$

Then, we will find the transfer fn's between u and y_d and d:

$$u = C(y_d - y) = C(y_d - d - Gu)$$

$$\hookrightarrow (I + CG)u = Cy_d - Cd$$

$$\hookrightarrow u = (I + CG)^{-1} C\, y_d - (I + CG)^{-1} C d$$

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Exercise 6 (cont.)

a. cont.) So, we have:

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} (I+GC)^{-1}GC & (I+GC)^{-1} \\ (I+CG)^{-1}C & -(I+CG)^{-1}C \end{bmatrix} \begin{bmatrix} y_d \\ d \end{bmatrix}$$

b) Yes, the transfer function between y and d is the sensitivity function $S(s) = (I+GC)^{-1}$.
This function measures the sensitivity of the output y to changes in the output disturbance d.

c) Yes, the transfer function between y and y_d is the complementary sensitivity function $T(s) = (I+GC)^{-1}GC$.
This function measures the sensitivity of the output y to changes in the set point y_d.

d) $S(s)$ and $T(s)$ sum to equal the identity, as shown:

$$(I+GC)^{-1} + (I+GC)^{-1}GC = (I+GC)^{-1}(I+GC) = I$$

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Exercise 7 (18.21 in SEMD)

See SEMD for problem context.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-0.04e^{-0.1s}}{0.11s+1} & \dfrac{0.0005e^{-0.15s}}{0.21s+1} \\ \dfrac{0.22}{0.12s+1} & \dfrac{-0.02}{0.21s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where y_1 = normalized (dimensionless) blood pressure
y_2 = normalized urine production rate
u_1 = rate of hydrochlorothiazide ingestion
u_2 = rate of oxybutynin ingestion

a) Calculate the relative gain array
b) What loop pairing would you suggest?

Solution:

a) To calculate the RGA, we first compute the steady-state gain matrix K, as follows:

$$K = \lim_{s \to 0} \begin{bmatrix} \dfrac{-0.04e^{-0.1s}}{0.11s+1} & \dfrac{0.0005e^{-0.15s}}{0.21s+1} \\ \dfrac{0.22}{0.12s+1} & \dfrac{-0.02}{0.21s+1} \end{bmatrix}$$

$$= \begin{bmatrix} -0.04 & 0.0005 \\ 0.22 & -0.02 \end{bmatrix}$$

Then, since this gain matrix is invertible, we can compute the RGA with the formula: $K \otimes (K^{-1})^T$ where $\otimes$ is the Schur product (component-wise multiplication).
Computing this in MATLAB, we find:

$$\Lambda = \begin{bmatrix} 1.1594 & -0.1594 \\ -0.1594 & 1.1594 \end{bmatrix}$$

b) Since the RGA is already close to the identity matrix, it clearly suggests the loop pairings y_1-u_1 and y_2-u_2.

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Exercise 8 (18.22 in SEMD)

A rapid thermal processing system for microelectronics manufacturing uses three concentric lamp heater arrays to keep the wafer temperature uniform. The gain matrix for the system is:

$$k = \begin{bmatrix} 3.38 & 2.50 & 0.953 \\ 3.20 & 2.38 & 0.986 \\ 3.13 & 2.33 & 1.054 \end{bmatrix}$$

The system experiences difficulties controlling all 3 temperatures uniformly. Examine possible control difficulties using RGA analysis.

Solution:

Since the given gain matrix is invertible, we can compute the RGA as follows:

$$\Lambda = K \otimes (K^{-1})^T$$ where $\otimes$ is the Schur product (component-wise multiplication)

Using MATLAB to perform this computation, we find:

$$\Lambda = \begin{bmatrix} 210.3310 & -211.1848 & 1.8538 \\ -390.9319 & 406.5781 & -14.6462 \\ 181.6009 & -194.3933 & 13.7924 \end{bmatrix}$$

Here, we can see that there is a very high level of interaction because the RGA is not close to being diagonal, and no swapping of rows and columns will be able to change that. Furthermore, most of the entries have very large magnitudes, indicating a high degree of sensitivity to uncertainty in the elements of k, since K will become singular if k_{ij} is changed to $k_{ij}\left(1-\frac{1}{\lambda_{ij}}\right)$. This means very small changes to elements in K can create significant changes in the system's controllability. Clearly, a MIMO controller will be needed due to the interactions, but this choice still won't alleviate the controllability concerns.