CH E 152B Homework 5

Connor Hughes

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Exercise 11: Identification of a SISO system from a PRBS test

Consider the dataset in the file `data-exercise-siso-1.mat` uploaded on the class website.

(a)

Plot the input and output data. By inspection from the initial few samples, can you tell what type of a system might create this behavior?

(b)

Estimate the transfer function using the function `tfest` from the `Matlab ID` toolbox. What order do you choose? Try different orders and pick an appropriate order based on confidence intervals. Plot the model fit to the dataset of your identified model.

(c)

Plot the step response of your identified model.

Solution:

(a)

The system is most likely second order (or higher) as we see from the first few time units of data that y has a steady state value of approximately -5 for an input of $u = -1$, but when the input switches to 1 and back to -1, the response y reaches values near 10 and -10, respectively, revealing significant overshoot and ruling out the possibility that the system is first order.

(b)

Estimating the system's transfer function using `tfest` in `MATLAB` confirms that the system is likely second-order. When estimates were made under the assumption that the system was first, second, third, and fourth order, the standard deviation of the estimated parameters was much smaller for the second-order model estimate than for the other options. See Figure 1 below for a comparison of the simulated response of the estimated model with the actual measured system response.

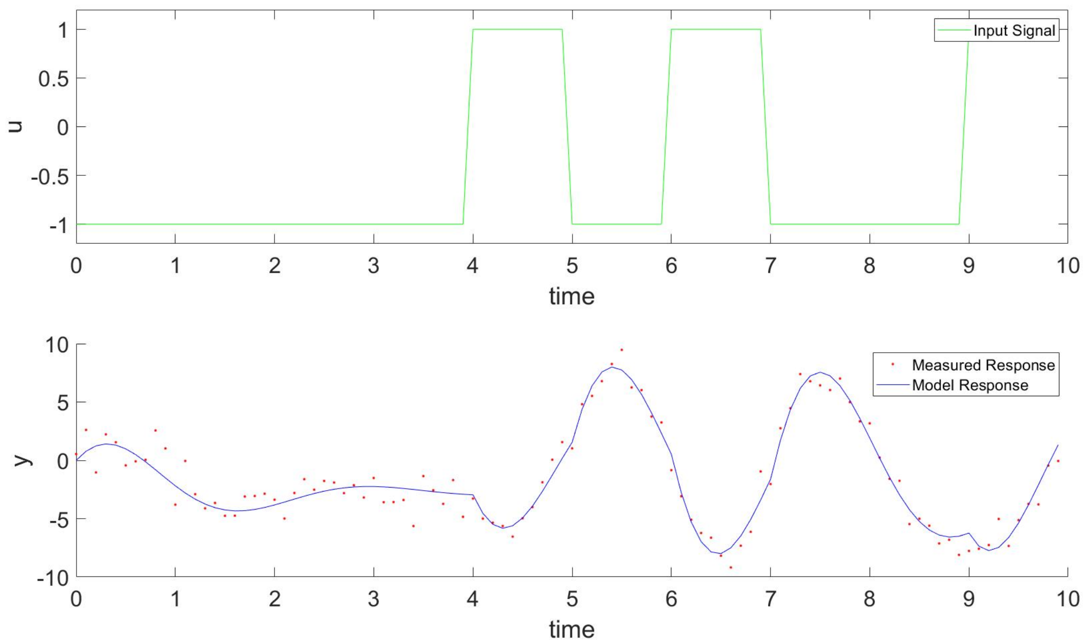

Figure 1: Plots of input signal, measured system response, and simulated response of the system model.

(c)

The step response of the system model can be seen in Figure 2 below.

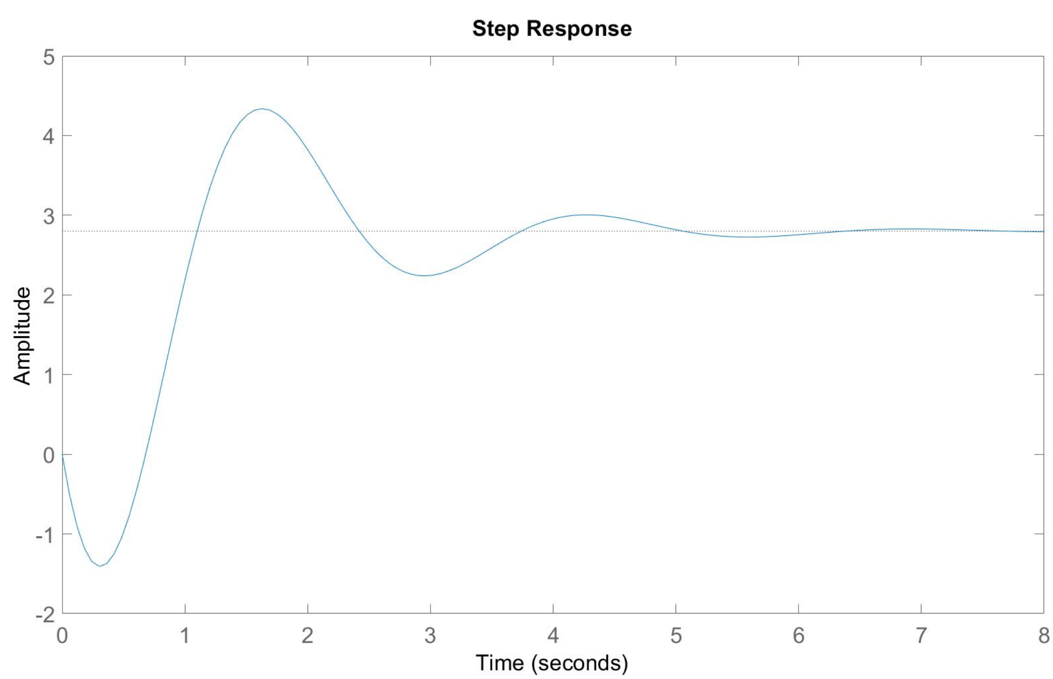

Figure 2: Step response of the estimated system model.

Exercise 12: Identification of a second SISO system from a PRBS test

Consider the dataset in the file `data-exercise-siso-2.mat` uploaded on the class website.

(a)

Plot the input and output data. By inspection from the initial few samples, what type of a system do you think might create this behavior? Is there any delay in the system from input to output?

(b)

Estimate the transfer function using the function `tfest` from the `Matlab ID` toolbox. What order you choose? Try different orders and pick an appropriate order, make sure to pass the delay to the `Matlab ID` toolbox. Plot the model fit to the dataset of your identified model.

(c)

Plot the step response of your identified model.

Solution:

(a)

From initial inspection of the data, it appears that the system is first-order, as we can see from the first few time units that the steady state response to the input $u = -1$ is approximately -3, which is the same value that y appears to asymptotically approach when u switches from 1 back to -1 later on in the PRBS test. There also appears to be a time delay, as the response seems to lag the input by about 5 units of time.

(b)

Estimating the system's transfer function using `tfest` in `MATLAB` confirms that the system is likely first-order with a time delay. When estimates were made under the assumption that the system was first, second, third, and fourth order, and the standard deviation of the estimated parameters was comparable between the first-order model estimate and the other options. However, when a time delay was introduced, the first-order model fit improved significantly and the standard deviations for the parameter values reduced to well below that of the other options. See Figure 3 below for a comparison of the simulated response of the estimated model with the actual measured system response.

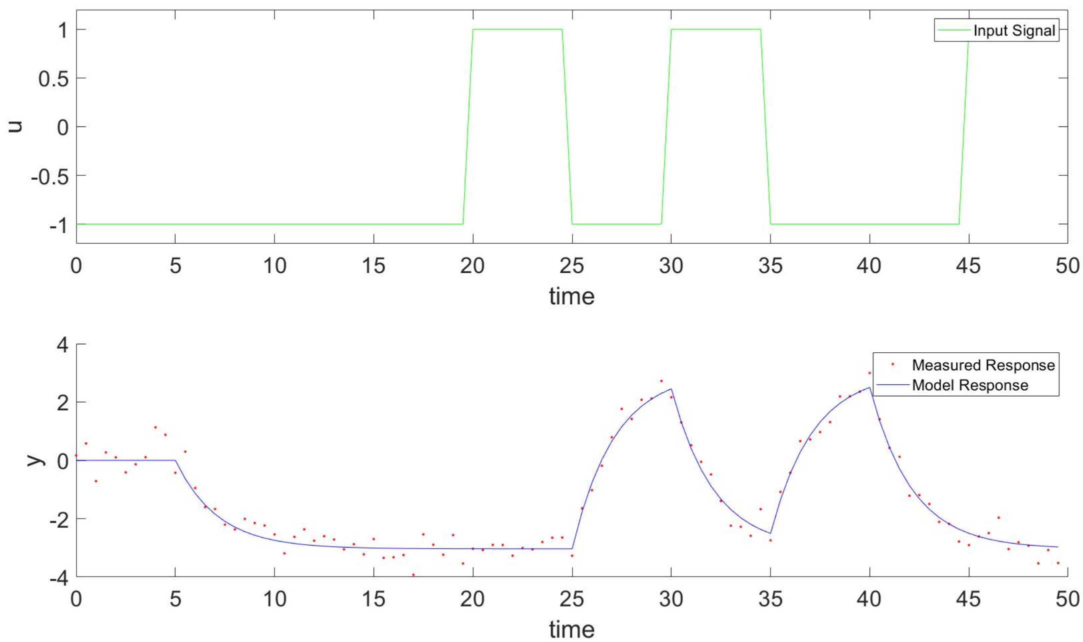

Figure 3: Plots of input signal, measured system response, and simulated response of the system model.

(c)

The step response of the system model can be seen in Figure 4 below.

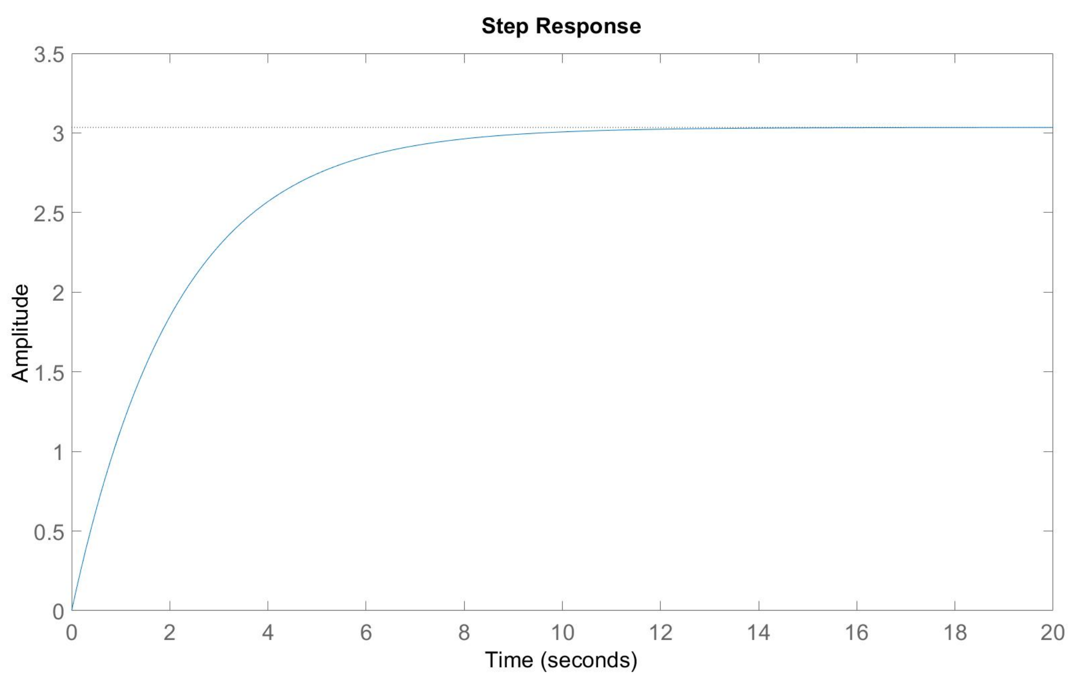

Figure 4: Step response of the estimated system model.

Exercise 13: Identification of a MIMO system from a PRBS test

Consider the dataset in the file `data-exercise-mimo.mat` uploaded on the class website.

(a)

Plot the input and output dataset. We have two inputs and two outputs, and we will identify a 2x2 transfer function for this system.

(b)

Clearly we cannot easily estimate these four transfer functions by graphical methods and we need a systematic computational procedure to perform identification on this problem. Estimate a transfer function and a state-space model from this input and output dataset. Use the functions `tfest` and `ssest` from the `Matlab ID` toolbox.

(c)

Give the transfer function parameters and show the 4 step tests of the four transfer functions.

Solution:

(a)

See plots of input and output data below in Figure 5.

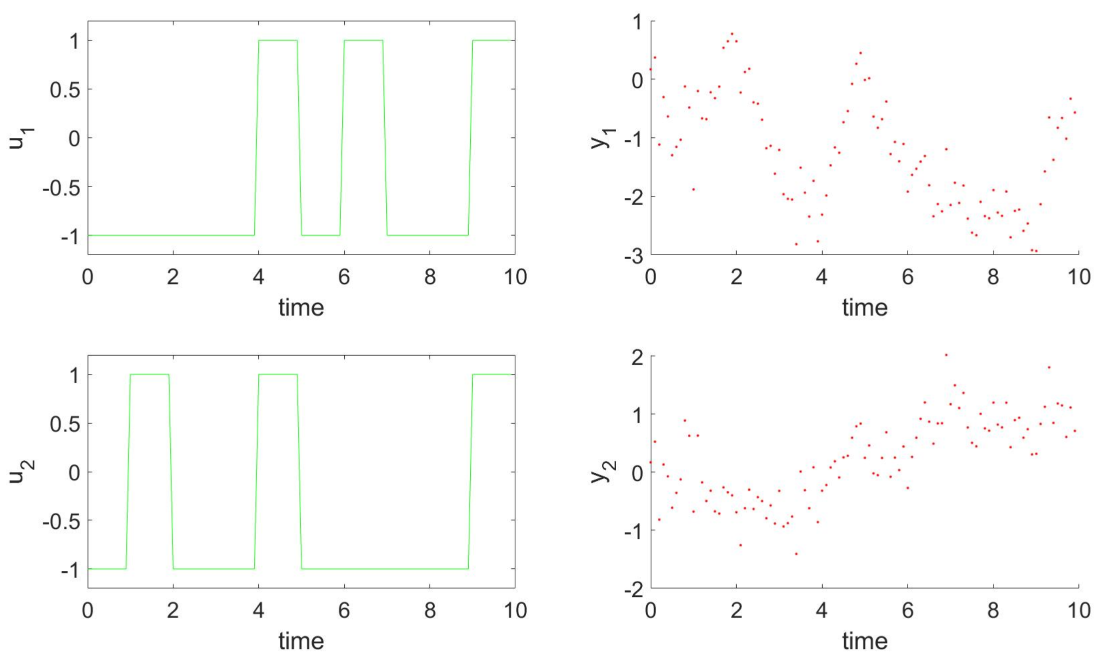

Figure 5: Input and output data for the MIMO system.

(b)

We can estimate the 4 system transfer functions, under the assumption they are each first-order, by calling `tfest(data, 1)`. Then, we can use `lsim` to simulate the response of the 2x2 system of first-order transfer functions to the given input data. The model response is plotted in blue, and superimposed over the measured system response data in the plots below in Figure 6.

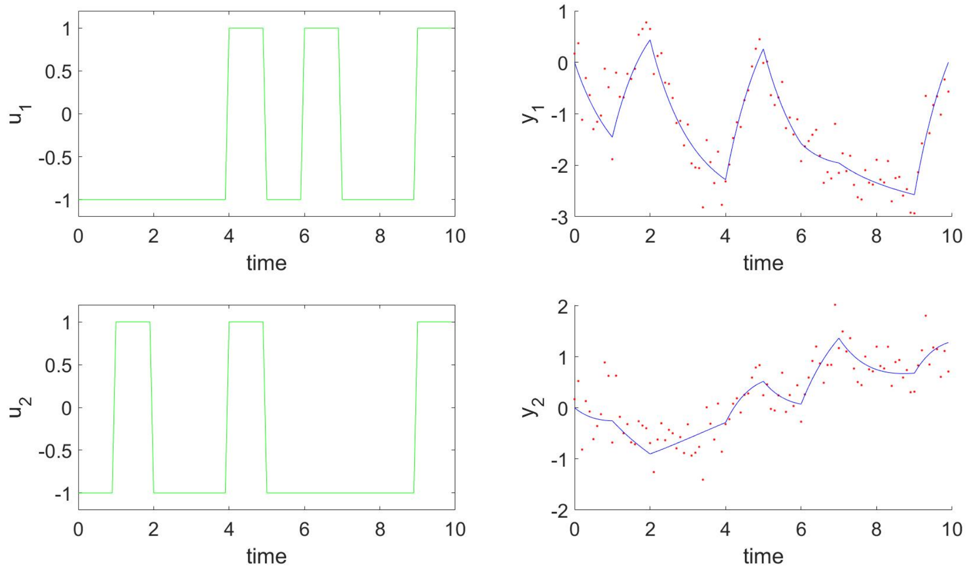

Figure 6: Input and output data for the MIMO system, along with model response to input.

Similarly, we can directly estimate a state-space model with a 2-dimensional state vector, by calling `ssest(data, 2)`. Then, we can again simulate the response of the model to the given inputs with `lsim`, which was done in order to generate the blue curves in the plots below in Figure 7. Notice that it seems the estimation of 4 first-order transfer functions has yielded the better fit to the data.

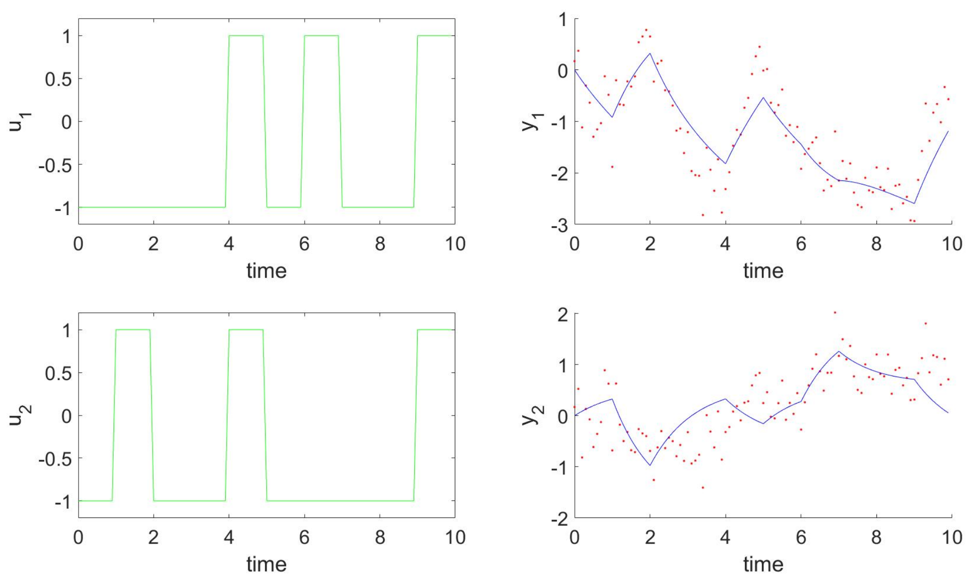

Figure 7: Input and output data for the MIMO system, along with state space model response to input.

(c)

The four first-order transfer functions estimated from the data are as follows: $G_{11}(s) = \frac{0.2}{s+0.1}$, $G_{12}(s) = \frac{2}{s+1}$, $G_{21}(s) = \frac{1}{s+1}$, and $G_{22}(s) = \frac{-0.4}{s+0.1}$, where $G_{ij}(s)$ is the transfer function from u_j to y_i. The step responses for each of these transfer functions are shown in Figure 8 below.

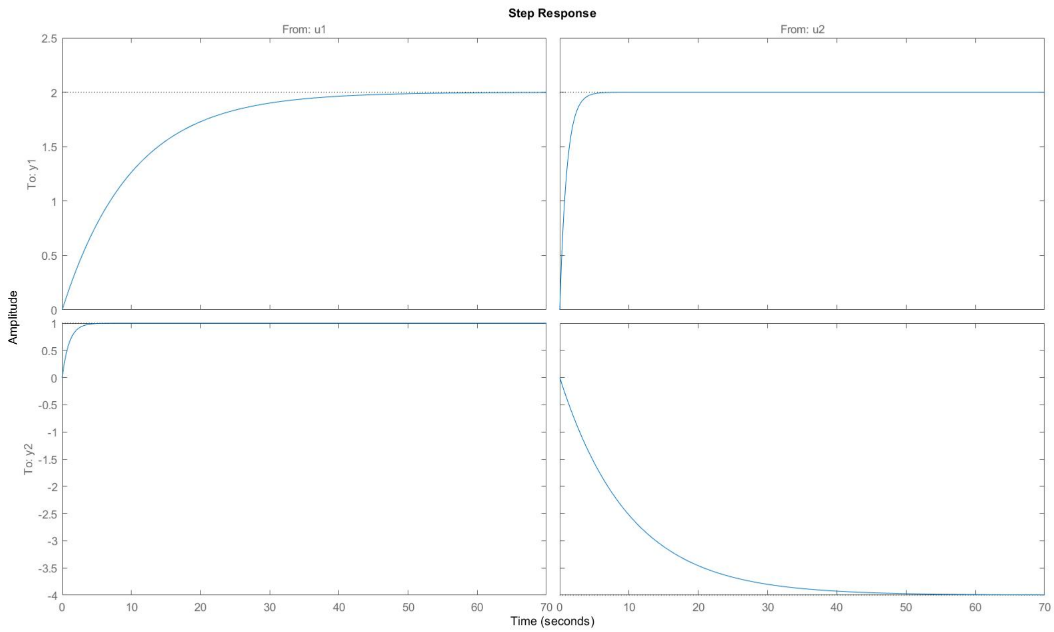

Figure 8: Step responses for the four estimated transfer functions.

MATLAB Code:

The following MATLAB code was used to generate all the above results.

```
%Connor Hughes
%CH E 152B
%HW 5

%% Exercise 11
clear, clc, close all;
load data-exercise-siso-1
figure
subplot(2, 1, 1)
plot(tsim, u, 'g')
ax = gca;
ax.FontSize = 20;
xlabel('time')
ylabel('u')
ylim([-1.2, 1.2])
lgd = legend('Input Signal');
lgd.FontSize = 14;
subplot(2, 1, 2)
scatter(tsim, ym, 'Marker', '.', 'MarkerEdgeColor', 'r')
ax = gca;
ax.FontSize = 20;
xlabel('time')
ylabel('y')
hold on

dat = iddata(ym', u', 0.1);
% sys = tfest(dat, 1);
%[pv1, pvsd1] = getpvec(sys)
sys = tfest(dat, 2);
[pv2, pvsd2] = getpvec(sys)
% sys = tfest(dat, 3);
% [pv3, pvsd3] = getpvec(sys)
% sys = tfest(dat, 4);
% [pv4, pvsd4] = getpvec(sys)

yest = lsim(sys, u, tsim)
plot(tsim, yest, 'b')
lgd = legend('Measured Response', 'Model Response');
lgd.FontSize = 14;

figure
sys_tf = tf(sys.Numerator, sys.Denominator);
step(sys_tf)
ax = gca;
ax.FontSize = 20;
ylabel('Amplitude', 'FontSize', 20)
xlabel('Time', 'FontSize', 20)
title('Step Response', 'FontSize', 20)
```

```matlab
%% Exercise 12
clear, clc, close all;
load data-exercise-siso-2
figure
subplot(2, 1, 1)
plot(tsim, u, 'g')
ax = gca;
ax.FontSize = 20;
xlabel('time')
ylabel('u')
ylim([-1.2, 1.2])
lgd = legend('Input Signal');
lgd.FontSize = 14;
subplot(2, 1, 2)
scatter(tsim, ym, 'Marker', '.', 'MarkerEdgeColor', 'r')
ax = gca;
ax.FontSize = 20;
xlabel('time')
ylabel('y')
hold on

dat = iddata(ym', u', 0.5);
sys = tfest(dat, 1, 0, NaN);
[pv1, pvsd1] = getpvec(sys)
% sys = tfest(dat, 2);
% [pv2, pvsd2] = getpvec(sys)
% sys = tfest(dat, 3);
% [pv3, pvsd3] = getpvec(sys)
% sys = tfest(dat, 4);
% [pv4, pvsd4] = getpvec(sys)

yest = lsim(sys, u, tsim);
plot(tsim, yest, 'b')
lgd = legend('Measured Response', 'Model Response');
lgd.FontSize = 14;

figure
sys_tf = tf(sys.Numerator, sys.Denominator);
step(sys_tf)
ax = gca;
ax.FontSize = 20;
ylabel('Amplitude', 'FontSize', 20)
xlabel('Time', 'FontSize', 20)
title('Step Response', 'FontSize', 20)
```

```
%% Exercise 13
clear, clc, close all;
load data-exercise-mimo.mat
figure
subplot(2, 2, 1)
plot(tsim, u(1, :), 'g')
ylabel('u_1')
xlabel('time')
ylim([-1.2, 1.2])
ax = gca
ax.FontSize = 20;
subplot(2, 2, 3)
plot(tsim, u(2, :), 'g')
ylabel('u_2')
xlabel('time')
ylim([-1.2, 1.2])
ax = gca
ax.FontSize = 20;
subplot(2, 2, 2)
scatter(tsim, ym(1, :), 'Marker', '.', 'MarkerEdgeColor', 'r')
ylabel('y_1')
xlabel('time')
hold on
ax = gca
ax.FontSize = 20;
subplot(2, 2, 4)
scatter(tsim, ym(2, :), 'Marker', '.', 'MarkerEdgeColor', 'r')
ylabel('y_2')
xlabel('time')
ax = gca
ax.FontSize = 20;
hold on

id = iddata(ym', u', 0.1)
Gsys = tfest(id, 1)


%figure
G = ssest(id, 2);
L = lsim(Gsys, u, tsim);
subplot(2, 2, 2)
plot(tsim, L(:, 1), 'b')
subplot(2, 2, 4)
plot(tsim, L(:, 2), 'b')

figure
step(Gsys)
```