

Model solving

Ellen Vitercik

ML to speed up MILP solvers

- Modern MILP solvers expose **hundreds** of parameters

CPX_PARAM_NODEFILEIND 100	CPX_PARAM_TRELIM 160	CPX_PARAM_RANDOMSEED 130	CPXPARAM_MIP_Pool_RelGap 148	CPX_PARAM_FLOWCOVERS 70
CPX_PARAM_NODELIM 101	CPX_PARAM_TUNINGDETTILIM 160	CPX_PARAM_REDUCE 131	CPXPARAM_MIP_Pool_Replace 151	CPX_PARAM_FLOWPATHS 71
CPX_PARAM_NODESEL 102	CPX_PARAM_TUNINGDISPLAY 160	CPX_PARAM_RELINV 121	CPXPARAM_MIP_Strategy_Branch 20	CPX_PARAM_FPHEUR 72
CPX_PARAM_NODEFILEIND 100	CPXPARAM_MIP_Limits_RampupTimeLimit 128	CPXPARAM_OptimalityTarget 106	CPXPARAM_TimeLimit 159	CPX_PARAM_MIPSEARCH 92
CPX_PARAM_NODELIM 101	CPXPARAM_MIP_Limits_Solutions 79	CPX_PARAM SOLUTIONTYPE 152	CPXPARAM_Tune_DetTimeLimit 160	CPX_PARAM_MIQCPSTRAT 93
CPX_PARAM_PRICELIM 123	CPXPARAM_MIP_Limits_StrongCand 154	CPX_PARAM_STARTALG 139	CPXPARAM_Tune_Display 162	CPX_PARAM_MIRCUTS 94
CPX_PARAM_PROBE 123	CPXPARAM_MIP_Limits_StrongIt 154	CPX_PARAM_STRONGCANDLIM 154	CPXPARAM_Tune_Measure 163	CPX_PARAM_MPULONGNUM 94
CPX_PARAM_PROBEDETTIME 124	CPXPARAM_MIP_Limits_TreeMemory 160	CPX_PARAM_STRONGITLIM 154	CPXPARAM_Tune_Repeat 164	CPX_PARAM_NETDISPLAY 95
CPX_PARAM_PROBETIME 124	CPXPARAM_MIP_OrderType 91	CPX_PARAM_SUBALG 99	CPXPARAM_Tune_TimeLimit 165	CPX_PARAM_NETEPOPT 96
CPX_PARAM_QPMAKEPSDIND 125	CPXPARAM_MIP_Pool_AbsGap 146	CPX_PARAM_SUBMIPNODELIMIT 155	CPXPARAM_WorkDir 167	CPX_PARAM_NETEPRHS 96
CPX_PARAM_QPMETHOD 138	CPXPARAM_MIP_Pool_Capacity 147	CPX_PARAM_SYMMETRY 156	CPXPARAM_WorkMem 168	CPX_PARAM_NETFIND 97
CPX_PARAM_QPNZREADLIM 126	CPXPARAM_MIP_Pool_Intensity 149	CPX_PARAM_THREADS 157	CraInd 50	CPX_PARAM_NETITLIM 98
		CPX_PARAM_TILIM 159		CPX_PARAM_NETPPRIIND 98

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CPX_PARAM_NODESEL 102	CPX_PARAM_TUNINGDISPLAY 160	CPX_PARAM_RELNIV 121	CPXPARAM_MIP_Strategy_Branch 20	CPX_PARAM_FPHEUR 72
CPX_PARAM_NODESTAT 103				FRACCAND 73
CPX_PARAM_NODESTAT 104				FRACCUTS 73
CPX_PARAM_NODESTAT 105				FRACPASS 74
CPX_PARAM_NODESTAT 106				GUBCOVERS 75
CPX_PARAM_NODESTAT 107				HEURFREQ 76
CPX_PARAM_NODESTAT 108				IMPLBD 76
CPX_PARAM_NODESTAT 109				INTSOLFILEPREFIX 78
CPX_PARAM_NODESTAT 110				INTSOLLIM 79
CPX_PARAM_NODESTAT 111				ITLIM 80
CPX_PARAM_NODESTAT 112				LANDPCUTS 82
CPX_PARAM_NODESTAT 113				LBHEUR 81
CPX_PARAM_NODESTAT 114				LPMETHOD 136
CPX_PARAM_NODESTAT 115				MCFCUTS 82
CPX_PARAM_NODESTAT 116				MEMORYEMPHASIS 83
CPX_PARAM_NODESTAT 117				MIPCREDLDP 84
CPX_PARAM_NODESTAT 118				MIPDISPLAY 85
CPX_PARAM_NODESTAT 119				MIPEMPHASIS 87
CPX_PARAM_NODESTAT 120				MIPINTERVAL 88
CPX_PARAM_NODESTAT 121				MIPKAPPASTATS 89
CPX_PARAM_NODESTAT 122				MIPORDIND 90
CPX_PARAM_NODESTAT 123				MIPORDTYPE 91
CPX_PARAM_NODESTAT 124				CPX_PARAM_MIPSEARCH 92
CPX_PARAM_NODESTAT 125				CPX_PARAM_MIQCPSTRAT 93
CPX_PARAM_NODESTAT 126				CPX_PARAM_MIRCUTS 94
CPX_PARAM_NODESTAT 127				CPX_PARAM_MPULONGNUM 94
CPX_PARAM_NODESTAT 128				CPX_PARAM_NETDISPLAY 95
CPX_PARAM_NODESTAT 129				CPX_PARAM_NETEPOPT 96
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CPX_PARAM_NODESTAT 131				CPX_PARAM_NETFIND 97
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CPX_PARAM_NODELIM 101	CPX_PARAM_TUNINGDETTILIM 160	CPX_PARAM_REDUCE 131	CPXPARAM_MIP_Pool_Replace 151	CPX_PARAM_FLOWPATHS 71
CPX_PARAM_NODESEL 102	CPX_PARAM_TUNINGDISPLAY 160	CPX_PARAM_RELNIV 121	CPXPARAM_MIP_Strategy_Branch 20	CPX_PARAM_FPHEUR 72
CPX_PARAM_NODESTL 103				FRACCAND 73
CPX_PARAM_NODESTL 104				FRACCUTS 73
CPX_PARAM_NODESTL 105				FRACPASS 74
CPX_PARAM_NODESTL 106				GUBCOVERS 75
CPX_PARAM_NODESTL 107				HEURFREQ 76
CPX_PARAM_NODESTL 108				IMPLBD 76
CPX_PARAM_NODESTL 109				INTSOLFILEPREFIX 78
CPX_PARAM_NODESTL 110				INTSOLLIM 79
CPX_PARAM_NODESTL 111				ITLIM 80
CPX_PARAM_NODESTL 112				LANDPCUTS 82
CPX_PARAM_NODESTL 113				LBHEUR 81
CPX_PARAM_NODESTL 114				LPMETHOD 136
CPX_PARAM_NODESTL 115				MCFCUTS 82
CPX_PARAM_NODESTL 116				MEMORYEMPHASIS 83
CPX_PARAM_NODESTL 117				MIPCREDLDP 84
CPX_PARAM_NODESTL 118				MIPDISPLAY 85
CPX_PARAM_NODESTL 119				MIPEMPHASIS 87
CPX_PARAM_NODESTL 120				MIPINTERVAL 88
CPX_PARAM_NODESTL 121				MIPKAPPASTATS 89
CPX_PARAM_NODESTL 122				MIPORDIND 90
CPX_PARAM_NODESTL 123				MIPORDTYPE 91
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CPX_PARAM_NODESTL 124				
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CPXPARAM_MIP_Limits_StrongCand 154	CPXPARAM_MIP_Limits_StrongIt 154	CPX_PARAM_SOLUTIONTYPE 152	CPXPARAM_Tune_DetTimeLimit 160	CPX_PARAM_MIQCPSTRAT 93
CPXPARAM_MIP_Limits_TreeMemory 160	CPXPARAM_MIP_OrderType 91	CPX_PARAM_STARTALG 139	CPXPARAM_Tune_Display 162	CPX_PARAM_MIRCUTS 94
CPXPARAM_MIP_Pool_AbsGap 146	CPXPARAM_MIP_Pool_Capacity 147	CPX_PARAM_STRONGCANDLIM 154	CPXPARAM_Tune_Measure 163	CPX_PARAM_MPSLONGNUM 94
CPXPARAM_MIP_Pool_Intensity 149	CPXPARAM_MIP_Pool_Replace 151	CPX_PARAM_STRONGITLIM 154	CPXPARAM_Tune_Repeat 164	CPX_PARAM_NETDISPLAY 95
		CPX_PARAM_SUBALG 99	CPXPARAM_Tune_TimeLimit 165	CPX_PARAM_NETEPOPT 96
		CPX_PARAM_SUBMIPNODELIMIT 155	CPXPARAM_WorkDir 167	CPX_PARAM_NETEPRHS 96
		CPX_PARAM_SYMMETRY 156	CPXPARAM_WorkMem 168	CPX_PARAM_NETFIND 97
		CPX_PARAM_THREADS 157	CraInd 50	CPX_PARAM_NETITLIM 98
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- Small changes can yield **large runtime** differences
- **Many** ways to integrate ML into solvers

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CPX_PARAM_NODESEL 102	CPX_PARAM_TUNINGDISPLAY 160	CPX_PARAM_RELNIV 121	CPXPARAM_MIP_Strategy_Branch 20	CPX_PARAM_FPHEUR 72
CPX_PARAM_FLOWCOVERS 70	CPX_PARAM_FLOWPATHS 71	CPX_PARAM_FPHEUR 72	FRACCAND 73	FRACCUTS 73
CPX_PARAM_FLOWPATHS 71	CPX_PARAM_FPHEUR 72	FRACPASS 74	GUBCOVERS 75	HEURFREQ 76
CPX_PARAM_FPHEUR 72	FRACPASS 74	IMPLBD 76	INTSOLFILEPREFIX 78	INTSOLLIM 79
FRACCAND 73	GUBCOVERS 75	INTSOLLIM 79	ITLIM 80	LANDPCUTS 82
FRACCUTS 73	HEURFREQ 76	ITLIM 80	LBHEUR 81	LBHEUR 81
FRACPASS 74	IMPLBD 76	LANDPCUTS 82	LMETHOD 136	LMETHOD 136
GUBCOVERS 75	INTSOLFILEPREFIX 78	MCFCUTS 82	MCFCUTS 82	MEMORYEMPHASIS 83
HEURFREQ 76	INTSOLLIM 79	MIPCBREDLP 84	MIPCBREDLP 84	MIPDISPLAY 85
IMPLBD 76	ITLIM 80	MIPDISPLAY 85	MIPDISPLAY 85	MIPEMPHASIS 87
INTSOLFILEPREFIX 78	LANDPCUTS 82	MIPEMPHASIS 87	MIPEMPHASIS 87	MIPINTERVAL 88
INTSOLLIM 79	LBHEUR 81	MIPINTERVAL 88	MIPINTERVAL 88	MIPKAPPASTATS 89
ITLIM 80	LMETHOD 136	MIPKAPPASTATS 89	MIPKAPPASTATS 89	MIPORDIND 90
LANDPCUTS 82	MCFCUTS 82	MIPORDIND 90	MIPORDIND 90	MIPORDTYPE 91
LBHEUR 81	MIPCBREDLP 84	MIPORDTYPE 91	MIPORDTYPE 91	CPX_PARAM_MIPSEARCH 92
LMETHOD 136	MIPDISPLAY 85	CPX_PARAM_MIPSEARCH 92	CPX_PARAM_MIPSEARCH 92	CPX_PARAM_MIQCPSTRAT 93
MCFCUTS 82	MIPEMPHASIS 87	CPX_PARAM_MIQCPSTRAT 93	CPX_PARAM_MIQCPSTRAT 93	CPX_PARAM_MIRCUTS 94
MEMORYEMPHASIS 83	MIPINTERVAL 88	CPX_PARAM_MIRCUTS 94	CPX_PARAM_MIRCUTS 94	CPX_PARAM_MPSLONGNUM 94
MIPCBREDLP 84	MIPKAPPASTATS 89	CPX_PARAM_MPSLONGNUM 94	CPX_PARAM_MPSLONGNUM 94	CPX_PARAM_NETDISPLAY 95
MIPDISPLAY 85	MIPORDIND 90	CPX_PARAM_NETDISPLAY 95	CPX_PARAM_NETDISPLAY 95	CPX_PARAM_NETEPOPT 96
MIPEMPHASIS 87	MIPORDTYPE 91	CPX_PARAM_NETEPOPT 96	CPX_PARAM_NETEPOPT 96	CPX_PARAM_NETEPRHS 96
MIPINTERVAL 88	CPX_PARAM_MIPSEARCH 92	CPX_PARAM_NETEPRHS 96	CPX_PARAM_NETEPRHS 96	CPX_PARAM_NETFIND 97
MIPKAPPASTATS 89	CPX_PARAM_MIQCPSTRAT 93	CPX_PARAM_NETFIND 97	CPX_PARAM_NETFIND 97	CPX_PARAM_NETITLIM 98
MIPORDIND 90	CPX_PARAM_MIRCUTS 94	CPX_PARAM_NETITLIM 98	CPX_PARAM_NETITLIM 98	CPX_PARAM_NETPPRIIND 98
MIPORDTYPE 91	CPX_PARAM_MPSLONGNUM 94	CPX_PARAM_NETPPRIIND 98	CPX_PARAM_NETPPRIIND 98	
CPX_PARAM_MIPSEARCH 92	CPX_PARAM_NETDISPLAY 95			
CPX_PARAM_MIQCPSTRAT 93	CPX_PARAM_NETEPOPT 96			
CPX_PARAM_MIRCUTS 94	CPX_PARAM_NETEPRHS 96			
CPX_PARAM_MPSLONGNUM 94	CPX_PARAM_NETFIND 97			
CPX_PARAM_NETDISPLAY 95	CPX_PARAM_NETITLIM 98			
CPX_PARAM_NETEPOPT 96	CPX_PARAM_NETPPRIIND 98			
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- Modern MILP solvers expose **hundreds** of parameters
- Defaults are rarely optimal: instance families vary widely in structure
- Small changes can yield **large runtime** differences
- **Many** ways to integrate ML into solvers
 - Potential for ML to provide **significant** speedups over defaults

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CPX_PARAM_FLOWPATHS 71	CPX_PARAM_FPHEUR 72	FRACPASS 74	GUBCOVERS 75	HEURFREQ 76
CPX_PARAM_FPHEUR 72	FRACPASS 74	IMPLBD 76	INTSOLFILEPREFIX 78	INTSOLLIM 79
FRACCAND 73	GUBCOVERS 75	INTSOLLIM 79	ITLIM 80	LANDPCUTS 82
FRACCUTS 73	HEURFREQ 76	ITLIM 80	LBHEUR 81	LPMETHOD 136
FRACPASS 74	IMPLBD 76	LANDPCUTS 82	MCFCUTS 82	MEMORYEMPHASIS 83
GUBCOVERS 75	INTSOLFILEPREFIX 78	LBHEUR 81	MIFCBREDLP 84	MIPDISPLAY 85
HEURFREQ 76	INTSOLLIM 79	MCFCUTS 82	MIPEMPHASIS 87	MIPINTERVAL 88
IMPLBD 76	ITLIM 80	MIFCBREDLP 84	MIPKAPPASTATS 89	MIPORDIND 90
INTSOLFILEPREFIX 78	LANDPCUTS 82	MIPDISPLAY 85	MIPORDTYPE 91	MIPORDTYPE 91
INTSOLLIM 79	LBHEUR 81	MIPEMPHASIS 87	CPX_PARAM_MIPSEARCH 92	CPX_PARAM_MIPSEARCH 92
ITLIM 80	MCFCUTS 82	MIPINTERVAL 88	CPX_PARAM_MIQCPSTRAT 93	CPX_PARAM_MIQCPSTRAT 93
LANDPCUTS 82	MIFCBREDLP 84	MIPKAPPASTATS 89	CPX_PARAM_MIRCUTS 94	CPX_PARAM_MIRCUTS 94
LBHEUR 81	MIPDISPLAY 85	MIPORDIND 90	CPX_PARAM_MPULONGNUM 94	CPX_PARAM_MPULONGNUM 94
LPMETHOD 136	MIPEMPHASIS 87	MIPORDTYPE 91	CPX_PARAM_NETDISPLAY 95	CPX_PARAM_NETDISPLAY 95
MCFCUTS 82	MIPINTERVAL 88	CPX_PARAM_NETEPOPT 96	CPX_PARAM_NETEPOPT 96	CPX_PARAM_NETEPOPT 96
MEMORYEMPHASIS 83	MIPKAPPASTATS 89	CPX_PARAM_NETEPRHS 96	CPX_PARAM_NETEPRHS 96	CPX_PARAM_NETEPRHS 96
MIFCBREDLP 84	MIPORDTYPE 91	CPX_PARAM_NETFIND 97	CPX_PARAM_NETFIND 97	CPX_PARAM_NETFIND 97
MIPDISPLAY 85	CPX_PARAM_NETITLIM 98	CPX_PARAM_NETITLIM 98	CPX_PARAM_NETPPRIIND 98	CPX_PARAM_NETPPRIIND 98
MIPEMPHASIS 87	CPX_PARAM_NETPPRIIND 98	CPX_PARAM_TILIM 159	CraInd 50	
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Outline

1. Background
 - i. **Branch-and-cut (B&C) algorithm**
 - ii. Brief overview of ML for B&C: non-GenAI
2. LLMs for speeding up solvers
3. Takeaways

Background: Branch-and-cut algorithm

Uses guidance from LP relaxations to guide search



Background: Branch-and-cut algorithm

Uses guidance from LP relaxations to guide search

- Explore **search tree** of restricted MILP subproblems



Background: Branch-and-cut algorithm

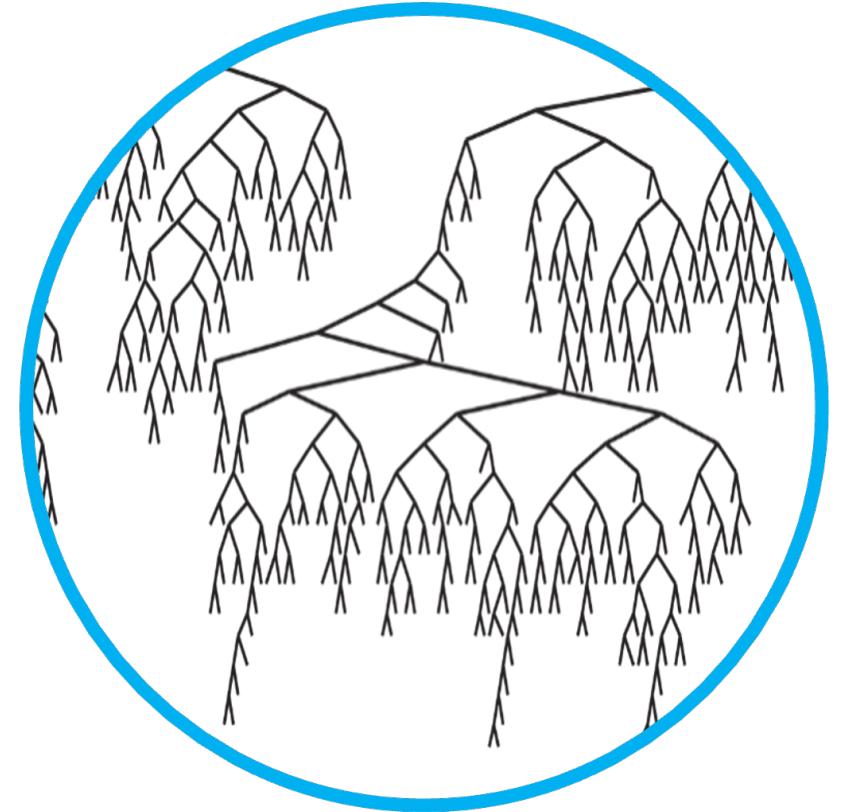
Uses guidance from LP relaxations to guide search

- Explore **search tree** of restricted MILP subproblems
 - Each node adds bounds on integer variables



Background: Branch-and-cut algorithm

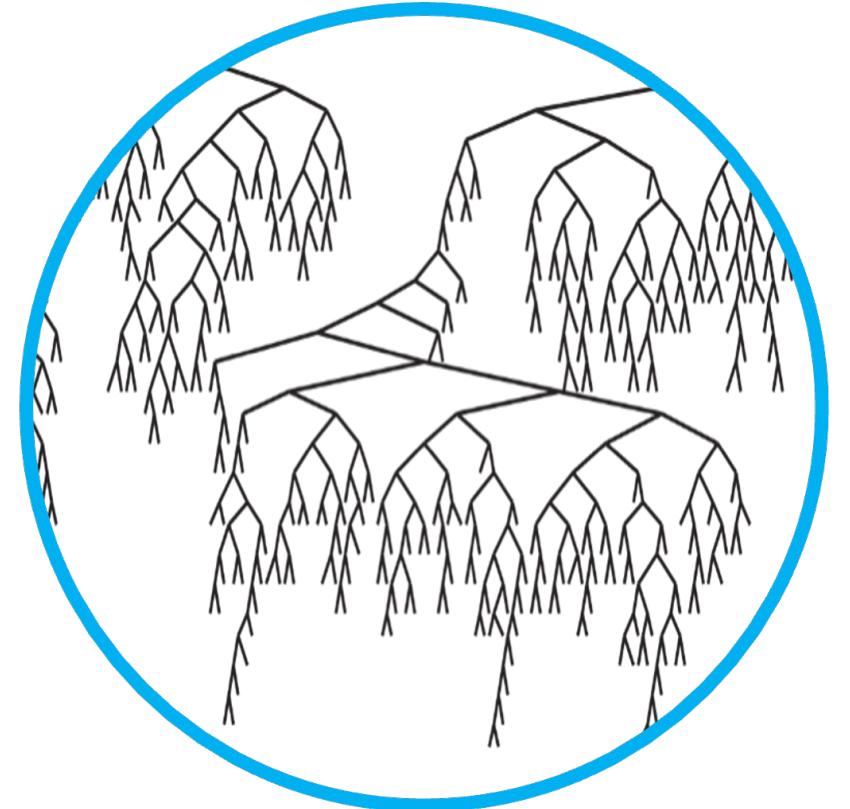
Uses guidance from LP relaxations to guide search



- Explore **search tree** of restricted MILP subproblems
 - Each node adds bounds on integer variables
- Solve **LP relaxation** to upper bound subproblem's integer-feasible solution

Background: Branch-and-cut algorithm

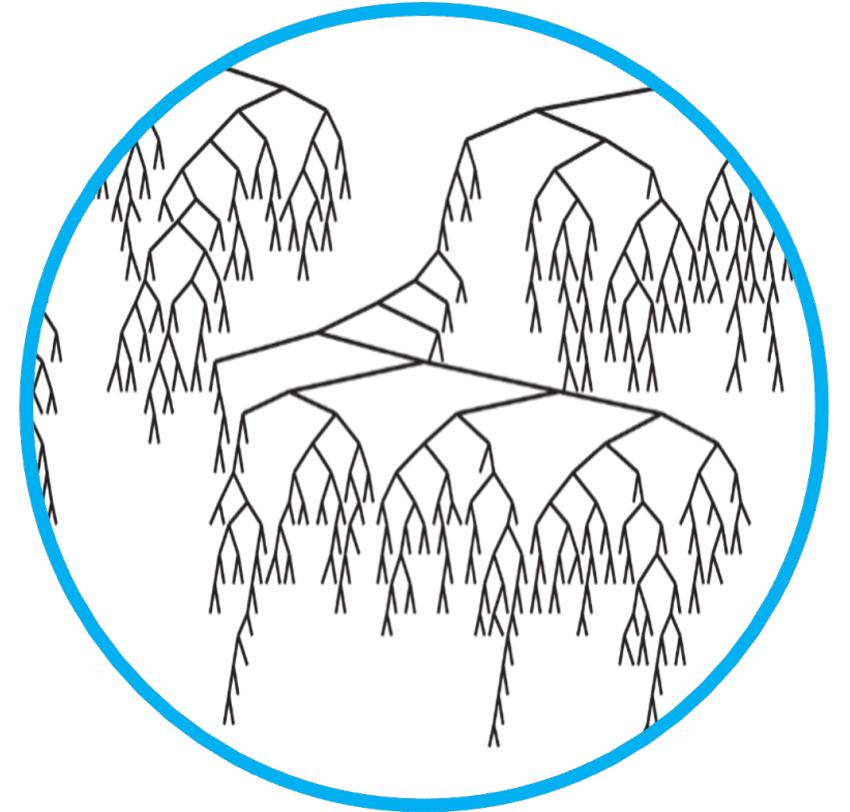
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- Explore **search tree** of restricted MILP subproblems
 - Each node adds bounds on integer variables
- Solve **LP relaxation** to upper bound subproblem's integer-feasible solution
 - If integer-feasible: incumbent solution

Background: Branch-and-cut algorithm

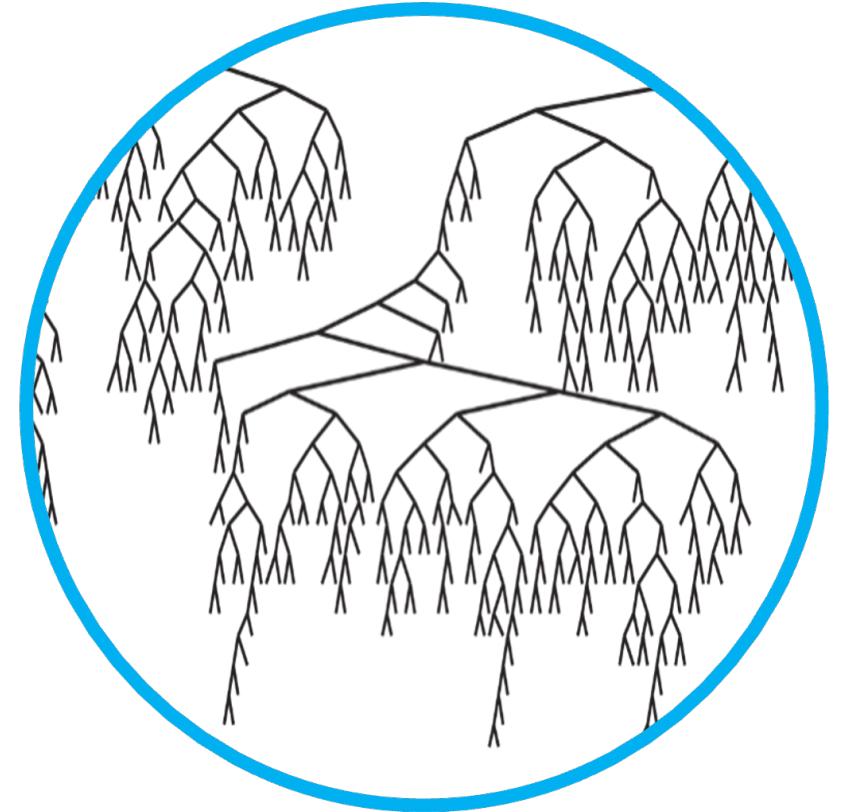
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- Explore **search tree** of restricted MILP subproblems
 - Each node adds bounds on integer variables
- Solve **LP relaxation** to upper bound subproblem's integer-feasible solution
 - If integer-feasible: incumbent solution
- **Branch** by choosing a variable and splitting its domain

Background: Branch-and-cut algorithm

Uses guidance from LP relaxations to guide search

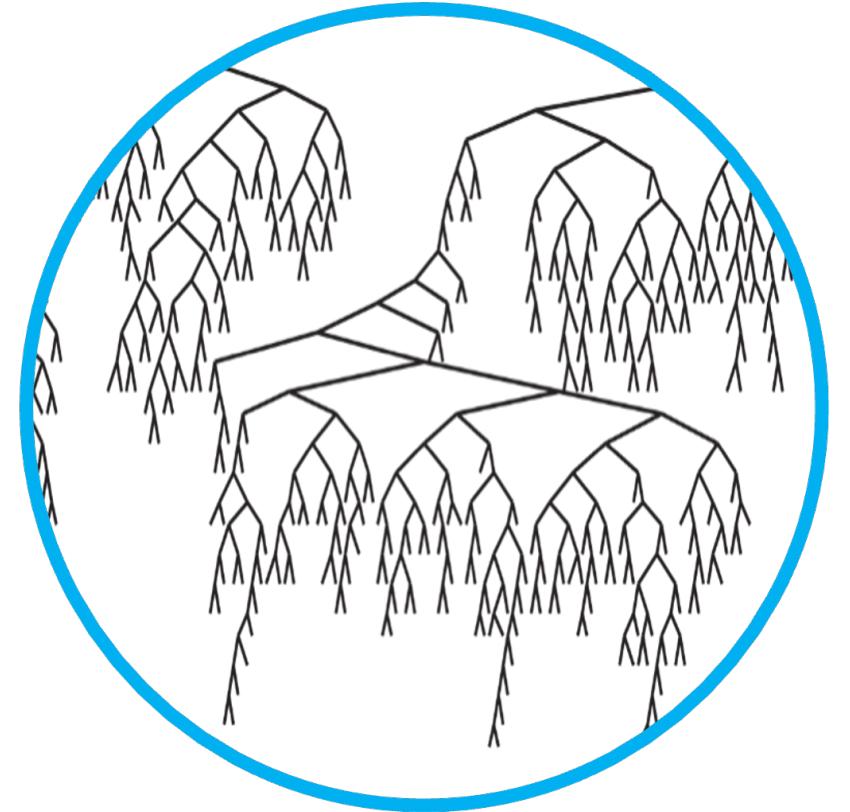


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Background: Branch-and-cut algorithm

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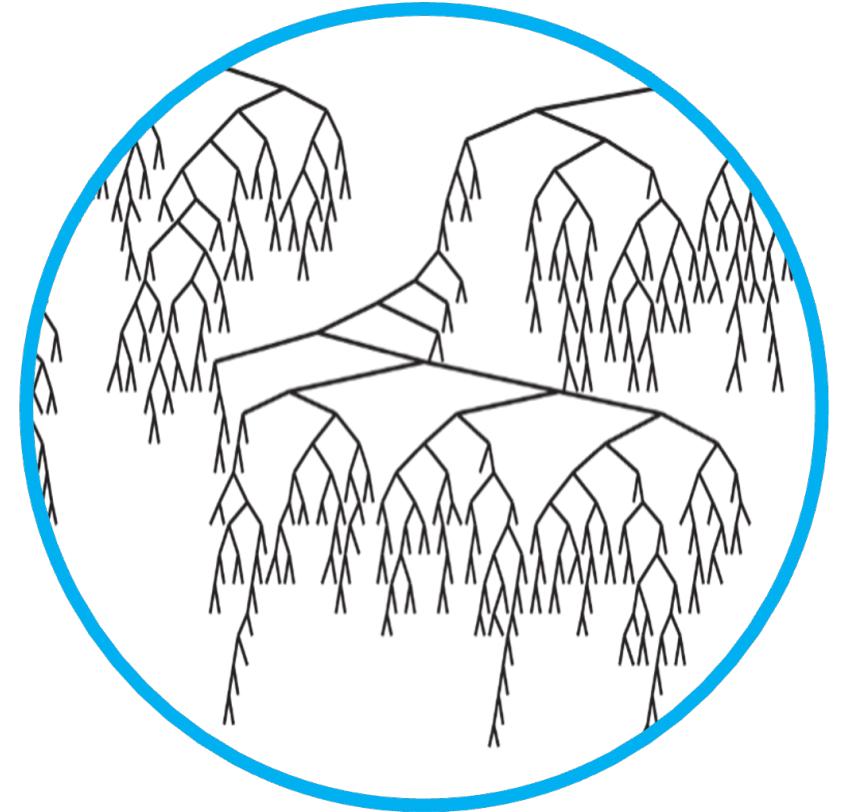


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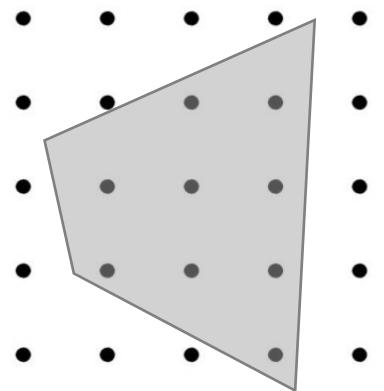
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MILP

maximize $c^T x$

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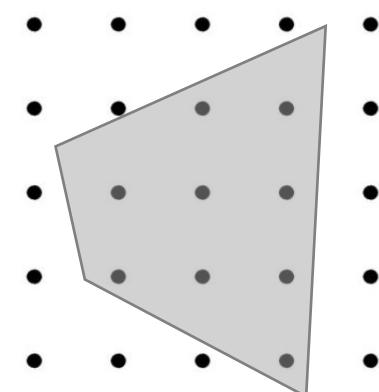
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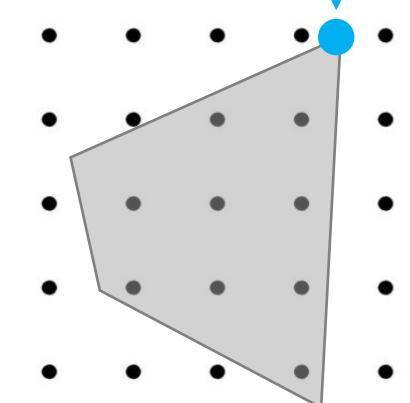
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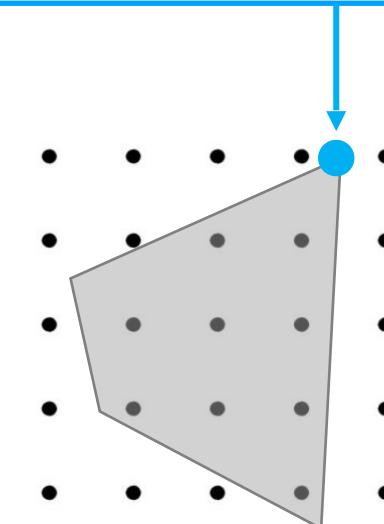
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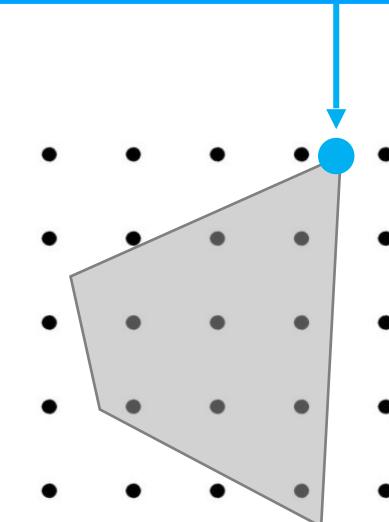
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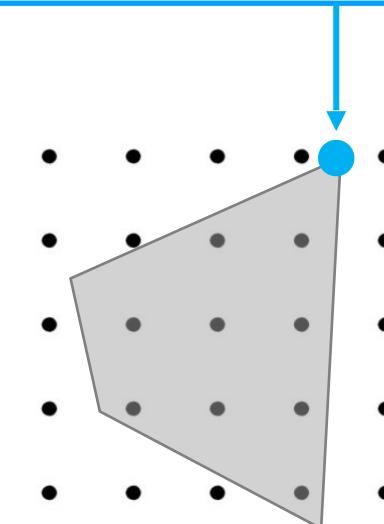
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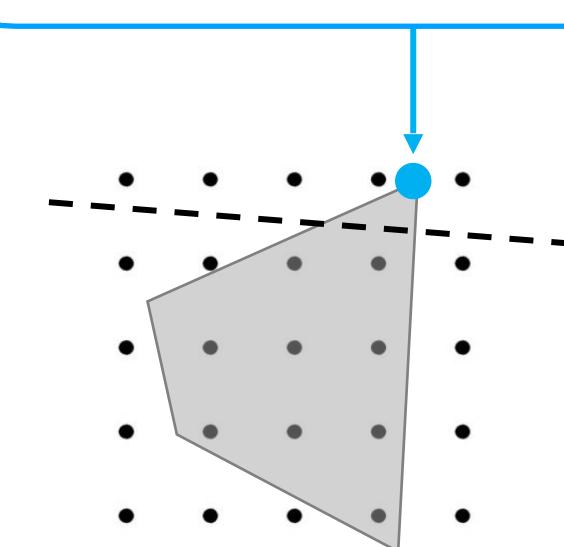
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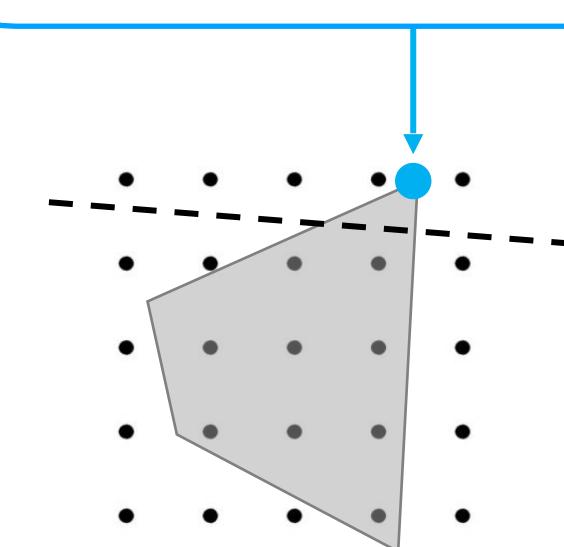
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Many different families of CPs; which to use when?

LP optimal solution



Outline

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 - i. Branch-and-cut (B&C) algorithm
 - ii. **Brief overview of ML for B&C: non-GenAI**
2. LLMs for speeding up solvers
3. Takeaways

General setup

ML for MILP solvers

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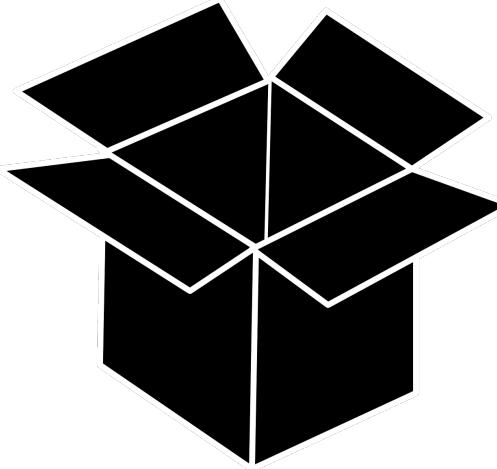
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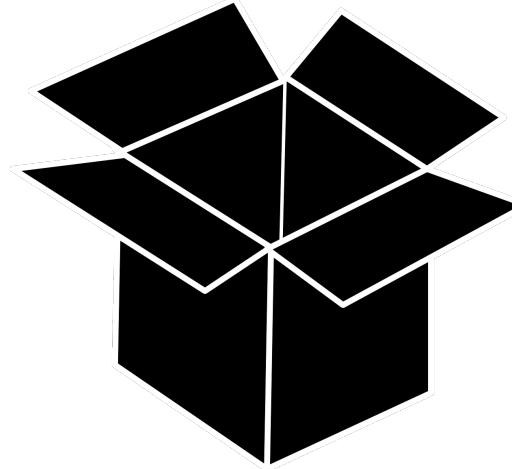
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 - Can learn *offline* θ or *instance-aware* $\theta(z)$ configuration

Blackbox algorithm configuration



- Early work: treat solver (largely) as a black-box; learn from evaluations

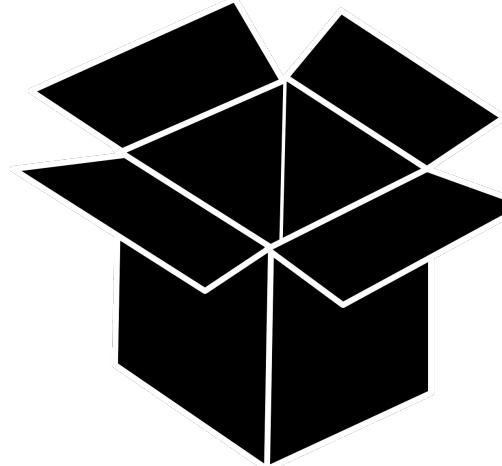
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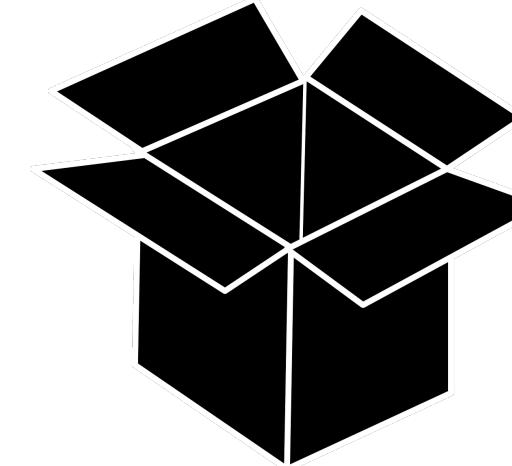
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Lobjois, Lemaître, AAAI'98; Gomes, Selman, AI'01; Xu, Hoos, Leyton-Brown, AAAI'10; Kadioglu et al., ECAI'10, Sandholm, Handbook of Market Design'13

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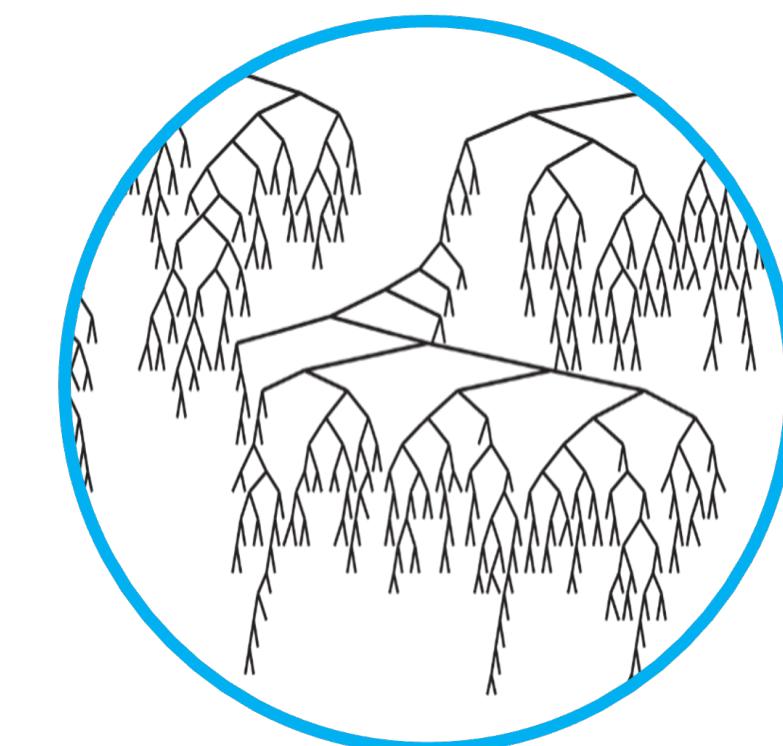


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Configuration of solver components

Next gen: don't treat solver as blackbox; adapt to solver components. E.g.:

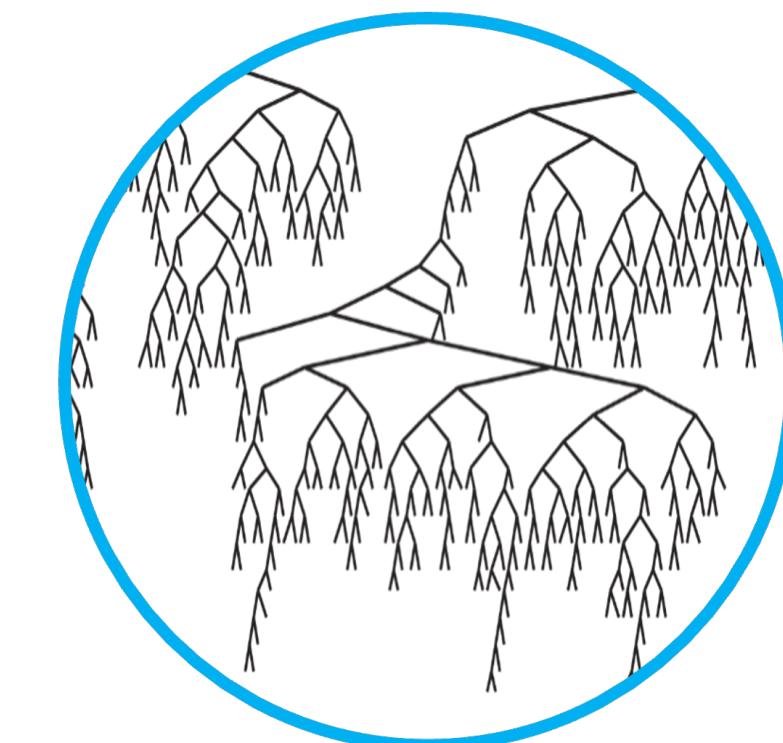
Cut selection	Variable selection	Node selection
Tang et al., ICML'20	Khalil et al., AAAI'16	He et al., NeurIPS'14
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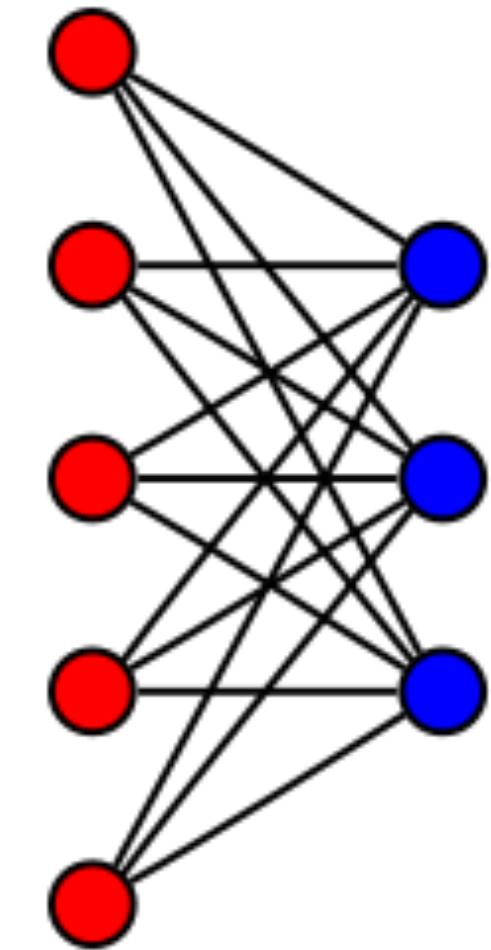
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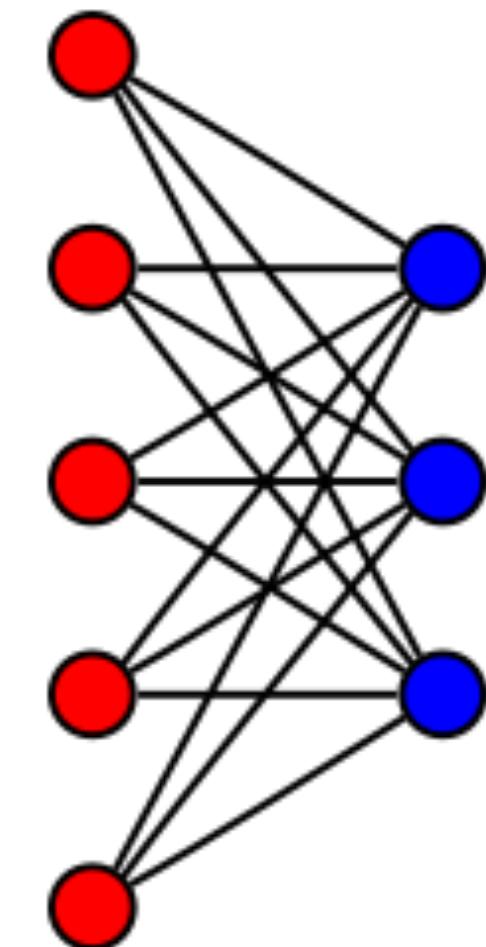


Graph neural networks for variable selection



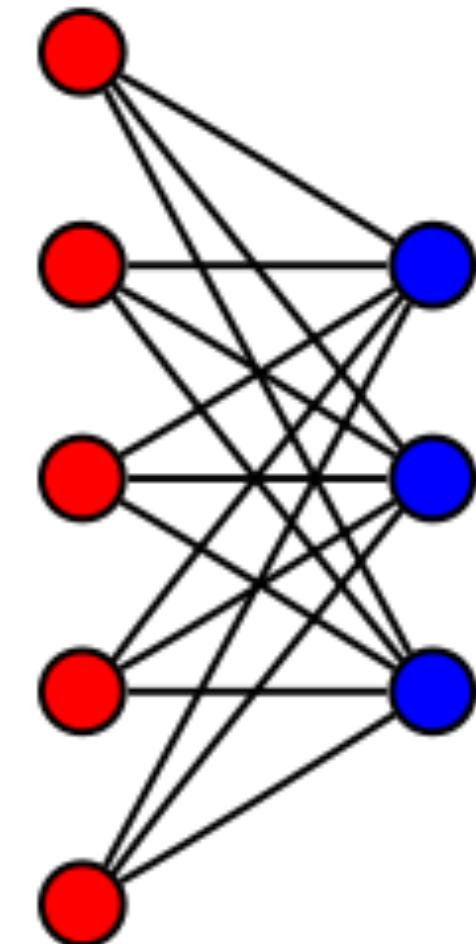
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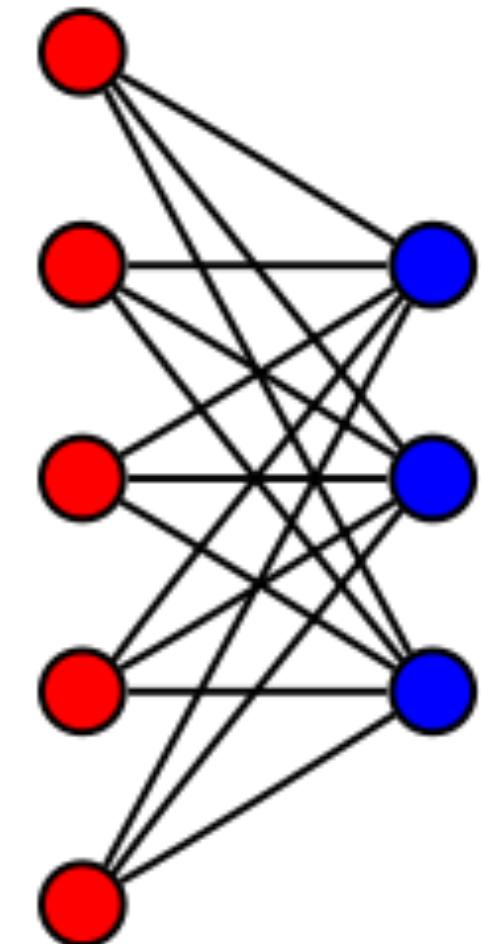
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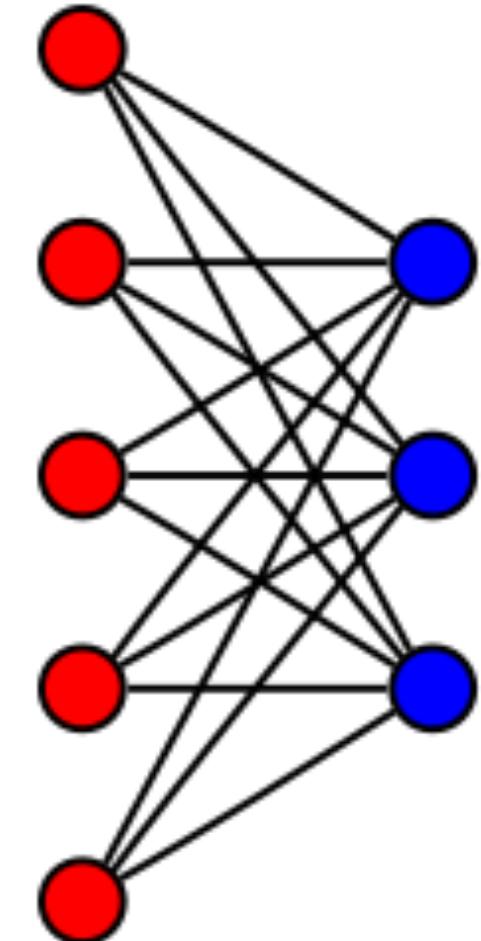
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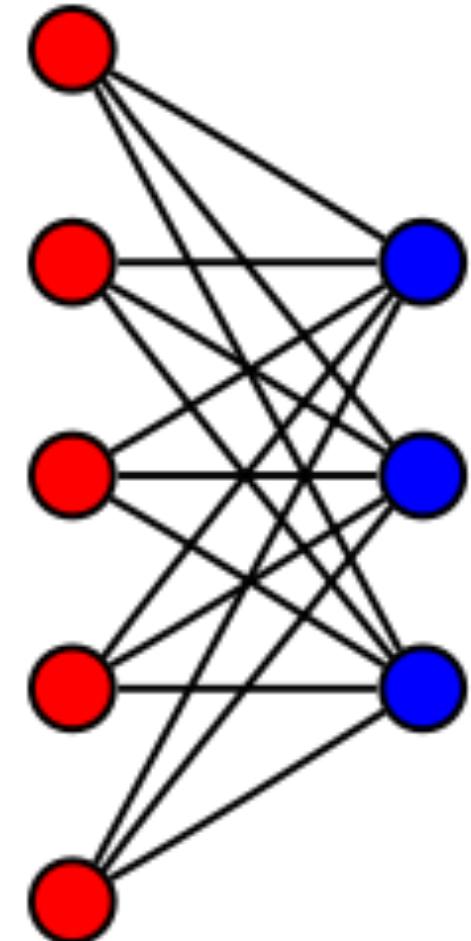
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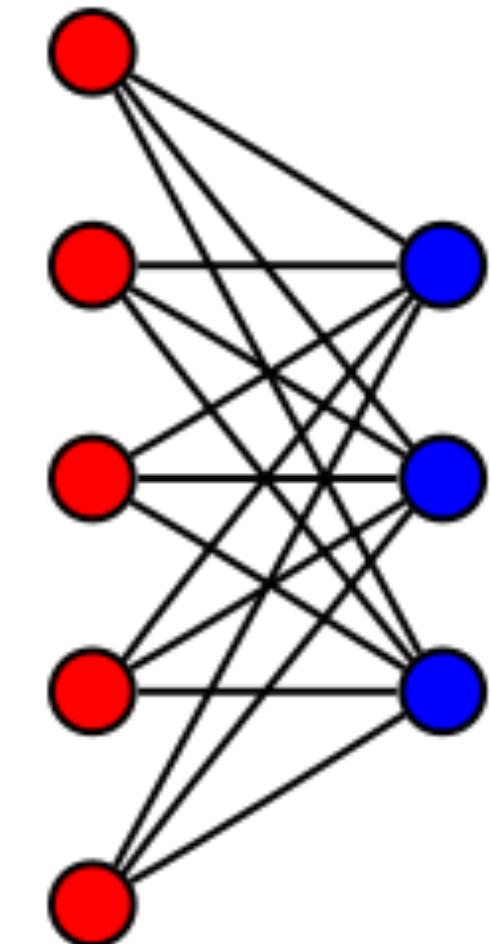
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2. LLMs for speeding up solvers
 - i. **Solver configuration**
 - ii. Evolutionary search for MILP heuristics
3. Takeaways

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Key question: Can we generate problem-specific cutting plane configurations
with **little to no historical data and compute?**

Our contributions

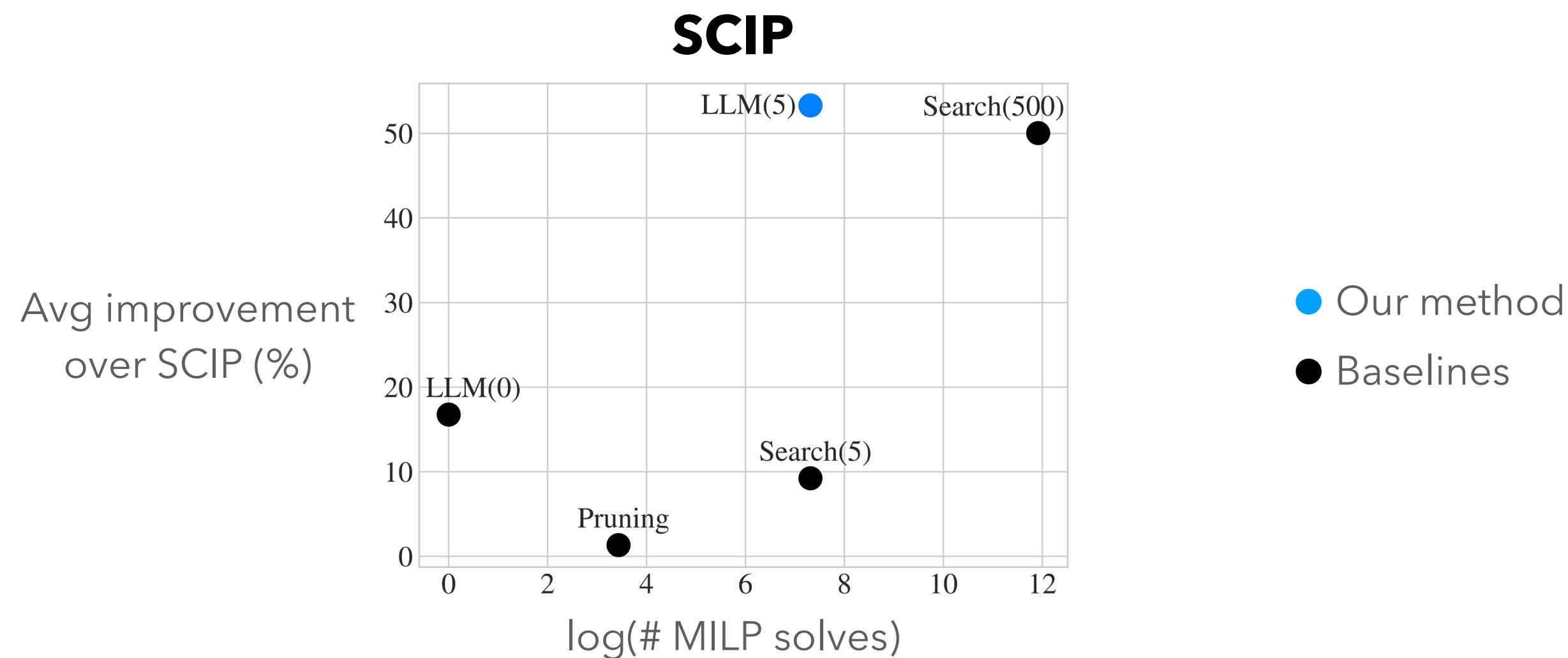
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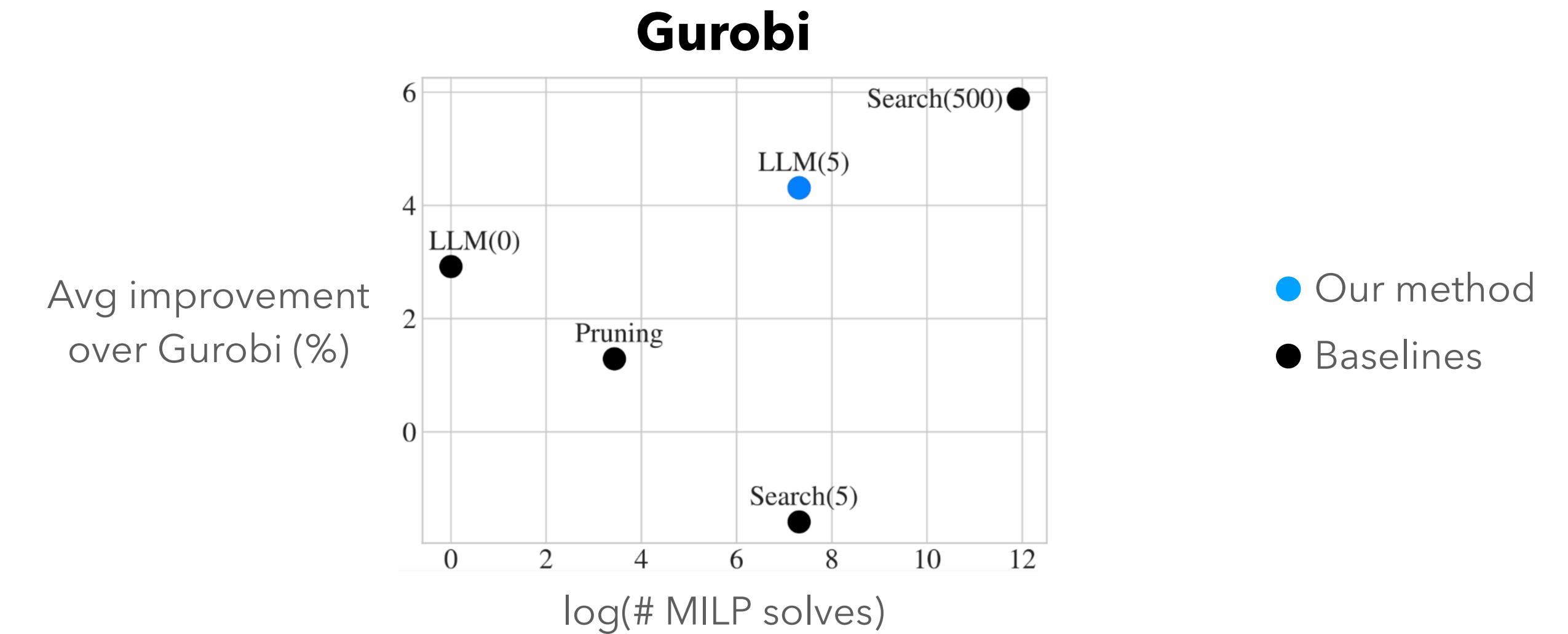
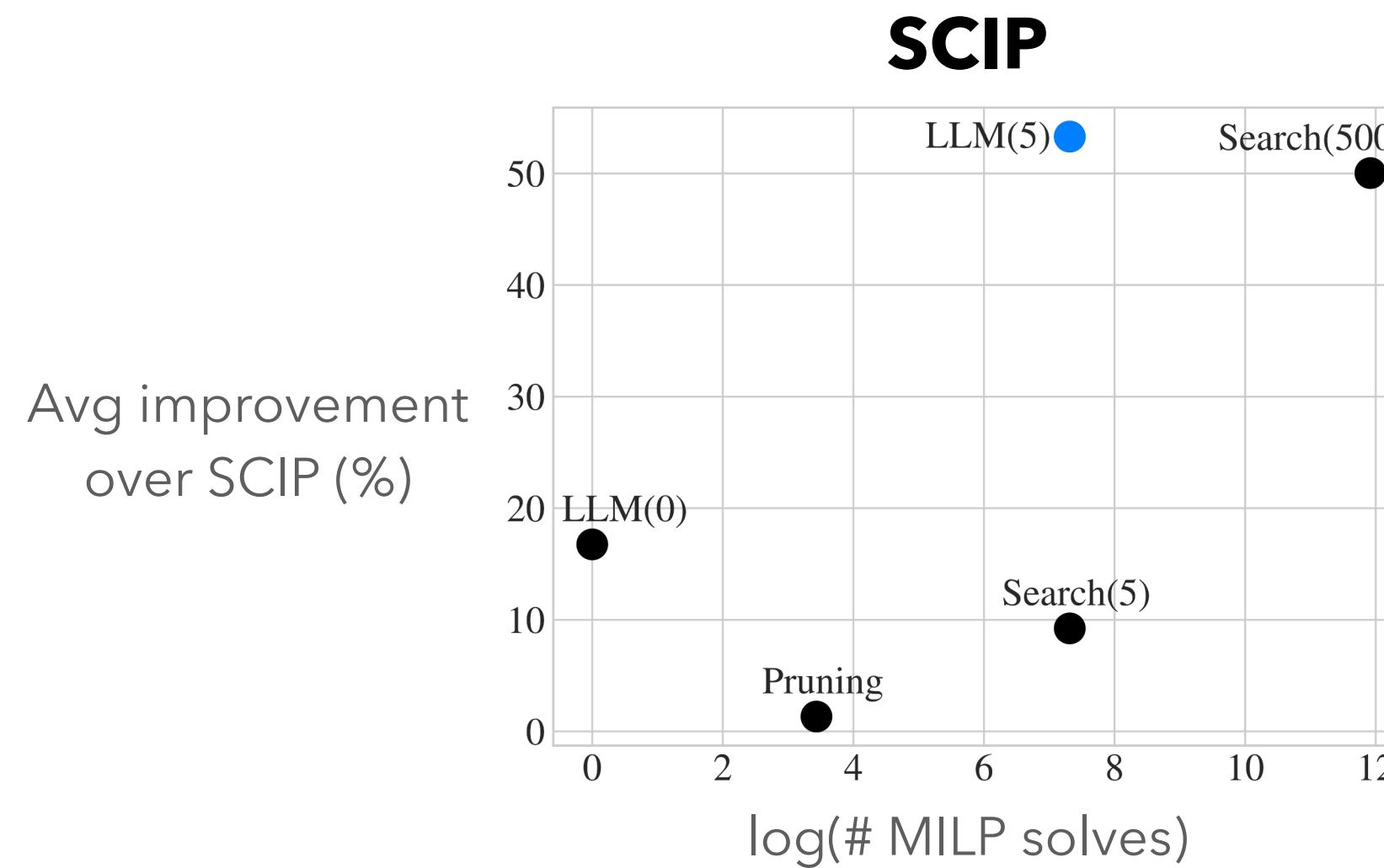
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- LLM output can be highly **unstable**
- Cutting plane separators are **solver-specific**
 - Details of solver separators are not always available

› CoverCuts

Cover cut generation

- Type: `int`
- Default value: `-1`
- Minimum value: `-1`
- Maximum value: `2`

Controls cover cut generation. Use 0 to disable these cuts, 1 for moderate cut generation, or 2 for aggressive cut generation. The default -1 value chooses automatically. Overrides the [Cuts](#) parameter.

LLM for cold-start configuration pipeline

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Problem description

Text summary

LaTeX model

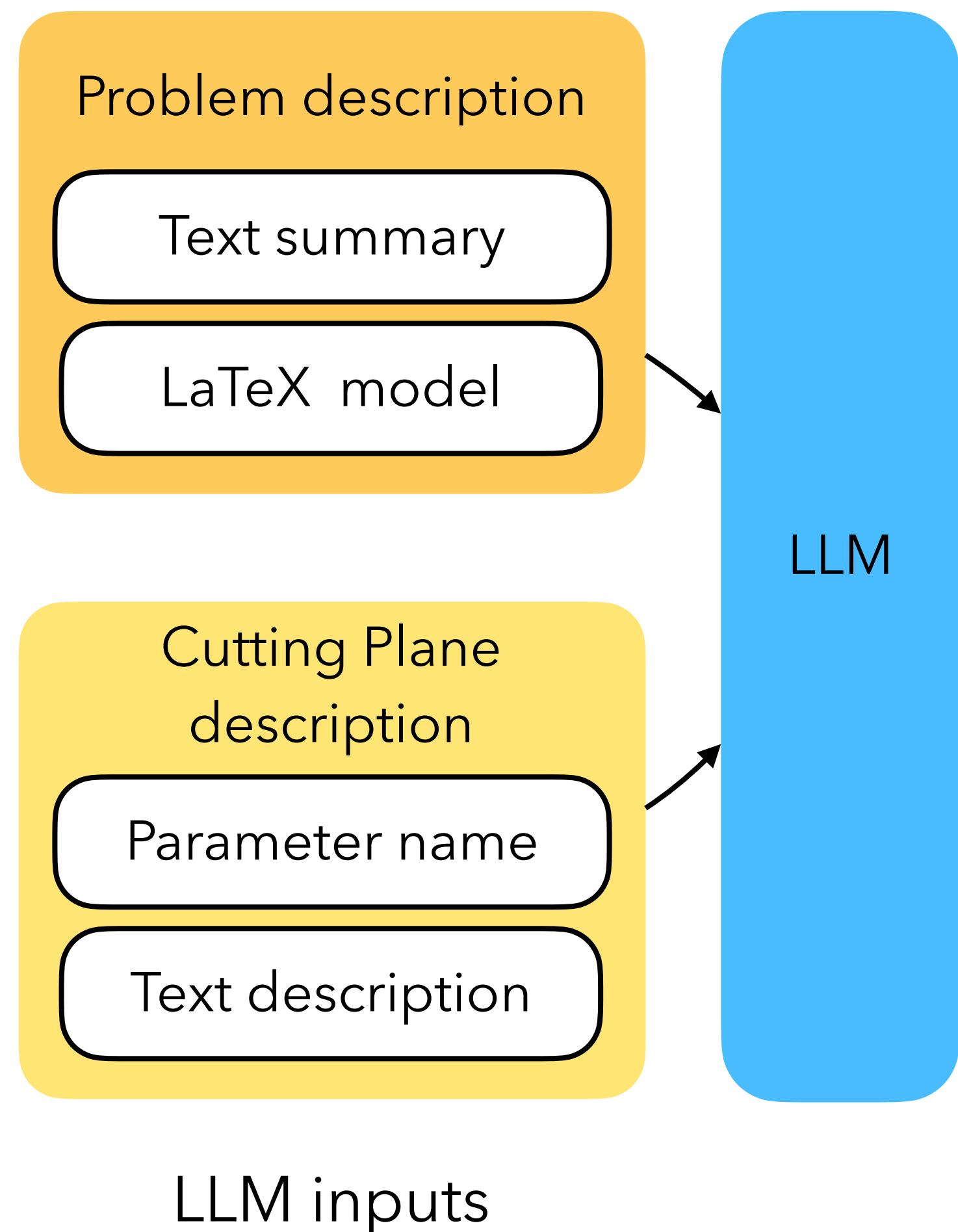
Cutting Plane
description

Parameter name

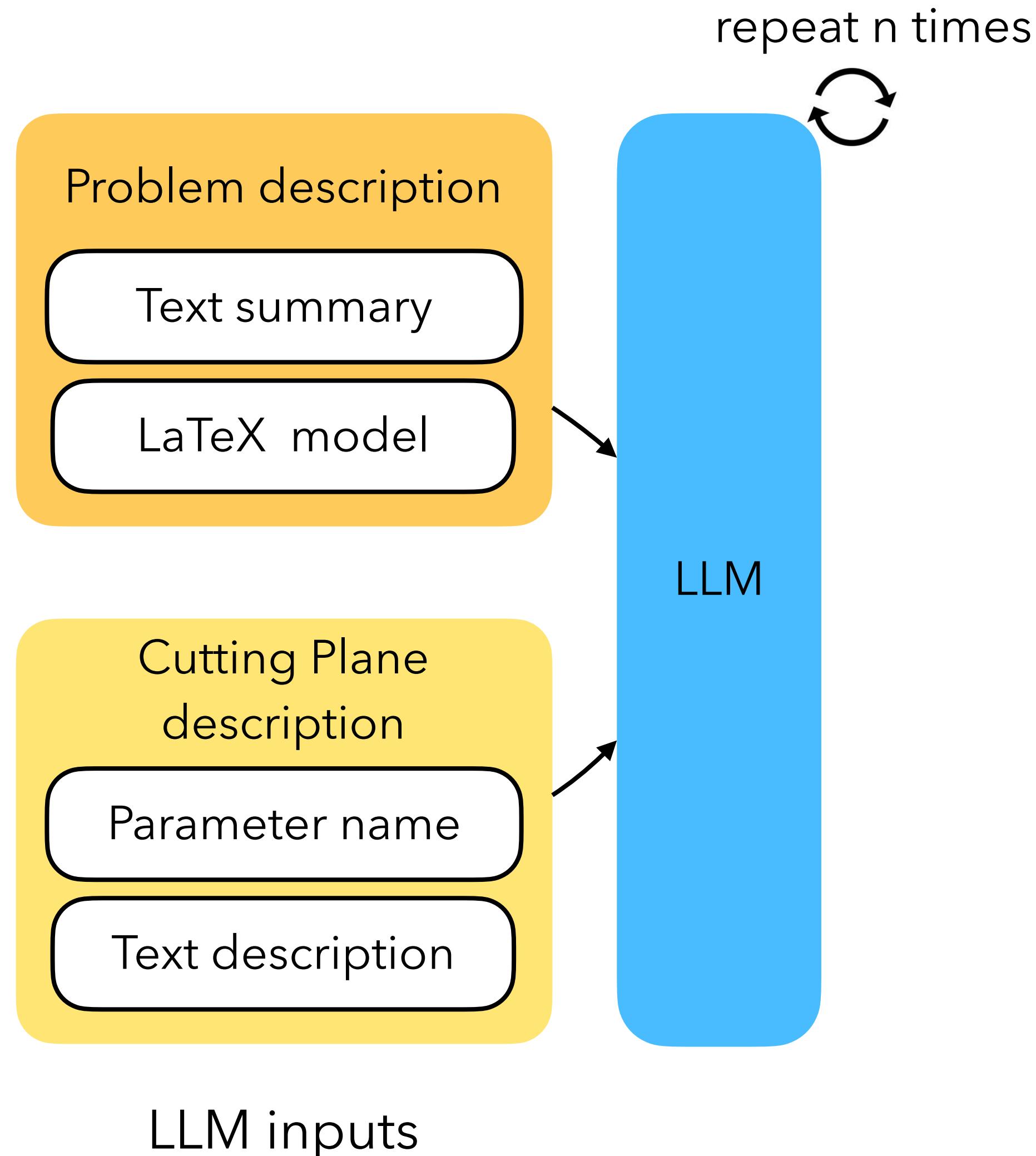
Text description

LLM inputs

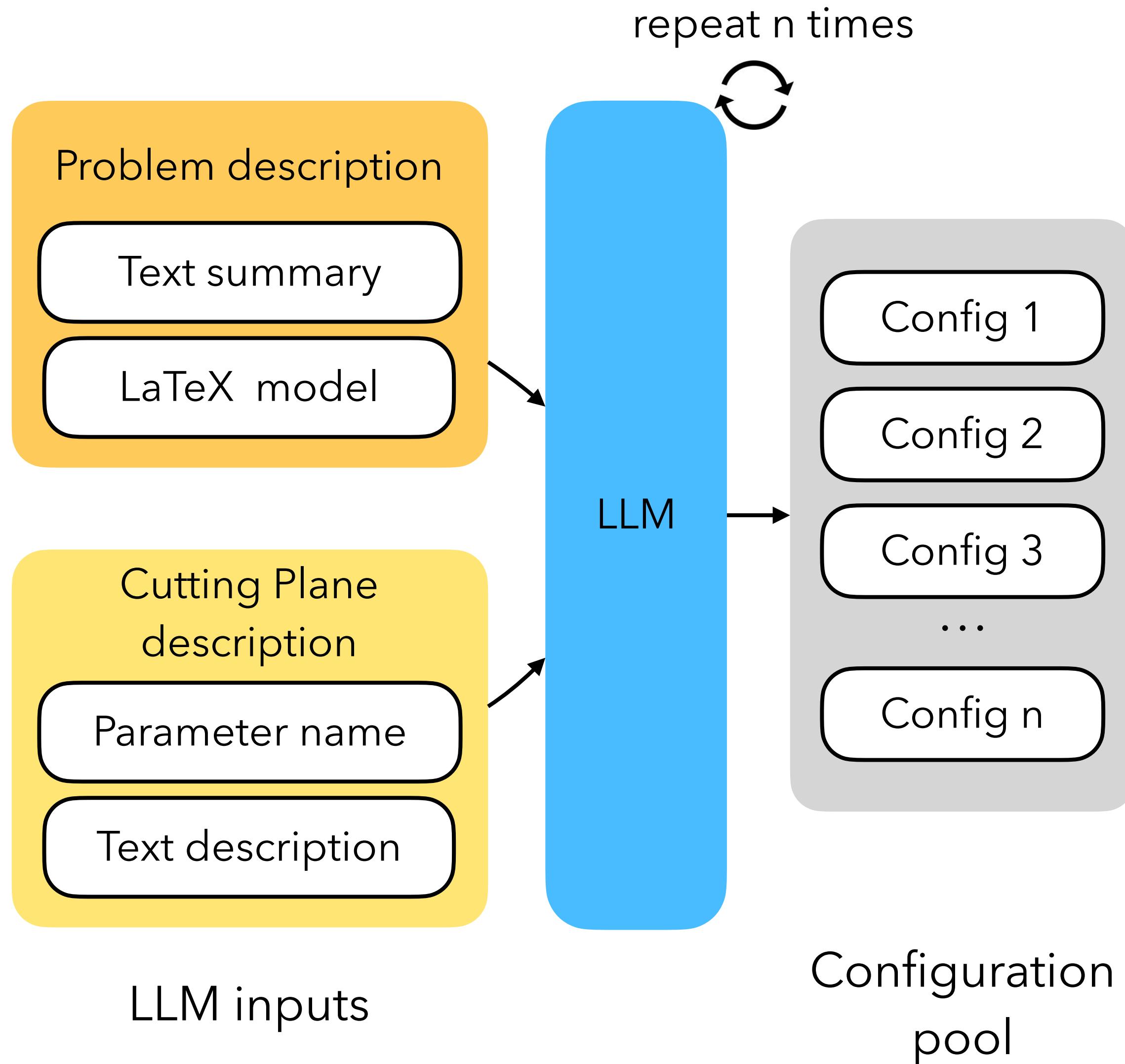
LLM for cold-start configuration pipeline



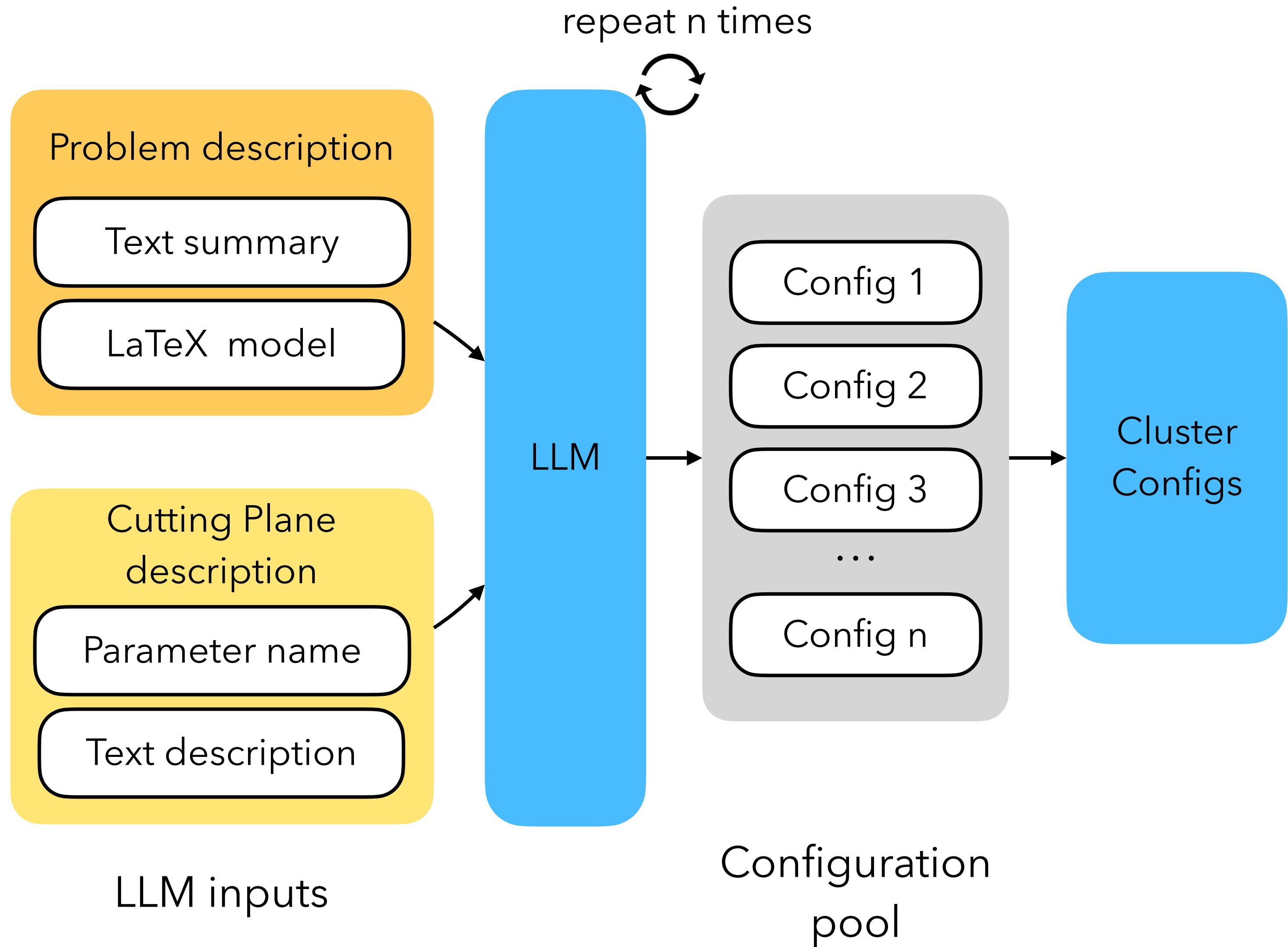
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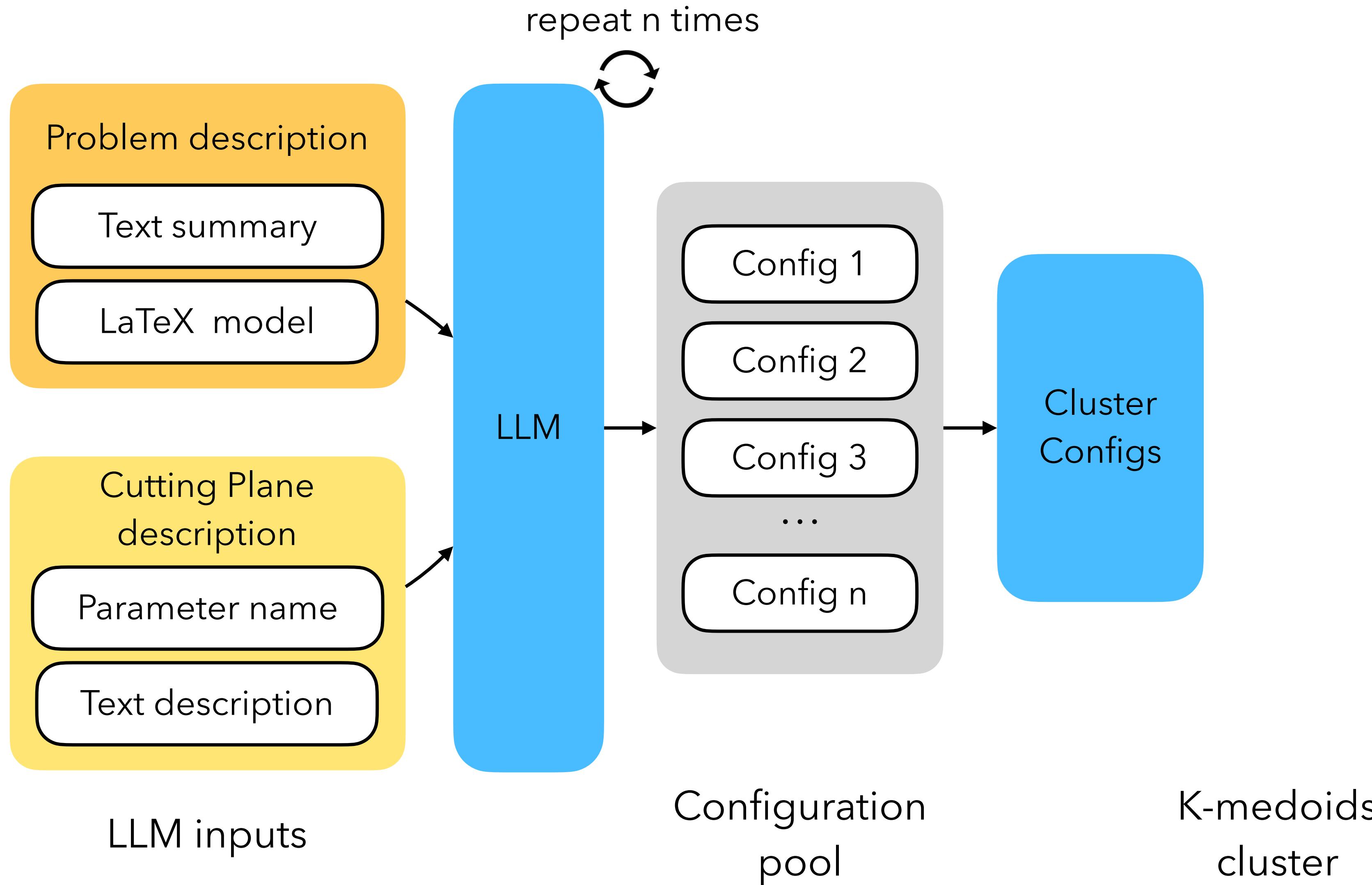
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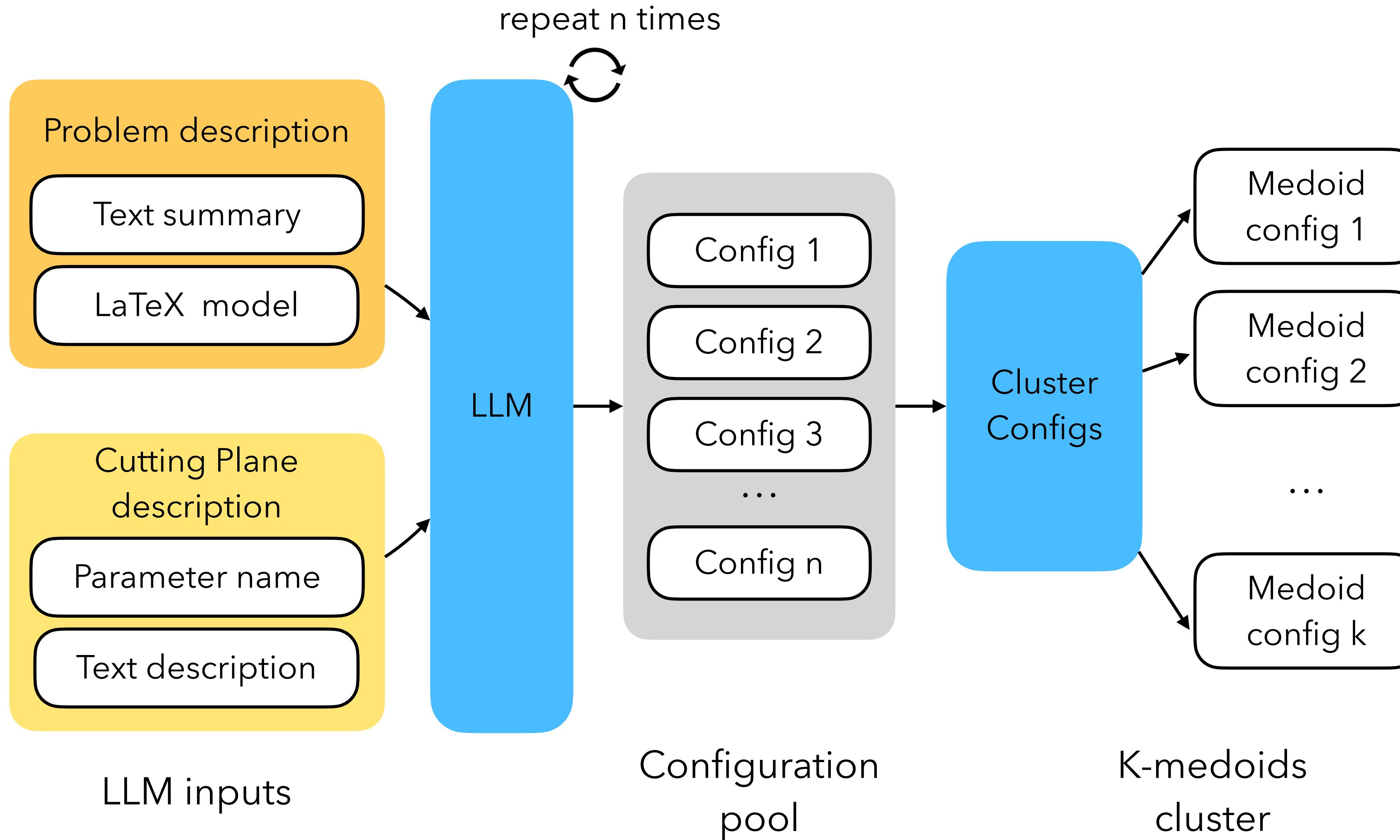
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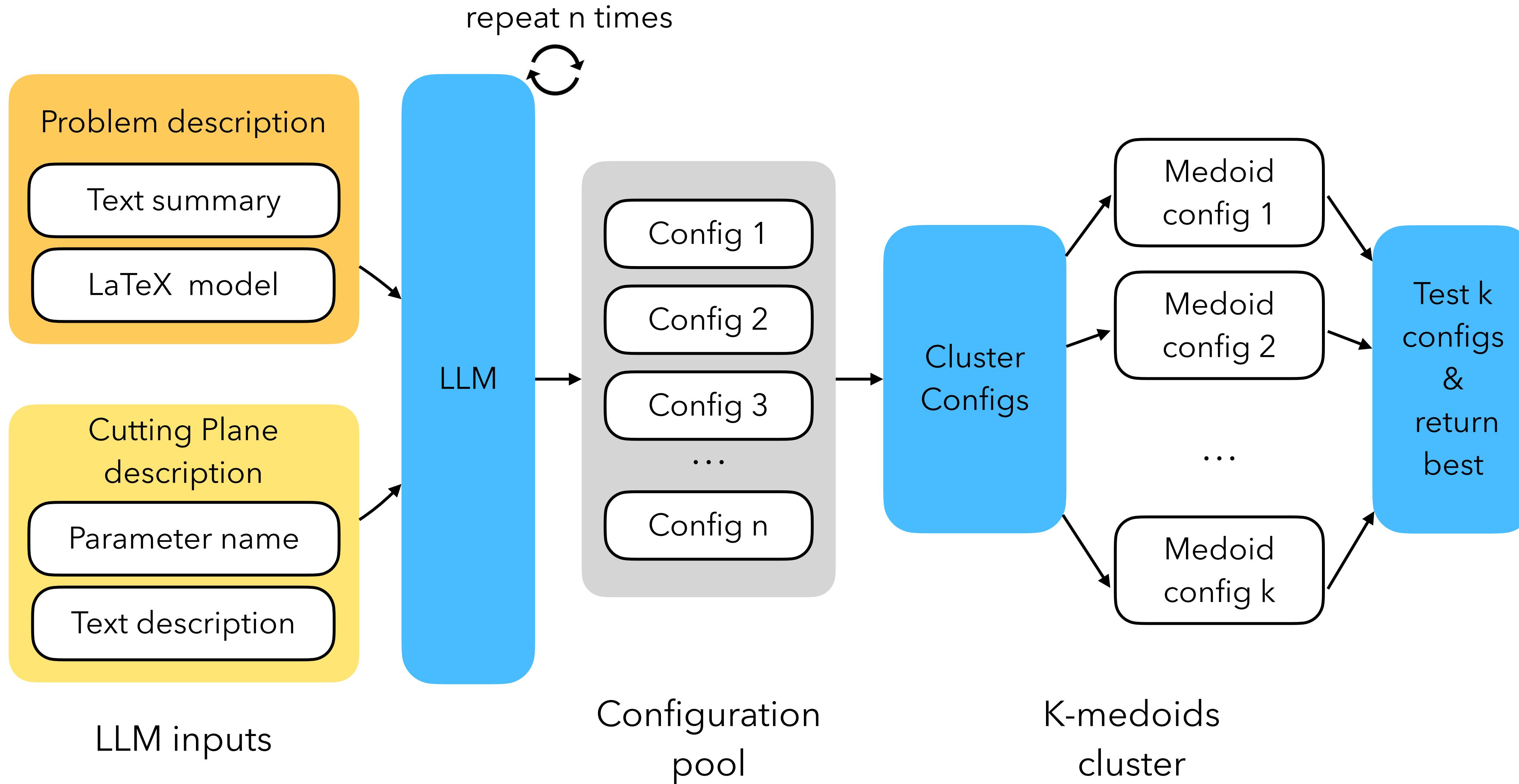
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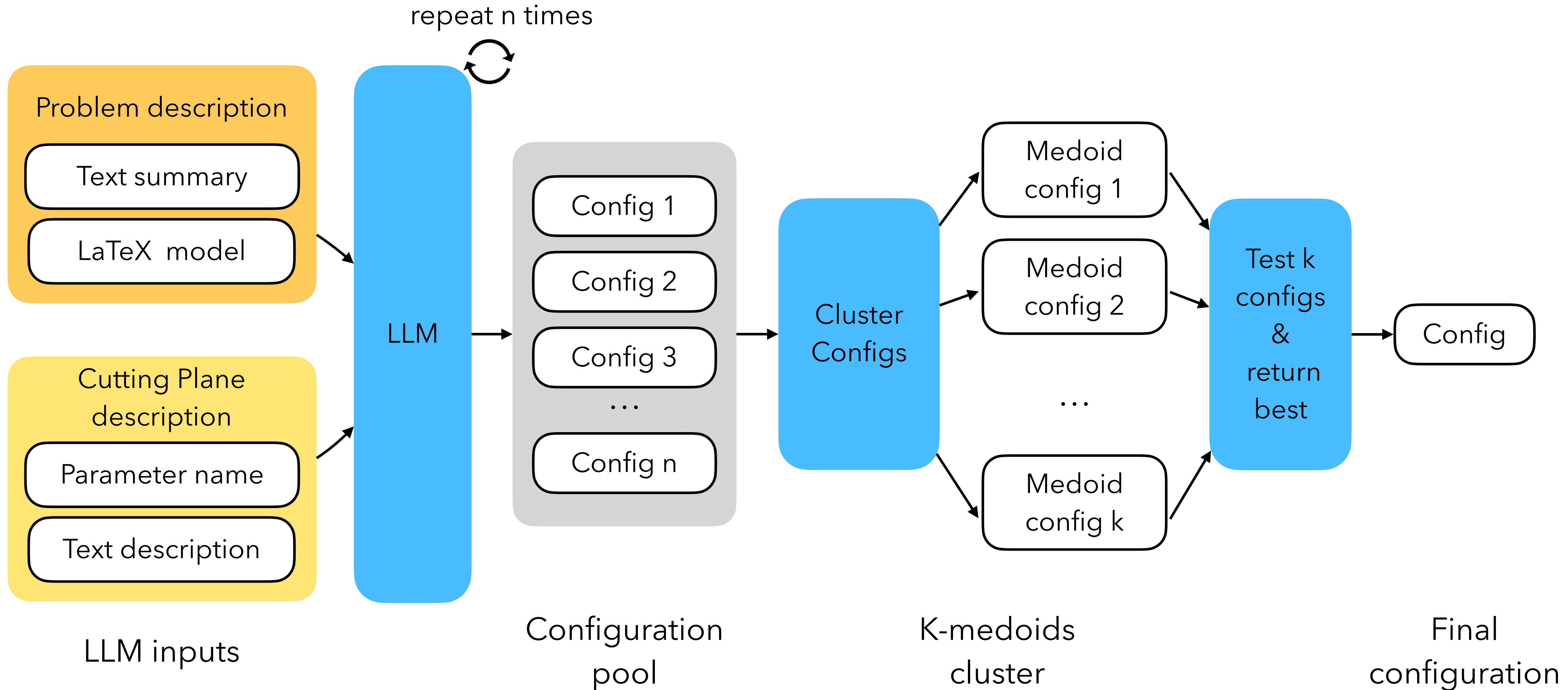
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Experimental set-up

Baselines and our method

- **Pruning**: turns off all CPs not used while solving validation set

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 2. Select the best performing medoid on the validation set

Experimental set-up

Datasets, model

Dataset	# vars	# constrs
Binary packing	300	300
Capacitated facility location	100	100
Combinatorial auction	100	500
Maximum independent set	500	1088
Max cut	54	134
Packing	60	60
Set cover	500	250
Load balancing	64340	61000
Middle-mile consolidation network design (MM)	569	248

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Complex real-world MILP families	Load balancing	64340	61000
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Experimental set-up

Datasets, model

	Dataset	# vars	# constrs	Model: GPT-4o
Classic MILP families	Binary packing	300	300	Training set size: 100 Val set size: 30 Evaluation metric: % improvement over default solve time
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	Combinatorial auction	100	500	
	Maximum independent set	500	1088	
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Complex real-world MILP families	Load balancing	64340	61000	
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Empirical results

Cold-start(5) yields 6-71% faster runtimes than SCIP's default

Problem	Pruning	Search(5)	Search(500)	Zero-shot	Cold-start(5)
Bin. pack.	1.33	9.23	39.3	16.76	38.35
Cap. fac.	-0.64	9.57	2.72	7.61	26.12
Comb. auc.	1.96	58.1	64.01	21.06	63.59
Ind. set	2.07	26.95	67.01	21.6	71.95
Max. cut	-2.18	17.72	69.63	71.43	71.01
Pack.	15.87	-13.81	24.49	15.09	25.51
Set cov.	6.62	-10.04	61.08	61.72	61.74
Load bal.	0.08	-150.01	-50.02	0.0	6.37
MM	-0.12	-8.83	50.03	-6.52	53.3

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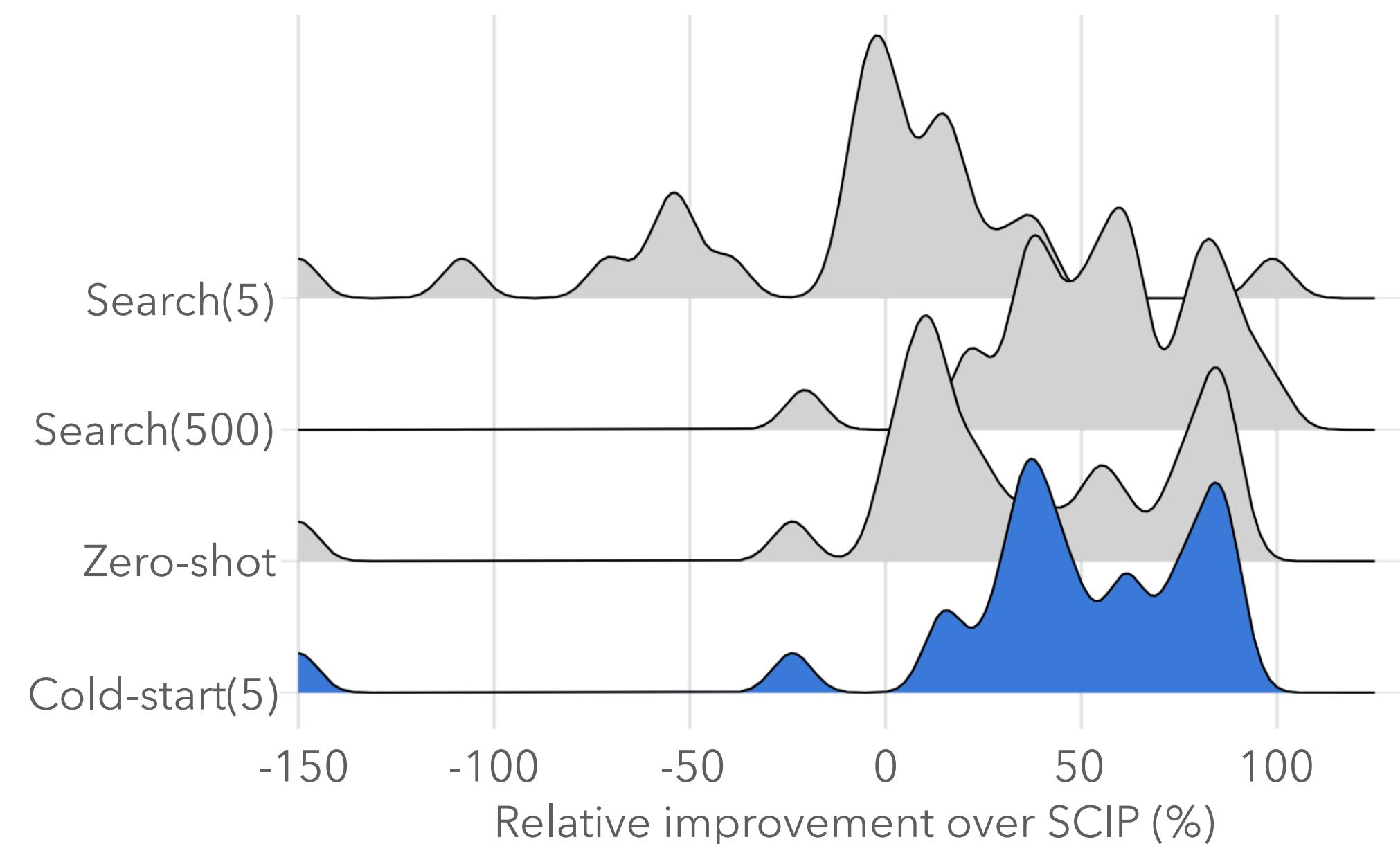
By testing only 5 configs,
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Without solving **any** MILPs,
we match/beat Search(500)
on **4/9 instances**

On out-of-distribution instances

25 families of problem from MILP-Evolve dataset [Li et al. ICLR'25]

- New dataset, “evolving” pipeline for generating new MILP families
- Designed to be highly diverse to mimic real-world optimization scenarios



Ablations

Our design choices are robust

Setting	Ind. set	Max cut	Bin. pack.	MM
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Ablations

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Disable cutting planes	-14.96	71.25	30.43	-150

Disabling CPs can reduce performance

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Setting	Ind. set	Max cut	Bin. pack.	MM
Ours	71.95	71.01	38.35	53.3
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No CP text descr.	72.27	71.49	16.85	9.29

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Our **CP descriptions** boost performance

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Ensembling strategies					
Average configuration	20.65	71.24	17.52	-11.08	
Mode configuration	21.08	71.44	18.11	-12.63	k-medoids outperforms simpler heuristics
Smallest configuration	20.83	70.91	17.42	-4.74	

Recap

Can we use LLMs to configure MILP solvers with minimal training data?

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- Requires **no custom solver interface**
- Competitive with existing configuration approaches
but only requires a **fraction of the training data and computation time**

Outline

1. Background
2. LLMs for speeding up solvers
 - i. Solver configuration
 - ii. **Evolutionary search for MILP heuristics**
 - a. **Background**
 - b. Cut selection
 - c. Large neighborhood search
 - d. Diving heuristics
3. Takeaways

LLMs for automated heuristic design

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- Automated heuristic design with LLMs:
 - LLMs generate heuristic code
 - Heuristic **fitness** scored automatically
 - **Evolutionary selection** keeps improved heuristics
 - E.g., FunSearch [Romera-Paredes, Nature'24], ReEvo [Ye et al., NeurIPS'24], AlphaEvolve [Novikov et al., '25], ...

Evolution of Heuristics

[Liu, Tong, Yuan, Lin, Luo, Wang, Lu, Zhang, ICML'24]

Example: Online bin packing

- Task: Pack items of varying sizes into fewest bins

Figure by Fawzi, Romera-Paredes



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Example: Online bin packing

- Task: Pack items of varying sizes into fewest bins
- Items arrive sequentially
 - Must be packed into a bin immediately
 - No knowledge of future arrivals
- Each bin has fixed capacity (experiments: $C = 100$)

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Example: Online bin packing

Heuristic representation

Heuristic

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Heuristic

Code

```
def heuristic(item, bins):
    """
    item: scalar item size
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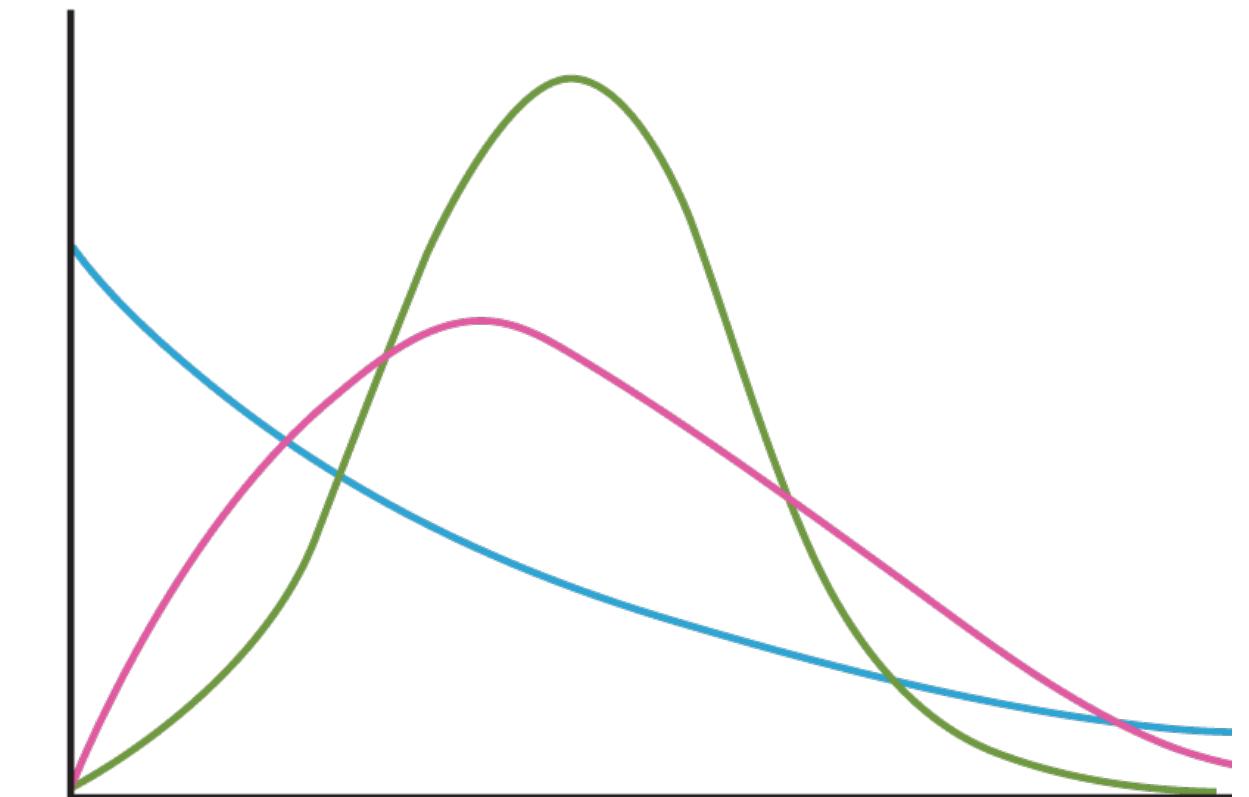
Incorporates a weighted average of the utilization ratio, dynamic adjustment, and an exponentially decaying factor, with different parameter settings to minimize the number of used bins.

Fitness: 0.0196

Example: Online bin packing

Fitness metric

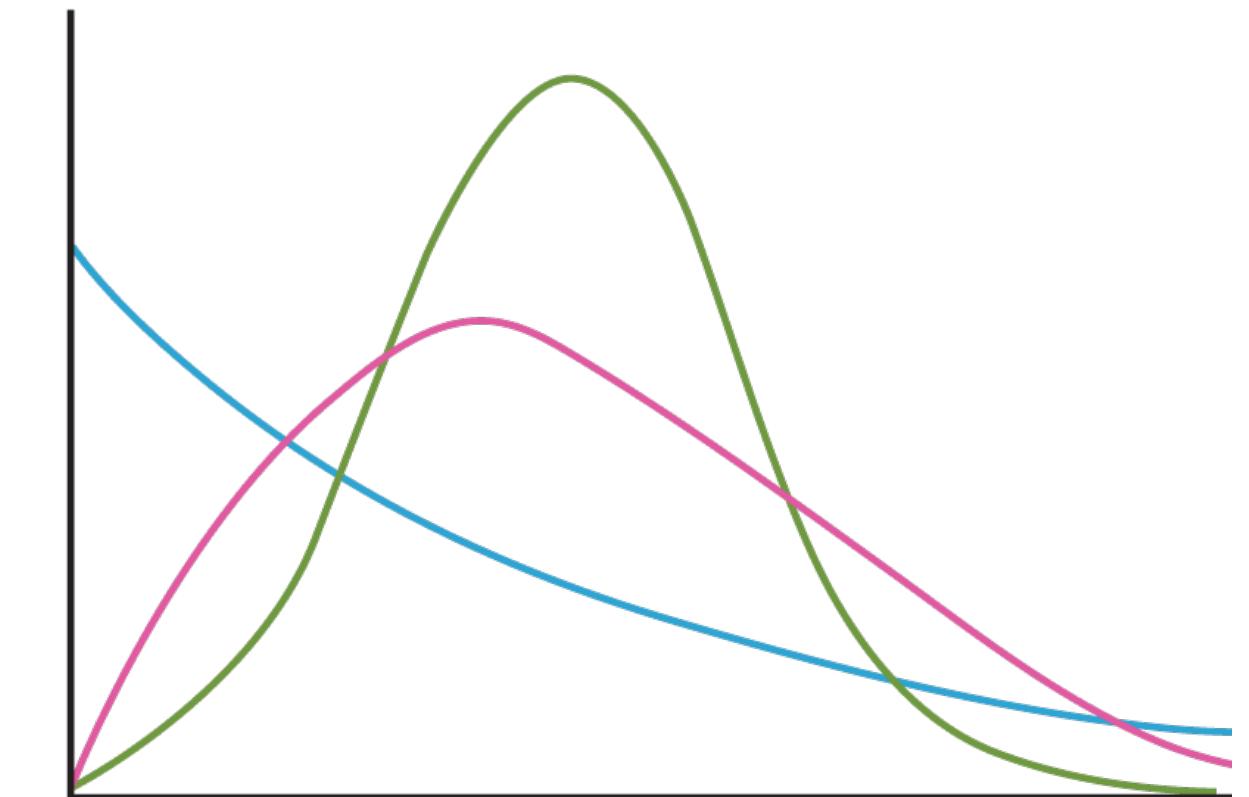
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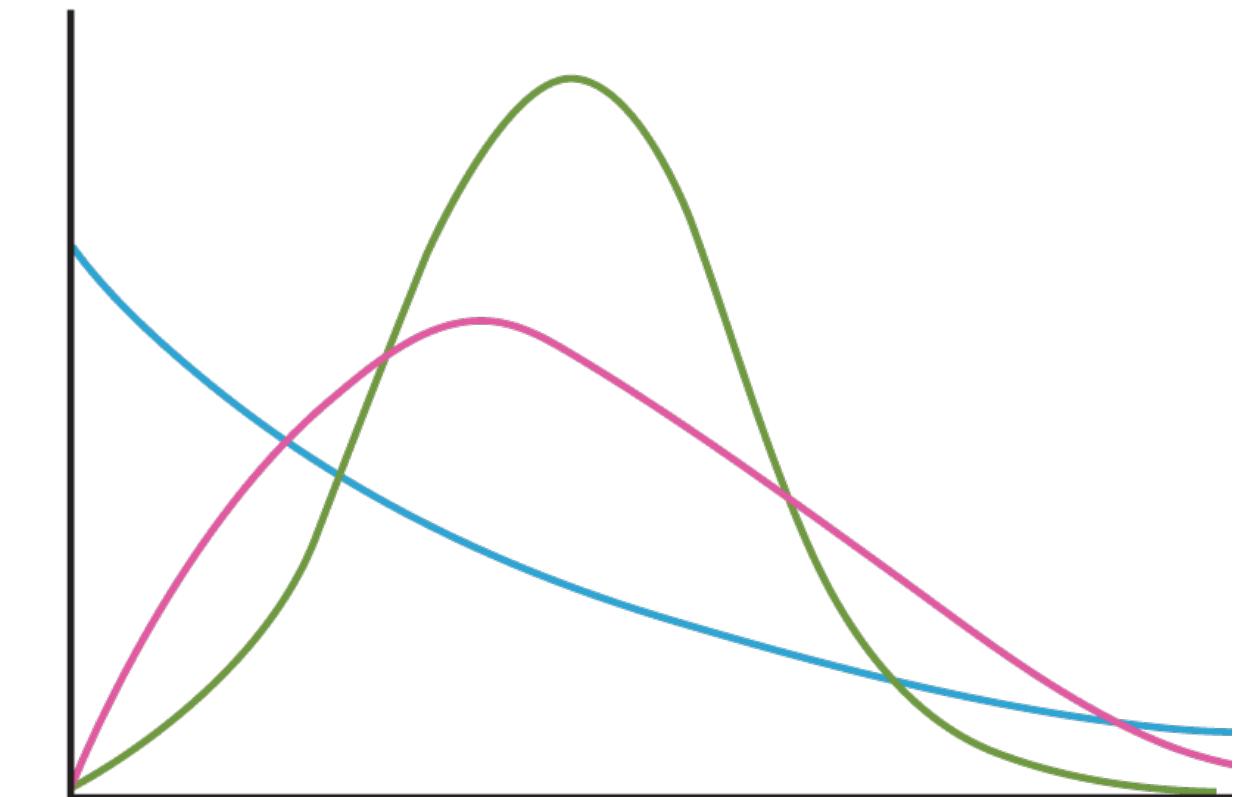
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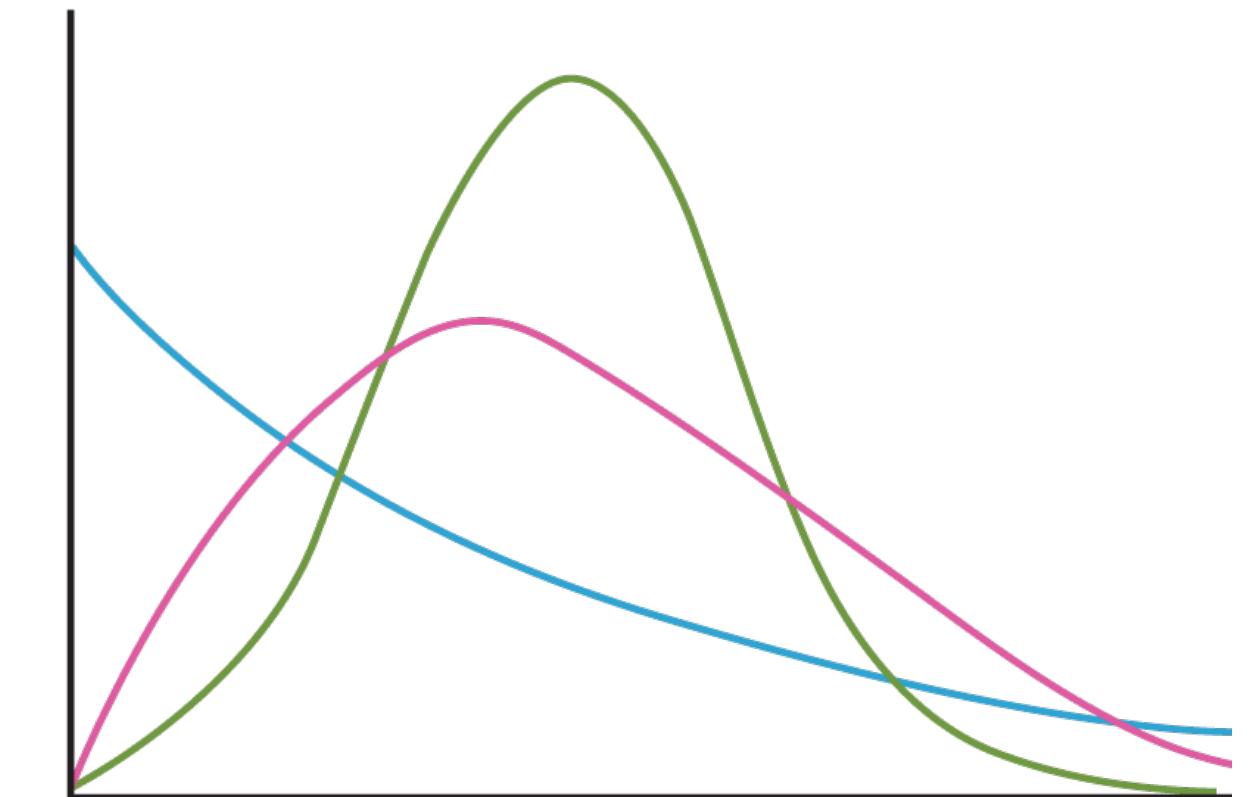
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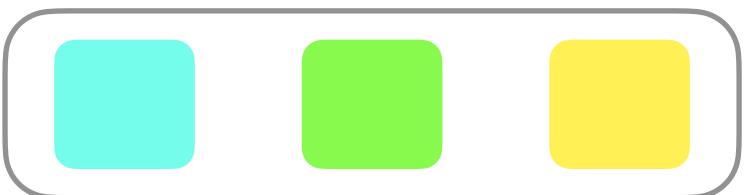
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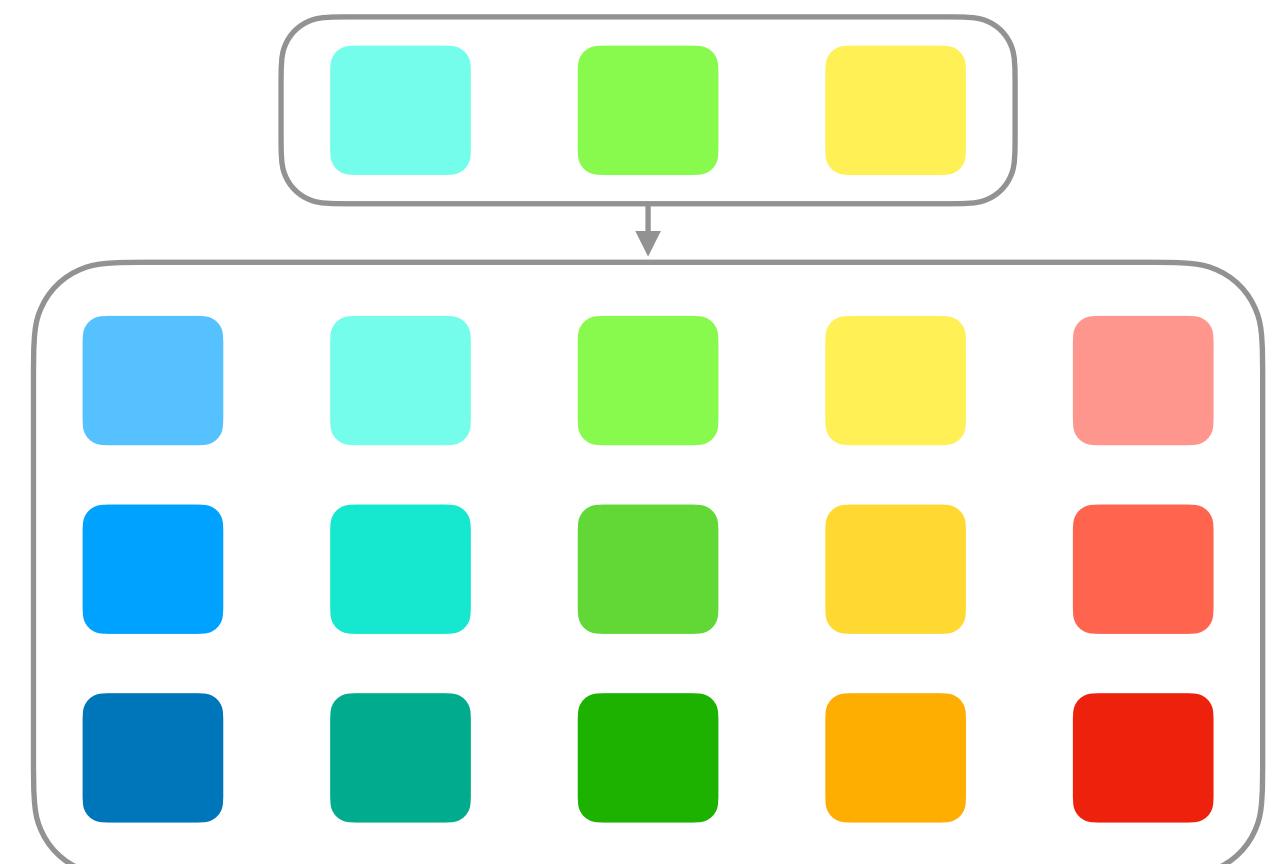
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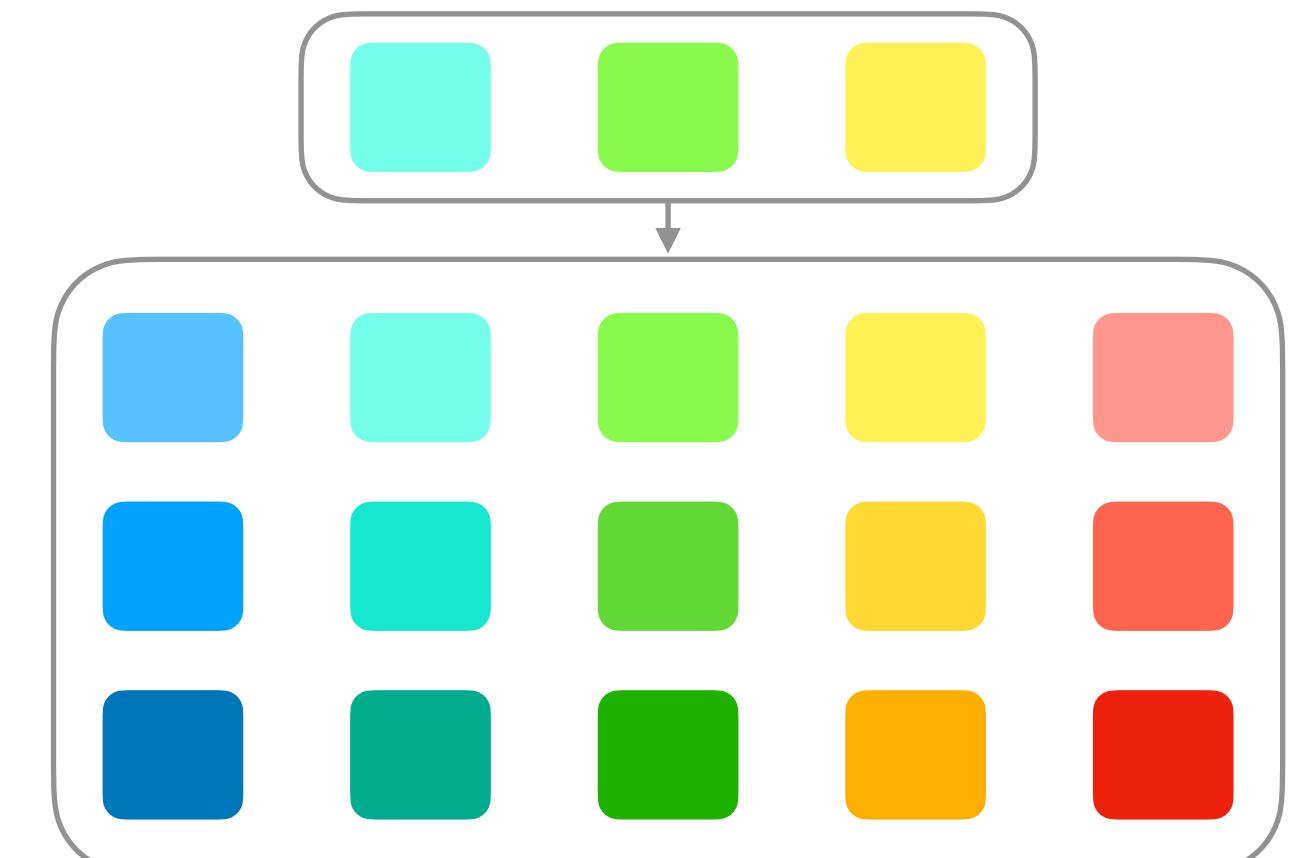
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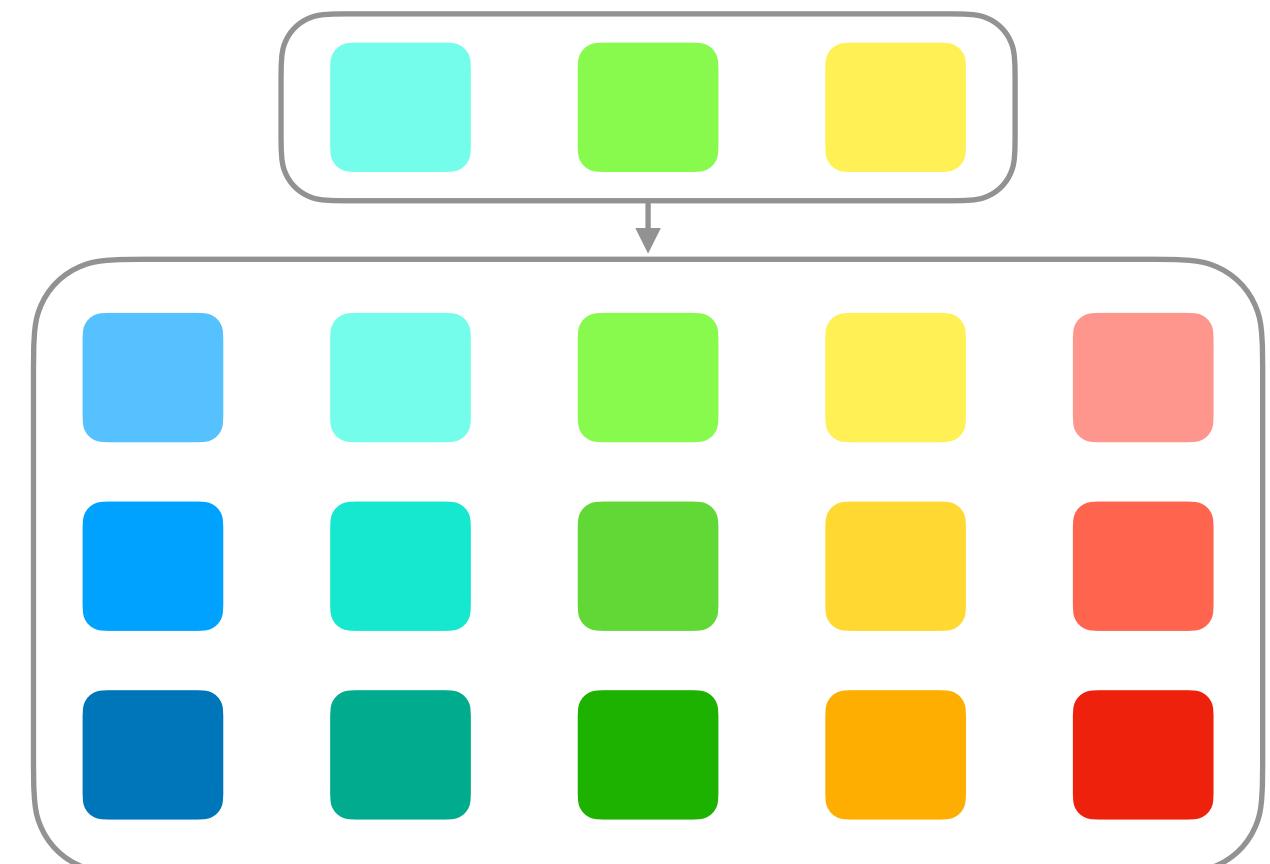


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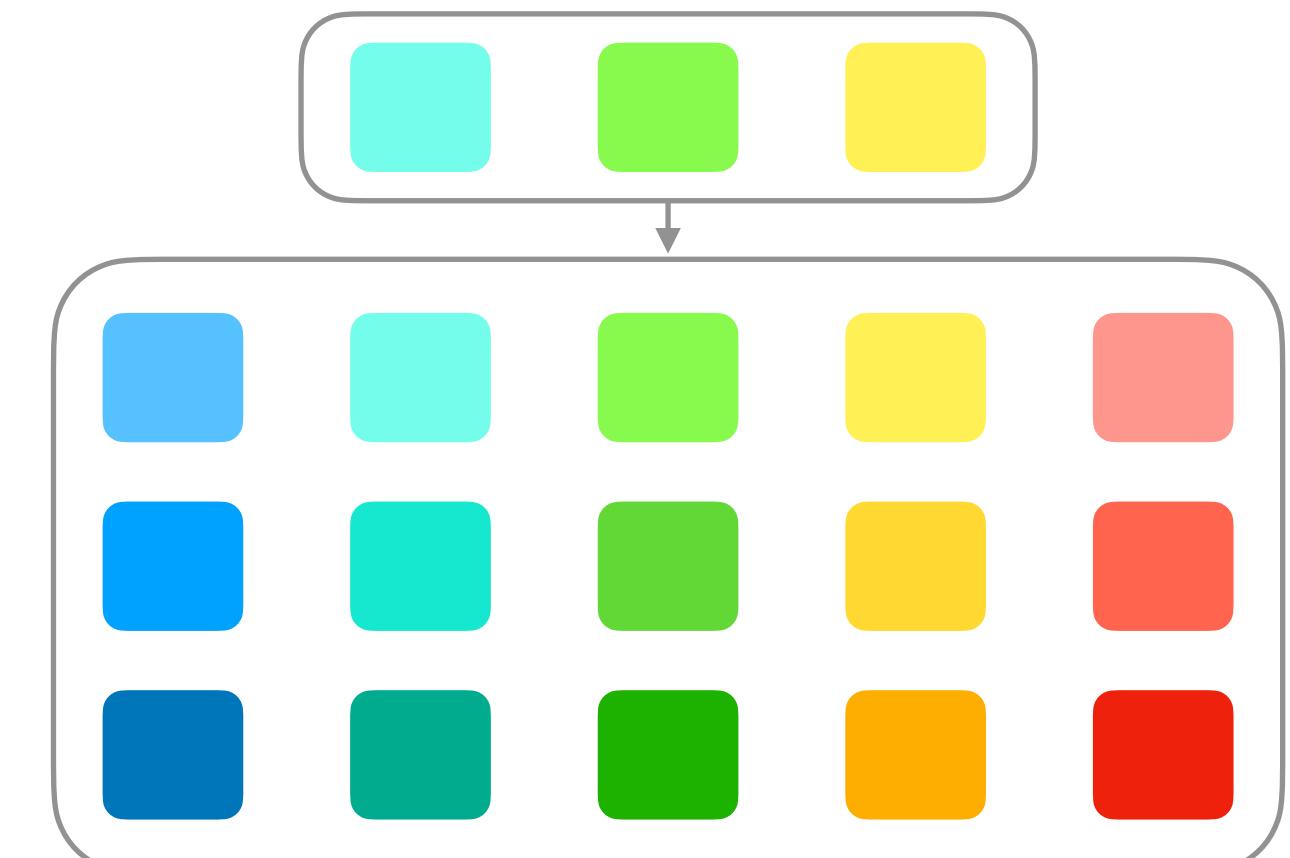


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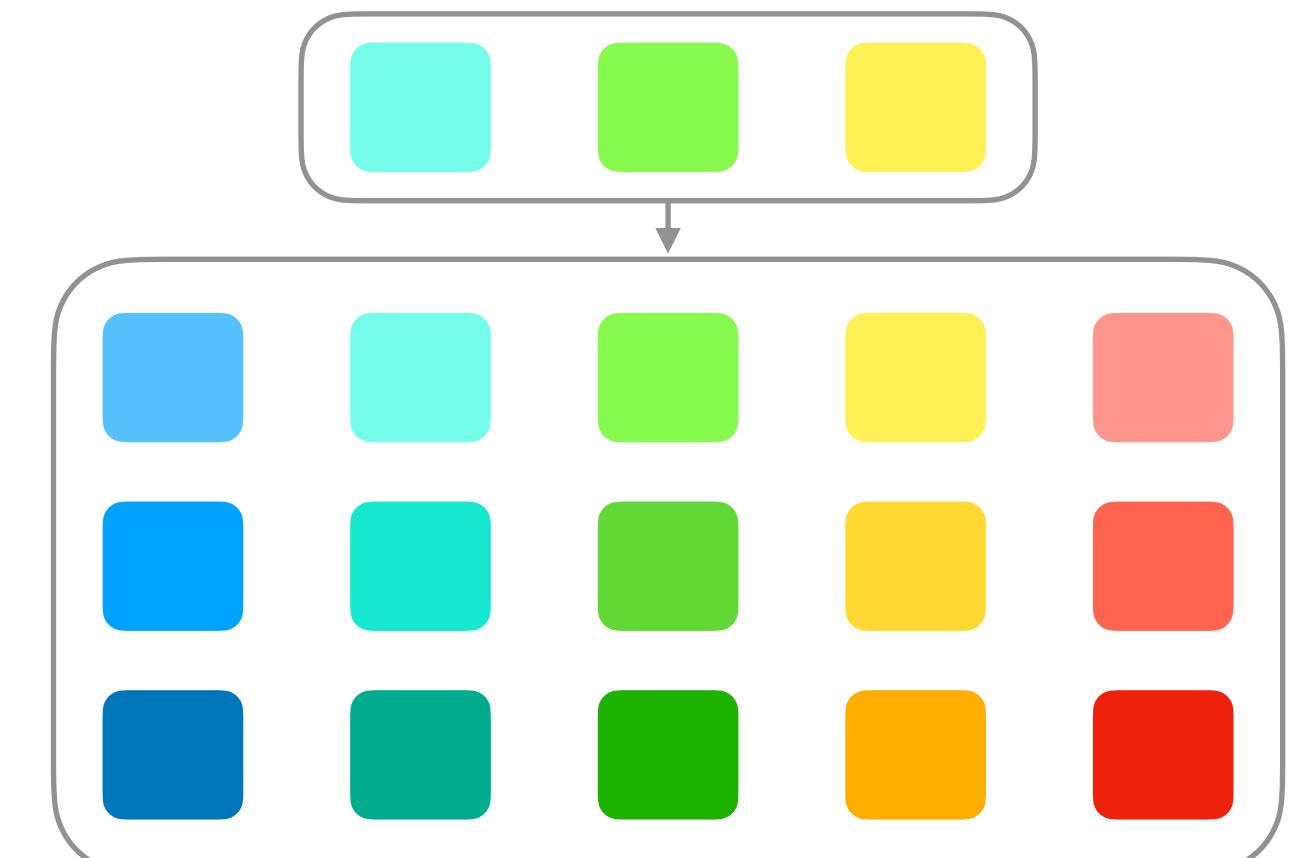


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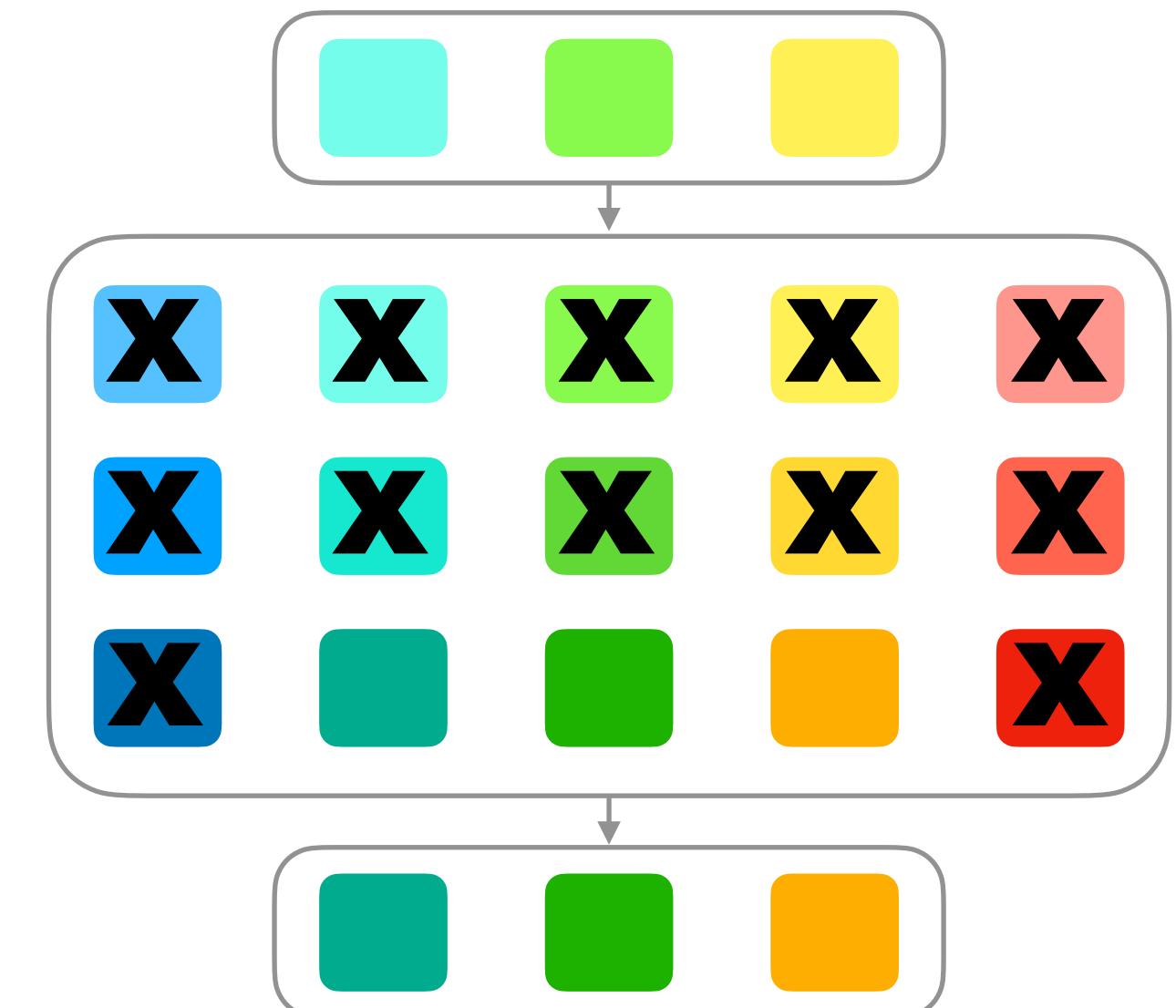
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3. **Retain** top N heuristics by fitness; return to Step 1



Example: Online bin packing

Initialization prompt

- Help design a new heuristic that scores a set of bins to assign an item

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Evolution prompts

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- **E2 - Shared-Idea variants:** generate based on high-performing “themes”

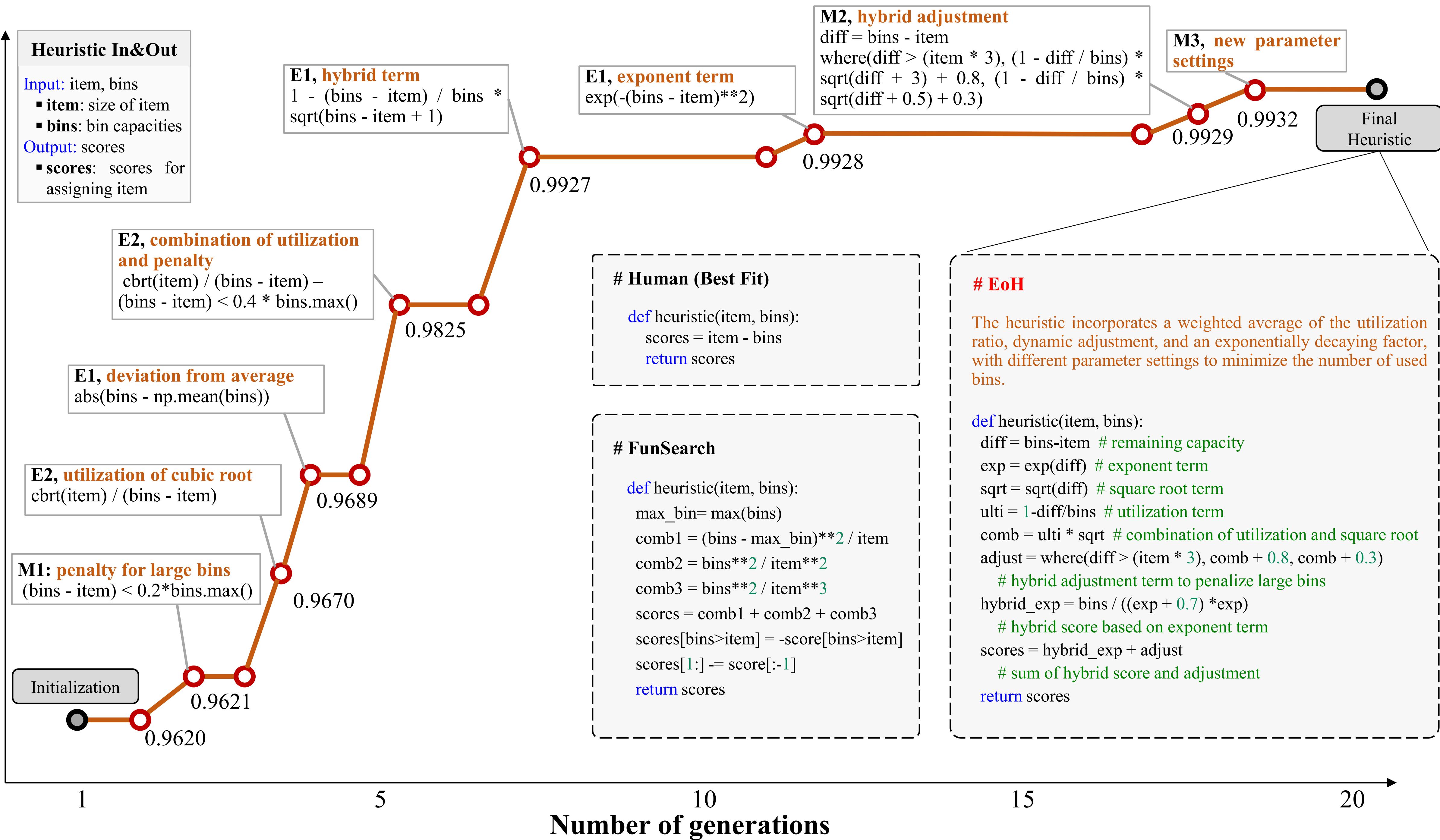
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- **M3 - simplification:** prune unnecessary components or redundant logic

Performance (objective)



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Experimental setup

- **Baselines:**
 - First Fit: place item in first bin that fits

Example: Online bin packing

Experimental setup

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- **Each setting:** 5 randomly generated instances

Bin packing results

Method	C=100			C=500		
	1k items	5k items	10k items	1k items	5k items	10k items
First Fit	5.32%	4.40%	4.44%	4.97%	4.27%	4.28%
Best Fit	4.87%	4.08%	4.09%	4.50%	3.91%	3.95%
FunSearch	3.78%	0.80%	0.33%	6.75%	1.47%	0.74%
EoH	2.24%	0.80%	0.61%	2.13%	0.78%	0.61%

Metric: average gap to lower bound [Martello & Toth '90]

Beyond EoH

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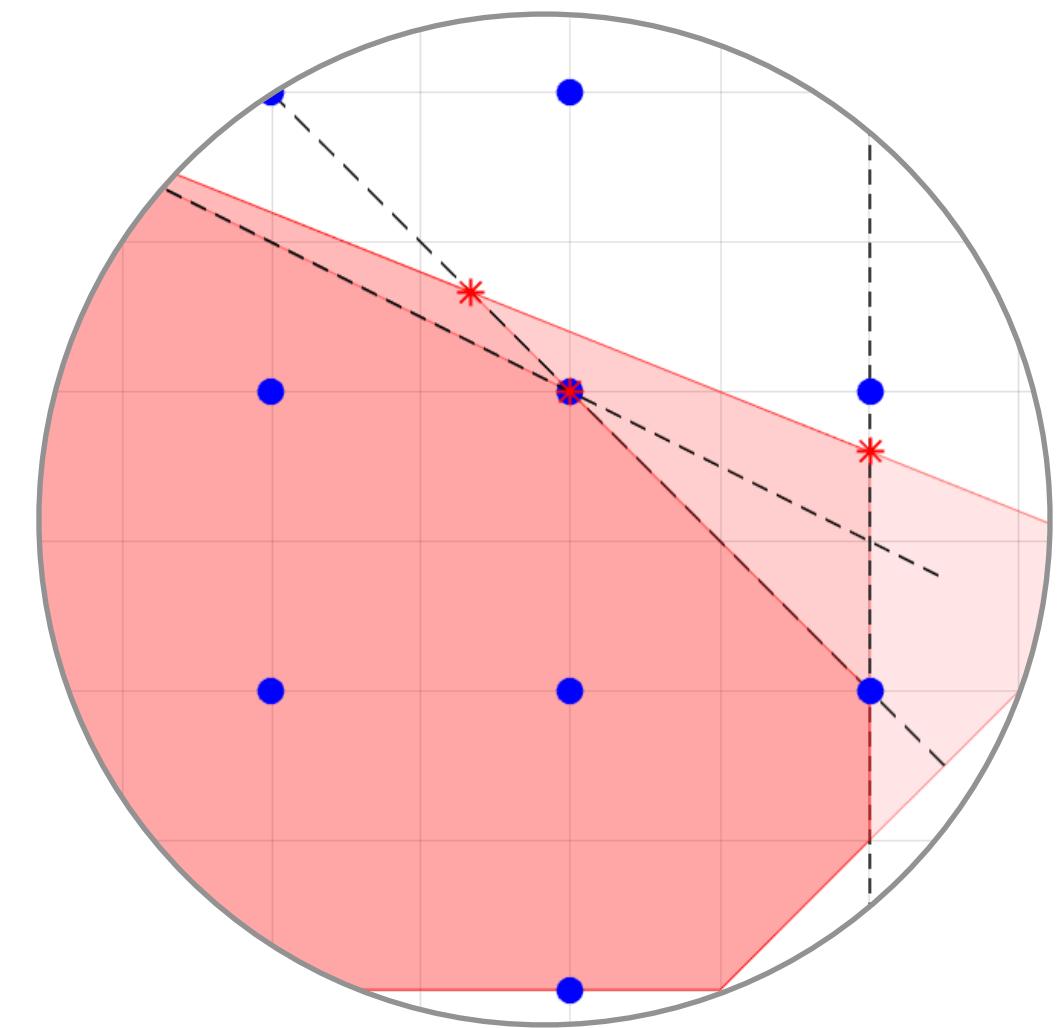
Outline

1. Background
2. LLMs for speeding up solvers
 - i. Solver configuration
 - ii. Evolutionary search for MILP heuristics
 - a. Background
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 - c. Large neighborhood search
 - d. Diving heuristics
3. Takeaways

Yazdani, Mostajabdeh, Aref, Zhou, arXiv'25

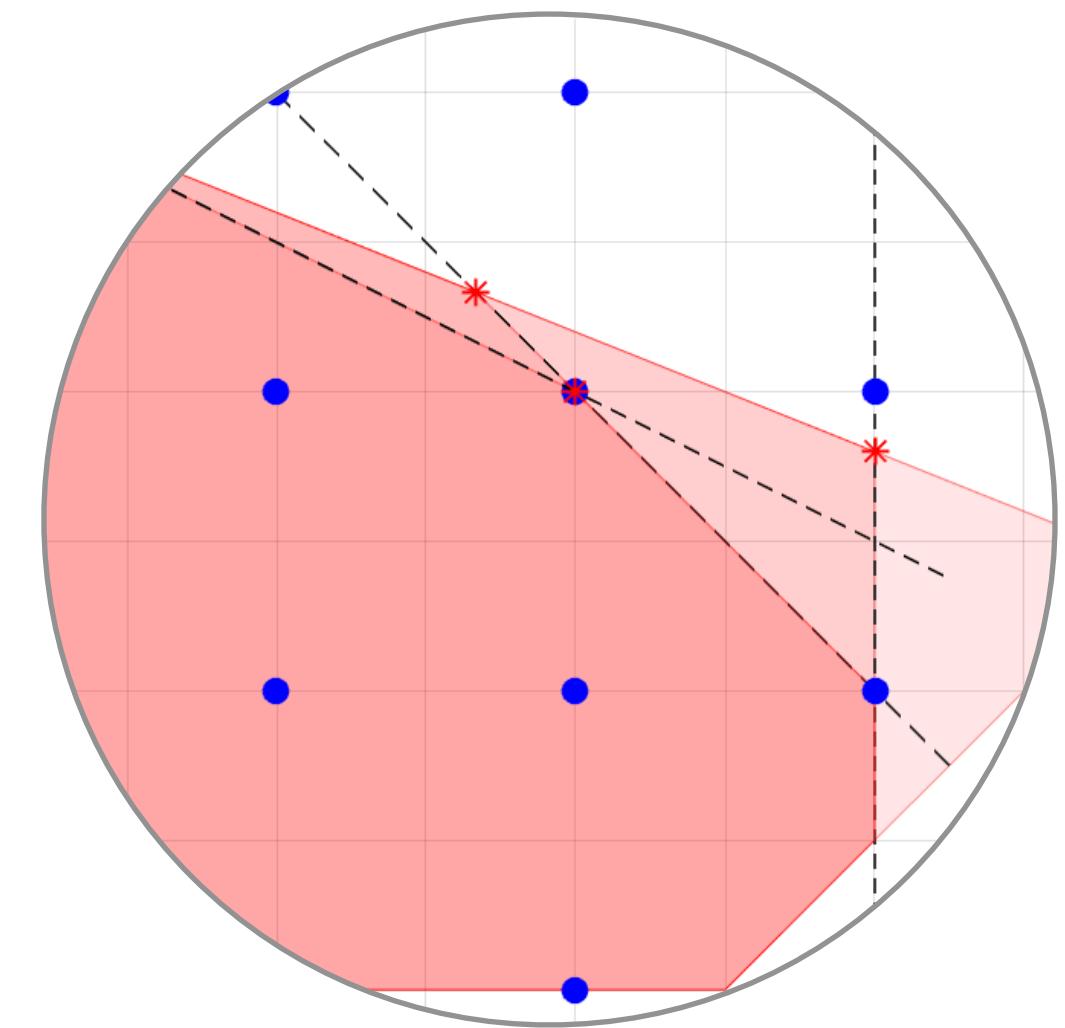
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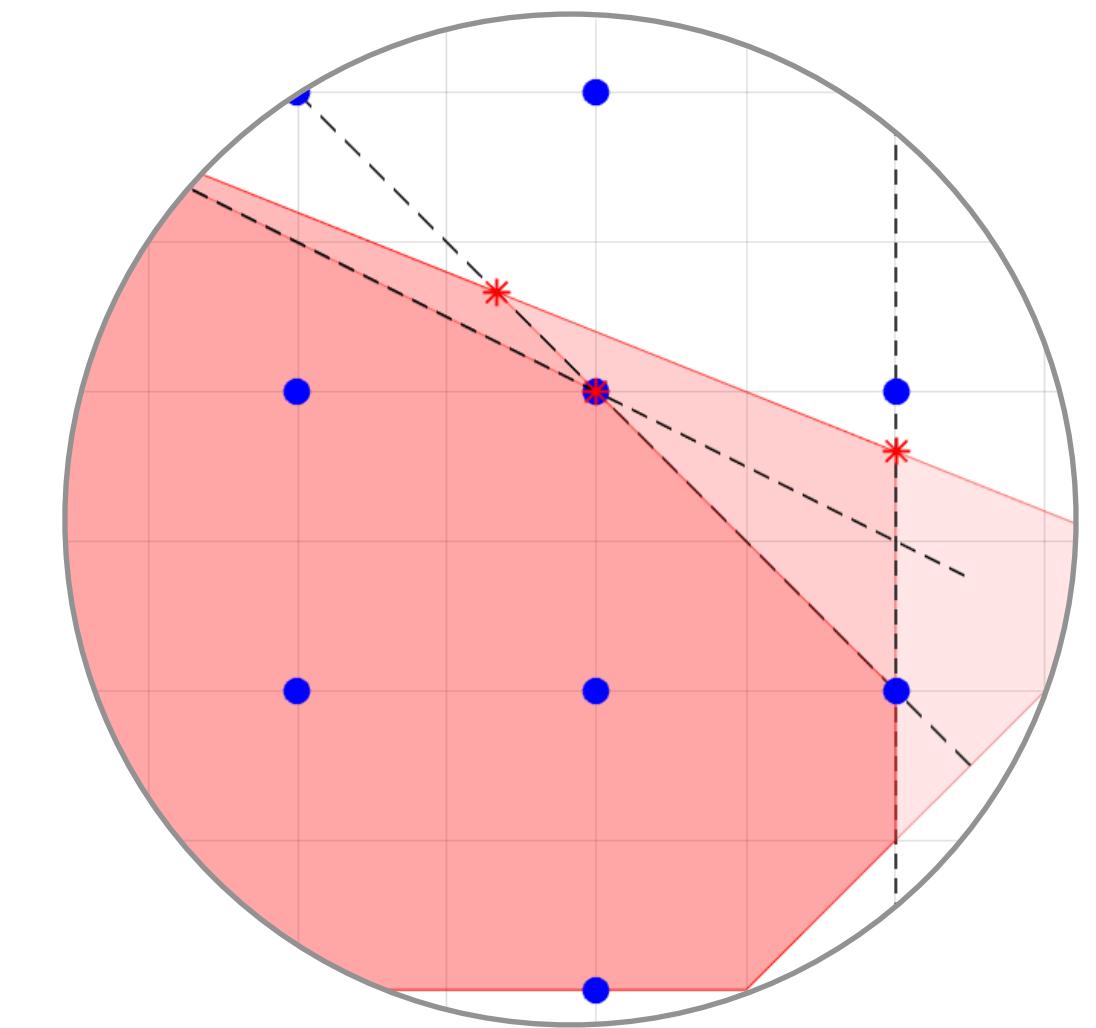
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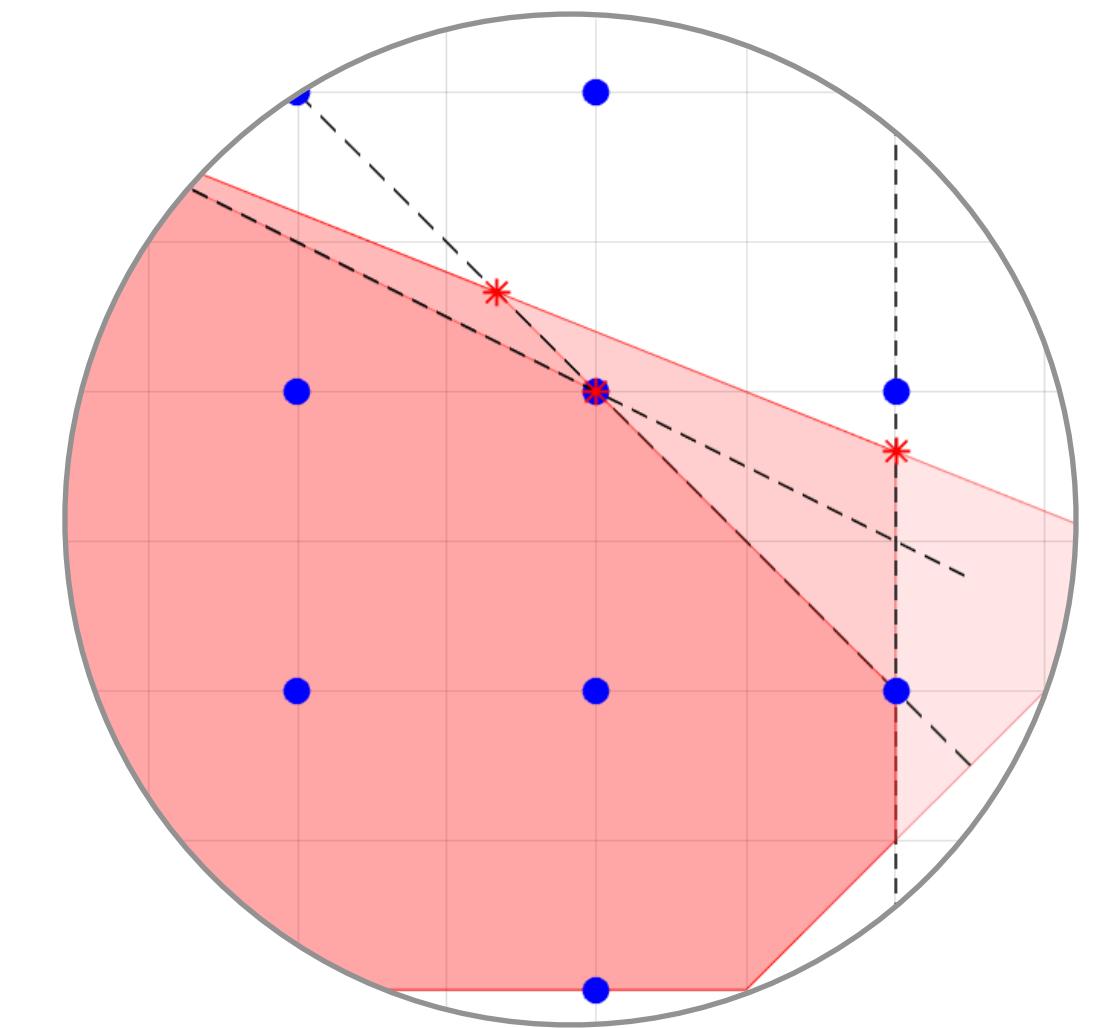
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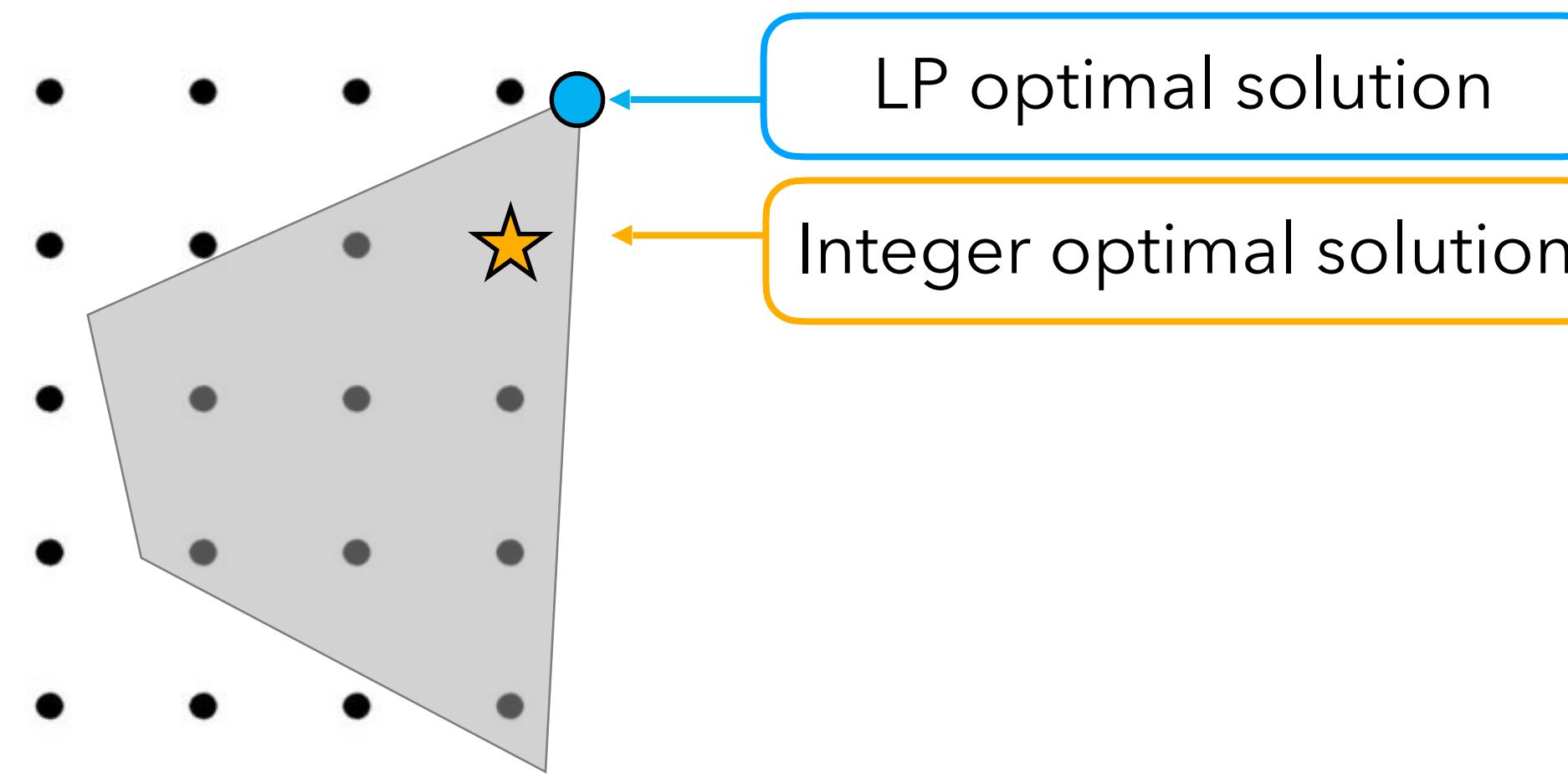
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Acceleration cuts

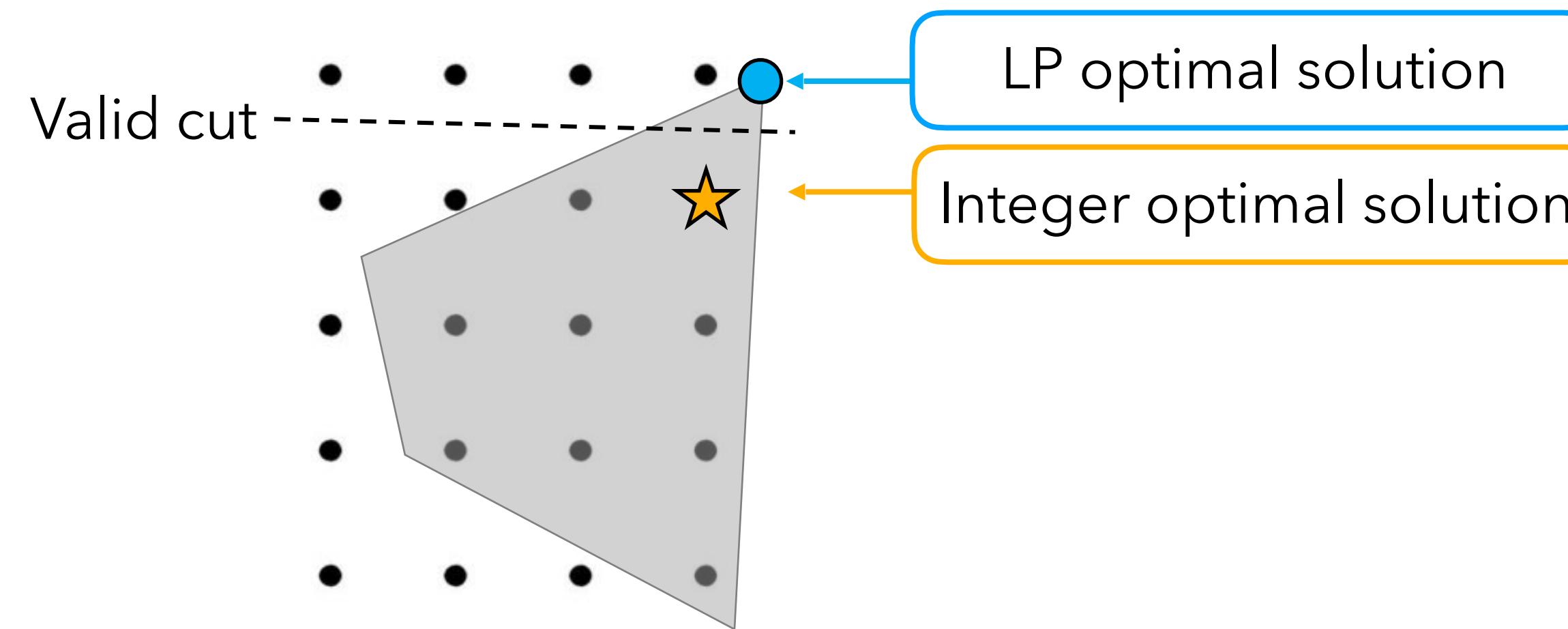
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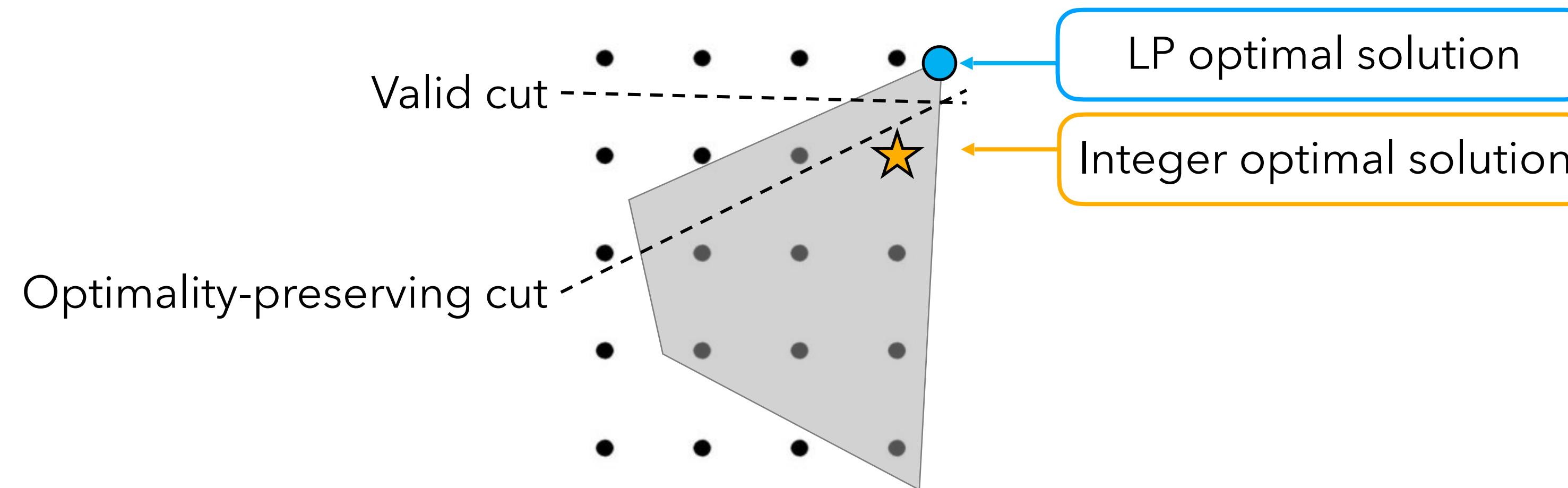
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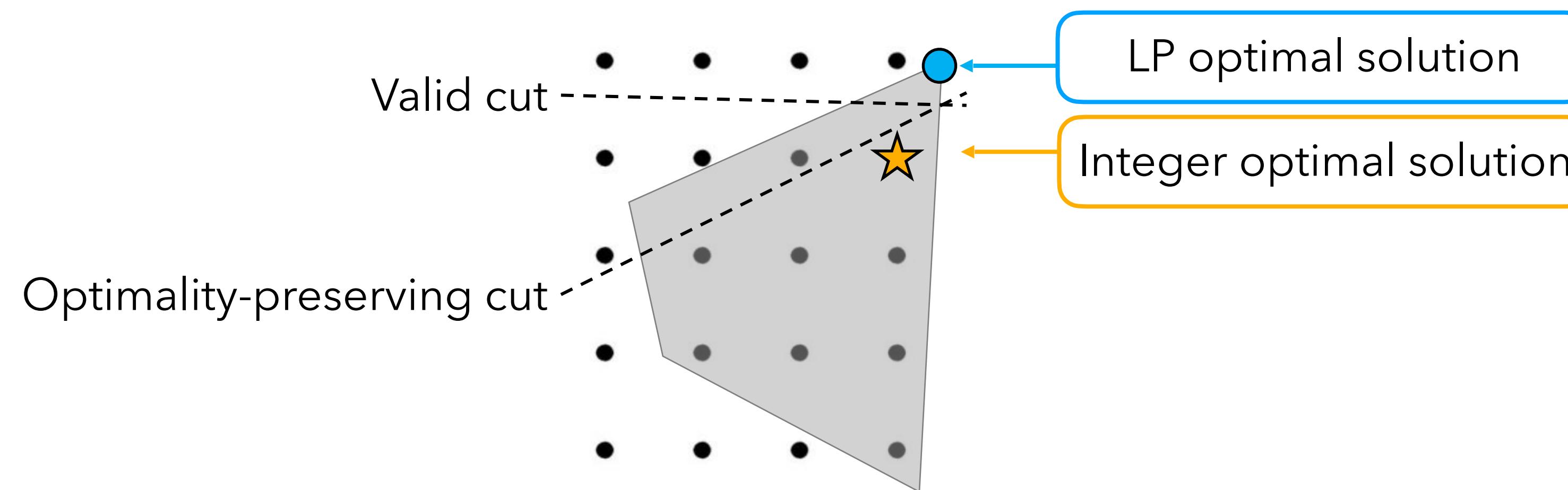
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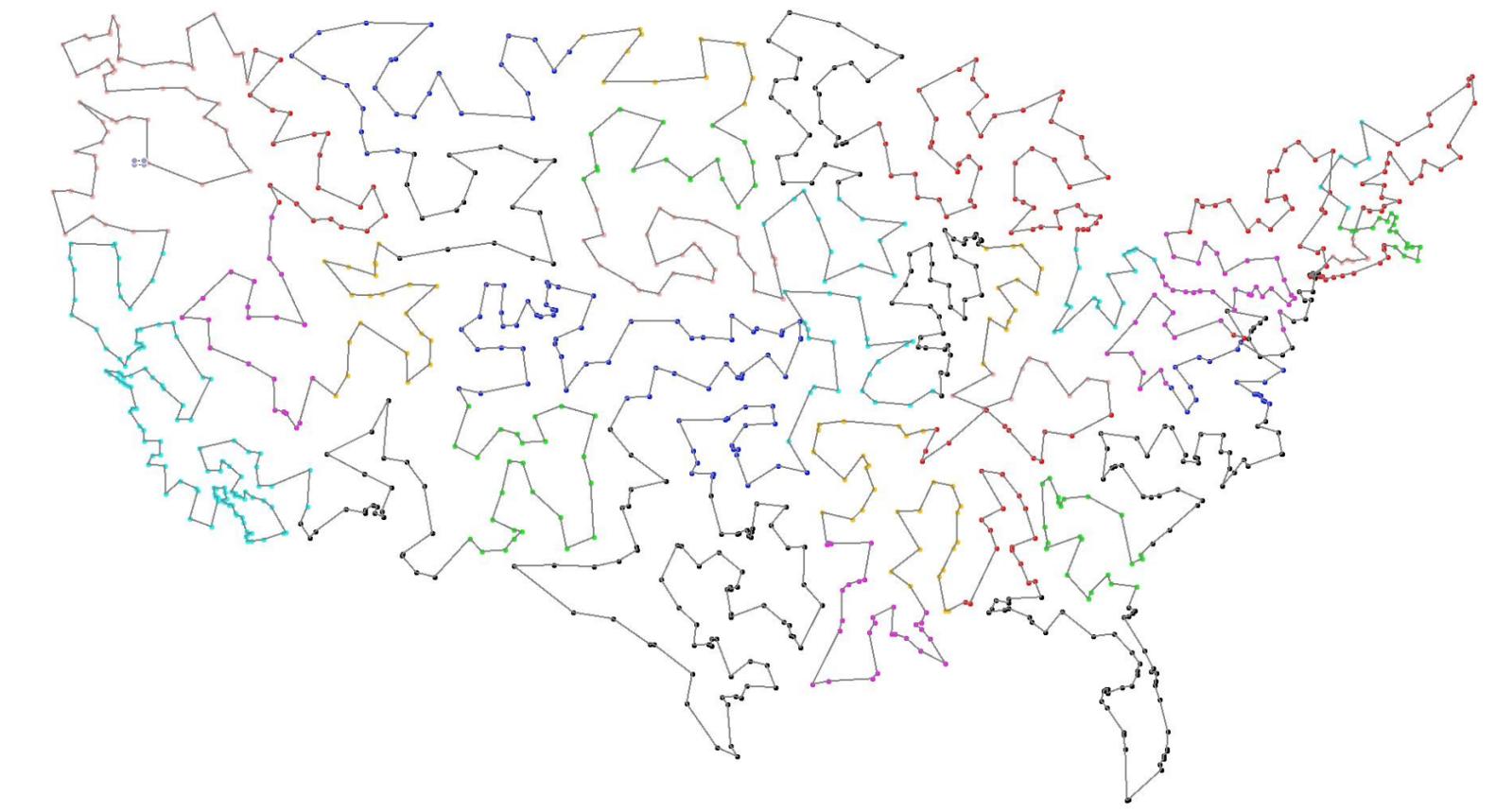
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- EvoCut cuts aren't proven optimality-preserving; empirically checked



Example: Traveling salesman problem

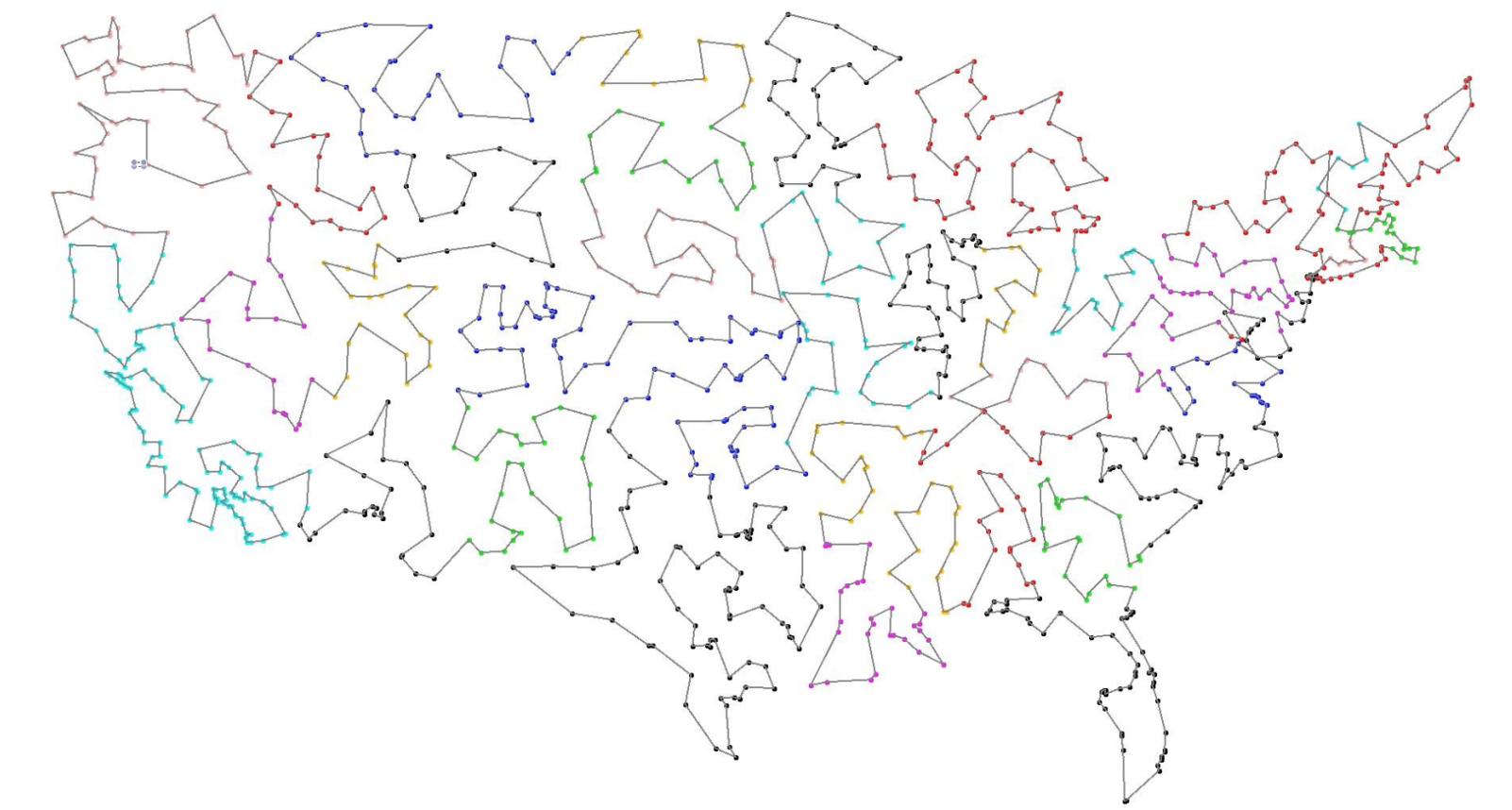
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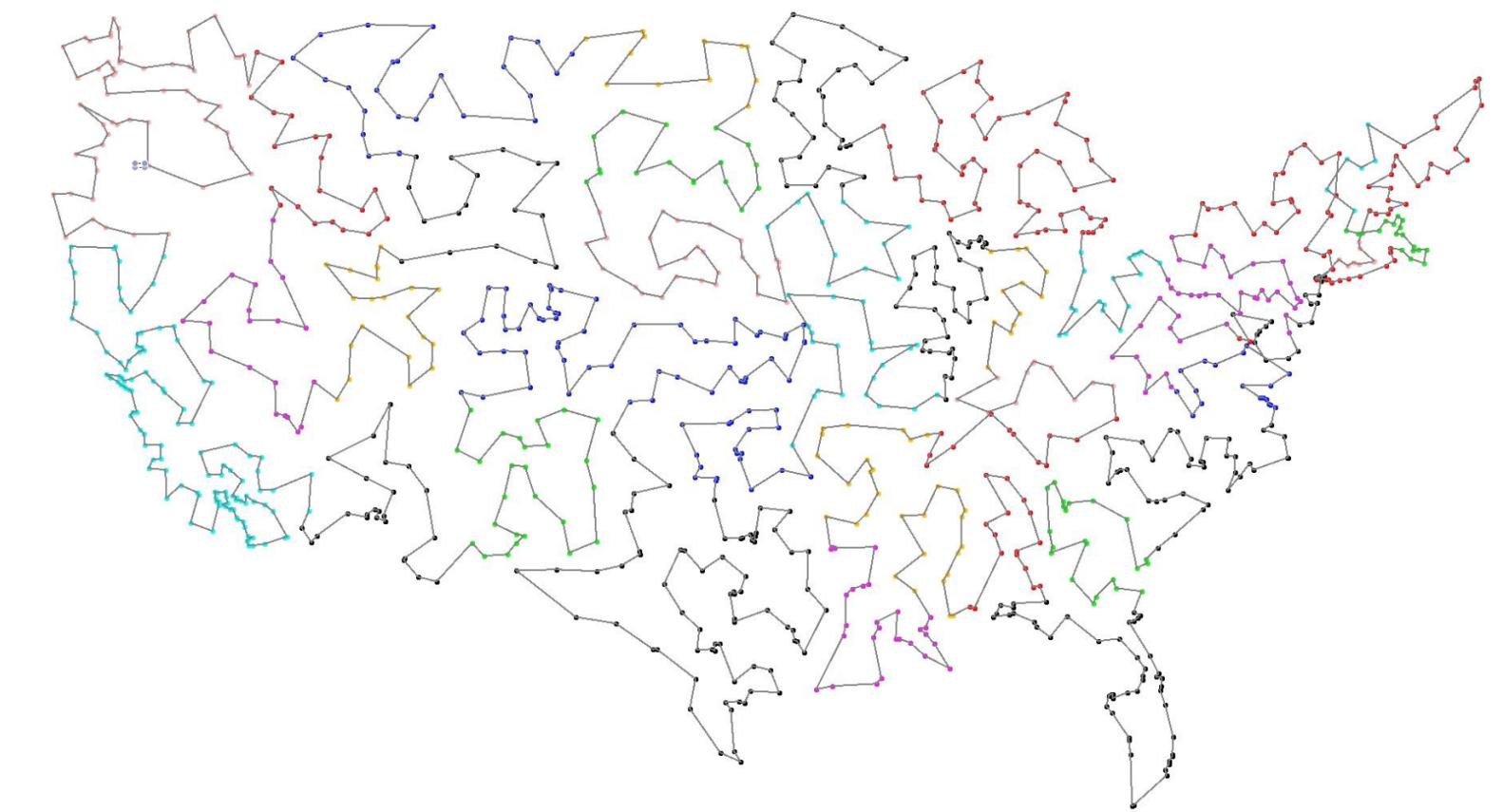
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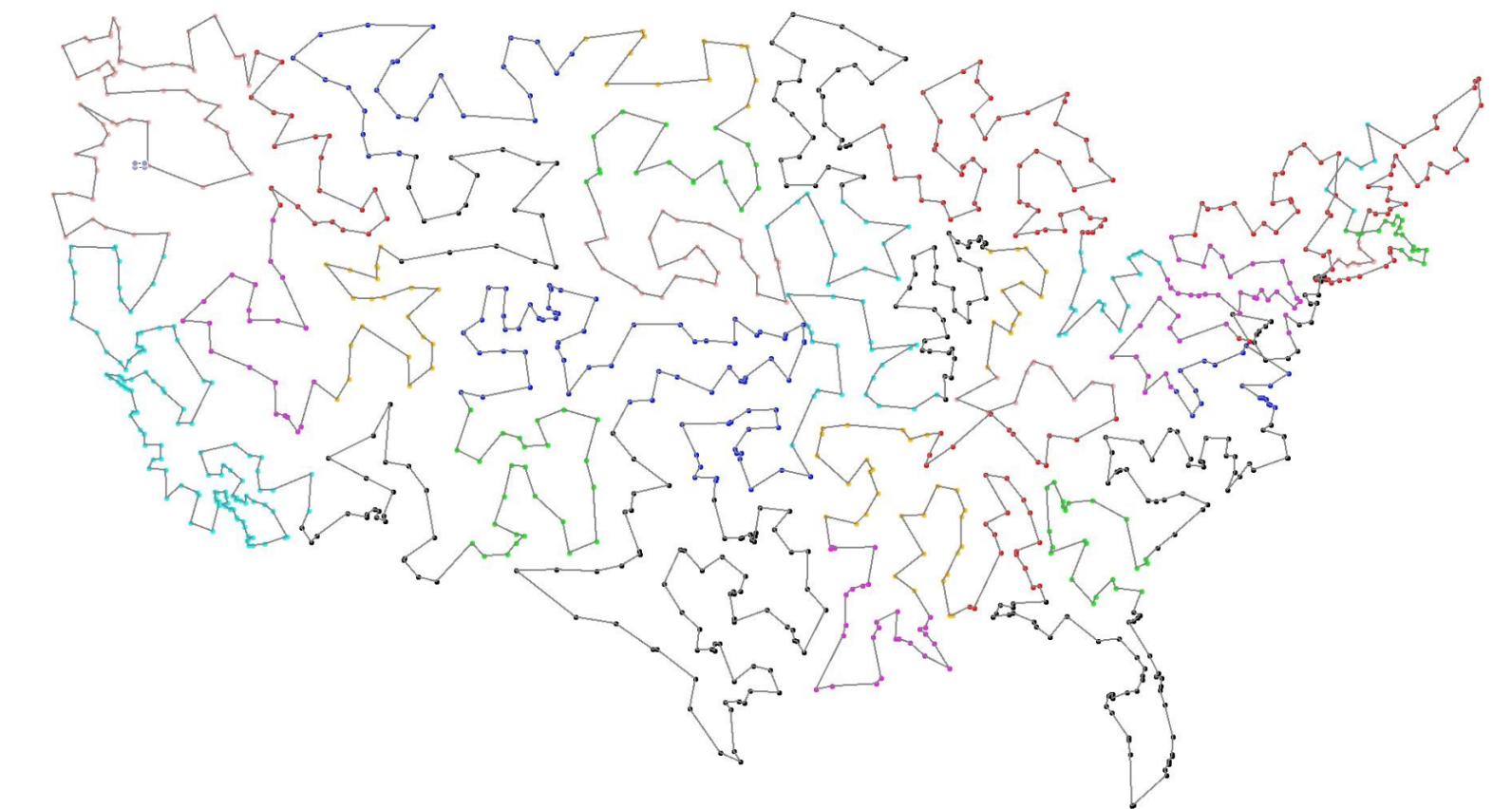
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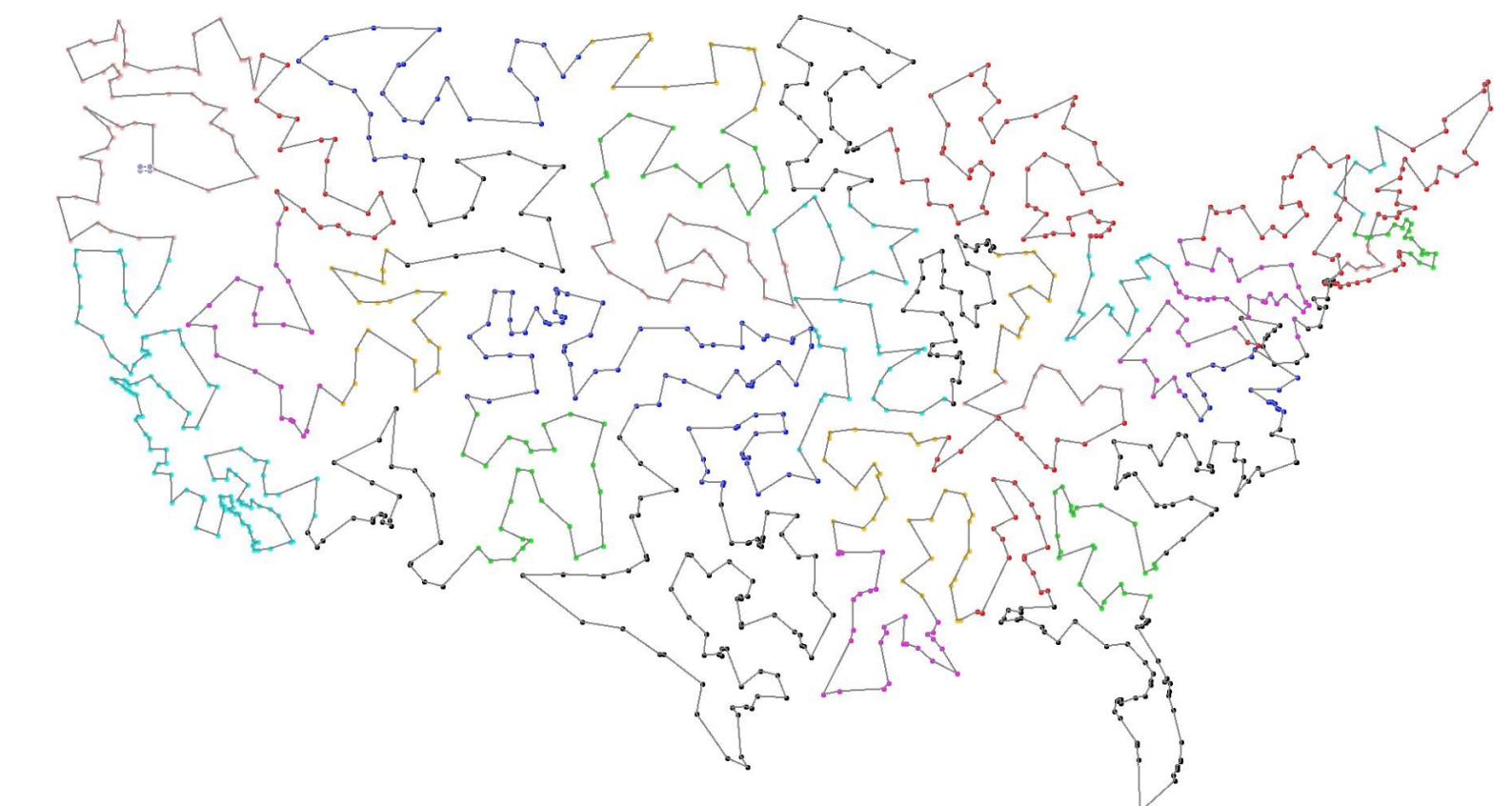
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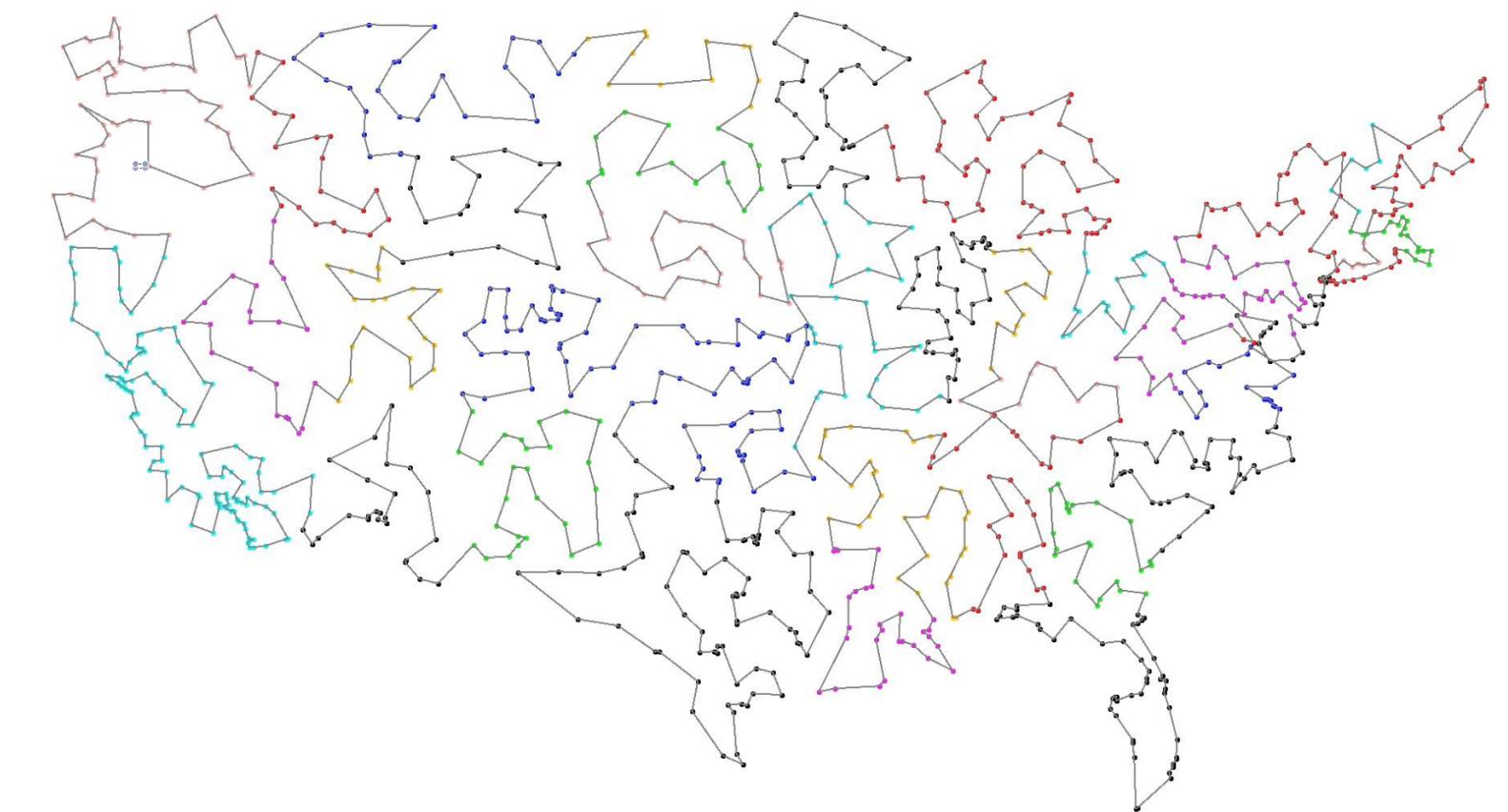
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Subtour-elimination: $u_i - u_j + nx_{ij} \leq n - 1, \quad \forall i \neq j, \quad i, j \in \{2, \dots, n\}$



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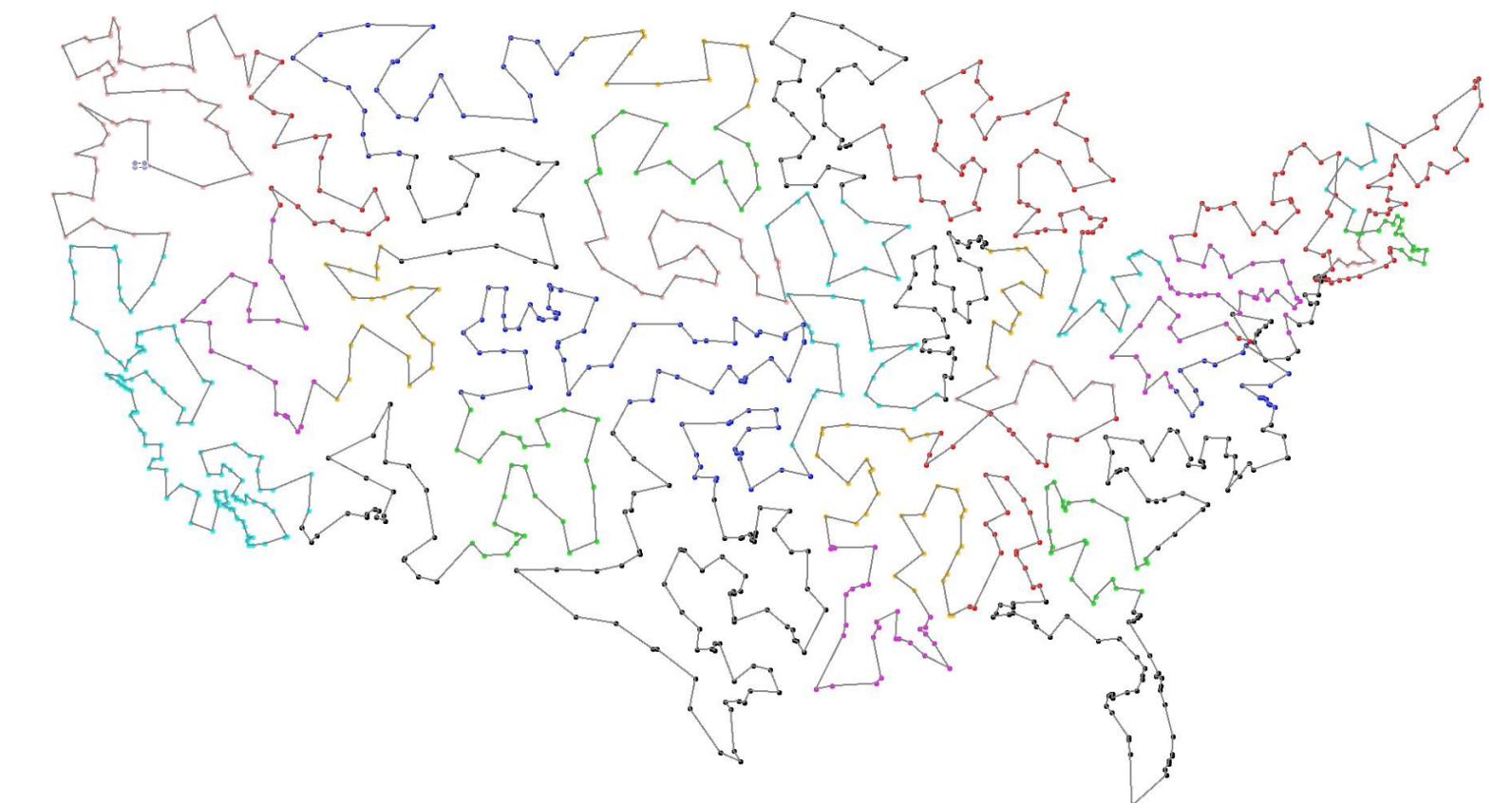
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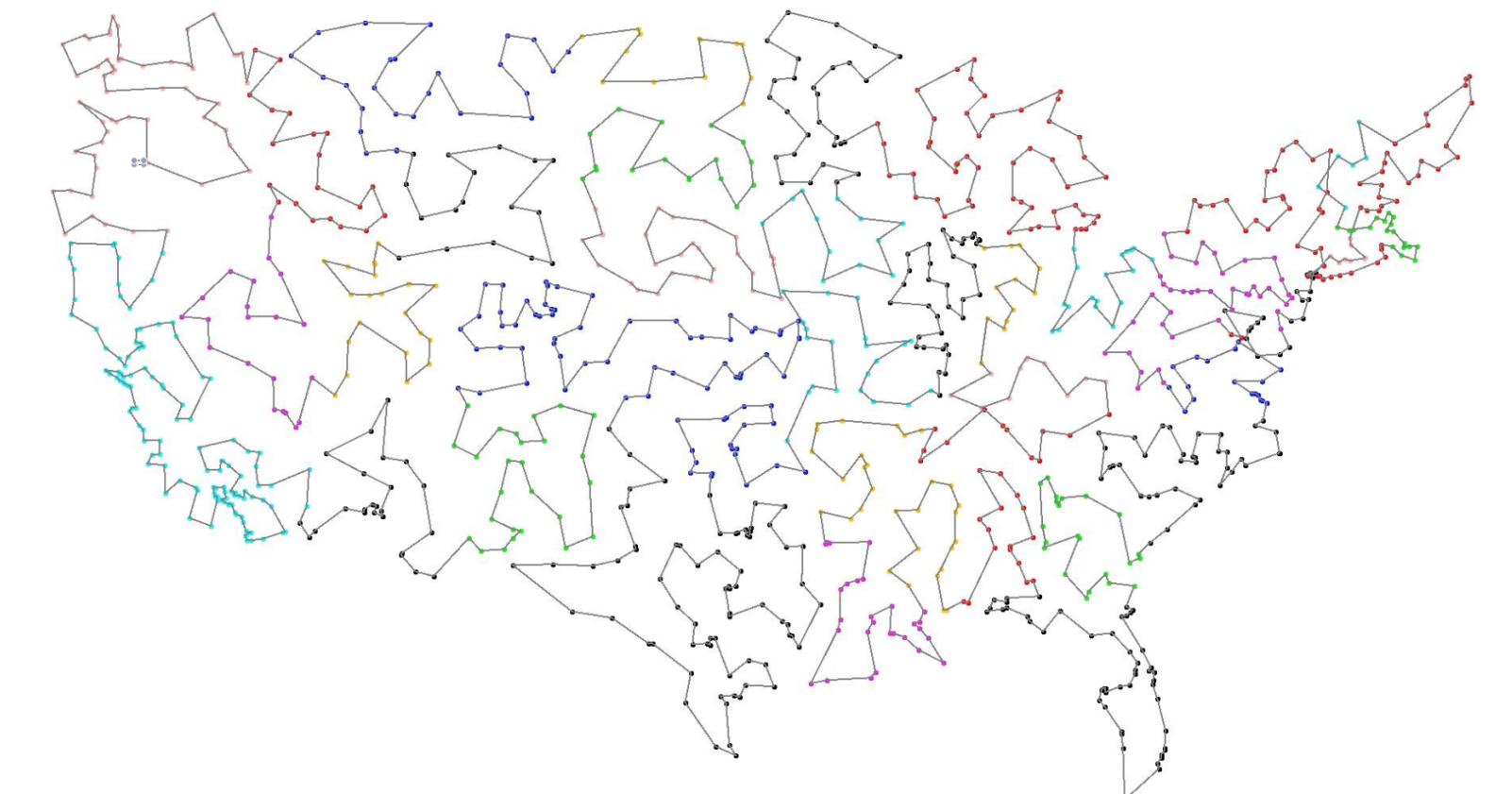
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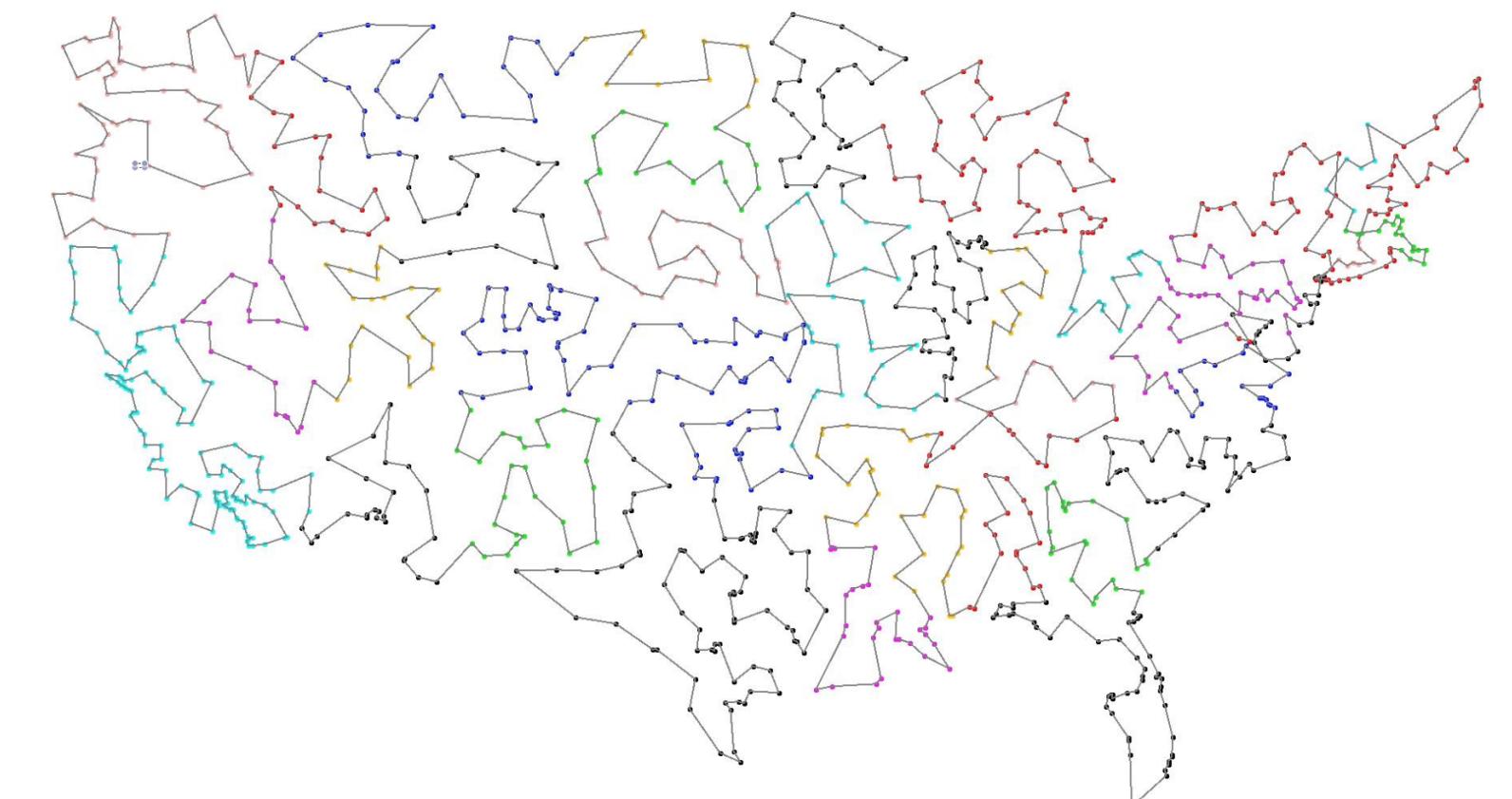
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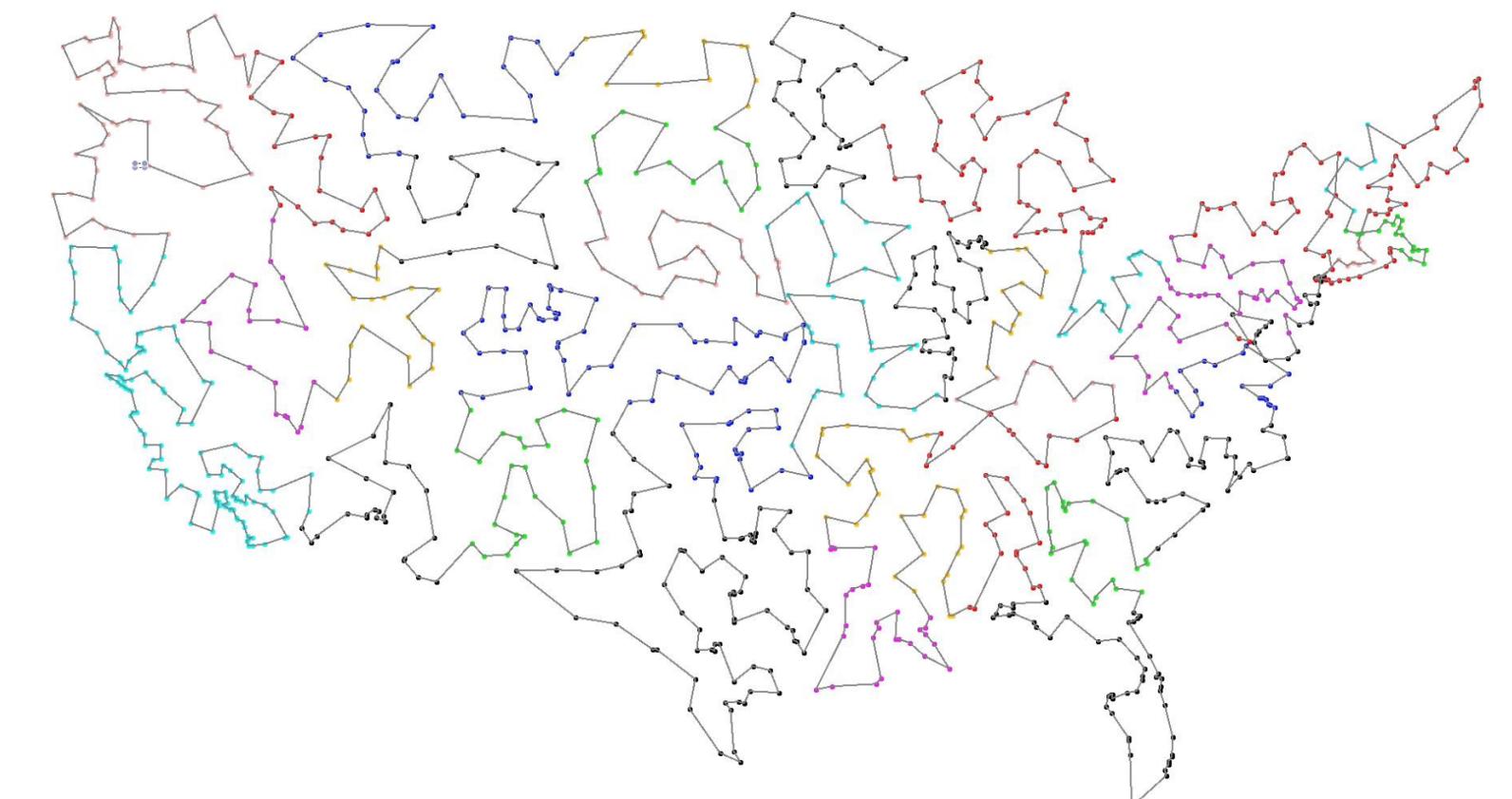
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Method overview

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- Construct **evaluation** set D_e and **verification** set D_v of MILPs

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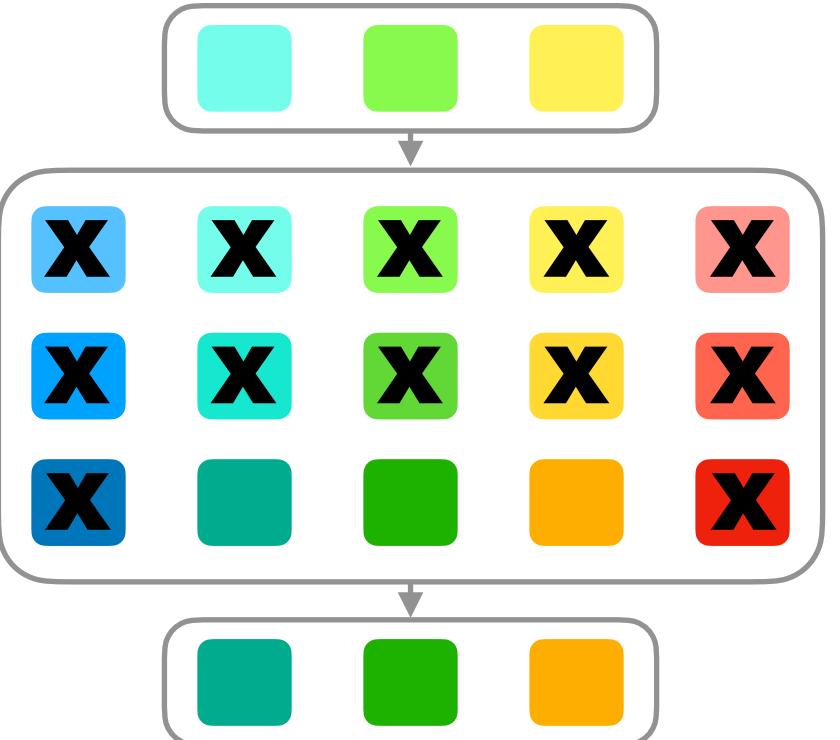
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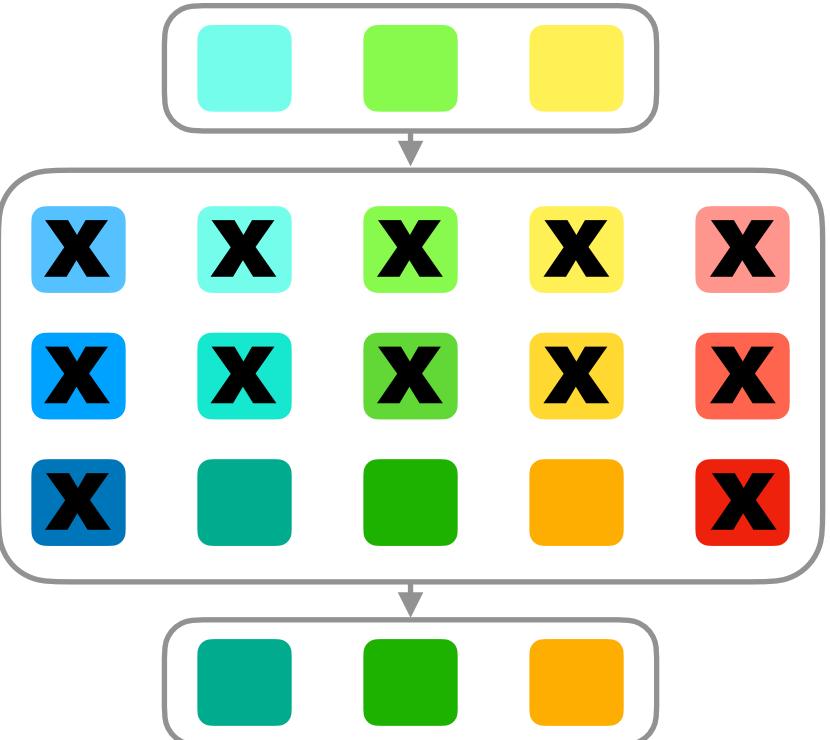
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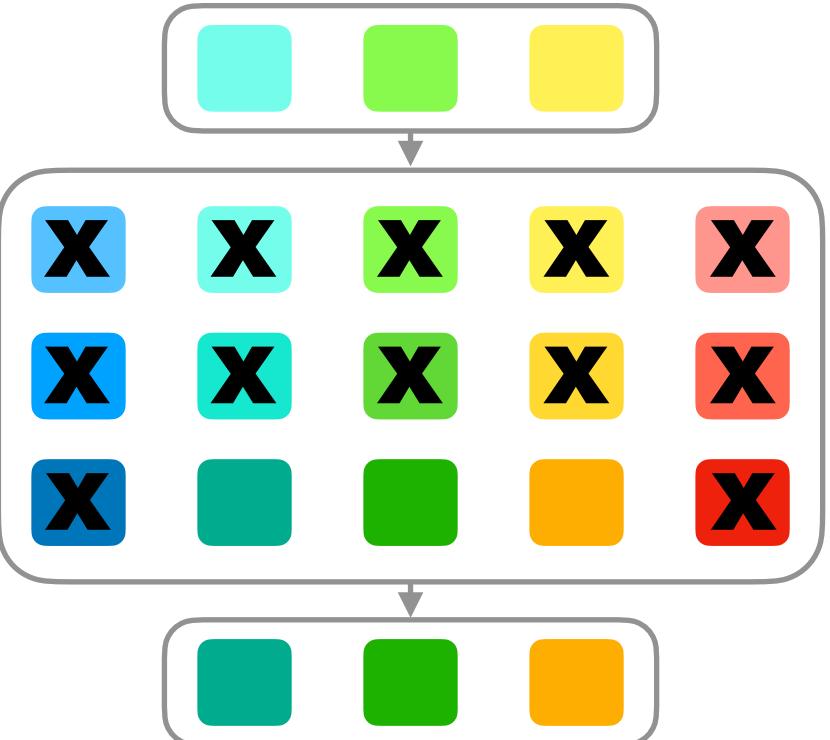
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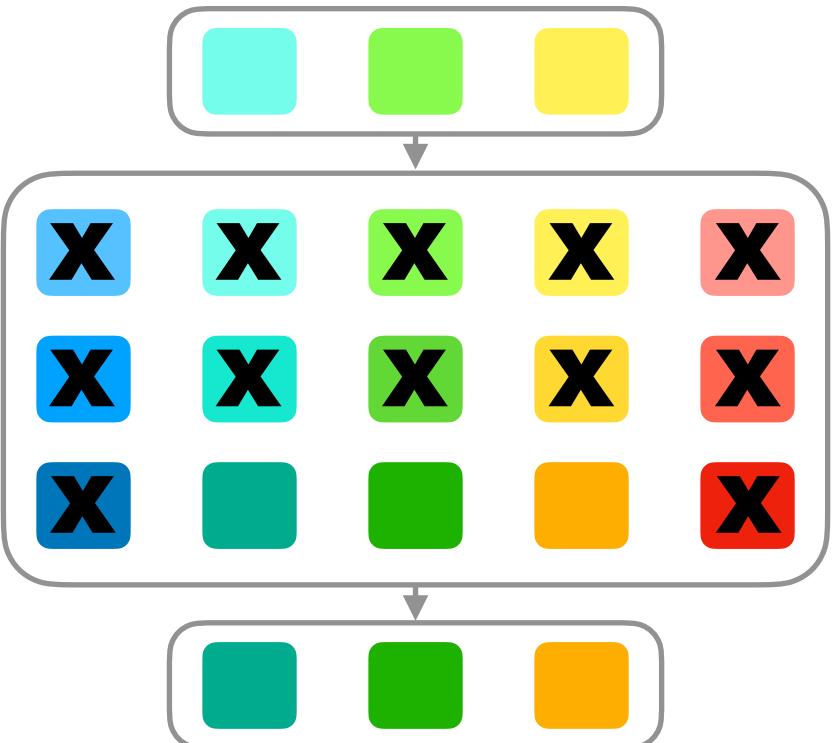
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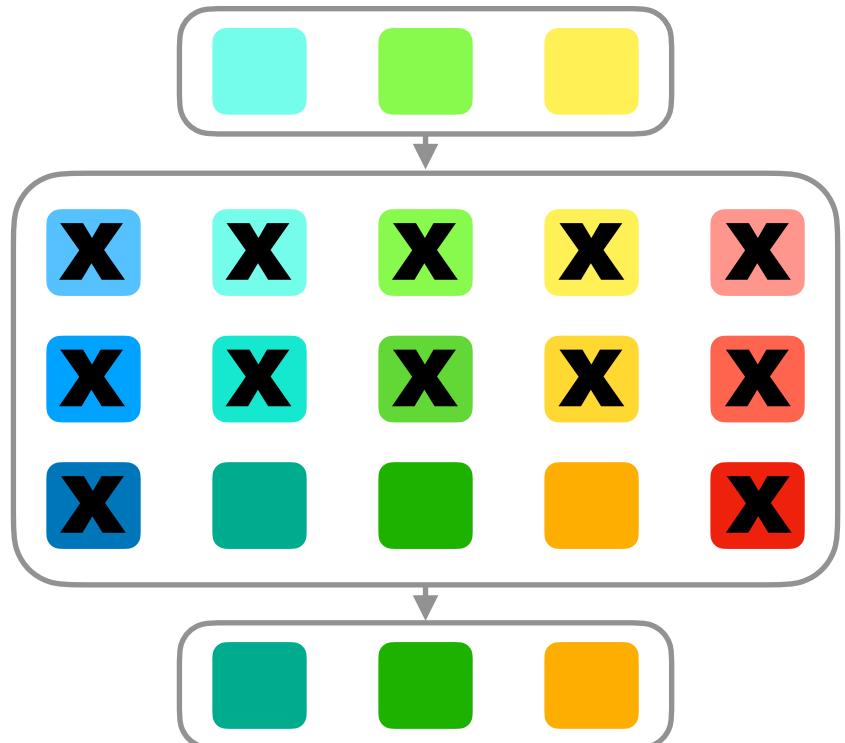
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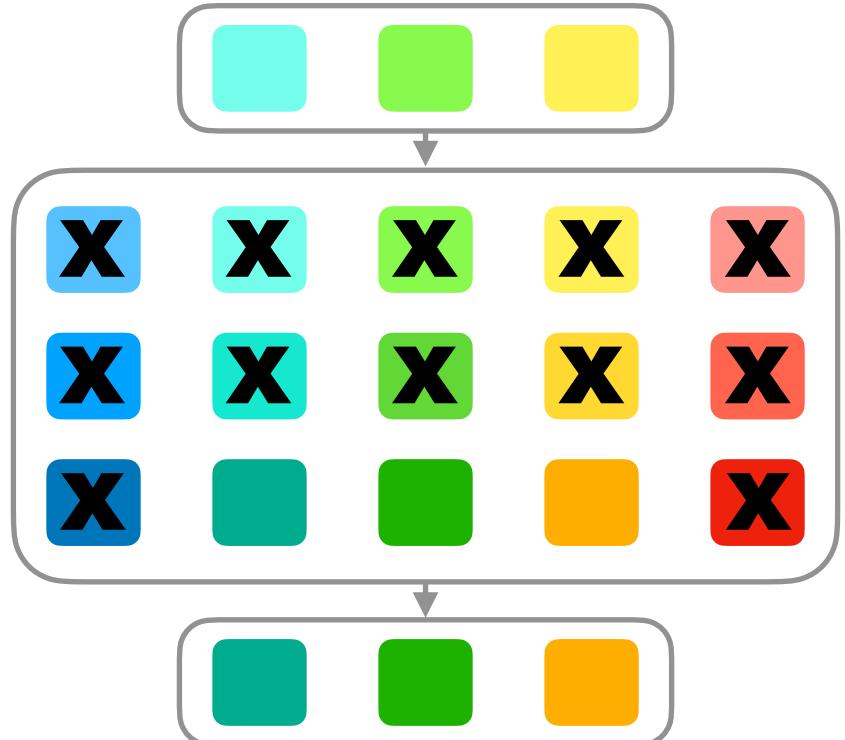
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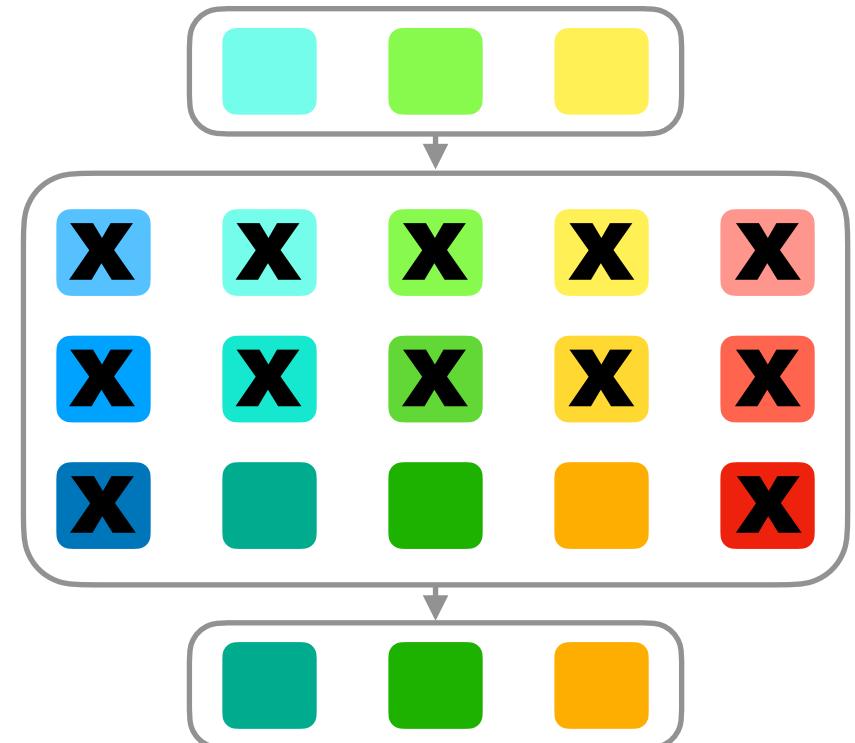
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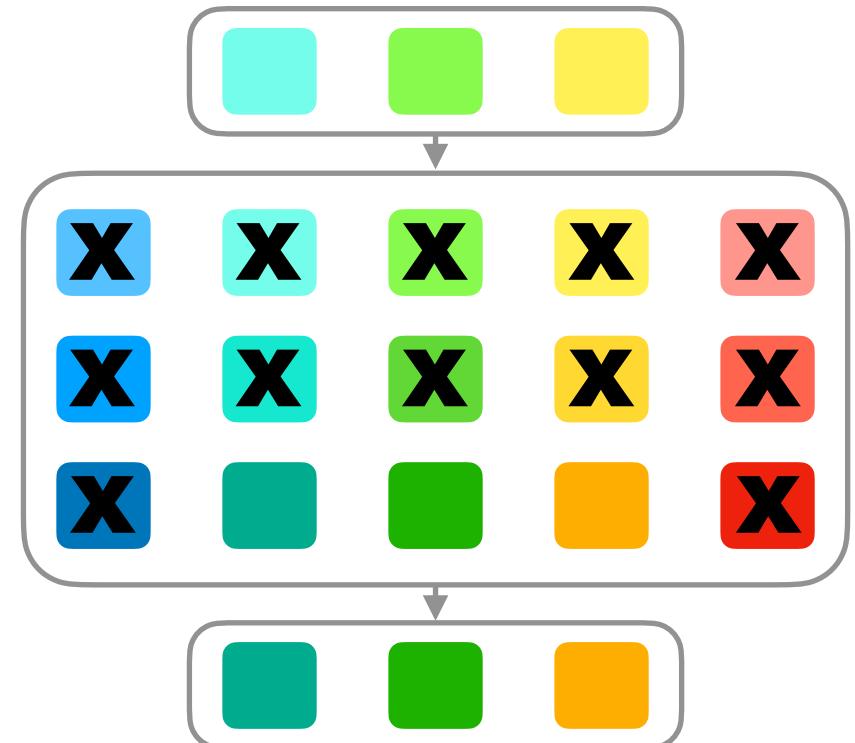
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- **Test set** D_t : public datasets; 40 hard medium/large instances each

Snapshot of results

Checkpoints	5 s	10 s	50 s	150 s	300 s	OSP rate (%)
TSP	16.3 ± 24.9	15.4 ± 27.3	27.7 ± 31.1	44.4 ± 27.7	57.4 ± 26.3	100
MCND	9.4 ± 21.1	6.3 ± 22.0	11.7 ± 19.1	10.4 ± 18.4	17.1 ± 20.2	100
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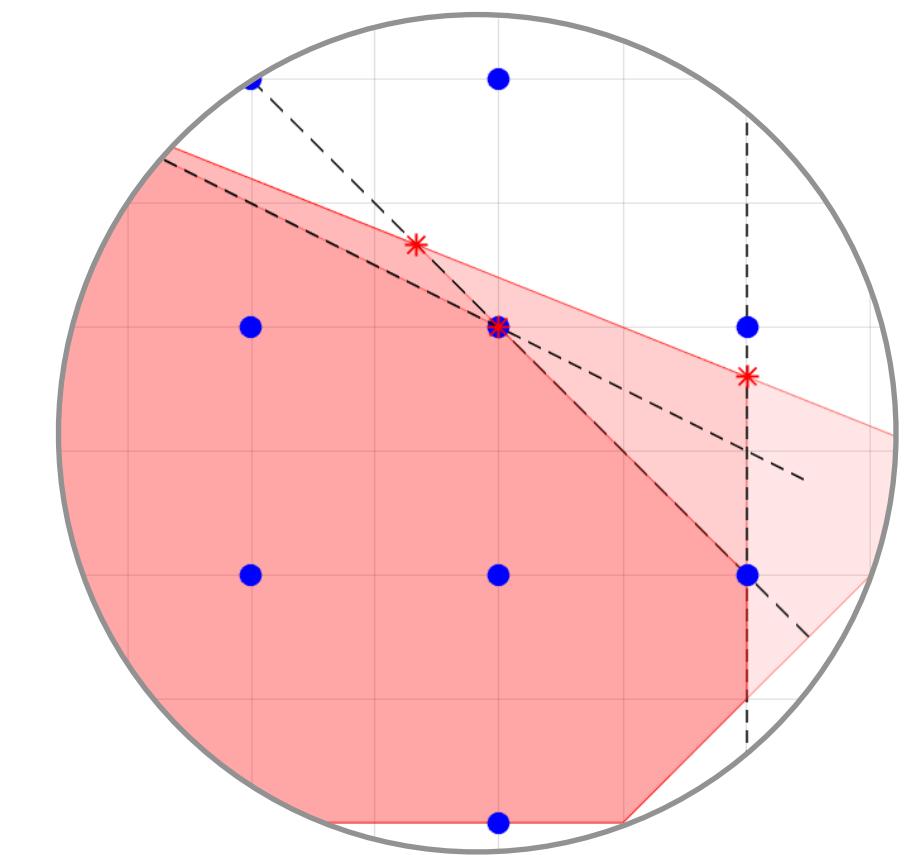
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- Mutation/crossover succeed 63–82% attempts; initializer 25.4%

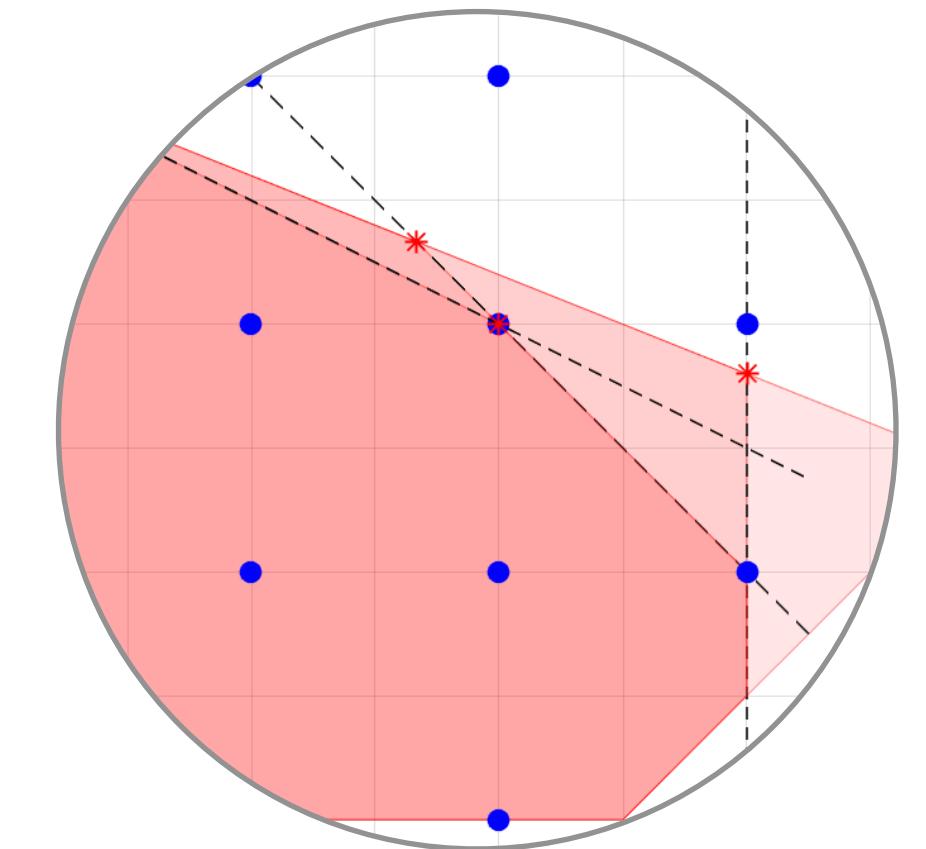
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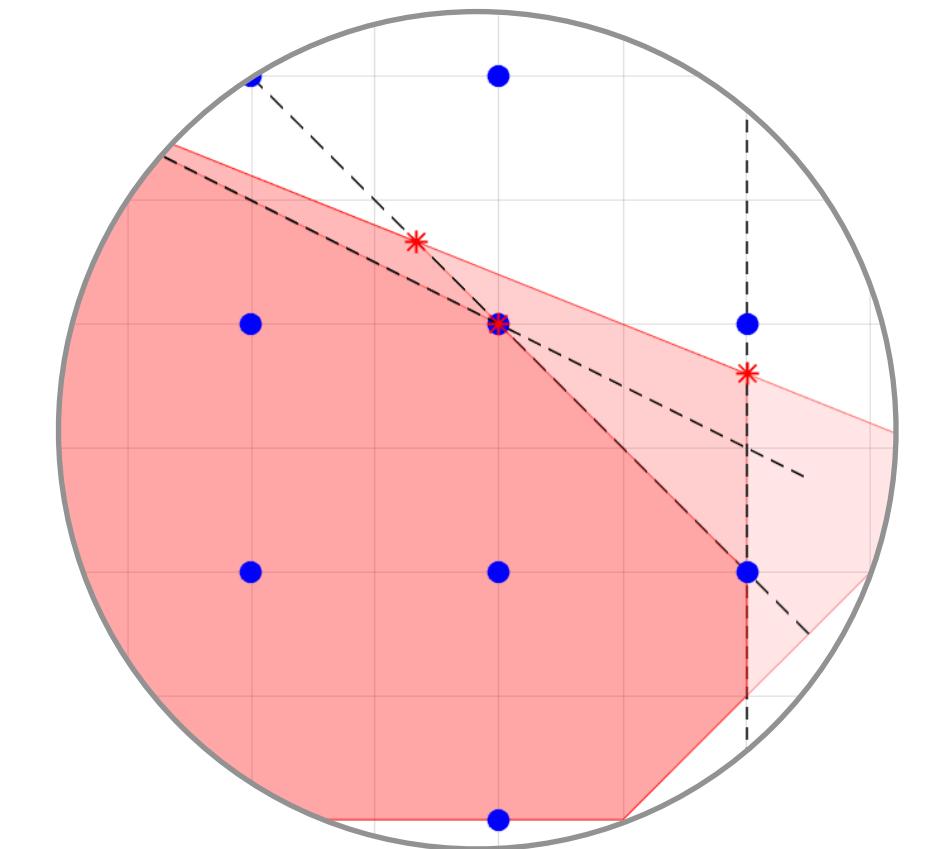
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- Cuts improve **optimality-gap reduction** throughout solve trajectory



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LLMs for large neighborhood search (LNS)

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- Propose **LLM-LNS**: LLM-guided neighborhood scoring for MILPs

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 - Training instances are roughly **100x smaller** than the test instances

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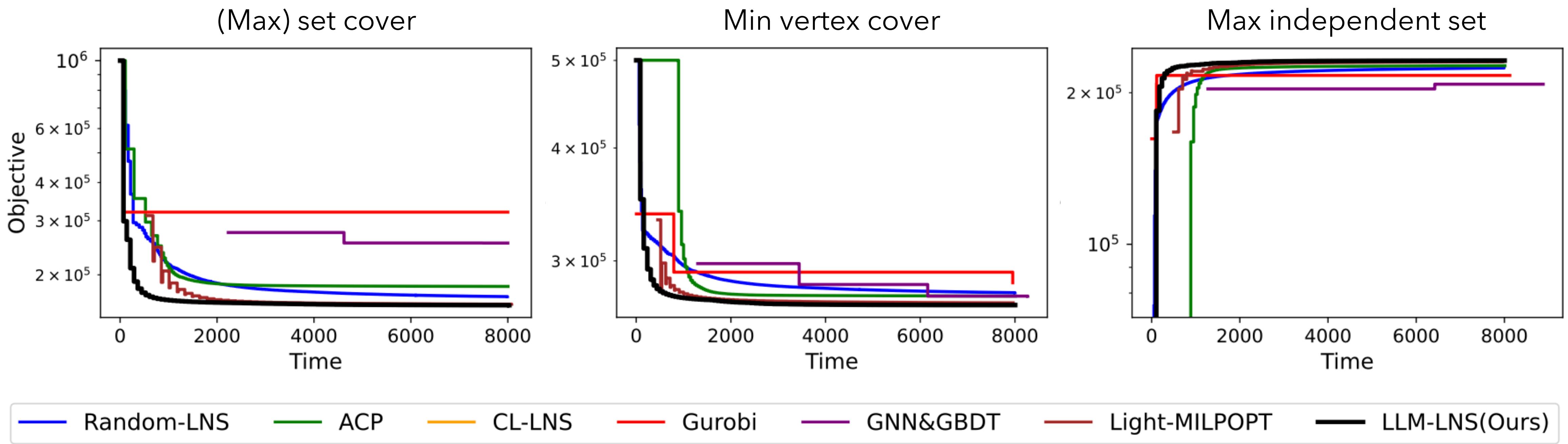
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- **Light-MILPopt**: lightweight ML framework for MILP optimization
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Experiments

Snapshot of results



- LLM-LNS is mostly **Pareto optimal** (see paper for many more experiments)

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- Inner-outer agent evolves neighborhood selection heuristics **and** prompts

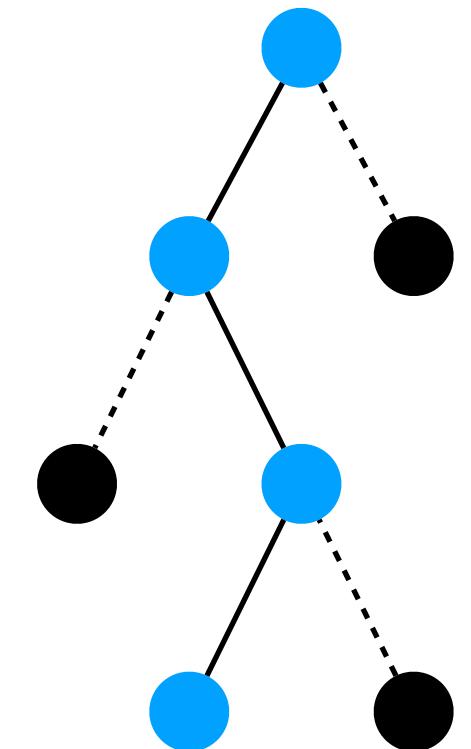
Outline

1. Background
2. LLMs for speeding up solvers
 - i. Solver configuration
 - ii. Evolutionary search for MILP heuristics
 - a. Background
 - b. Cut selection
 - c. Large neighborhood search
 - d. Diving heuristics**
3. Takeaways

Zhang, Li, Li, Liu, Chen, Li, Zhong, An, Liu, arXiv'25

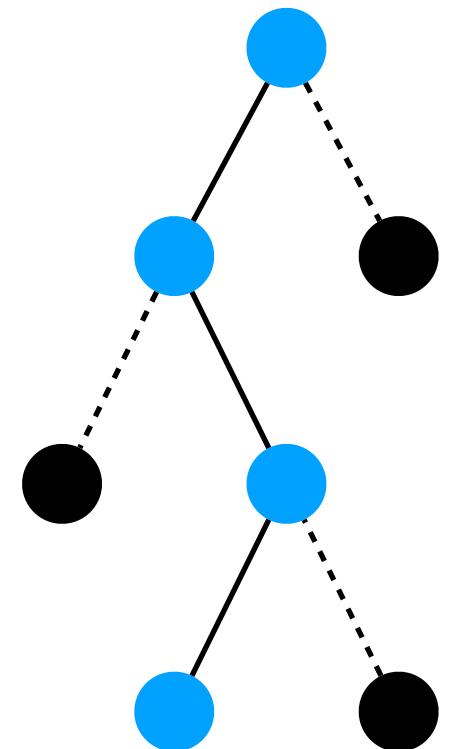
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- **Diving**: quickly explores a single path (a "dive") in the search tree



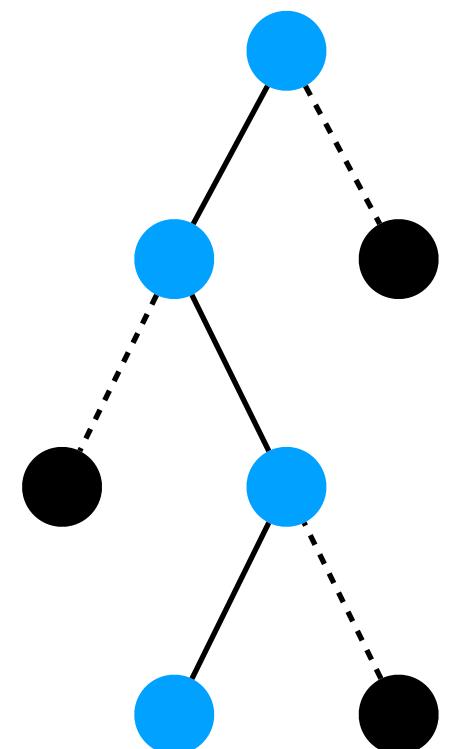
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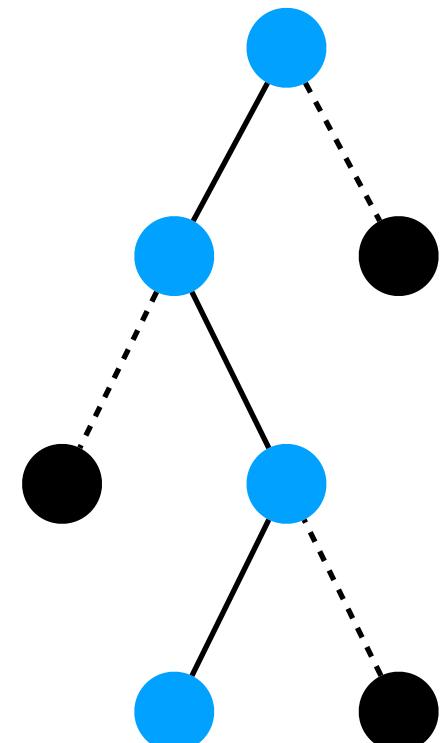
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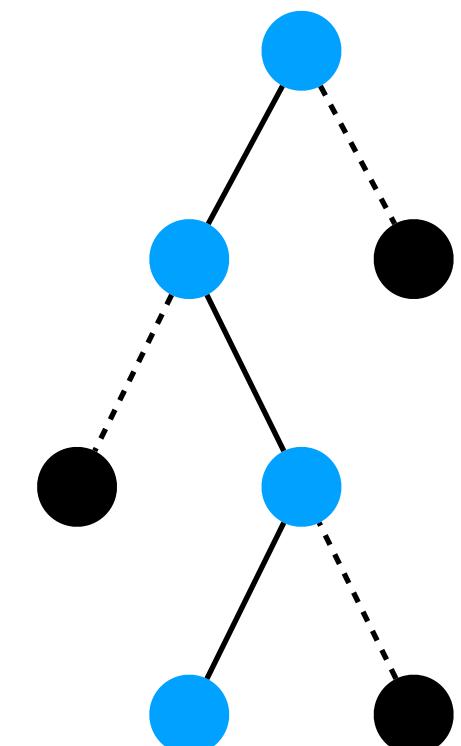
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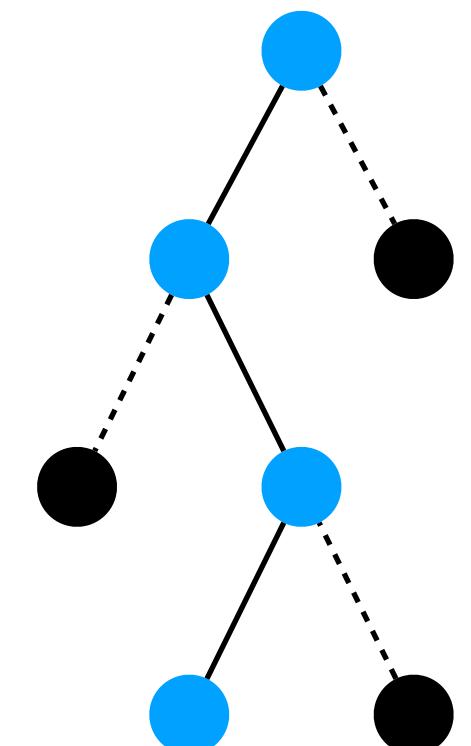
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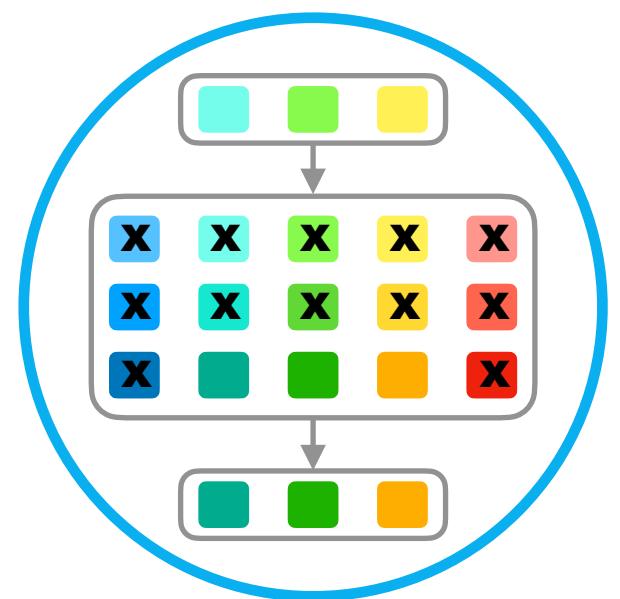
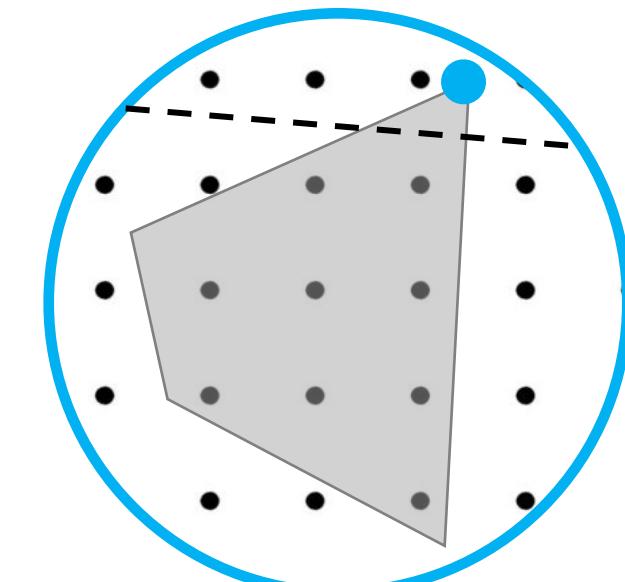
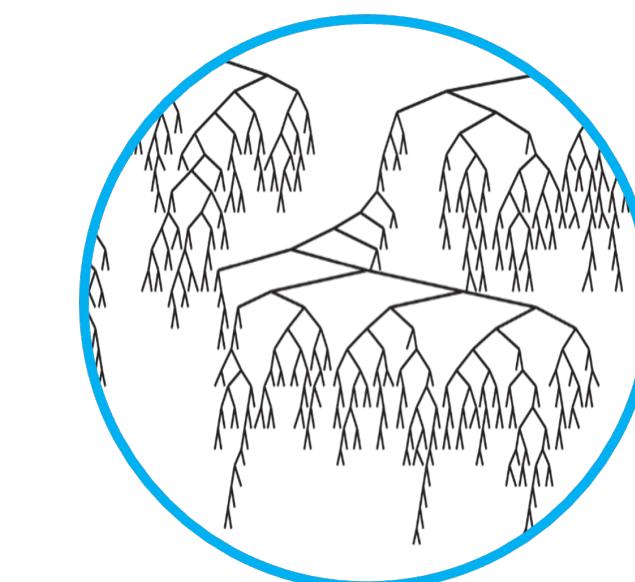
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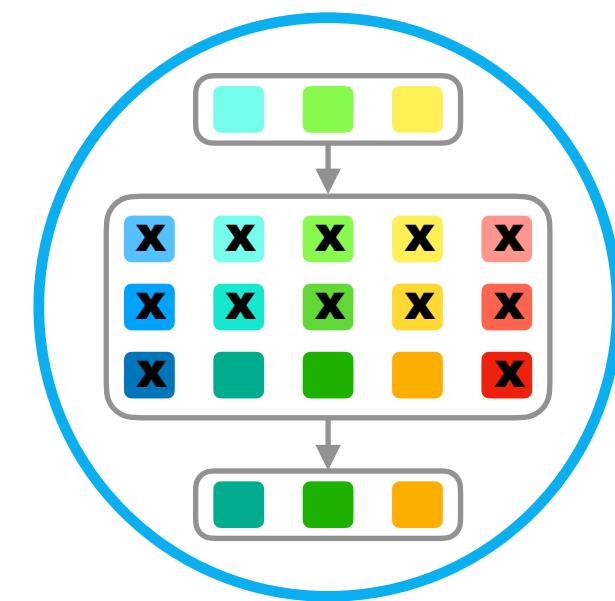
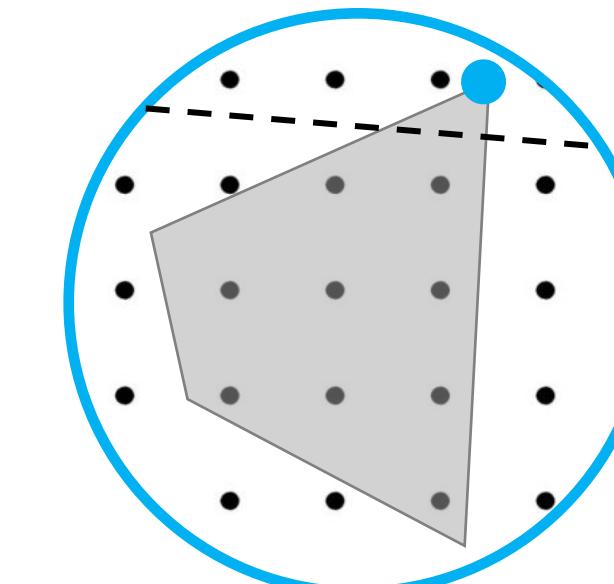
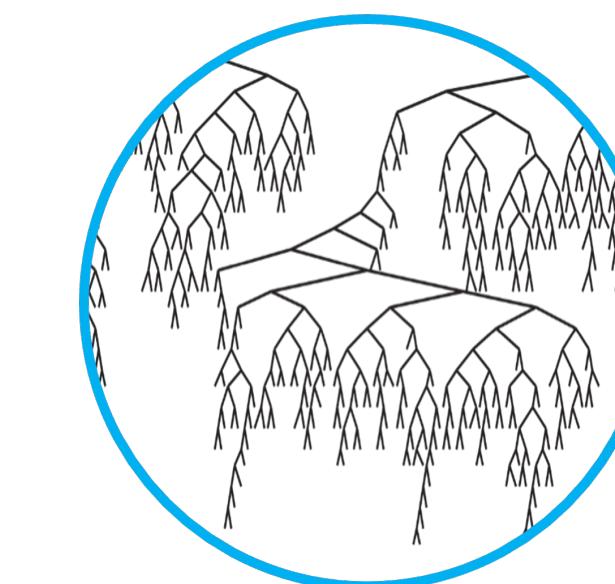
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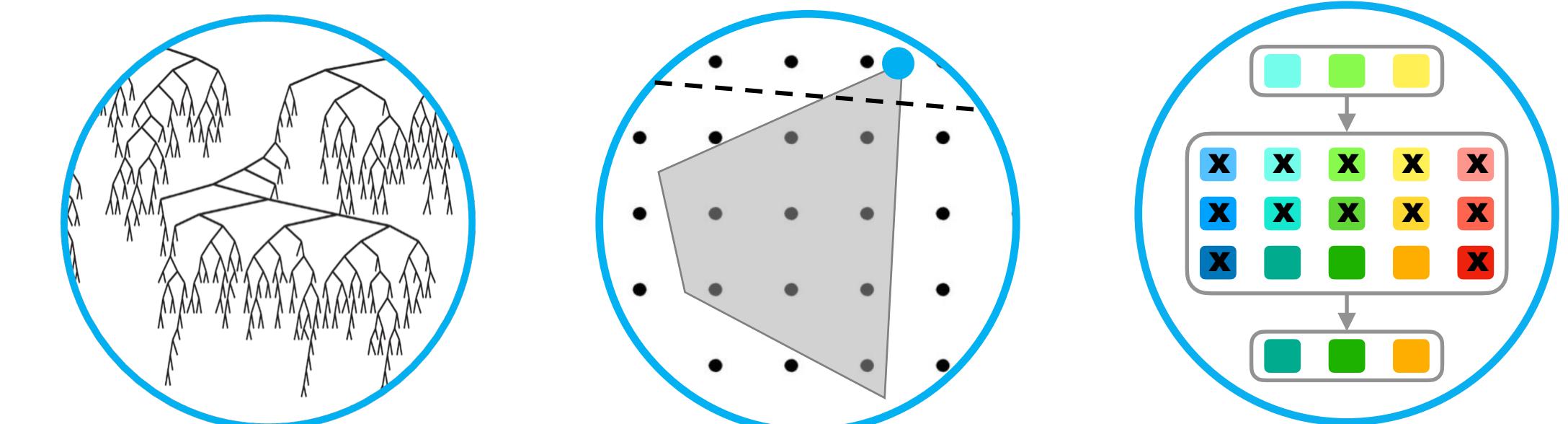
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Thank you! Questions?

