Semi- and Nonparametric Quantile Regression with Neural Networks

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`This is a live document and subject to change.`

Abstract

I consider the use of deep neural networks for quantile regression. The proposed framework allows for semi- and nonparametric estimation of the conditional quantile function.

1. Loss Functions

`In progress.`

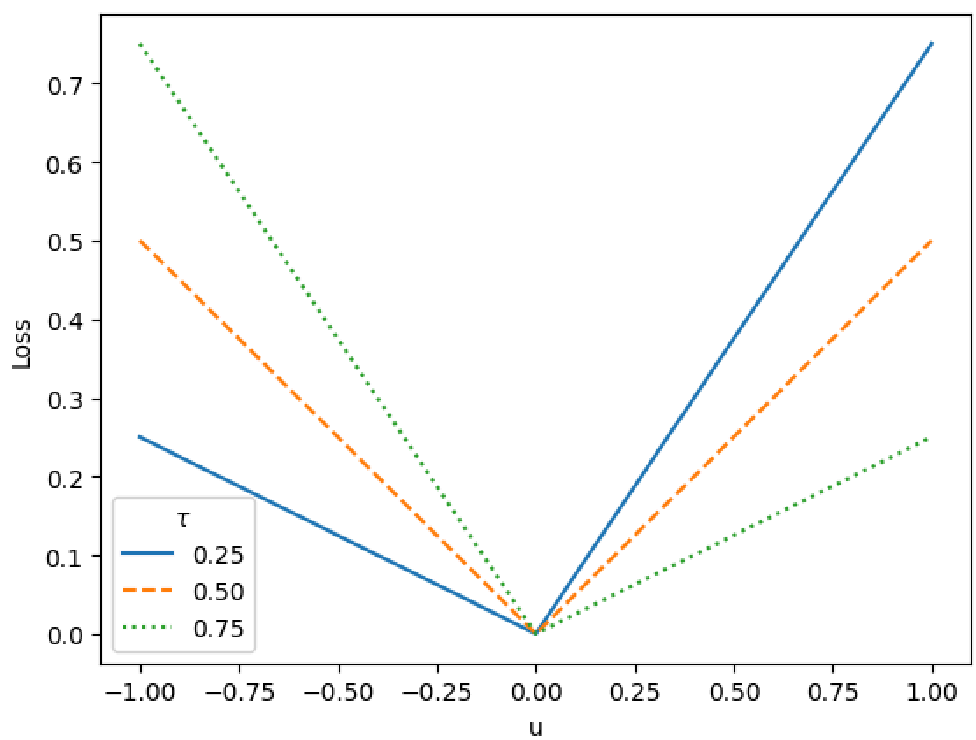
1.1. Standard Check Loss

The standard loss function used in quantile regression, often called the check loss, is defined

$$\ell(u;\tau) \stackrel{\text{def}}{=} (\tau - \mathbb{1}\{u < 0\})u. \tag{1}$$

I plot this function in Figure 1 for various quantiles τ.

Figure 1: Check Loss Function



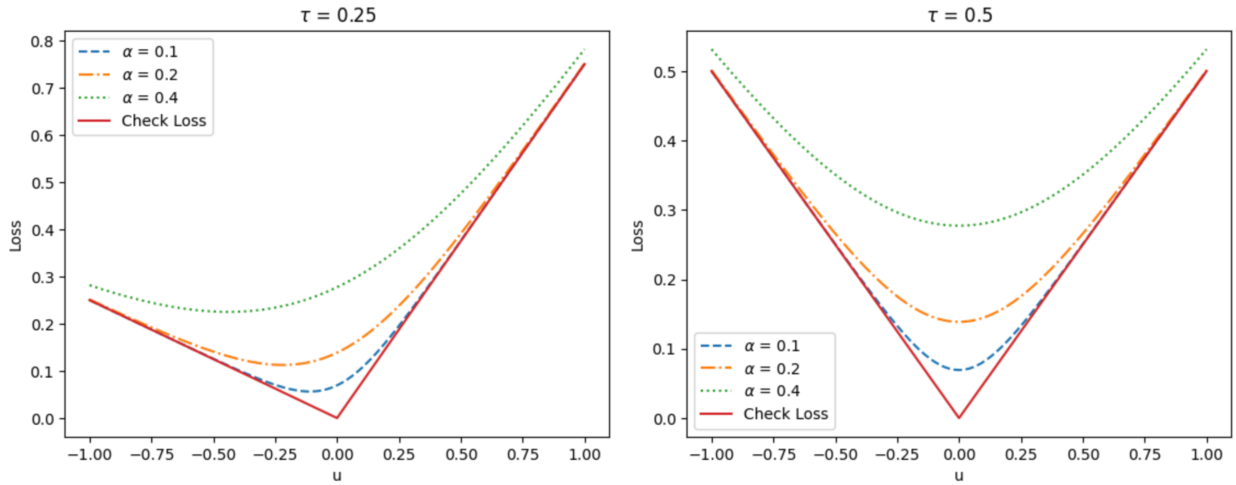
1.2. Smooth Check Loss

Next, I consider a smooth approximation to the check loss, proposed by Zheng (2011). I define

$$\ell^s(u; \tau, \alpha) \stackrel{\text{def}}{=} \tau u + \alpha \log\left(1 + \exp\left(-\frac{u}{\alpha}\right)\right), \tag{2}$$

where we call α the smoothing parameter. I plot the smooth check loss in Figure 2 for various quantiles τ and smoothing parameters α.

Figure 2: Check Loss Function



1.3. Simple Empirical Applications of (Smooth) Check Loss

I consider a simple empirical application of these two loss functions.

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In progress.
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Figure 3: Check Loss Function Performance on $X \sim U[0, 1]$

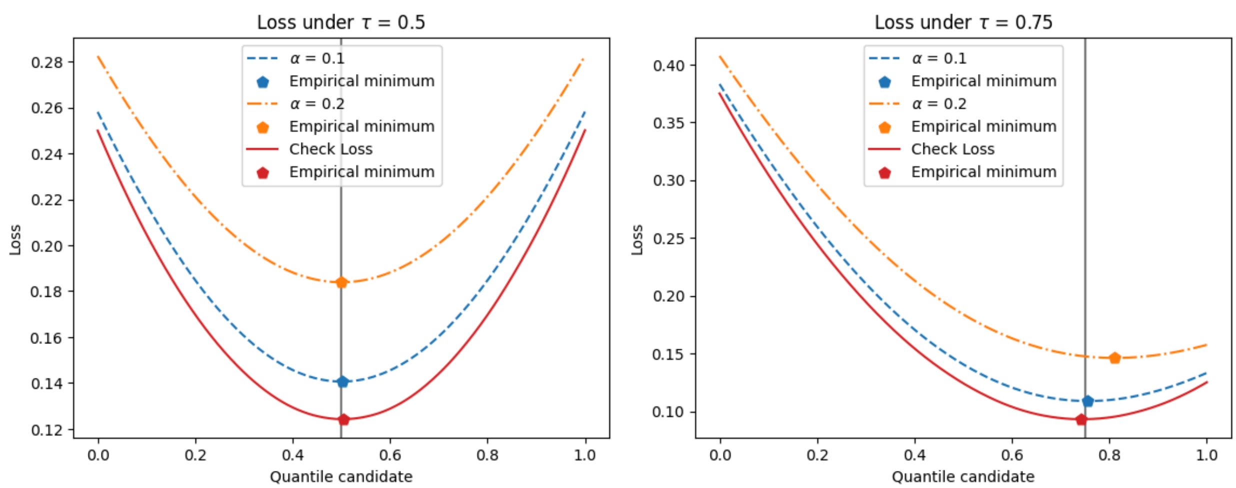
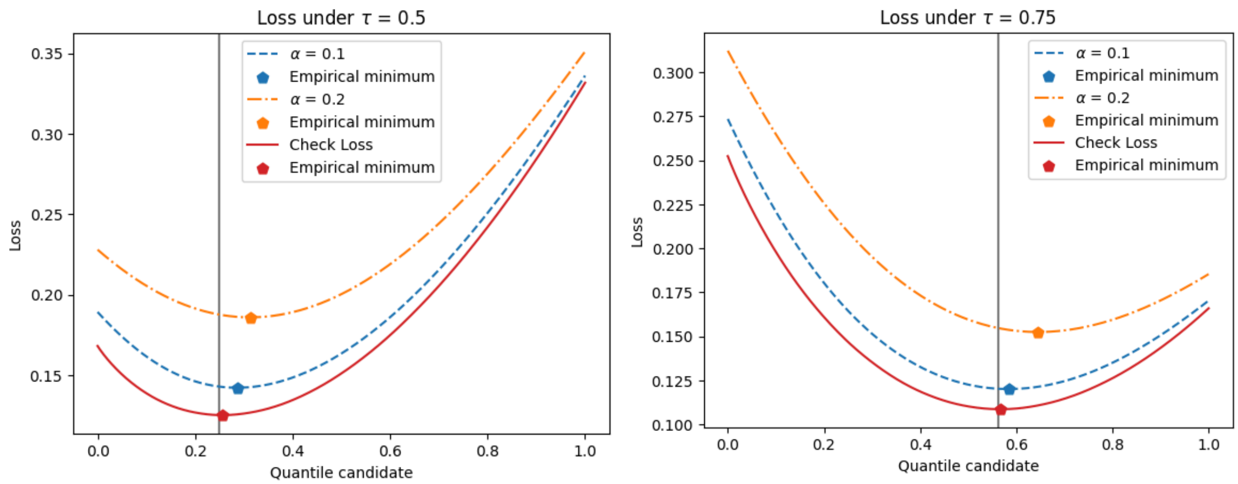


Figure 4: Check Loss Function Performance on $Y = X^2$, where $X \sim U[0, 1]$



2. Prediction Applications

We specify a generic data generating process (DGP).

$$Y = \mathbb{Q}_U(Y \mid \boldsymbol{X}), \quad \text{where } U \sim \text{Uniform}[0, 1]. \tag{3}$$

Bibliography

Zheng, Songfeng. 2011. "Gradient Descent Algorithms for Quantile Regression with Smooth Approximation". *International Journal of Machine Learning and Cybernetics* 2 (3): 191–207. https://doi.org/10.1007/s13042-011-0031-2