Discrete Structures

Assignment # CLO2

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Q # 1 (a)

i) $p \rightarrow q$
$\underline{p}$ ⇒ Modus Ponens
$\therefore q$ ⇒ Valid Argument.

ii) $p \rightarrow q$
$\underline{\sim r \rightarrow \sim q}$ or
$\therefore r$

$p \rightarrow q$ ⇒ Transitivity
$\underline{q \rightarrow r}$ ⇒ Invalid Argument
$\therefore r$ because truth value of p is unknown.

iii) $p \leftrightarrow r$ ⇒ Biconditional Rule
$\underline{r}$ ⇒ Valid Argument.
$\therefore p$

iv) $(p \vee r) \rightarrow q$ ⇒ Modus Ponens
$\underline{q}$ ⇒ Valid Argument
$\therefore p \rightarrow r$

v) $p \rightarrow (q \vee r)$ ⇒ Valid Argument.
$\underline{\sim q \wedge \sim r}$
$\therefore \sim p$

Q#1 (b)

1) • If 4gb is better than no memory at all, then
• we will buy new computer
• If 4GB is better than no memory at all, then we will by more memory

Conclusion: If 4GB is better than no memory at all, then we will buy more memory and new computer.

2) ⇒ If 4GB is better than no memory at all, then we will buy a new computer or more memory
⇒ If we will buy new computer than we will not buy more memory.

Conclusion: If 4GB is better than no memory at all, then we will buy a new computer.

3) ⇒ If 4GB is better than no memory at all, then we will buy a new computer.
⇒ If we will buy a new computer, then we will buy more memory.

Conclusion: We will buy more memory.

4) ⇒ If we don't buy a new computer, then ~~USB~~ 4GB is not better than no memory at all.
⇒ We will buy a new Computer

Conclusion : 4GB is better than no memory at all.

Q # 1 (c)

Conclusions :-

1) I took Thursday off and it rained on Thursday.
2) I did not eat spicy foods and there was no thunder while I slept.
3) I am clever and I won the lottery.

Q # 1 (d)

i) p
$\therefore p \vee q$
Addition

ii) $p \wedge q$
$\therefore p$
Simplification

iii) $p \to q$
p
$\therefore q$
Modus Ponens

iv) $p \to q$
$\sim q$
$\therefore p$
Modus Tollens

v) $p \to q$
$q \to r$
$\therefore p \to r$
Hypothetical Syllogism

e)

Q#2

(i)

$n = 2k+1$ for odd

$m = 2j$ for even

So,

$5n+3$ can be written as $5(2k+1)+3$

$= 5(2k+1)+3$

$= 10k+8$

$5n+3 = 2(5k+4)$

Taking $(5k+4)$ as j.

This means $(5n+3)$ is even.

(ii)

By Contrapositive :-

$3n-5$ is odd

Substitute $2j$ as n

$= 3(2j)-5$

$= 6j-5$

$= 2(3j)-5$

Substracting 5 from even number gives odd number.

Therefore, $3n-5$ is odd.

(iii)

If $7n-5$ is odd, then n is even.

$\Rightarrow$ let $7n-5$ be odd

$\Rightarrow$ then $7n$ is even (odd + odd = even), so n is even.

(iv)

Roots of $an^2+bn+c=0$ are not rational substituting odd integers for a, b, c leaves to a discriminant (b^2-4ac) that is not a perfect square. Hence, roots are irrational.

(v)

If n is even, then $3n+7$ is odd.

Proof by contradiction.

Assume n is even ($n=2k$) and $3n+7$ is not odd (i.e if it is even).

Substituting.

$$3n+7 = 3(2k)+7 = 6k+7$$

(vi)

If n is odd : $5n+3$

$$5(1)+3=8$$

$$5(3)+3=18$$

Hence, $5n+3$ is even.

If $5n+3$ is even : let $n=2k+1$

$$5(2k+1)+3$$

$$10k+5+3$$

$$10k+8$$

$$10(1)+8 = \text{even}$$

$$10(2)+8 = \text{even}$$

Hence, proved

(vii)

Counter Example : let $n = -2$

$n^3 = -2^3 = -8$

$n = -2 > n^3 = -8$

Hence, proved that the statement is "false".

(viii)

$3n^2 + 2y^2 = 30$

Put $n = 2$

$3(2)^2 + 2y^2 = 30$

$2y^2 = 30 - 12$

$2y^2 = 18$

$y^2 = 18/2$

$y^2 = 9$

$\boxed{y = 3}$

So, Solution pair is $(2,3)$

Now put $(3,2)$:-

$3(3)^2 + 2(2)^2 = 30$

$27 + 8 = 30$

$35 \neq 30$

Hence, proved, no other pair than $(2,3)$ is solution set of $3n^2 + 2y^2 = 30$.

(ix)

The sq. of every integer ends in 4 or 6

let n be even $n = 2k$

sq. of $n = n^2 = (2k)^2 = 4k^2$

The last depends only on the unit place of $4k^2$

(x)

let $n = 2k$

$n^2 = 4k$

Put $k = 1 \Rightarrow n^2 = 4(1)^2$

$n^2 = 4$

$k = 2 \Rightarrow n^2 = 16$

$k = 4 \Rightarrow n^2 = 64$

Hence, proved

Q#3

(i)

$$\sum_{i=1}^{n+1} i2^i = n \cdot 2^{n+2} + 2$$

Put 0 for n

$$\sum_{i=1}^{0+1} i 2^i = 0 \cdot 2^{0+2} + 2$$

L.H.S $\sum_{i=1}^{1} (1) 2^1 = 2$, $0 \cdot 2^2 + 2 = 0 + 2 = 2 \rightarrow$ R.H.S

$\boxed{2 = 2}$ Hence, it is true for 0.
$\boxed{n = 0}$

Inductive Step:-

$$\sum_{i=1}^{k+1} i \cdot 2^i = k \cdot 2^{k+2} + 2$$

$$\sum_{i=1}^{k+2} i \cdot 2^i = (k+1) \cdot 2^{k+1+2} + 2$$

L.H.S becomes.

$$\sum_{i=1}^{k+1} i \cdot 2^i + (k+2) \cdot 2^{k+2} = (k \cdot 2+2) + (k+2) \cdot 2^{k+2}$$

$$= (k+k+2)\,2^{k+2}+2$$
$$= (2k+2)\,2^{k+2}+2$$
$$= k+1\cdot 2^{k+2+1}+2$$

So, L.H.S = R.H.S

ii

$$\left[1-\frac{1}{2^2}\right]\cdot\left[1-\frac{1}{3^2}\right]\cdots\left[1-\frac{1}{n^2}\right]=\frac{n+1}{2n}$$

let $n=2$, then

$$L.H.S = 1-\frac{1}{4} = \frac{3}{4}$$

$$R.H.S = \frac{3}{4}$$

Inductive steps:-

$$\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{k+1}\right)=\frac{(k+1)+1}{2(k+1)}$$

Using (i) & (ii)

$$= \left(\frac{k+1}{2n}\right)\left(1-\frac{1}{(k+1)^2}\right)$$

$$= \left(\frac{k+1}{2n}\right)\left(\frac{(k+1)^2-1}{(k+1)^2}\right)$$

$$= \left(\frac{1}{2n}\right)\left(\frac{k^2+2k+0}{k+1}\right)$$

$$= \frac{k^2+2k}{2k(k+1)}$$

$$= \frac{k+1+1}{2(k+1)}$$ $\boxed{L.H.S = R.H.S}$

$\therefore$ proved

(iii)

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1$$

Put $n = 1$

$$\sum_{i=1}^{1} 1(1!) = (1+1)! - 1$$

$\boxed{1 = 1}$ proved

Suppose for $n = k$.

$$\sum_{n=1}^{k} i(i!) = (k+1)! - 1$$

Put $n = k+1$ in (2)

$$\sum_{i=1}^{k+1} i(i!) = (k+2)! - 1$$

$$\sum_{i=1}^{k} i(i!) + (k+1)(k+1)! = (k+2)! - 1$$

$$(k+1)! - 1 + (k+1)(k+1)! = (k+2)! - 1$$

$$(k+1)!\left(1+(k+1)\right) - 1 = (k+2)! - 1$$

$$(k+1)!(k+2) - 1 = (k+2)! - 1$$

$$\Rightarrow (k+2)! - 1 = (k+2)! - 1$$

$$\boxed{L.H.S = R.H.S}$$

Hence, proved

(iv)

$$\overline{\bigcap_{i=1}^{k} Ai} = \bigcup_{i=1}^{k} \overline{A_i} \text{ ......}$$

P(2) is true.

$$\text{L.H.S of } P(2) = \overline{\bigcap_{i=1}^{2} Ai} = \overline{A_1 \cap A_2}$$

$$= \overline{A_1} \cup \overline{A_2} \quad \because \text{De Morgan's law}$$

$$= \bigcup_{i=1}^{2} \overline{A_i} = \text{R.H.S of } P(2)$$

Inductive step:-

Assume P(k) is true.

$$\overline{\bigcap_{i=1}^{k} Ai} = \bigcup_{i=1}^{k} \overline{Ai}$$

Now Put $n-k=1$

$$\overline{\bigcap_{i=1}^{k+1} Ai} = \overline{\left(\bigcap_{i=1}^{k} Ai\right) \cap A_{k+1}}$$

$$= \left(\overline{\bigcap_{i=1}^{k} Ai}\right) \cup \overline{A_{k+1}} \quad \because \text{De morgan's law}$$

$$= \left(\bigcup_{i=1}^{k} \overline{Ai}\right) \cup \overline{A_{k+1}}$$

$$= \bigcup_{i=1}^{k+1} \overline{Ai} \Rightarrow R.H.S$$

$$\boxed{L.H.S = R.H.S}$$

Proved

(5)

$n^3 - n$ is divisible by 3. $n \geq 1$

Put $n=1$

$$(1)^3 - 1 = 0$$ which is divisible by 3.

Suppose $n^3 - n$ for $n=k$ is true

$$K^3 - k = 3m \quad \therefore m = 1, 2, 3 \ldots$$

Now put $n = k+1$

$$(K+1)^3 - (k+1) = k^3 + 3k^2 + 3k + 1 - k - 1$$

$$= k^3 + 3k^2 + 3k - k$$
$$= k^3 + 3k^2 + 2k$$
$$= k(k^2 + 3k + 2)$$
$$= k(k^2 + 2k + k + 2)$$
$$= k(k(k+2) + 1(k+2))$$
$$= k(k+2) \cdot (k+1)$$

Hence, for every integer k.

$$k(k+2)(k+1) = 3k$$

Hence proved

(vi)

$2^{2n} - 1$ is divisible by 3.

For $n=1$

$$2^{(4)} - 1 = 4 - 1 = \boxed{3}$$ which is divisible by 3.

Inductive step:-

$2^{2k} - 1 = 3m$ for some integer m

Put $(k+1)$ for k

$2^{2(k+1)} - 1$:

$2^{2k+2} - 1$

$2^{2k} \cdot 2^{2} \cdot 1$

$(2^{2k} \cdot 4) - 1 \rightarrow$ (i)

Since,

$2^{2k} - 1 = 0$

$2^{2k} = 1$

Put it in eqn (i)

$= (2^{2k} \cdot 4) - 1$

$= (1 \cdot 4) - 1$

$= 4 - 1$

$= \boxed{3}$ which is ÷ by 3.

(vii)

$n^3 - n$ is divisible by 6.

$n = 2$

$n^3 - n = 8 - 2$

$= \boxed{6}$ which is ÷ by 6.

Inductive step:-

Put $n = k$

$k^3 - k$ is divisible by $6 \rightarrow$ (i)

$= (k+1)^3 - (k+1)$

$= k^3 + 3k^2 + 3k + 1 - k - 1$

$= k^3 + 3k^2 + 3k - k$

$(k^3 - k) + 3(k^2 - k) \rightarrow$ (ii)

$\Rightarrow$ From eqn (i), $k^3 - k$ is divisible by 6.

$\Rightarrow$ From eqn (ii), $3(k^2+k)$, can be written as $3(k(k+1))$, which is also divisible by 6.

Hence, proved

(ix)

$x^n - y^n$ is divisible by $x-y$

For $n=1$,

$$x^1 - y^1 \Rightarrow x - y \text{ (divisible)}$$

Induction Rule :-

$x^k - y^k$ divisible by $x-y$

Now, put $n = k+1$

$$x^{k+1} - y^{k+1} = x^k \cdot x - y^k \cdot y$$

$$\cancel{= (x^k - y^k) + (x^k \cdot x}$$

$$= (x^k - y^k) + (x \cdot x^k - x \cdot y^k)$$

$$= (x^k - y^k) + x(x^k - y^k)$$

$\Rightarrow (x^k - y^k)$ is divisible by $x-y$

$\Rightarrow x(x^k - y^k)$ is divisible by 6.

(x)

$2n+1 < 2^n$

For $n = 3$,

$2(3)+1 = 6+1 = 7$ which is less than $2^3 = \boxed{8}$

Induction Rule:-

Put ~~ke~~ $\boxed{n = k}$.

$2(k)+1 < 2^k$

now, $k = k+1$

$2(k+1)+1 < 2^{k+1} \rightarrow$ (i)

L.H.S $\quad 2k+2+1 = 2k+3$

We need to show that $2k+3 < 2^{k+1}$

$2k+3 < \boxed{2^k+2} \rightarrow$ (ii)

Now, put all these terms together. (i) & (ii)

$2^k+2 < 2^{k+1}$

$2^k+2 < 2 \cdot 2^k$

Since $2^{k+1} = 2 \cdot 2^k$, we know that $k \geq 3$.

Hence proved