Assignment # 1

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Course = Discrete Structures (CSC - 102)

Q=1

Prepositions	Truth	Values
1) $p \vee q$	T	
2) $\sim p \vee \sim q$	T	
3) $\sim p \vee q$	T	
4) $\sim p \vee \sim (q \wedge r)$	T	
5) $\sim (p \vee q) \wedge (\sim p \vee r)$		F
6) $(p \vee \sim r) \wedge \sim ((q \vee r) \leftrightarrow \sim (r \vee p))$		F
7) $\sim p \vee \sim (q \wedge r) \rightarrow (\sim p \vee q)$	T	
8) $(q \vee \sim r) \oplus (p \wedge r)$	T	

Q=2

i) Exclusive Or
ii) Inclusive or
iii) Inclusive or
iv) Exclusive or
v) Inclusive or

Q=3

1) $\sim p \wedge (q \vee r)$

In Words : You cannot use wireless network and either you pay the semester fees or you are a subscriber to the service.

Truth Value : F

2) $\sim(p \vee q) \wedge r$

In Words : It is not the case that you can use the wireless network or you can pay the semester fee, and you are a subscriber to the service.

Truth Value : F

3) $(p \wedge q) \wedge \sim(r \vee p)$

In Words : You can use the wireless network and you can pay the semester fees, and it is not the case that you are a subscriber or you can use the wireless network.

Truth Value : F

4) $(p \wedge q) \vee \sim r$

In Words : You can use the wireless network and you pay the semester fee, or you are not a subscriber to the service.

Truth Value : T

5) $\sim(p \vee q) \leftrightarrow r$

In Words : It is not the case that you can use the wireless network or you pay the semester fee iff you are a subscriber to the service.

Truth Value : T

6) $(p \wedge (q \vee r)) \rightarrow (r \vee p)$

In Words: If you can use the wireless network and either you pay the semester fee or you are a subscriber to the service, then you are a subscriber to the service or you can use the wireless network.

Truth Value: T

Q=4

i) $q \rightarrow p$

ii) $q \wedge \sim p$

iii) $q \rightarrow p$

iv) $\sim q \rightarrow \sim p$

Truth Table:-

p	q	$\sim p$	$\sim q$	$q \rightarrow p$	$q \wedge \sim p$	$\sim q \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	F	T

Q=5

⇒ Restating:-

1) If Javed studies hard, then he will pass the Discrete Structure exam

2) If Ayesha passes 133 credit hours, then she may graduate.

3) If Farid buys a computer, then he has obtained \$2000.

4) If Kiran passes discrete structure, then can take algorithm course.

5) If Buick will build them, then better cars are built.

6) If the chairperson gives the lecture, then the audience will go to sleep.

7) If the program is readable, then it is well ,structured.

⟹ Part B:-

1) Converse : If Javed passes the ~~Disto~~ Discrete Structure exam, then he studied hard.

Inverse : If Javed do not study hard, then he will not pass Discrete Structure exam.

Contrapositive: If Javed did'nt pass Discrete Structure exam, then he did not study hard.

2) Converse : If Ayesha may graduate, then she passes 133 credit hours.

Inverse : If Ayesha does not pass 133 credit hours, then she may not graduate.

Contrapositive : If Ayesha does not graduate, then she did not pass 133 credit hours.

3) Converse : If Farid has obtained $ 2000, then he buys a computer.

Inverse : If Farid does not buy a computer, then he has not obtained $2000.

Contrapositive : If Farid has not obtained $2000, then he does not buy a computer.

4) Converse : If Kiran takes the algorithm course, then she passes discrete structure.

Inverse : If Kiran does not passes discrete structure, then she cannot take the algorithm course.

Contrapositive : If Kiran does not take the algorithm course, then she did not pass discrete structure.

5) Converse : If betters cars are built, then Buick will build them.

Inverse : If Buick will not build them, then better cars are not built.

Contrapositive : If better cars are not built, then Buick does not build them.

6) Converse: If audience goes to sleep, then the chairperson gives the lecture.

Inverse: If the chairperson does not gives the lecture, then the audience will not go to sleep.

Contrapositive: If the audience does not go to sleep, then the chairperson did'nt give the lecture.

7) Converse: If the program is well structured, then it is readable.

Inverse: If the program is not readable, then it is not well structured.

Contrapositive: If the program is not well structured, then it is not readable.

$\longleftrightarrow$

Q=6

i) The router can send packets to the edge system only if it supports the new address space $= s \rightarrow a$

ii) For the router to support the new address space, it is necessary that the latest softare release be installed. $= a \rightarrow r$

iii) The router can send address packets to the edge system if the latest software release is installed. $= r \rightarrow s$

iv) The router does not support the new address space $= \sim a$

So, from (iv) statement, we came to know that $\boxed{a = F}$. By putting value of a in (i) & (iii), we will get the truth values. Therefore,

i) $s \rightarrow a$

If a is F, s is also F.

iii) $r \rightarrow a$

If a is F, r is also F.

Truth Values:-

$s = F$

$a = F$

$r = F$.

Q-7 a) ii)

Laws of logic

Start with L.H.S:-

$\sim[r \vee (q \wedge (\sim r \rightarrow \sim p))]$

Rewrite:-

$\sim[r \vee (q \wedge (r \vee \sim p))]$

Distribute:-

$q \wedge (r \vee \sim p) \equiv (q \wedge r) \vee (q \wedge \sim p)$

$\Rightarrow \sim[r \vee (q \wedge r) \vee (q \wedge \sim p)]$

De-Morgan's law:-

$\sim r \wedge \sim(q \wedge r) \wedge \sim(q \wedge \sim p)$

De Morgan to Components:-

For $\sim(q \wedge r)$:

$\sim q \vee \sim r$

For $\sim(q \wedge \sim p)$:

$\sim q \vee p$

Substituting:-

$\sim r \wedge (\sim q \vee \sim r) \wedge (\sim q \vee p)$

Combine:-

Since, $\sim r$ & $\sim q$ is common.

So,

$\boxed{\sim r \wedge (\sim q \vee p)}$

This shows that both expressions are equivalent.

Q = 7 a) (i)

p	q	r	$\sim r$	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim r \rightarrow \sim p$	$q \wedge (\sim r \rightarrow \sim p)$	$\sim r \wedge (p \vee \sim q)$	$r \vee (q \wedge (\sim r \rightarrow \sim p))$	$\sim(r \vee (q \wedge (\sim r \rightarrow \sim p)))$	$\leftrightarrow$ / $\equiv$
T	T	T	F	F	F	T	T	T	F	T	F	T
T	T	F	T	F	F	T	F	F	T	F	T	T
T	F	T	F	F	T	T	T	F	F	T	F	T
T	F	F	T	F	T	T	F	F	T	F	T	T
F	T	T	F	T	F	F	T	T	F	T	F	T
F	T	F	T	T	F	F	T	T	F	T	F	T
F	F	T	F	T	T	T	T	F	F	T	F	T
F	F	F	T	T	T	T	T	F	T	F	T	T

Thus, both are equivalent

Q-7 (B):-

a) $(p \land q) \rightarrow p$

i)

p	q	$p \land q$	$(p \land q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

ii) $(p \land q) \rightarrow p$

$\sim(p \land q) \lor p$

$\sim p \lor \sim q \lor p$

$p \lor \sim p \lor \sim q$ $\quad (p \lor \sim p$ is always true$)$

Therefore, " $(p \land q) \rightarrow p$ is a <u>tautology</u> "

b) $p \rightarrow (p \lor q)$

i)

p	q	$(p \lor q)$	$p \rightarrow (p \lor q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

ii) $p \rightarrow \cancel{p}(p \lor q)$

$\sim p \lor (p \lor q)$

$(\sim p \lor p) \lor q$ $\quad (\sim p \lor p)$ is always true

Therefore, " $p \rightarrow (p \lor q)$ is a <u>tautology</u> "

c) $\sim p \rightarrow (p \rightarrow q)$

i)

p	q	$\sim p$	$p \rightarrow q$	$\sim p \rightarrow (p \rightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

ii) $\sim p \rightarrow (p \rightarrow q)$

$\sim p \rightarrow (\sim p \vee q)$

$p \vee \sim p \vee q$ "($p \vee \sim p$ is always true)".

Therefore, "$\sim p \rightarrow (p \rightarrow q)$ is a tautology".

d) $(p \wedge q) \rightarrow (p \rightarrow q)$

i)

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

ii) $(p \wedge q) \rightarrow (p \rightarrow q)$

$\sim (p \wedge q) \vee (p \rightarrow q)$

$\sim p \vee \sim q \vee (\sim p \vee q)$

$\sim p \vee \sim q \vee q$ "($\sim q \vee q$) is always true."

Therefore, "$(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology".

e) $\sim(p \to q) \to p$

i)

p	q	$p \to q$	$\sim(p \to q)$	$\sim(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

ii) $\cancel{\sim p(}\ \sim(p \to q) \to p$

$\sim(\sim p \vee q) \to p$

$(p \wedge \sim q) \to p$

$\sim(p \wedge \sim q) \vee p$

$\sim p \vee q \vee p$

$p \vee \sim p \vee q$ "($p \vee \sim p$ is always true)"

Therefore, $(\sim p \to q) \to p$ is a <u>tautology</u>"

f) $\sim(p \to q) \to \sim q$

i)

p	q	$\sim q$	$p \to q$	$\sim(p \to q)$	$\sim(p \to q) \to \sim q$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T

ii) $\sim(p \to q) \to \sim q$

$\sim(\sim p \vee q) \to \sim q$

$(p \wedge \sim q) \to \sim q$

$\sim(p \wedge \sim q) \vee \sim q$

$\sim p \vee q \vee \sim q$ "($q \vee \sim q$) is always true"

Therefore, "$\sim(p \to q) \to \sim q$ is a <u>tautology</u>"

Q=8 (ii):-

i) $P(11)$

In Words : "11 divides 77"

T or F : True, because $77 \div 11 = 7$

ii) $P(1)$

In Words : "1 divides 77"

T or F : True, because any positive integer is divisible by 1.

iii) $P(3)$

In Words : "3 divides 77"

T or F : False, because $77 \div 3$ does not yeild an integer.

iv) $\forall n\, P(n)$

In Words : "For all positive integers n, n divides 77."

T or F : False, since there are positive integers (like 3) that do not divide 77.

v) $\exists n\, P(n)$.

In Words : "There exists positive integers n such that n divides 77."

T or F : True, because integers like 1, 7, 11 divides 77.

vi) $\forall n \sim P(n)$

In Words : "For all positive integers n, n does not divide 77."

T or F : False, since some integers (like 1, 7, 11) do divide 77.

vii) $\exists n \sim P(n)$

In Words : There exists a positive integer n such that n does not divide 77.

T or F : True, integers like 3 do not divide 77.

viii) $\sim(\forall n\ P(n))$

In Words: "It is not true that all positive integers n, n divides 77."

T or F: True.

ix) $\sim(\exists n\ (P(n))$

In Words: "It is not true that there exists a positive integer n such that n divides 77."

T or F: False.

Q = 8 (iii) :-

i) $\forall x\ (P(x) \rightarrow Q(x))$

In Words: "For all people x, if x is a profesional athalete, then x plays soccer."

T or F: False, not all athletes play soccer.

ii) $\exists x\ (P(x) \rightarrow Q(x))$

In Words: "There exists people x such that if x is a profesional athlete, then x plays soccer."

T or F: True.

iii) $\forall x\ (Q(x) \rightarrow P(x))$

In Words: "For all people, if x plays soccer, then x is a profesional athlete."

T or F: False. Many people play soccer recreationally.

iv) $\exists x\,(Q(x) \to P(x))$

In Words: "There exists a person x such that if x plays soccer, then x is a professional athlete."

T ~~F~~ or F : True.

v) $\forall x\,(P(x) \lor Q(x))$

In Words: "For all people x, x is a professional athlete or x plays soccer."

T or F : False. There are many who are nither professional athletes nor soccer players.

vi) $\exists x\,(P(x) \lor Q(x))$

In Words: "There exists a person x such that x is a profesional athlete or x plays soccer."

T or F : True.

vii) $\forall x\,(P(x) \land Q(x))$

In Words: For all people x, x is a professional athlete and x plays soccer.

T or F : False.

viii) $\exists x\,(P(x) \land Q(x))$

In Words: There exists a person x such that x is a ~~p~~ professional athlete and x plays soccer.

T or F : True.

The End

Q-8 (i) :-

i) $(2n+1)^2$ is an odd integer:
Propositional Function: Yes, it depends upon variable "n".
Domain : The set of integers ($\mathbb{Z}$).

ii) Choose an integer b/w 1 & 10.
Propositional Function: Yes, it implies a variable (the chosen integer).
Domain : The integers (1-9)

iii) Let n be real number.
Propositional Function: Yes, it defines a variable "n".
Domain : The set of Real number ($\mathbb{R}$).

iv) The movie won the Academy Award as the best picture of 1995.
Propositional Function: No, it is a statement about particular event, not dependent on a variable.

v) $1+3=4$.
P.F : No, it is a mathematical statement, that is always true and no mathematical statement is involved.

vi) There exists n such that $n < y$ (n, y real numbers).
Propositional Function: Yes, it depends on y.
Domain : The set of Real numbers ($\mathbb{R}$).