Discrete Structures CSC 102

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Assignment # 3

⇒ Question 1 :-

a) $A \cap B = \{e, x\}$

b) $B' \cap C' = \{a, b, z, y\}$

c) $A \cap B \cup C = \{a, b, d, x, e\} \cap \{h, e, m, k, x\}$
$\cup \{d, n, k, x\}$

$= \{a, b, d, x, e\} \cap \{h, k, e, m, x, n, d\}$
$= \{x, e, d\}$ Ans

d) $A \cup C' = \{a, b, d, e, h, x, z, m, y\}$

e) $B \cup A' = \{e, k, h, n, m, x, z, y\}$

⇒ Question 2 :-

(a)

Two valid pairs of a & b are :-

1) $a = 8$, $b = 10$ (Because in $8 < 9 < 10$, $9 \notin A$)

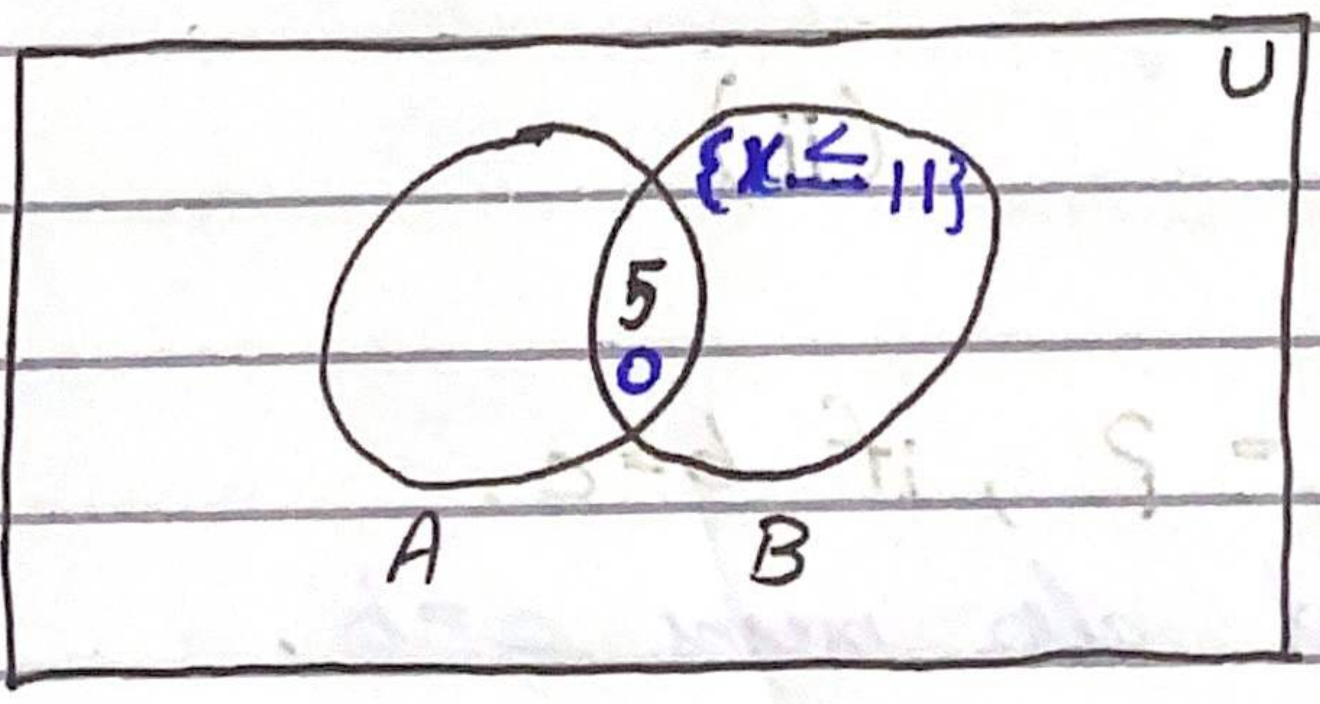
2) $a = 3$, $b = 5$ (Because in $3 < 4 < 5$, $4 \notin A$)

(b) (i)

Universal Set $= \{1 \ldots\ldots 12\}$ // $U < 12$

B set $= \{1, 2 \ldots\ldots, 11\}$ // $\{x \leq 11\}$

A set $= \{0, 1, 2 \ldots\ldots 5\}$



$B = \frac{1}{3}x - 1 < 3\frac{1}{3}$

$= \frac{1}{3}x - 1 < \frac{10}{3}$

xing by 3 on b.s

$x - 3 < 10$

$x < 13$

Since, $U < 12$, so

$B \leq 11$

(ii)

a) $A \cap B = \{0,5\} \cap \{x \le 11\} = \{0,5\}$ Ans

b) $A \cup B' = \{0,5\} \cup \emptyset = \{0,5\}$ Ans

Question # 3:-

(i)

$\therefore U = 120$ members.

$\therefore X = 80$ members.

$\therefore Y = 48$ members.

X includes $a+b$, so $a+b = 80$ members.

Y includes $b+c$, so $b+c = 48$ members.

Therefore,

$$(X+Y) = (a+b)+(b+c)$$

$$a+b+b+c = 80+48$$

$$\boxed{a+2b+c = 128 \text{ members}} \rightarrow (i)$$

$$\therefore U = (a+b+c)+d = 120$$

Since, $d = 0$

Therefore, $\boxed{U = a+b+c = 120} \rightarrow (ii)$

Subtracting eq (ii) from eq (i).

$$(a+2b+c)-(a+b+c) = 128-120$$

$$a+2b+c-a-b-c = 8$$

$$\boxed{b = 8}$$ Ans

(ii)

Given Data:-

$d = ?$, if $b=c$.

If $b=c$, that also means $c=b$.

Solution:-

$$X = a+b = 80 \rightarrow (i)$$
$$Y = b+c = 48 \rightarrow (ii)$$
$$b+b = 48$$
$$2b = 48$$
$$\boxed{b = 24} \quad \text{or} \quad \boxed{c = 24} \qquad \because b=c$$

Putting value of b in eq (i):-

$$X = a+24 = 80$$
$$a = 80-24$$
$$\boxed{a = 56}$$

Now, put all values together and subtract.

$$U = (X \cup Y) + d$$
$$(a+b+c)+d = 120$$
$$(56+24+24)+d = 120$$
$$(104)+d = 120$$
$$d = 120-104$$
$$\boxed{d = 16} \quad \text{Ans}$$

(iii)

To Find d_{max}, we use this formula :-

$$d_{max} = U-(X+Y-b)$$
$$\because X = 80, \quad U = 120, \quad Y = 48$$

$\because$ b = maximum number will be used for d_{max}, so, we can use (48) for this. $\boxed{b = 48}$

So,

$$d_{max} = U-(X+Y-b)$$
$$= 120-(80+48-48)$$
$$= 120-80$$
$$\boxed{d_{max} = 40}$$

"40 members are neither batsmen or bowlers."

Q=4

(i)

i) $\underline{A'}$:-

$A = \{-15, -14 \ldots\ldots\ldots 14, 15\}$

$\because U = \{-15, -14 \ldots\ldots\ldots 14, 15\}$

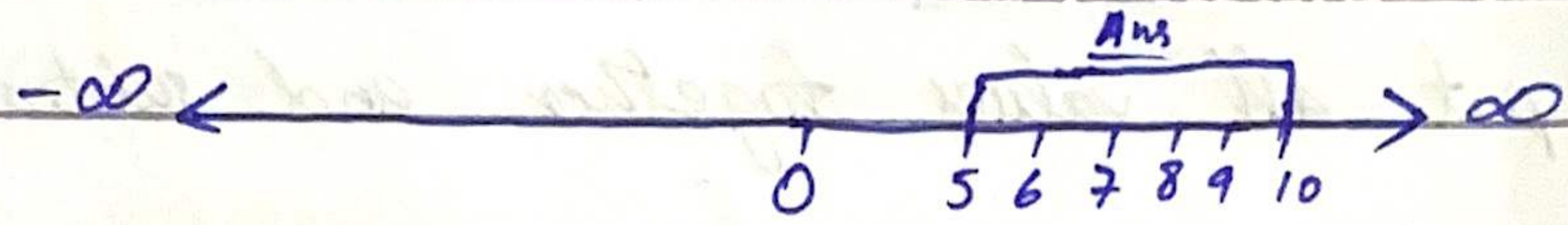
So,:.

$A' = \{\phi\}$ "As there are no elements left."

ii) $\underline{B \cap C}$:-

$B \cap C = {}^{B}\{5, 6, \ldots\ldots, 15\} \cap {}^{C}\{-10, -9 \ldots\ldots, 10\}$

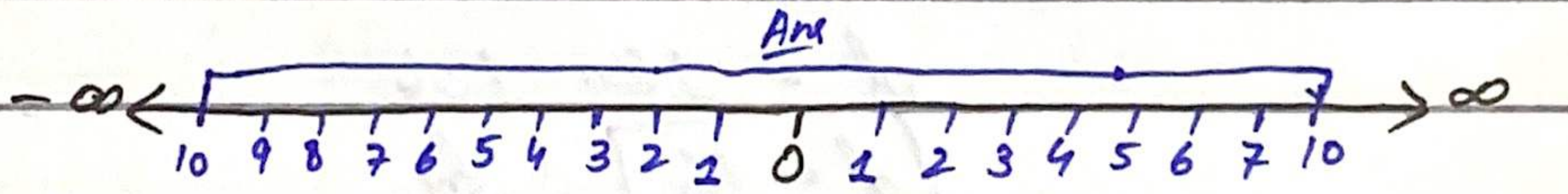
$= \{5, 6, 7, 8, 9, 10\}$ Ans



iii) $\underline{C \cap D}$:-

$C \cap D = {}^{C}\{-10, -9 \ldots\ldots, 10\} \cap {}^{D}\{-15, -14 \ldots\ldots 8\}$

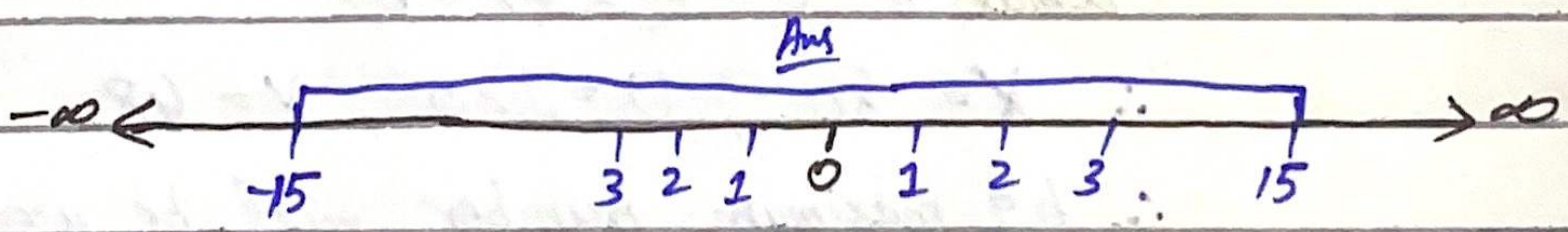
$C \cap D = \{-10, -9 \ldots\ldots\ldots, 7, 8\}$ Ans



iv) $\underline{B \cup D}$:-

$B \cup D = {}^{B}\{5, 6 \ldots\ldots, 15\} \cup {}^{D}\{-15, -14 \ldots\ldots, 8\}$

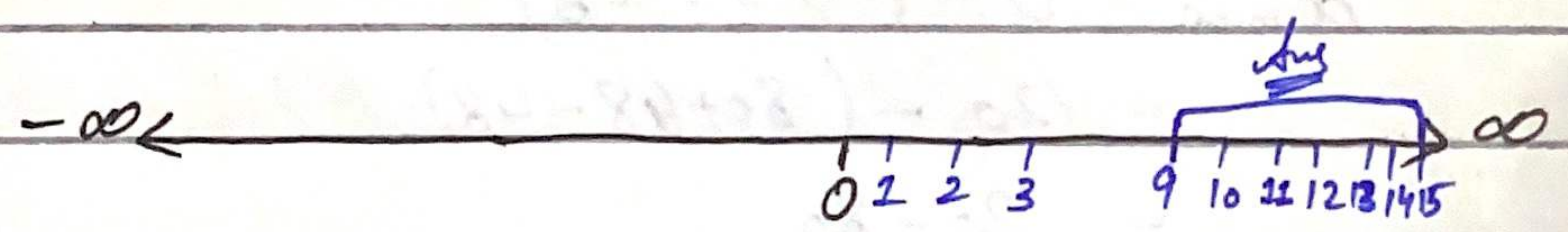
$B \cup D = \{-15, -14 \ldots\ldots\ldots, 14, 15\}$ Ans



v) $\underline{B \cap D'}$:-

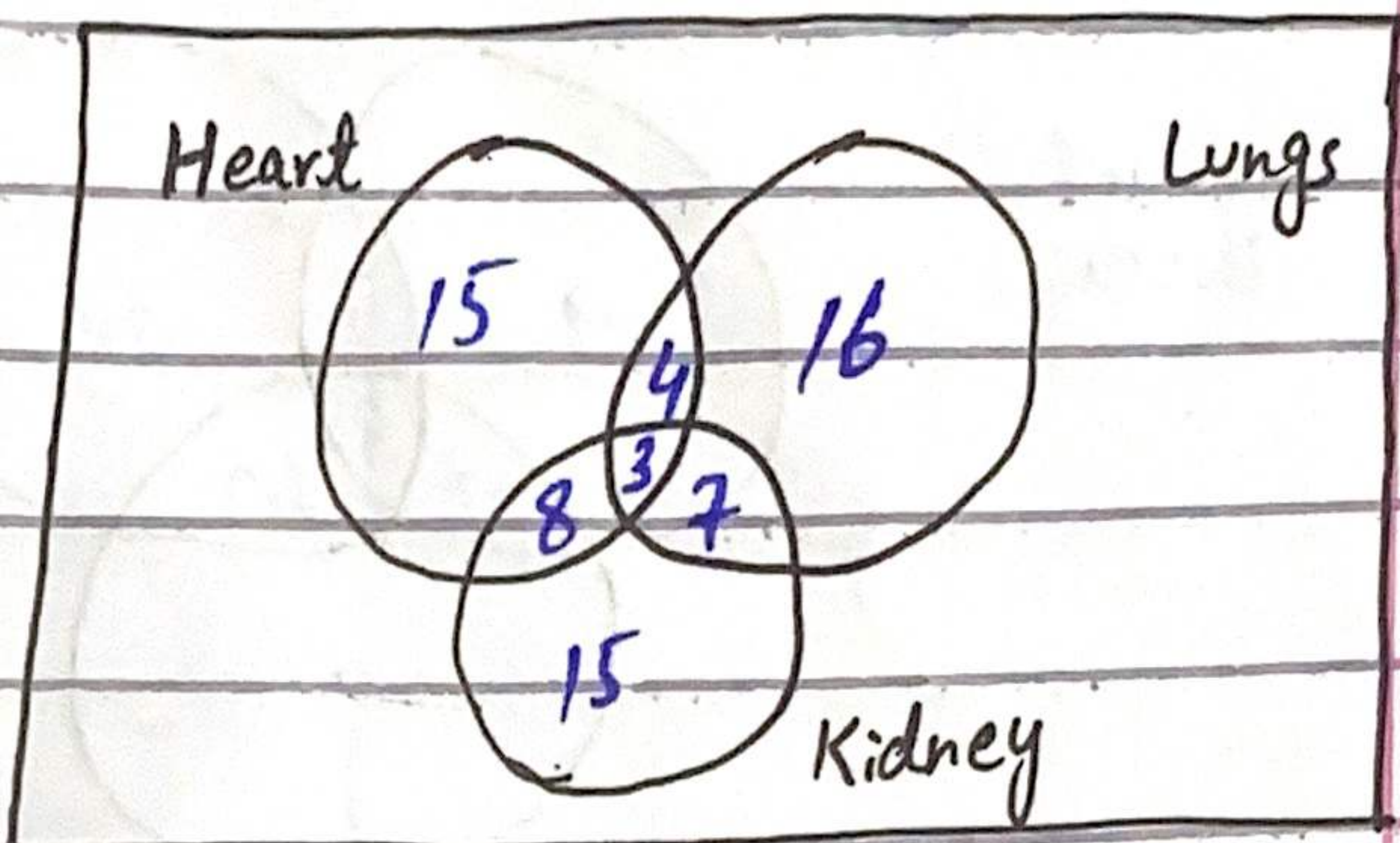
$B \cap D' = {}^{B}\{5, 6 \ldots\ldots, 15\} \cap {}^{D'}\{9, 10 \ldots\ldots 15\}$

$B \cap D' = \{9, 10 \ldots\ldots\ldots\ldots, 15\}$ Ans



Question #5:-

i) Total Heart Diseases = 30
T. Lung Diseases = 30
T. Kidney Diseases = 33



Both Lung and Kidney = 10
Both kidney and heart = 11
Both lung and heart = 7

Now, We have to find patients with all three diseases.

Let those patients $= x$

So,

$$68 = 30+30+33-7-11-10+x$$
$$68 = 65+x$$
$$68-65 = x$$

$\boxed{x = 3}$ → These patients are suffering from all diseases.

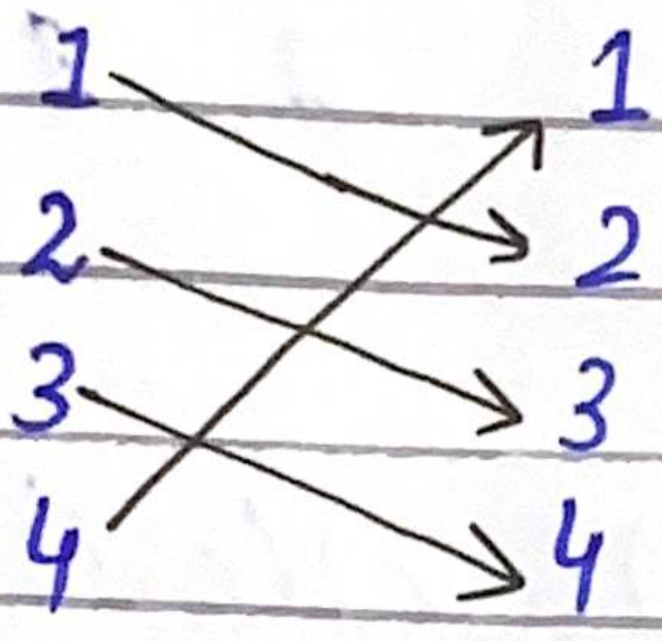
ii) Only lung diseases $= 30-7-10+3$

$= \boxed{16}$ → These are the patients suffering from lung disease only.

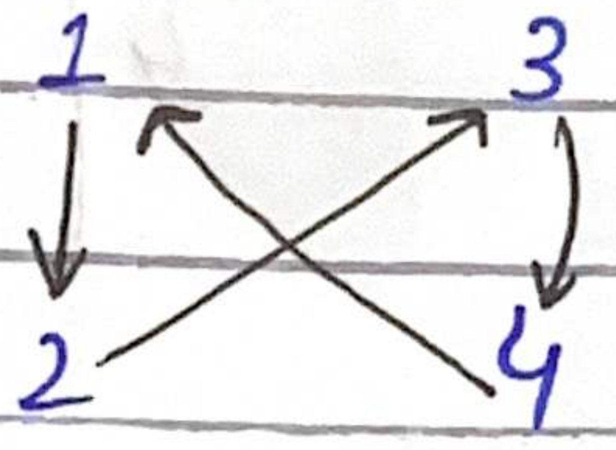
Q#6:-

a) i) The Relation $\{(1,2)(2,3)(3,4)(4,1)\}$ on $\{1,2,3,4\}$:

Arrow Diagram



Diagraph

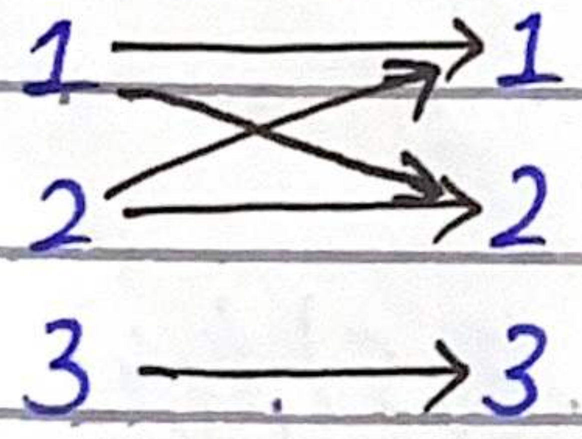


Matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

ii) The Relation $\{(1,2)(2,1)(3,3)(1,1)(2,2)\}$ on $X=\{1,2,3\}$

Arrow Diagram



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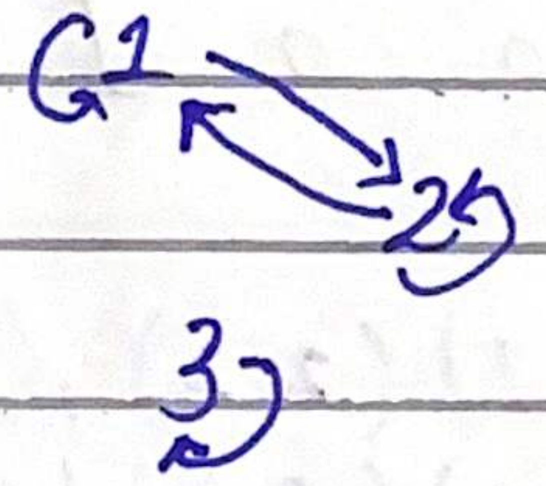


Matrix

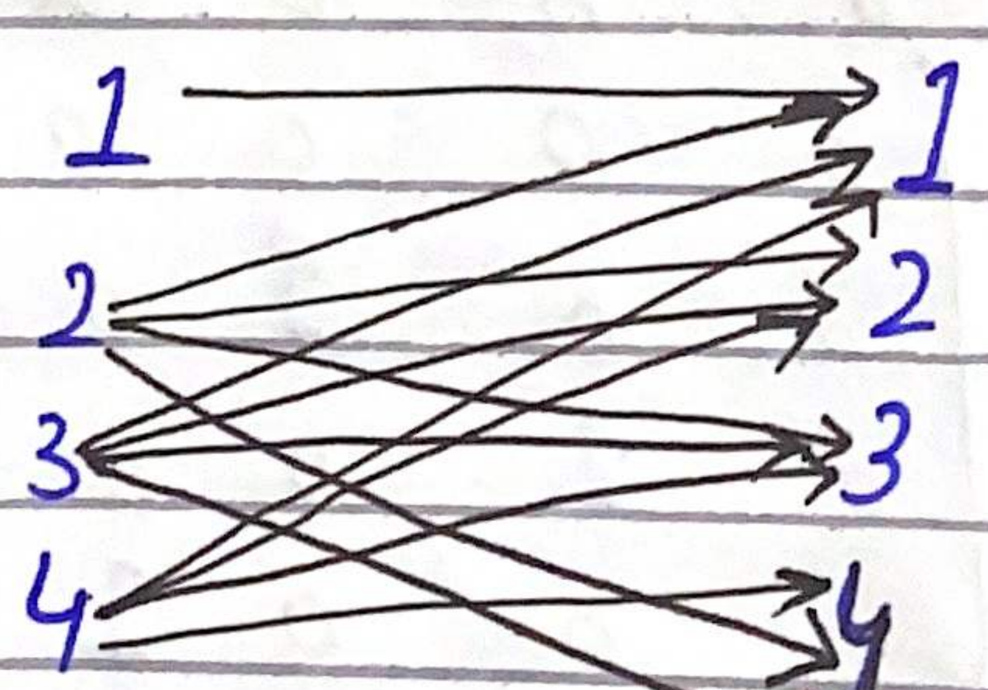
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

iii) The Relation R on $\{1,2,3,4\}$ defined by $(x,y) \in R$ if $x^2 \geq y$.

So, according to the given condition, we have to choose ordered pairs (x,y) in which the value of y is less than the square of x value. According to it, these are the ordered pairs:-

$R = \{(1,1)(2,1)(2,2)(2,3)(2,4)(3,1)(3,2)$
$(3,3)(3,4)(4,1)(4,2)(4,3)(4,4)\}$

Arrow Diagram :



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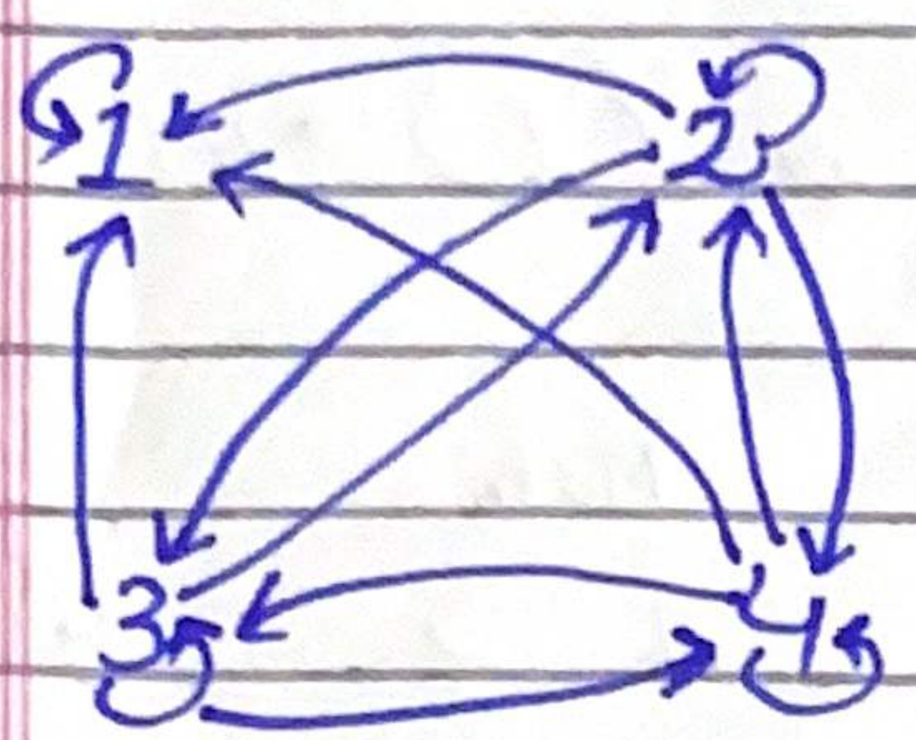

Matrix

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

b) 1) Ordered Pairs : $R = \{ (a,b)(b,a)(a,c)(c,c)(c,d)(d,b) \}$

Matrix :

$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

2) Ordered Pairs : $R = \{ (1,1)(2,2),(3,3),(3,5),(4,4)(4,3)(5,4)(5,5) \}$

Matrix :

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

3) Ordered Pairs : $\{ (b,c)(c,b)(d,d) \}$

Matrix :

$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

c) 1) Ordered Pairs = {(a,w),(a,y),(c,y),(d,w),(d,x)
(d,y),(d,z)}

Domain : a, c, d   Range : w, x, y, z

2)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 & 1 \end{array}$$

Ordered Pairs : {(1,1),(1,3),(2,2),(2,3),(2,4)}

Domain : 1, 2   Range : 1, 2, 3, 4

3)

$$\begin{array}{c|cccc} & w & x & y & z \\ w & 1 & 0 & 1 & 0 \\ x & 0 & 0 & 0 & 0 \\ y & 1 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 1 \end{array}$$

Ordered pairs : {(w,w) (w,y) (y,w) (y,y) (z,z)}

Domain : w, y, z   Range : w, y, z.

d) (1)

$$\begin{array}{c|ccc} & a & b & c \\ a & \underline{1} & 1 & 0 \\ b & 0 & \underline{1} & 1 \\ c & 1 & 0 & \underline{1} \end{array}$$

Reflexive

(2)

$$\begin{array}{c|ccc} & a & b & c \\ a & 1 & \underline{1} & 0 \\ b & 1 & 0 & 1 \\ c & \underline{0} & \underline{1} & 1 \end{array}$$

Symmetric.

(3)

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & \underline{1} & 1 & \underline{1} \\ b & \underline{0} & 1 & 1 \\ c & 1 & \underline{0} & \underline{1} \end{array}$$

Transitive.

e) $R_1 = \{(1,1),(1,2),(3,4),(4,2)\}$
$R_2 = \{(1,1),(2,1),(3,1),(4,4),(2,2)\}$

i) $R_1 \cup R_2 = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,4)$
$(4,2),(4,4)\}$

ii) $R_1 \cap R_2 = \{1,1\}$

iii) $R_1 \circ R_2 = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)$
$(4,2)\}$

iv) $R_2 \circ R_1 = \{(1,1),(1,2),(3,4),(4,1),(4,2)\}$

Q#7:-

We are asked that if a relation, which is based upon a set of all four bit string, is reflexive, symmetric, transitive?-

$X = \{0011, 0101, 1000\}$

⇒ Reflexive : Yes, it is a reflexive relation. Because each string is related to itself.

⇒ Symmetric : If 0101 have some common sub-string of length 2 with 0011, then 0011 is also related to 0101. By such means, we can say that this relation is ~~also~~ symmetric as well.

⇒ Transitive : A relation is transitive if $S_1 R S_2$ and $S_2 R S_3$ are related, then $S_1 R S_3$ are also related. But, in this case, S_1 doesn't share any common substring with S_3, so it is not a transitive relation.

Question # 8:-

i) The elements of R would form those pairs in which A and B have atleast one element of Y and all other elements of A and B outside Y is same.

ii) Taking Example $A = [2,4,5]$
$B = [2,3,5]$

$B \cup Y = A \cup Y$

but $B \cup Y \neq C \cup Y$ $\because [1,3] = C$

Therefore, the relation is not transitive and consequently it is ~~not~~ ~~eq~~ not equivalent as well.

Q-9:-

To show that this relation is in partial order, there are three conditions.

i) Reflexive
ii) Anti-symmetric
iii) Transitivity.

⇒ Since iRj and $i=j$
So every element is related to itself.
i.e Relation is Reflexive.

⇒ It is given that iRj and $i=j$
then it means $j=i$.
Hence, it is antisymmetric.

⇒ Taking iRj and jRK

then either $i=j$ and $j=k$

or

i is performed before j, and j before k.

Thus,

i performed first, then j, then k.

Relation is Transitive.

i) Reflexive : It is reflexive as $i=i$,

ii) Antisymmetric : It is not symmetric.

iii) Transitive : It is transitive.

$\boxed{\text{Hence, it is a Partial Order Relation}}$

Assignment 3A

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Question 1 :- Counting functions using Combinations

i) For each of the m element of domain, there are n choices in co-domain.
So, total function = n^m

ii) (a) $n=2, m=10$
$n^m = 2^{10} = 1024$
(b) $n^m = 3^{10} = 59049$
(c) $n^m = 4^{10} = 1048576$
(d) $n^m = 5^{10} = 9765625$

iii) The function is one-one if $m < n$
The count is $P(n,m) = \frac{n!}{(n-m)!}$

iv) $n=4$, Not possible as $m > n$
$n=5$, $P(5,5) = 5! = 120$
$n=6$, $P(6,5) = 6!/(6-5)! = 720$

v) 2^n

vi) As codomain < domain, so not possible
vii) We have formula $(n+1)^5$ since there are 5 element and for each element, there are $(n+1)$ choices.

(a) $n=1 = (1+1)^5 = 2^5 = \boxed{32}$

ⓑ $n=2 = (2+1)^5 = 3^5 = 243$
ⓒ $n=5 = (5+1)^5 = 6^5 = 7776$
ⓓ $n=9 = (9+1)^5 = 10^5 = 100000$

viii) 2^{100} (since there are 100 elements and each elements, has two choices include in subset or not.)

$1+100 = 101$ (since 1 subset with 0 and 100 subsets with one element

$\boxed{\text{Final Ans} = 2^{100} - 101}$

Question #3:- (Permutation & Combinations)

i) $P(n,3) = 12\ P(n/2, 3)$

$$\frac{n!}{(n-3)!} = 12 \cdot \frac{(n/2)!}{(n/2-3)!}$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 12 \cdot \frac{(n/2)(n/2-1)(n/2-2)\cancel{(n/2-3)!}}{\cancel{(n/2-3)!}}$$

$$8\cancel{n}(n-1)\cancel{(n-2)} = 12\cancel{n}\ \cancel{(n-2)}(n-4)$$

$$8n-8 = 12n-48$$
$$4n = 40$$
$$\boxed{n=10}$$

ii) Formula of permutation is,
$$P(n,r) = \frac{n!}{(n-r)!}$$

We are given,

$$\cancel{\frac{P(2n+1)(n-1)}{(2n-1),(n)} = \frac{3}{5}}$$

$$P(2n+1, n-1) : P(2n-1, n) = 3:5$$

$$\frac{\frac{(2n+1)!}{(n+2)!}}{\frac{(2n-1)!}{(n-1)!}} = 3/5$$

$$\frac{(2n+1)!\,(n-1)!}{(n+2)!\,(2n-1)!} = 3/5$$

$$\frac{(2n+1)(2n)\cancel{(2n-1)!}\,\cancel{(n-1)!}}{(n+2)(n+1)n\,\cancel{(n-1)!}\,\cancel{(2n-1)!}} = 3/5$$

$$5(2n+1)(2n) = 3n(n+1)(n+2)$$

$$10n^2 + 10n = 3n^3 + 9n^3 + 6n$$

$$3n^2 - n^2 - 4n = 0$$

$$n(3n^2 - n - 4n) = 0$$

$$3n^2 - n - 4n = 0$$

$$\boxed{n = 4, \quad n = -3}$$ Ans

iii) $C(18, r) = C(18, r+2)$, Find $C(r, 5)$

The Combination Formula:-

$$C(n, r) = C(n, n-r)$$

$$C(18, r) = C(18, r+2)$$

$$r = 18 - (r+2)$$

$$r = 16 - r$$

$2r = 16$

$\boxed{r = 8}$

Find $C(r,5)$ when $r = 8$

$$C(8,5) = \frac{8!}{5!(8-5)!}$$

$$= \frac{8!}{5!\;3!} = \frac{8\cdot 7\cdot 6\cdot 5!}{\cancel{5!}\;3!}$$

$$= \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}$$

$$= \frac{336}{6} = \boxed{56}$$

Symmetry check.

$$C(8,5) = C(8,3)$$

$$C(8,3) = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56$$

$$\boxed{C(8,5) = 56}$$ Ans

iv)

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

$$= \frac{n\cdot n!}{(r-1)\cdot[n-(r-1)]!}$$

$$n = r(r+1)!$$

v) Total number of ways to select 4 members = $^{12}C_4$

Number of ways with no Processors = $^{8}C_4$

By Substraction = $^{12}C_4 - ^{3}C_4$

= 495 − 70

= $\boxed{425 \text{ ways}}$

vi) Average = 7 letters = a → 2 times R → 2 times

Total Permutations = $\frac{7!}{2!\cdot 2!}$ = 1260

a) Total Arrangements = 1260

→ 1 Block of R.

→ 5 Remaining letter a, a, v, g, e 5+1=6

Permutation = $\frac{6!}{2!}$ = 360

Subtraction = 1260 − 360

= $\boxed{900 \text{ ways}}$

b) 1 block of A & 1 block of R

3 remaining = 5! = 120

= 360 − 120

= $\boxed{240 \text{ ways}}$

c) Word ⇒ Arrange

Total Arrangements = 1260

Arrangements with a together = 360

Arrangement with r together = 360

Permutation = 120

Required Arrangements $= 1260 - (360+360) + 120$
$\boxed{= 660}$ Ans

vii) Given Digits:-
3, 4, 5, 6, 5, 4, 3
Even digits = 4, 6 (3 even place 2, 4, 6)

Permutation $= \frac{3!}{2!} = 3$

Remaining odd digits (3, 5, 5, 3) odd position (1, 3, 5, 7)

$= \frac{4!}{2! \cdot 2!} = 6$

Total Permutation $= 3 \times 6 = 18$ ways

Question # 3:-

⇒ Part A

1) $A(0,7)$:
Since $m = 0$
$A(0,7) = 7+1 = 8$

2) $A(1,1)$
$A(1,1) = A(0, A(1,0))$
$A(1,0) = A(0,1) = 1+1 = 2$
$A(1,1) = A(0,2) = 2+1 = 3$

3) $A(2,2)$

$A(2,2) = A(1, A(2,1))$

$A(2,1) = A(1, A(2,0)) = A(1,1) = 3$

$A(2,2) = A(1,3) = 5$

(ii) Prove for $n > 0$

$A(1,n) = n+2$

Base case ($n=0$):

$A(1,0) = A(0,1) = 1+1 = 2$

Inductive step:-

Assume $A(1,k) = k+2$

$n = k+1$

$A(1,k+1) = A(0, A(1,k))$

$A(1,k) = k+2$

$A(1,k+1) = A(0,k+2) = (k+2)+1 = k+3$

⇒ Part B:-

⇒ Compute each stirling Number.

a) $S(2,2)$

since $r = n$:

$S(2,2) = 1$

b) $S(5,2)$

Using formula.

$S(5,2) = S(4,1) + 2 \cdot S(4,2)$

$S(4,1) = 1,\ S(4,2) =$
$S(3,1) + 2 = S(2,2)$
$S(5,2) = 1+2\cdot(1+2\cdot 3) = 1+2\cdot 7 = \boxed{15}$

⇒ Part C:-

i) $\gcd(28,18) = \gcd(18,10)$

$\gcd(18,10) = \gcd(10,8)$
$\gcd(10,8) = \gcd(8,2)$
$\gcd(8,2) = \gcd(2,0)$
$\boxed{g(2,0) = 2}$

ii) $\gcd(24,75)$

$\gcd(24,75) = \gcd(75,24)$
$\gcd(75,24) = \gcd(24,3)$
$\gcd(24,3) = \gcd(3,0)$

$\boxed{\gcd(3,0) = 3}$ Ans

⇒ Part D:-

i) $f(99)$
$f(99) = f(f(110)) = f(100)$
$f(99) = 91.$

ii) $f(98)$
$f(98) = f(f(109))$
$f(f(109)) = f(99)$
$f(99) = 91$

⇒ Part E:-

$L_1 = 1$

$L_2 = 3$

$L_3 = L_2 + L_1$

$= 3 + 1$

$= 4$

$L_4 = 4 + 3$

$= 7$

$L_5 = 7 + 4$

$= 11$

$L_6 = 11 + 7$

$= 18$

The first six lucus numbers are:-

1, 3, 4, 7, 11, 18

⇒ Part F:-

⇒ (a) $t(5)$:-

Break $n = 5$ into $(5/2) = 2$
and $(5/2) = 3$

$$t(5) = t(2) + t(3) + 5b \quad \rightarrow (i)$$

$$t(2) = t(1) + t(1) + 2b$$

$$t(2) = a + a + 2b = \boxed{2a + 2b}$$

$$t(3) = t(1) + t(2) + 3b$$

$$t(3) = a + (2a + 2b) + 3b$$

$$\boxed{t(3) = 3a + 5b}$$

Putting values of $t(2)$ & $t(3)$ in eqn (i)

$$t(5) = 2a + 2b + 3a + 5b + 5b$$

$$\boxed{t(5) = 5a + 12b}$$ Ans

(b) $t(6)$

$$t(6) = t(3) + t(3) + 6b$$

$$t(6) = 3a + 5b + 3a + 5b + 6b$$

$$\boxed{t(6) = 6a + 16b}$$ Ans