6.1) $m(x) = \frac{1}{N} \sum_{i=1}^{N} x_i$

$y_i = a + bx_i$

$$m(a + bx) = \frac{1}{N} \sum_{i=1}^{N} (a + bx_i)$$

Split using rules of summation:

$$m(a+bx) = \frac{1}{N} \left( \sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i \right)$$

$\sum a = Na$ so leave constant b out:

$$m(a+bx) = \frac{Na}{N} + b\left(\frac{1}{N} \sum x_i\right)$$

Substitute definition of $m(x)$:

$$m(a+bx) = a + b \cdot m(x)$$

6.2) $\operatorname{cov}(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$

$$s^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$$

plug in x for y for covariance formula:

$$\operatorname{cov}(x,x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(x))$$

$$\operatorname{cov}(x,x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$$

This is the same as the formula for $s^2$

6.3) $m(a+bY) = a + b \cdot m(Y)$

$cov(x, a+bY) = \frac{1}{N} \sum (x_i - m(x))([a+by_i] - m(a+bY))$

substitute $m(a+bY)$ with $a + b \cdot m(Y)$:

$cov(x, a+bY) = \frac{1}{N} \sum (x_i - m(x))(a + by_i - a - b \cdot m(Y))$

A terms cancel out:

$cov(x, a+bY) = \frac{1}{N} \sum (x_i - m(x))(b(y_i - m(Y)))$

b goes out:

$cov(x, a+bY) = b \cdot cov(x, Y)$

6.4) Substitute, so constants a cancel out and b factors out

$cov(a+bX, a+bY) = \frac{1}{1} \sum (b(x_i - m(x)) \cdot b(y_i - m(Y)))$

factor out b terms:

$cov(a+bX, a+bY) = b^2 \cdot cov(X, Y)$

The reason is because in Q6.2, $cov(x,x) = s^2$. Therefore

$cov(bX, bX) = b^2 s^2$

6.5) $x_1, x_2, x_3 \dots x_N$

$y_i = a + bx_i$

Since $b > 0$, the transformation is increasing. The order stays the same. Additionally, the middle values of Y come from the middle values of X. So if the median of X is med(X) the median of Y is $med(Y) = a + b\,med(X)$. Therefore, we can conclude that:

$$med(a+bX) = a + b\,med(X).$$

So, the median of $a+bX$ equals to $a + b\,med(X)$.

$IQR(X) = Q_3(X) - Q_1(X)$

Quartiles transform the same way.

$Q_1(Y) = a + bQ_1(X) \quad Q_3(Y) = a + bQ_3(X)$

$$\begin{aligned} IQR(Y) &= Q_3(Y) - Q_1(Y) \\ &= (a + bQ_3(X)) - (a + bQ_1(X)) \\ &= b(Q_3(X) - Q_1(X)) = b\,IQR(X) \end{aligned}$$

So it's true that $IQR(a+bX) = b\,IQR(X)$

However, it is not true that $IQR(a+bX)$ is equal to $a + b\,IQR(X)$.

6.6)

Data:

$x = \{1, 7\}$, $N = 2$

Mean of $X$:

$$m(X) = \frac{1+7}{2} = 4$$

Mean of $X^2$:

$$m(x^2) = \frac{1^2 + 7^2}{2} = \frac{1+49}{2} = 25$$

Square of the mean:

$$(m(X))^2 = 4^2 = 16$$

$$25 \neq 16$$

Mean of $\sqrt{X}$:

$$m(\sqrt{x}) = \frac{\sqrt{1} + \sqrt{7}}{2} = \frac{1 + 2.646}{2} = 1.823$$

Square root of the mean:

$$\sqrt{m(X)} = \sqrt{4} = 2$$

$$1.823 \neq 2$$

Therefore:

$$m(X^2) \neq (m(X))^2 \text{ and } m(\sqrt{X}) \neq \sqrt{m(X)}$$