Geometric Representation of Little’s Law – CSC148

**Def** - Ti = total time there were i cj in S

**Def** – the wj the 1X1 tiles, each representing 1 t.u. in which there was one cj in S

Example: (w2+w4) = residence duration for c2

**Def** – A is the area under function L(t) vs. t = (the number of cj in S at time t) vs. t

**Def** – T = total time elapsed during cj processing

**Def** - n = total number of ck that arrived during time [0,T]

Consider the simple events: a1=1, a2 = 3, a3 = 4, and s1=1, s2=2, s3=1; in this model let T = 6

*Note: In this document, each term of the form x^ is a* ***statistic*** *obtained from measurements during execution of the above model*

**L(t)**

Figure – the geometry: 2 **---------**

T1 T1 w3 T1 *< - - Example Ti for I = 1*

1 **----------** **---------**  **---------**

w1 w2 w4 w5

0 --------1--------2--------3--------4--------5--------6 **--> (t <=6)**

a1 a2 a3

(sum over all i) (i \* **Ti**/T) = A/T = avg(L(t)), abbreviated as avg(L)^ = (0\*2 + 1\*3 + 2\*1) /6 = (5/6)

{ i\*Ti = total area occupied by tiles when there are i cj in S} *< -- the “L dimension”*

Area A is also = ( total number of wk tiles) *< - - the t dimension*

However, (total residence duration over all cj)/n = avg(cj residence duration) = A/n = avg(Wcj)^

In the Figure above, A/n = (sum of all cj residence durations)/3 = ( w1 + (w2+w4) + (w3+w5) )/3 =

(1 + 2 + 2)/3 = 5/3

Combining above equalities: T \* avg(L)^ = A = n \* avg(Wcj)^, and isolating avg(L)^ on L.H.S.,

we get: avg(L)^ = (n/T) \* avg(Wcj)^ **. . . (1)**

As T - - > ∞ , (n/T) - ->  and the “^” terms in (1) - -> avg(L) and avg(Wcj) respectively.

Also, drop the cj subscript in Wcj because Wcj means cj (and only cj) residence duration so we get

* Little’s Law is: avg(L) =  \* avg(W) **. . . (2)**

*Notes – (1) is treating (n/T) as constant, meaning that the arrivals rate must be constant during [0,T].*

***An overall note*** *– the diagram and event calculations are greatly simplified by using integer time values. But this is no loss in generality because fractional times simply change 1X1 tiles to rectangle-shaped tiles;*

*the geometry and all terms in (1) and (2) remain correct.*