The Forward-Backward Algorithm

We have a hidden Markov model (HMM) with

N hidden states $\{q_1\ q_2 \ldots q_N\}$
M emission states $\{y_1, y_2, \ldots y_M\}$

And where

π_i is the initial probability for hidden state q_i
a_{ij} is the transition probability for moving from hidden state q_i to q_j
$b_i(y_j)$ is the emission probability for hidden state q_i and observation y_j

We are given a sequence of observed emission states $y_1, y_2, \ldots, y_t, \ldots y_T$

We have seen the forward algorithm, which involves calculating the probability of being in hidden state i after the first t observations (the forward probability), as well as the backward algorithm, which calculates the probability of observing the remaining observations from time $t+1$ to T (the backward probability).

So far, we've solved problems that consider the entire sequence of emission states (eg. scoring, decoding, and filtering).

By combining the forward and backward algorithms, we can also answer the question, what is the distribution of hidden states in the middle of the sequence at time $t, t < T$. This is the filtering problem.

Exercises

Theory
1.1 Write the forward-backward algorithm.
1.2 Given our weather HMM and the emission sequence

umbrella, umbrella, hat, sunglasses, hat, <u>sunglasses</u>, sunglasses, hat, hat, hat

What is the most likely hidden state at the underlined $t = 6$?
(For a description of the weather HMM, refer to ScratchSheet.pdf)

Bonus
2.0 Learn the Baum-Welch algorithm
2.1 How are the forward and backward probabilities used in the Baum-Welch algorithm?
2.1 Write the algorithm steps
2.2 What are the limitation of the Baum-Welch algorithm?