PROBLEM SET 5

I PLEDGE my honor that I have abided by the Stevens Honor System.

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## Problem 2

let $B = \{0^i 1^j : i \neq j\}$. We can see that $\overline{B} \cap 0^* 1^*$. This is equal to $\{0^k 1^k$, for all $k \geq 0\}$. If B were regular, then $\overline{B} \cap 0^* 1^*$ would be as well. But this is not a regular language $\{0^k 1^k$, where $k \geq 0\}$, so, hence, B cannot be regular.

## Problem 3

1. $p = 4$. The string 000 is in the language but cannot loop. If s had 4 or more, it must contain 1's. $s = xyz$ and $x = 000$, y is the first 1 and z is any leftover.
2. $p = 1$. The $\epsilon$ cannot be pumped. Every nonempty string can be divided up where ~~xyz and z~~ $x = \epsilon$, $y$ = first character, and $z$ = the rest.
3. $p = 3$. 11 is in the language & cannot be pumped (that would be $p=2$). If s is from the left side, $x = \epsilon$, y = first symbol, and z is the rest. → (and s begins w/ either a 0 or 11, $s = xyz$ where) If s is from the left and begins with 10, $s = xyz$ where $x = 10$ y is the next symbol, and z = the rest. If s is [illegible] the right side, let $s = xyz$ where $x = 1$, $y = 0$, z = the rest.
4. $p = 2$ $\epsilon$ [illegible] cannot be pumped. Every nonempty string $(\geq 2)$ can be [illegible] up where $s = xyz$ and $x = \epsilon$, $y = 01$, and z = the rest.
5. $p = 3$ 00 is in the language cannot pump. let s be xyz and $x = \epsilon$ or 1, $y = 010$ or $y = 00$ or $y = 01^+0$ and let z = the rest.

## Problem 4

1. $\{w : w = w^R\}$, the language of palindromes

$$S_0 \rightarrow 0S_00 \mid 1S_11 \mid \epsilon \mid 0$$
$$S_1 \rightarrow 0S_10 \mid 1S_21 \mid 1$$
$$S_2 \rightarrow 0S_20 \mid 1S_01$$

2. $\{w : w$ starts and ends with the same symbol$\}$

~~[illegible]~~
$$S \rightarrow 0T0 \mid 1T1$$
$$T \rightarrow 0T \mid 1T \mid \epsilon$$

3. $\{w : w$ contains more 0s than 1s$\}$

$$S \rightarrow TS \mid 1T \mid 1S$$
$$T \rightarrow TT \mid 0T1 \mid 1T0 \mid \epsilon$$

## Problem 1

~~[illegible]~~

3 COND. OF PUMPING LEMMA:

1. $|y| > 0$
2. $|xy| \leq p$
3. $xy^iz \in L \ \forall i \geq 0$

let $p = 2$ and we know $p < 2^p$ so $|y| < 2^p$. The second condition is already fulfilled $|xy| \leq 2$ because. Hence $|xy^2z| = |xyz| + |y|$. This is $< 2^p + 2^p$ which is equivalent to $|xyz| + |y| < 2^{p+1}$. The first cond. $|y| > 0$ and this holds true, picking x to be an $\epsilon$ string and y must have a length of @ least 1. We know $2^p < |xyyz| < 2^{p+1}$. The issue is that $|xyyz|$ is not possible to make w/ this language. Hence, not regular.