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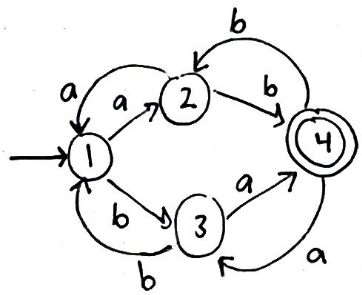
PROBLEM set 4 Constance Xu

I pledge my honor that I have abided by the Stevens honor system.

Problem 1

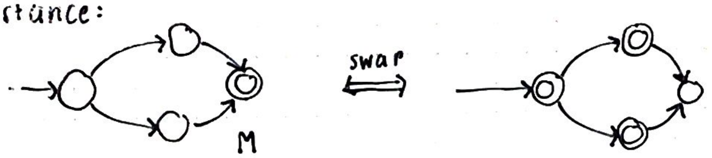
	1	2	3	4	5	6	7	8	9
1		✓	✓	✓		✓		✓	
2			✓	✓	✓	✓	✓		✓
3				✓	✓		✓	✓	✓
4					✓	✓	✓	✓	✓
5						✓		✓	
6							✓	✓	✓
7								✓	
8									✓
9									



Problem 2

A. Let M be a DFA that accepts a language L. Let $M = \{Q, \Sigma, q_0, \delta, A\}$. Create another DFA that accepts the complement only by taking all the accept states in this DFA and changing them with reject states. Furthermore, take all reject states and make them accept states.

For instance:



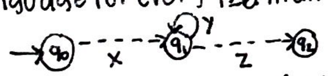
This will accept the complement of the language L.

B. Prove that if A, B are each regular, then so is A−B, the diff. of A and B.

A−B is equivalent to $A \cap \overline{B}$. Languages are considered a set, because of this, all normal rules apply that apply to sets. Therefore, De Morgan's law is applicable which is why A−B is equivalent to $A \cap \overline{B}$. From above we know that the complement of a language is also regular. A is a regular language (that is given) and $\overline{B}$ is as well. An intersection is the same as saying "and" and both are regular languages.

Hence, A−B is regular.

C. The pumping lemma is saying for any string s in a particular language, if it has a greater length than the "pumping length" p, then $s = xyz$ and $|xy| \le p$ such that xy^*z is also in the language for every $i \ge 0$ in an example:



where y is the "pumping" (or pumping threshold)

For this PARTICULAR LANGUAGE, L, we can say $p = 2$. This is the pumping length. Take any string $a^i b^j c^k$, where $i, j, k \ge 0$. If $i = 1$ or $i > 2$, we can take ϵ on x and $y = a$.

If $i = 1$, $j = k$ because that is the only way it would satisfy the requisites. The language will still be accepted with any number of a's because $j = k$.

For $i > 2$, all strings will be accepted because y will be used via THE PUMPING LEMMA.

For $i = 2$, there will be an even # of a's. OR $x = \epsilon$ and $y = aa$. These are both accepted in the language.

When $i = 0$: $x = \epsilon$ and $y = b$ if $j > 0$ and if not, $y = c$. When removing a^i, the string will be $y^j z^k$ which follows the form $b^j c^k$ and this is accepted.

Hence, this satisfies the 3 conditions of the PUMPING LEMMA.

D. L is not regular. L can be described as $b^*c^* \cup aaa^*b^*c^* \cup \{ab^ic^i : i \ge 0\}$.

Problem 2

E. The pumping lemma is only applicable to regular languages. From part(d) we know that L is not a regular language. Hence, the pumping lemma DOES NOT APPLY... which is why it does not contradict. For part (c), we know that the pumping lemma conditions apply, it happened to apply to a non-regular language. This is not always the case.

OPTIONAL PROBLEM

supriya.pai.2@credit-suisse.com