

Physics Equations Live Here

Physics 4A

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Chapter 1

Equations for Moment of Inertia Lab

Theoretical Times

Theoretical time for solid cylinder at 5°

$$\sqrt{\frac{2(1 + 0.5) \cdot 1.000}{9.8 \sin(5)}} = 1.8741 \text{ sec}$$

Theoretical time for hollow cylinder at 5°

$$\sqrt{\frac{2(1 + 1) \cdot 1.000}{9.8 \sin(5)}} = 2.1641 \text{ sec}$$

Theoretical time for sphere at 5°

$$\sqrt{\frac{2(1 + \frac{2}{5}) \cdot 1.000}{9.8 \sin(5)}} = 1.8106 \text{ sec}$$

Theoretical time for solid cylinder at 10°

$$\sqrt{\frac{2(1 + 0.5) \cdot 1.000}{9.8 \sin(10)}} = 1.3330 \text{ sec}$$

Theoretical time for hollow cylinder at 10°

$$\sqrt{\frac{2(1 + 1) \cdot 1.000}{9.8 \sin(10)}} = 1.5392 \text{ sec}$$

Theoretical time for sphere at 10°

$$\sqrt{\frac{2(1 + \frac{2}{5}) \cdot 1.000}{9.8 \sin(10)}} = 1.2878 \text{ sec}$$

RSS Error

Greatest contributors are used to calculate RSS error

$$\text{RSS} = \sqrt{\left(\frac{0.00005 \text{ sec}}{1.2037 \text{ sec}}\right)^2 + \left(\frac{0.0005m}{1.000m}\right)^2 + \left(\frac{0.00005m}{0.0255m}\right)^2} \times 100\% = 1.02\%$$

Back-End Error

Percent error should be used because we are calculating a theoretical value and comparing an experimental value with that number rather than comparing two unknown experimental values.

$$\% \text{error} = \frac{|E - K|}{K} \times 100\%$$

Where E = experimental value and K = theoretical value.

$$\% \text{error for solid cylinder at } 5^\circ = \frac{|1.9279 \text{ sec} - 1.8741 \text{ sec}|}{1.8741 \text{ sec}} \times 100\% = 2.87\%$$

$$\% \text{error for hollow cylinder at } 5^\circ = \frac{|2.1946 \text{ sec} - 2.1641 \text{ sec}|}{2.1641 \text{ sec}} \times 100\% = 1.41\%$$

$$\% \text{error for sphere at } 5^\circ = \frac{|1.8141 \text{ sec} - 1.8106 \text{ sec}|}{1.8106 \text{ sec}} \times 100\% = 0.19\%$$

$$\% \text{error for solid cylinder at } 10^\circ = \frac{|1.2564 \text{ sec} - 1.3330 \text{ sec}|}{1.3330 \text{ sec}} \times 100\% = 5.75\%$$

$$\% \text{error for hollow cylinder at } 10^\circ = \frac{|1.4367 \text{ sec} - 1.5392 \text{ sec}|}{1.5392 \text{ sec}} \times 100\% = 6.66\%$$

$$\% \text{error for sphere at } 10^\circ = \frac{|1.2151 \text{ sec} - 1.2878 \text{ sec}|}{1.2878 \text{ sec}} \times 100\% = 5.65\%$$

1.1 More Analysis (Q4)

Using:

$$\Delta t = \frac{1}{R_2} \sqrt{\frac{D(3R_2^2 + R_1^2)}{g \sin \theta}}$$

Calculated Δt for hollow cylinder

Trial using $\theta = 5^\circ$:

$$\Delta t = \frac{1}{0.0285} \sqrt{\frac{1.000(3 \cdot (0.0255)^2 + 0.0285^2)}{9.8 \sin(5)}} = 1.9957 \text{ sec}$$

Trial using $\theta = 10^\circ$:

$$\Delta t = \frac{1}{0.0285} \sqrt{\frac{1.000(3 \cdot (0.0255)^2 + 0.0285^2)}{9.8 \sin(10)}} = 1.4138 \text{ sec}$$

Comparing Values				
	Theoretical using C as 1	Theoretical using an R_1 & R_2	Experimental	%error
$\theta = 5^\circ$	2.1641 sec	1.9957 sec	2.1946 sec	1.41%
$\theta = 10^\circ$	1.5392 sec	1.4138 sec	1.4367 sec	6.66%

Analyzing these results show that using $C = 1$ was closer to the experimental value than using two radii for the 5° trial. However, for the 10° trial, the result is the opposite. Factoring in the %error, our error was much higher for the 10° trial. This tells me that with minimal error, using $C = 1$ is more accurate in this context.

4 - Procedural Errors and Improvements

Some of the contributing factors the errors in this lab are, inclometer limitations, calliper limitations, and photogate limitations. Though the goal of these devices are to attempt to give the best reading possible, the limitation to a specific decimal places allows room for error, which is why we allowed our data to be \pm a certain amount. A big contributor to error could be releasing the ball. It is difficult to release the ball exactly straight which not doing so would default the goal of the lab leading to error. Another factor for error is the initial velocity of the rolled object. It is difficult to release the object from rest without triggering the photogate timer, which also could lead to error. I would suggest that students or lab participants use something to help keep the object to roll in a straight line without disrupting its velocity.

Chapter 2

Conservation of Energy and Momentum

RSS Error

Greatest contributors are used to calculate RSS error

$$\text{RSS\% Error} = \sqrt{\left(\frac{0.00005\text{kg}}{0.2156\text{kg}}\right)^2 + \left(\frac{0.005\text{m/s}}{0.18\text{m/s}}\right)^2} \times 100\% = 2.78\%$$

Back-End Error % Difference

$$\% \text{ Difference} = \frac{|E_1 - E_2|}{\frac{(E_1 + E_2)}{2}} \times 100\% = \frac{|0.11 \frac{\text{kgm}}{\text{s}} - 0.09 \frac{\text{kgm}}{\text{s}}|}{\frac{(0.11 \frac{\text{kgm}}{\text{s}} + 0.09 \frac{\text{kgm}}{\text{s}})}{2}} \times 100\% = 19.50\%$$

Percent difference should be used to calculate error because we are comparing the initial momentum versus the final momentum. This explicit calculation was done for Trial#1's momentum values. This was an elastic collision where theoretically the initial momentum and final momentum should be the same, but as shown this is not the case. This shows that there was definitely error involved.

4 - Procedural Errors and Improvements

Procedural Errors

- In regards to errors, there are factors that limit the most precise and accurate readings. One of these factors are the software limiting us to specific decimal places
- Additionally, the scale is limited to four places past the decimal point allowing for error.
- Another factor is the air track we used to simulate a frictionless surface for the carts to glide upon. Of course it would be impossible to get a perfectly frictionless surface in the context of this lab, but without that preciseness, this lab is error prone.
- Finally, in the software itself of data studio, a different selection of data would yield different results. This could lead not only errors, but a biased selection of data.
- ★ It is noted that our error was awful for this lab, we re-ran trials multiple times included with your supervision, but still we were getting extremely high error. You advised us to not use this lab as a formal write up.

Improvements

- If I could recommend something to be improved, it would be a better printed procedural section. Some parts of the print were messed up make it difficult to read and understand.

Rotational Inertia of a Point Mass Lab

“Freebie”

Centripetal Force Lab

“Freebie”

Simple Pendulum Lab

RSS Error

Greatest contributors are used to calculate RSS error

$$\text{RSS\% Error} = \sqrt{\left(\frac{0.00005kg}{0.0104kg}\right)^2 + \left(\frac{0.0005m}{0.193m}\right)^2 + \left(\frac{0.00005s}{0.8991s}\right)^2} \times 100\% = 0.55\%$$

Back-End Error % Difference

$$\% \text{ Difference} = \frac{|E_1 - E_2|}{\frac{(E_1 + E_2)}{2}} \times 100\% = \frac{|1.3836s - 1.3978s|}{\frac{(1.3836s + 1.3978s)}{2}} \times 100\% = 1.01\%$$

Percent difference should be used to calculate error between the times of different mass, amplitudes, and lengths, because we are calculating multiple experimental values and comparing their differences.

Back-End Error % Error

$$\% \text{ error} = \frac{|E - K|}{K} \times 100\% = \frac{|9.8m/s^2 - 9.6369m/s^2|}{9.6369m/s^2} = 1.69\%$$

Percent error should be used to calculate error between the $g_{\text{experimental}}$ and the $g_{\text{theoretical}}$, because g is a scientifically known and accepted value. That being said, we want to know how off we were when attempted to achieve this value experimentally.

4 - Procedural Errors and Improvements

Procedural Errors

- In regards to errors, there are factors that limit the most precise and accurate readings. One of these factors are the timer limiting the time to four decimal places.
- Additionally, the scale is limited to four places past the decimal point allowing for error.
- Another factor is actually measuring the length of the string. We had to rely on a meter stick and were only accurate within three decimal places. We actually ran into an issue where our initial data was skewed by a bad reading of the length of the string, leading us to have our gravitational acceleration as around $35 \frac{m}{s^2}$. Once we re-measured, we were able to get a much more accurate representation of $g_{experimental}$.

Improvements

- If I could recommend something to be improved, it would be to be much more careful when measuring the length of the string to the center of the mass. My partner and I really suffered the consequences due to this negligence.

Archimede's Principle Lab

RSS Error

Greatest contributors are used to calculate RSS error.

$$\text{RSS\% Error} = \sqrt{\left(\frac{0.00005\text{kg}}{0.0505\text{kg}}\right)^2 + \left(\frac{0.00005\text{m}}{0.0355\text{m}}\right)^2} \times 100\% = 0.17\%$$

Back-End Error % Error

$$\% \text{error} = \frac{|E - K|}{K} \times 100\%$$

$$\% \text{error}_{\text{Small Can}} = \frac{|0.1536 \text{ kg} - 0.1592 \text{ kg}|}{0.1592 \text{ kg}} = 3.52\%$$

$$\% \text{error}_{\text{Large Can}} = \frac{|0.2583 \text{ kg} - 0.2761 \text{ kg}|}{0.2761 \text{ kg}} = 6.45\%$$

I decided to use percent error, because instead of comparing two experimental values, we are comparing a calculated value with a measured value. That being the value derived from Archimede's Principle, and the actually value we measured when placing mass into the cylinder. However, the Theory section of the lab has noted that "the experimental mass needed to sink the can will be the calculated mass and the theoretical mass will be the measured one."

4 - Procedural Errors and Improvements

Procedural Errors

- In regards to errors, there are factors that limit the most precise and accurate readings. One of these factors are the callipers limiting measurements to four decimal places.
- Additionally, the scale is limited to four places past the decimal point allowing for error.
- Another factor that could have led to error is the force used to place a mass inside the cans while measuring how much mass it took to sink. It is entirely possible that we used too much force which could have led to the can filling with water and sinking prematurely.

Improvements

- If I could recommend something to be improved, it would be to be much more careful when putting the mass inside the can. Due to my partner and I taking a lot of time with another lab, we were really rushed with time. Due to this I would make sure we ensure there is adequate time to finish both labs.

Random

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$$

Here, $f^{(n)}(a)$ is the n th derivative of $f(x)$ evaluated at a , and $n!$ is the factorial of n . The series is centered at a , and the terms in the series involve raising $x - a$ to increasingly higher powers.

Schrödinger Equation

The Schrödinger equation is a fundamental equation of quantum mechanics that describes how a wave function changes over time:

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H} \Psi(\mathbf{r}, t)$$

In this equation, $\Psi(\mathbf{r}, t)$ is the wave function, i is the imaginary unit, \hbar is the reduced Planck constant, and \hat{H} is the Hamiltonian operator.

Heisenberg Uncertainty Principle

The Heisenberg uncertainty principle is a fundamental principle of quantum mechanics that states that the position and momentum of a particle cannot be measured with arbitrary precision:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

In this equation, Δx and Δp are the uncertainties in the position and momentum of the particle, respectively, and \hbar is the reduced Planck constant.

The Pauli Exclusion Principle

The Pauli exclusion principle is a fundamental principle of quantum mechanics that states that no two particles can occupy the same quantum state simultaneously:

No two electrons can have the same set of quantum numbers

The Standard Model Lagrangian

$$\mathcal{L} = \sum_{i=1}^3 [(\bar{\psi}_i i \gamma^\mu \partial_\mu \psi_i) + (m_i \bar{\psi}_i i \psi_i)] - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} (D_\mu H)^\dagger (D^\mu H) - V(H)$$

where $F^{\mu\nu}$ is the field strength tensor for the gauge fields, ψ_i are the three generations of fermions (quarks and leptons), H is the Higgs field, m_i are the fermion masses, $V(H)$ is the Higgs potential, and D_μ is the covariant derivative.

Greek Symbols

α alpha and Alpha A

β beta and Beta B

γ gamma and Gamma Γ

δ delta and Delta Δ

ϵ epsilon and Epsilon E

ζ zeta and Zeta Z

η eta and Eta H

θ theta and Theta Θ

ι iota and Iota I

κ kappa and Kappa K

λ lambda and Lambda Λ

μ mu and Mu M

ν nu and Nu N

ξ xi and Xi Ξ

π pi and Pi Π

ρ rho and Rho P

σ sigma and Sigma Σ

τ tau and Tau T

υ upsilon and Upsilon Υ

ϕ phi and Phi Φ

ψ psi and Psi Ψ

ω omega and Omega Ω

Phys 4A Final Exam Cheat Sheet Equations

Following this page lie equations I will be using on my cheat sheet for my final exam.

1-D Kinematics

Horizontal Equations of Motion

$$v_f = v_0 + at$$

$$x_f = x_0 + v_0t + \frac{1}{2}at^2$$

$$x_f = x_0 + \frac{1}{2}[v_0 + v_f]t$$

$$v_f^2 = v_0^2 + 2a[x_f - x_0]$$

Vertical Equations of Motion

$$v_f = v_0 - gt$$

$$y_f = y_0 + v_0t - \frac{1}{2}gt^2$$

$$y_f = y_0 + \frac{1}{2}[v_0 + v_f]t$$

$$v_f^2 = v_0^2 - 2g[y_f - y_0]$$

Torque

Torque

$$\tau = \vec{r} \times \vec{F}$$

$$\tau = I\alpha$$

- Need to know direction of torque
- Use right hand rule (fingers curve direction of rotation, torque is where thumb points)
- ω & α are the basics
- Torque is orthogonal to rotation
- Torque is the function of moment arm from origin

Gravitation

Gravitation

Kepler's 3rd Law

$$|\vec{F}| = G \frac{M_s M_p}{r^2} = \frac{M_p v^2}{r}$$

$$\frac{GM_s}{r^2} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}, \text{ where } v = \frac{2\pi r}{T}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3$$

Fluids

Continuity Equation

For incompressible flow ($\rho_1 = \rho_2$):

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

consequences: small tube, high speed
large tube, low speed

$$Av = \left[\frac{\text{volume}}{\text{time}} \right] = \text{volume flux} = \text{flow rate}$$

“what goes in must come out”

Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

OR

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

OR when fluid is at rest, $v_1 = v_2 = 0$

$$(P_1 - P_2) = \rho g y_2 - \rho g y_1 = \rho g (y_2 - y_1) = \rho g h$$

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

$$P_1 = P_2 + \rho g (y_2 - y_1)$$

Oscillations

Simple Harmonic Motion

For a horizontal spring:

Proof.

$$F = -kx = ma$$

$$a = -\frac{k}{m}x$$

■

For a vertical spring:

Proof.

$$F = mg - ky = ma \rightarrow -ky = m(a - g) = ma'$$

Where $(a - g) = a'$

$$a' = -\frac{k}{m}y$$

□

■

□

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2x$$

$$\text{Where } \omega^2 = \frac{k}{m}$$

Definition of Terms

$$x(t) = A \cos(\omega t + \delta)$$

$x(t)$: Displacement at time t

A : Amplitude

ω : Angular Frequency

t : Time

δ : Phase Constant

$(\omega t + \delta)$: Phase

The period, T , is the time for the particle to go through one full cycle of its motion.

The phase increases by 2π in a time T

$$\omega t + \delta + 2\pi = \omega(t + T) + \delta$$

$$2\pi = \omega T$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

The freq. is inverse of period $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$$\text{rearranging } \omega = 2\pi f = \frac{2\pi}{T}$$

Pendulum The simple pendulum is another mechanical system that exhibits periodic, oscillatory motion

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \quad \text{which is similar to } \frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega = \sqrt{\frac{g}{L}} \quad \text{and } T = 2\pi \sqrt{\frac{L}{g}}$$

Physical Pendulum A physical pendulum consists of any rigid body suspended from a fixed axis that does not pass through the body's center of mass

$$\frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\theta$$

$$\omega = \sqrt{\frac{mgd}{I}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{I}{mgd}}$$

What About A & δ

$$x(t) = A \cos(\omega t + \delta) \rightarrow x_0 = A \cos \delta$$

$$v(t) = -A\omega \sin(\omega t + \delta) \rightarrow v_0 = -A\omega \sin \delta$$

Dividing these EQ's

$$\frac{v_0}{x_0} = -\omega \tan \delta$$

$$\tan \delta = -\frac{v_0}{\omega x_0}$$

Taking sum of squares:

$$x_0^2 + \left(\frac{v_0}{\omega}\right)^2 = A^2 (\cos \delta)^2 = A^2 [(\sin \delta)^2 + (\cos \delta)^2]$$

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$