Exercise Set 3 (Total Points: 100)

The limit definition of the derivate (5 points each)

Problem (1)**.** Let $f(x) = x^2 + x + 2$. Using the limit definition of the derivative compute $f'(x)$. Show all the intermediate steps, this question will be graded for work.

Problem (2)**.** Fill in the two missing steps of the following proof that $(e^x)' = e^x$

Proof. At the end of this proof will appeal to the limit $\lim\limits_{h \to 0} \dfrac{e^h - 1}{h} = 0$. I will not prove this result. By the limit definition of the derivative we have

$$\begin{aligned} (e^x)' &= \lim_{h \to 0} \underline{\hspace{6cm}} \\ &= \lim_{h \to 0} \frac{e^x e^h - e^x \cdot 1}{h} \\ &= \lim_{h \to 0} \underline{\hspace{6cm}} \\ &= e^x \end{aligned}$$

$\square$

Tangent lines and linear approximation (5 points each)

For the next problems we will use the same f and g. Suppose we have the following table of values for f, f', g :

x	$f(x)$	$f(x)$	$g(x)$
-1	0	1	$\frac{1}{4}$
0	4	4	$\frac{1}{2}$
1	-1	-1	1
4	0	3	8

3. What is $f'(0)$?

4. Approximate $f(1.1)$ linearly using the derivative.

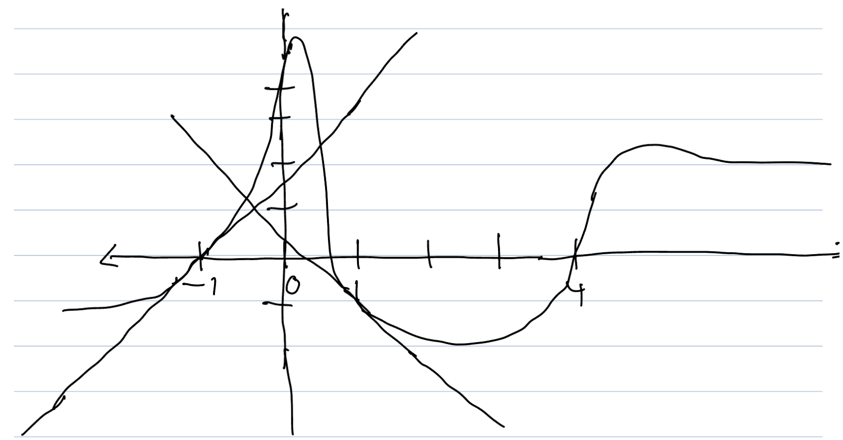
5. Approximate $f(0.2)$ linearly using the derivative.

6. Find the equation of the line that is tangent to $y = f(x)$ at $x = 4$.

7. Which of the following functions could represent $g(x)$:

$$(a) g(x) = x^2 + \frac{1}{2} \quad (b) g(x) = 2^{x-1} \quad (c) g(x) = \sin(\pi x) \quad (d) g(x) = \frac{7}{3}x - \frac{4}{3}$$

8. On the picture below label the following: the graph of $y = f(x)$, the tangent line to the graph at $x = -1$, the tangent line at $x = 1$, and the limit as f goes to infinity.



Computing Derivatives (5 points each)

9. Calculate $(x^5)'$

10. Calculate $(3 + 5x + 6x^2 - 2x^3)'$

11. Calculate $\frac{d}{dx}\left(\sqrt[4]{x} - \dfrac{1}{x^3}\right)$

12. Calculate $\frac{d}{dx}\left(2\sqrt{x} - \dfrac{3}{x} + x\right)$

13. Calculate $\frac{d}{dx}(4 + 3\sin(x) - 2\cos(x))$

14. Calculate $(e^t - ln(t) + t^{1}0)'$

15. Calculate $(\sin(x)e^x)'$

16. Calculate $\frac{d}{dy}(\cos(y^2) + y^3\ln(y))$

17. Calculate $\left((x-2)^6 + \dfrac{3-x}{x+7}\right)'$

18. Calculate $\left(\dfrac{\sin(t)}{t^2}\right)'$

19. Calculate $\left(e^{-\frac{x^2}{2}}\right)'$

20. Calculate $\ln(x\sin(x))'$