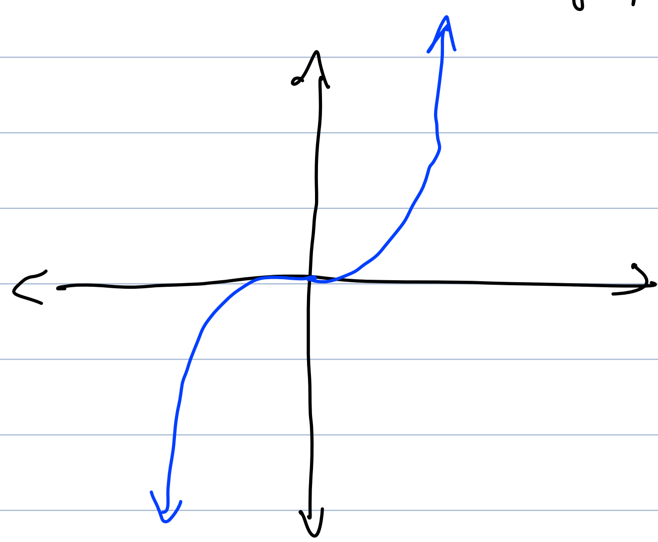
Piecewise Functions

A piecewise function is a function which is made out of different functions pasted together. The most common notation is for example $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0. \end{cases}$

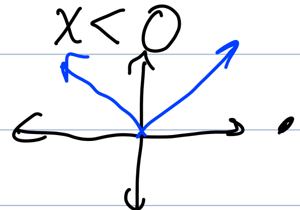
The two conditions, $(x \geq 0,\ x < 0)$ cover the whole number line which gives a definition of the function everywhere. Let's work out a table of values and graph this function:

x	x^2	$-x^2$	$f(x)$
0	0	0	0
1	1	-1	1
2	4	-4	4
-2	4	-4	-4
$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$
$-\frac{1}{3}$	$\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$



An important example is the absolute value function

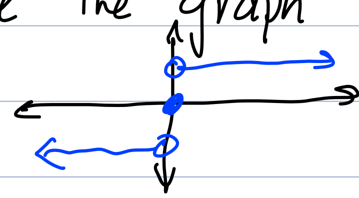
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

which has the graph  .

Another example is the sign function (not to be confused with the sine fuction)

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0. \end{cases}$$

Here the graph is



Limits

A limit is a way of filling in missing or "inconsistent" information on a function.

x^2+2x+4

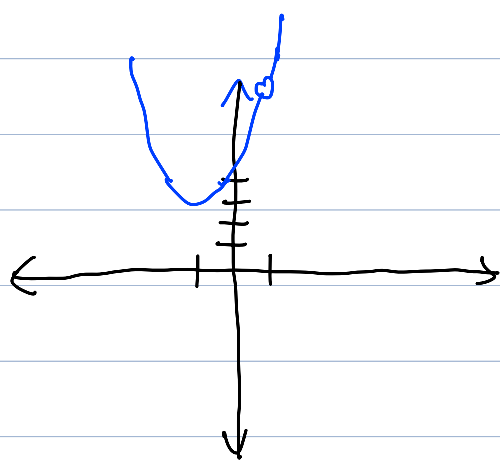
For example consider the function, $f(x) = \frac{x^3-8}{x-2}$. If we plug in $x=2$ we have $f(2) = \frac{2^3-8}{2-2} = \frac{8-8}{2-2} = \frac{0}{0}$ which is undefined. However, near 2 we can find a value

$$f(3) = \frac{3^3-8}{3-2} = 19$$
$$f(2.1) = \frac{9.261-8}{2.1-2} = 12.61$$
$$f(2.01) = \frac{8.120601-8}{2.01-2} = 12.0601$$
$$f(2.001) = 12.006001$$

Following this pattern we define $\lim_{x\to 2} f(x) = 12$.
You should read "$\lim_{x\to 2} f(x)$" as the limit of f of x as x approaches 2. Note that the approximation also works approaching in the other direction:

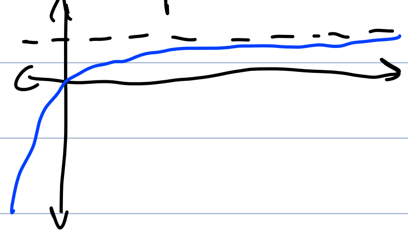
$$f(1.99) = \frac{7.880599-8}{1.99-2} = 11.9401 \approx 12$$

On a graph we draw



The open circle represents that the point (2, 12) is missing from the graph.

Limits can also be used to fill in how a function is going to infinity. For example consider $g(x) = 1-e^{-x}$. The graph of g is



. We say $\lim_{x\to\infty} g(x) = 1$ or in words the limit of g(x) as x goes to infinity is 1.

A common situation where limits to infinity show up is for expressions like $\frac{x^2+x+1}{2x^2-x+1}$. We can think of the fraction as comparing the two expressions, and the limit to infinity as being comparing the "eventual behavior" of the expressions. Here $\lim\limits_{x\to\infty} \frac{x^2+x+1}{2x^2-x+1} = \frac{1}{2}$. The reason for this is that only the fastest growing parts affect the answer so

$$\lim_{x\to\infty} \frac{x^2+x+1}{2x^2-x+1} = \lim_{x\to\infty} \frac{x^2}{2x^2} = \lim_{x\to\infty} \frac{1}{2} = \frac{1}{2}$$