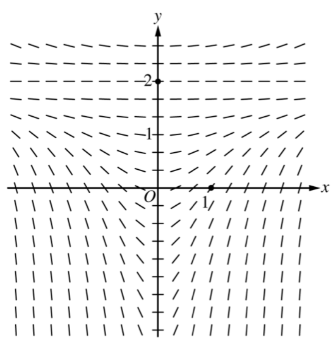
Tomorrow

- Discuss some other things for AB calc questions?
  - slope fields

- Work on Project?

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.

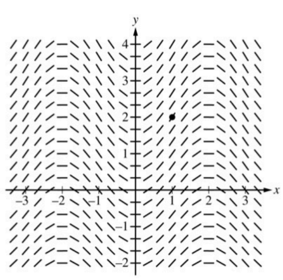


(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

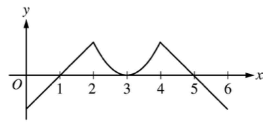
(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

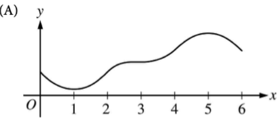
(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

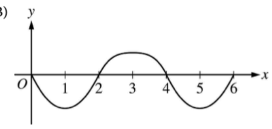
(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

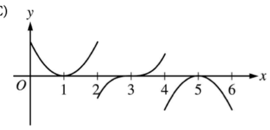


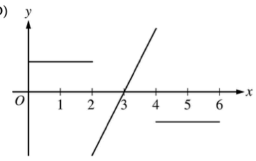
Graph of f'

11. The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f?

(A)


(B)


(C)


(D)


13. The temperature of a room, in degrees Fahrenheit, is modeled by H, a differentiable function of the number of minutes after the thermostat is adjusted. Of the following, which is the best interpretation of $H'(5) = 2$?

(A) The temperature of the room is 2 degrees Fahrenheit, 5 minutes after the thermostat is adjusted.

(B) The temperature of the room increases by 2 degrees Fahrenheit during the first 5 minutes after the thermostat is adjusted.

(C) The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Fahrenheit per minute.

(D) The temperature of the room is increasing at a rate of 2 degrees Fahrenheit per minute, 5 minutes after the thermostat is adjusted.

4. An ice sculpture in the form of a sphere melts in such a way that it maintains its spherical shape. The volume of the sphere is decreasing at a constant rate of 2π cubic meters per hour. At what rate, in square meters per hour, is the surface area of the sphere decreasing at the moment when the radius is 5 meters? (Note: For a sphere of radius r, the surface area is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.)

(A) $\frac{4\pi}{5}$

(B) 40π

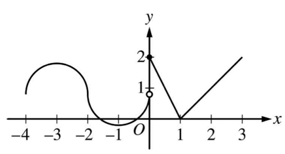
(C) $80\pi^2$

(D) 100π

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2



Graph of f

3. The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

6. Consider the curve given by the equation $6xy = 2 + y^3$.

(a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.

(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

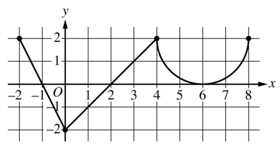
5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim\limits_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

8. Which of the following limits is equal to $\int_3^5 x^4\,dx$?

(A) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{k}{n}\right)^4 \frac{1}{n}$

(B) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{k}{n}\right)^4 \frac{2}{n}$

(C) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{2k}{n}\right)^4 \frac{1}{n}$

(D) $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \left(3 + \frac{2k}{n}\right)^4 \frac{2}{n}$

t	0	2
$f(t)$	4	12

10. Let $y = f(t)$ be a solution to the differential equation $\frac{dy}{dt} = ky$, where k is a constant. Values of f for selected values of t are given in the table above. Which of the following is an expression for $f(t)$?

(A) $4e^{\frac{t}{2}\ln 3}$

(B) $e^{\frac{t}{2}\ln 9} + 3$

(C) $2t^2 + 4$

(D) $4t + 4$

1. $\lim_{x \to \pi} \dfrac{\cos x + \sin(2x) + 1}{x^2 - \pi^2}$ is

(A) $\dfrac{1}{2\pi}$

(B) $\dfrac{1}{\pi}$

(C) 1

(D) nonexistent

$$f(x) = \begin{cases} 2x - 2 & \text{for } x < 3 \\ 2x - 4 & \text{for } x \geq 3 \end{cases}$$

6. Let f be the piecewise-linear function defined above. Which of the following statements are true?

I. $\lim_{h \to 0^-} \dfrac{f(3+h) - f(3)}{h} = 2$

II. $\lim_{h \to 0^+} \dfrac{f(3+h) - f(3)}{h} = 2$

III. $f'(3) = 2$

(A) None

(B) II only

(C) I and II only

(D) I, II, and III