Exercise Set 4 (Total Points: 100)

Derivatives

Problem (1)**.** (10 points) State the following derivate rules:

(a) $\frac{d}{dt}t^m$ where m is a constant.

(b) $\frac{d}{dt}a(t) + b(t)$ where a and b are differentiable functions.

(c) $\frac{d}{dt}ca(t)$ where c is a constant and a is a differentiable function.

(d) $\frac{d}{dt}a(t)b(t)$ where a and b are differentiable functions.

(e) $\frac{d}{dt}\frac{a(t)}{b(t)}$ where a and b are differentiable functions and b is nonzero everywhere.

(f) $\frac{d}{dt}b(a(t))$ where a and b are differentiable functions.

Problem (2)**.** (10 points) State the following standard derivates:

(a) $(1/x)'$

(b) $\sqrt{(}x)'$

(c) $(e^x)'$

(d) $\ln(x)'$

(e) $\sin(x)'$

(f) $\cos(x)'$

(g) $\tan(x)'$

(h) $\cot(x)'$

(i) $\csc(x)'$

(j) $\sec(x)'$

Problem (3)**.** (5 points) Compute the derivative of 5^x. (In the formula sheet there is a similar example)

Problem (4)**.** (10 points) Find the derivative of $\ln(\sec(x) + \tan(x))$. Make sure to fully simplify your answer.

Compute the following derivatives and second derivatives (5 points each)

5. $(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24})'$
6. $(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24})''$
7. $\frac{d}{dx}\frac{\sqrt{x}}{1+x^2}$
8. $\frac{d}{dx}\frac{1}{\sqrt{1-x^2}}$
9. $\frac{d^2x}{dx^2}(x\ln(x) - x)$.

Applications of Derivatives

Problem (10)**.** (15 points) The Acme Electric Car Company is developing an experimental ion powered car. In a test run the distance it travels along a road is $d(t) = 20t^{2.5} + 30t^2$ meters (t has units in seconds).

(a) Find the velocity of the ion car over time. Express your answer in terms of meters per second.

(b) Find the acceleration of the ion car over time. Express your answer in terms of meters per second squared.

Problem (10)**.** (15 points) A simple pendulum has a swing given by $\theta(t) = \theta_0 \cos(\omega t)$. Here t represents time, ω the frequency of the pendulum, θ_0 the initial angular displacement and $\theta(t)$ is the angular displacement over time (it has units as radians).

(a) Find the angular velocity of the pendulum.

(b) Find the angular acceleration of the pendulum.

Problem (12)**.** (10 points) In the past 45 days the value of the Euro against the US dollar can be accurately modelled by

$$\frac{1}{400}\Big(\Big(\frac{x}{12} - 1.9\Big)^3 - 1.1\Big(\frac{x}{12} - 1.9\Big)\Big) + 1.08.$$

Find the inflation rate of the Euro in terms of US dollars.