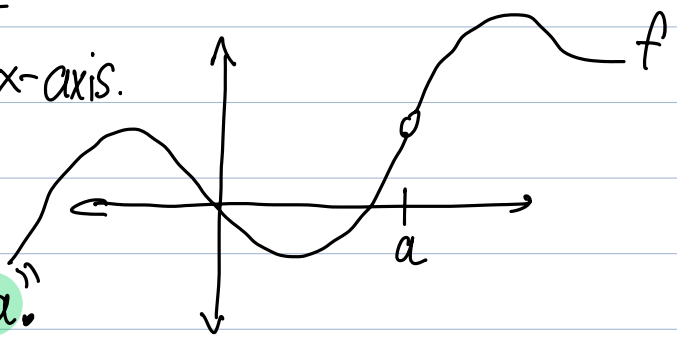
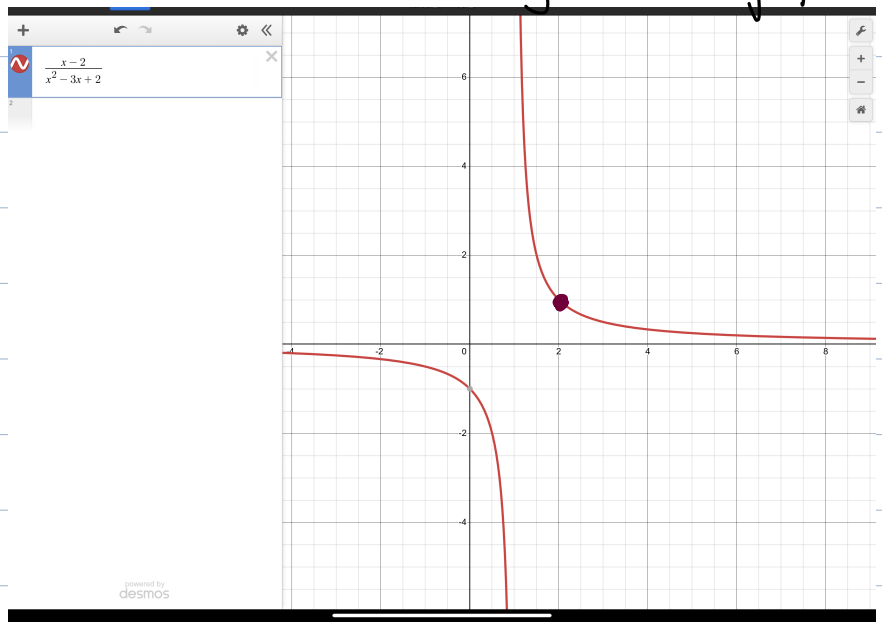
Warm up: Find the following, does not exist or ∞ are fine.

a) $0 \cdot 0 = ?$
b) $\frac{0}{0} = ?$
c) $0 + 1 = ?$
d) $1 \cdot \infty = ?$
e) $0 \cdot \infty = ?$
f) $\infty + 1 = ?$
g) $\infty - \infty = ?$
h) $(-1) \cdot \infty = ?$
i) $1^0 = ?$
j) $0^1 = ?$
k) $0^0 = ?$
l) $1^\infty = ?$
m) $2^\infty = ?$
n) $0^\infty = ?$

Definitions: Given a function f and a point a on the x-axis. there's $\lim_{x \to a} f(x)$ read as the "limit of $f(x)$ as x approaches a."



For example: Take $f(x) = \frac{x-2}{x^2-3x+2}$. $f(2) = \frac{2-2}{2^2-3\cdot 2+2} = \frac{0}{0}$ which is undefined. But looking at the graph



we see that near 2, $f(x)$ is about 1 so we should expect $\lim_{x \to 2} f(x) = 1$. Indeed, $\frac{x-2}{x^2-3x+2} = \frac{x-2}{x-2} \cdot \frac{1}{x-1}$

so as long as x is not 2, $f(x) = \frac{x-2}{x-2} \cdot \frac{1}{x-1} = \frac{1}{x-1}$

which means $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1} = 1.$

In the last part of the example, I substituted 2 into $\frac{1}{x-1}$ to say $\lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1}$. You can only do this when a function is "continuous" at a point. Formally a function f is **continuous at a** if $\lim_{x\to a} f(x) = f(a)$. If f is continuous at every point we say f is **continous.** Continous roughly means the function can be drawn without lifting your pencil.

One sided limits

Limits to infinity:

Example the sign function. • sgn

Projects: • Interpreting $\frac{0}{0}$: $\frac{x}{x}, \frac{2x}{x}, \frac{x^2}{x}, \frac{x}{x^3}, \frac{x}{\sqrt{x}}, \frac{\sqrt{x}}{x}, \frac{\sin(x)}{x}$

- Finding limits of type $\lim_{x\to 0} \frac{f(x)}{g(x)}$ where f and g are polynomials
- Compute $\lim_{x\to 0} \frac{\sin(x)}{x}$. Hint: the unit circle

• The δ-ε definition of the limit:

- check x^2 is continuous at 2 ie. $\lim_{x\to 2} x^2 = 4$

* Using the δ-ε definition to prove limit identities,
  • scaling • addition • multiplying • composing
- Failure of continuity: look up
  jump discontinuity, removable discontinuity, and essential discontinuity.
  Find examples

- Horizontal Asymptotes.
  - classify $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ for f, g polynomials.