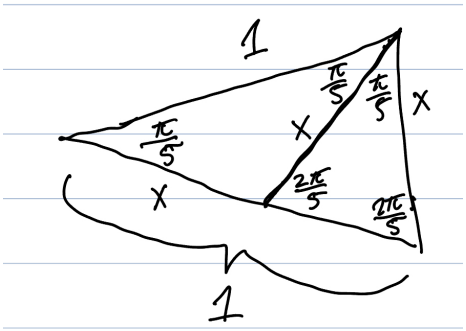
Problem Set 1 (Total Points: 100)

There are 10 problems. Each problem is worth 20 points, so there's up to 100 bonus points available on this assignment.

1. (This problem must be claimed.) Option (a) Learn about the types of discontinuities: removable discontinuities, jump discontinuities, poles, and essential singularities. Prepare a 5 to 10 minute presentation for Friday's class about them. Make sure to discuss the function $f(x) = \sin(1/x)$
   Option (b) Learn about the types of failures of differentiability: corners, cusps, and vertical tangents. Also discuss the examples of $f(x) = x\sin(1/x)$ and the Weierstrass function. Prepare a 5 to 10 minute presentation for Friday's class about this material.
   Option (c) Learn why $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$. Carefully write up a geometric argument on a piece of paper.
   Option (d) Learn about complex numbers. Include an explanation of how to add and multiply complex numbers, the complex conjugate, the absolute value of a complex number, and the argument. Prepare a 5 to 10 minute presentation for Friday's class about this material.

2. Find $\sin\left(\frac{2\pi}{5}\right)$. Here are some hints: use the unit circle and the special triangle with angles $\pi/2, 2\pi/5, \pi/10$. To find the side lengths of this triangle consider the following diagram, apply similar triangles, and use the quadratic formula.



3. Find $\tan(\pi/12)$. Fully simplify your answer.

4. George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first $\frac{1}{2}$ mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last $\frac{1}{2}$ mile in order to arrive just as school begins today?

5. How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

6. Points A and B lie on the graph of $y = \log_2 x$. The midpoint of $\overline{AB}$ is $(6, 2)$. What is the positive difference between the x-coordinates of A and B?

7. Positive real numbers x and y satisfy $y^3 = x^2$ and $(y - x)^2 = 4y^2$. What is $x + y$

8. For how many integers n does the expression

$$\sqrt{\frac{\log(n^2) - (\log n)^2}{\log n - 3}}$$

represent a real number, where log denotes the base 10 logarithm?

9. In the xy-plane, a circle of radius 4 with center on the positive x-axis is tangent to the y-axis at the origin, and a circle with radius 10 with center on the positive y-axis is tangent to the x-axis at the origin. What is the slope of the line passing through the two points at which these circles intersect?

10. Positive real numbers a and b have the property that

$$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$$

and all four terms on the left are positive integers, where log denotes the base 10 logarithm. What is ab?