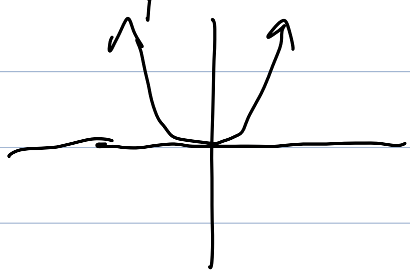
Graphing Functions, Mat 122 July 10^{th}

Say $f(x) = \cdots$ is some function.
The **graph of f** is a picture of all the values of f.
For example $f(x) = x^2$ has this graph.



In order to draw a graph you make a **table of values**

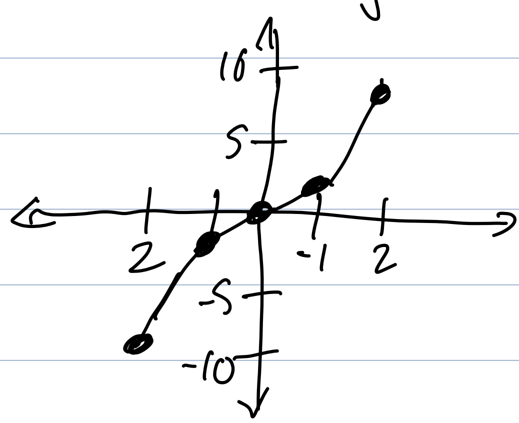
x	$f(x)$
0	$f(0)$
1	$f(1)$
2	$f(2)$
$\vdots$	

picking the inputs $x = 0, 1, 2 \cdots$ is a matter of style.
The you connect the dots

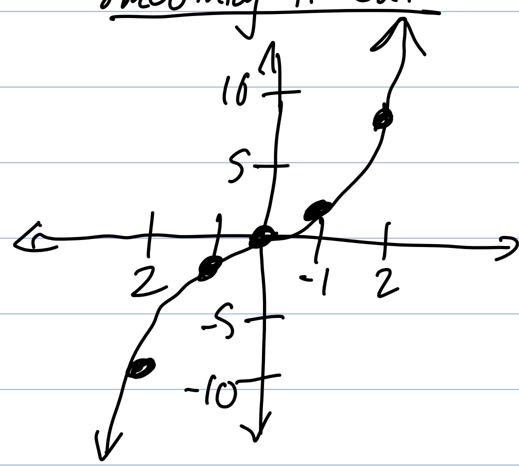
Example: Say $f(x) = x^3$.

x	x^3
0	0
1	1
-1	-1
2	8
-2	-8

Connecting the dots



Smoothing it out

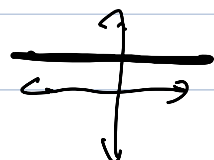


Formally graph(f) is defined as the set of all pairs $(x, f(x))$ where x ranges over the domain of f.
Remember a function is an assignment of elements of the domain to the codomain.
$f(x) = x^2$ has domain and codomain as the real numbers.

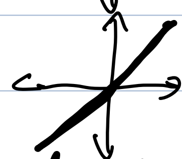
Parent Functions are standard functions which should be able to recognize. The graphs of other functions can be adapted from parent functions hence the name parent functions.

Group 1: Polynomial Curves

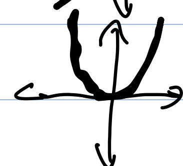
$f(x) = 1$ horizontal line



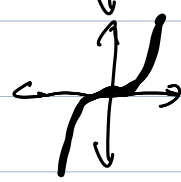
$f(x) = x$ diagonal line



$f(x) = x^2$ parabola

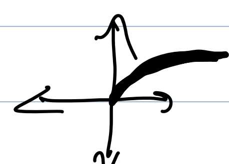


$f(x) = x^3$ cubic curve

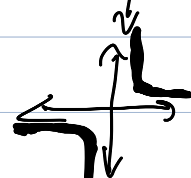


Group 2: Other powers of x

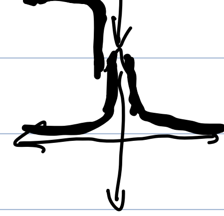
$f(x) = \sqrt{x} = x^{1/2}$ square root curve



$f(x) = \frac{1}{x} = x^{-1}$ hyperbola
one over x

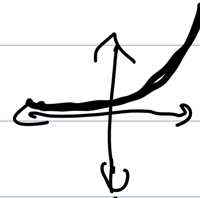


$f(x) = \frac{1}{x^2} = x^{-2}$ one over x^2

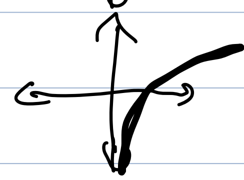


Group 3: Exponentials

$f(x) = e^x$ exponential curve

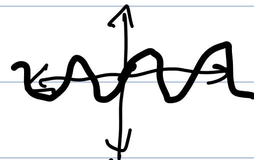


$f(x) = \ln(x)$ logarithmic curve

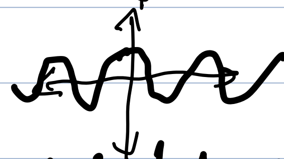


Group 4: Trigonometric Functions

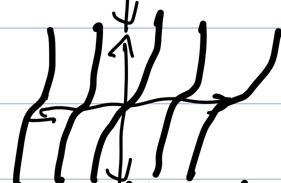
$f(x) = \sin(x)$ Sine curve



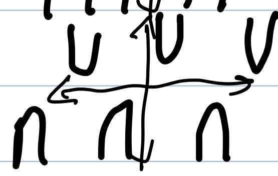
$f(x) = \cos(x)$ Cosine curve



$f(x) = \tan(x)$ tangent curve



$f(x) = \csc(x)$ cosecant curve



Modifying Functions: From parent functions you can get many other functions by modifying them.

Shifting If $f(x)$ is a function then

$f(x-a)$ shifts the graph a units to the right

$f(x+a)$ shifts the graph a units to the left

$f(x)+a$ shifts the graph a units up

$f(x)-a$ shifts the graph a units down

Stretching $f\left(\frac{x}{a}\right)$ stretchs the function in the x-direction by a factor of a.

$f(ax)$ compresses the function in the x-direction by a factor of a.

$af(x)$ stretchs the function in the y-direction by a factor of a.

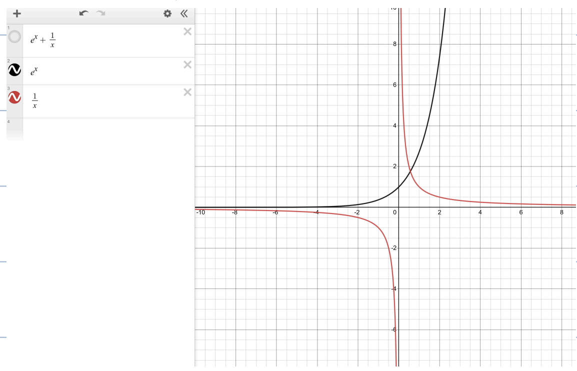
$\frac{1}{a}f(x)$ compresses the function in the y-direction by a factor of a.

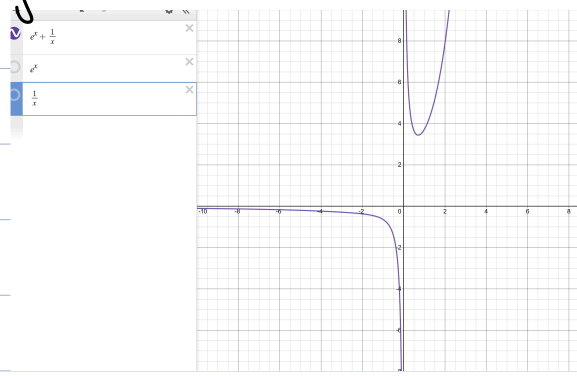
$f(-x)$ reverses the function in the x-direction

$-f(x)$ reverses the function in the y-direction

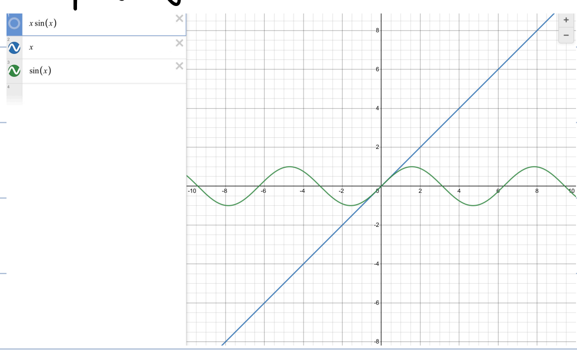
There are 3 general operations on functions which are generally pretty hard to visulize.

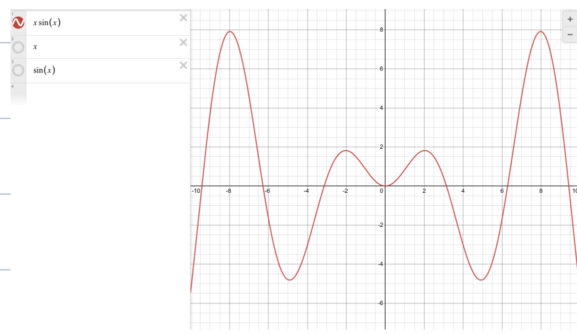
Adding functions $f(x)+g(x)$



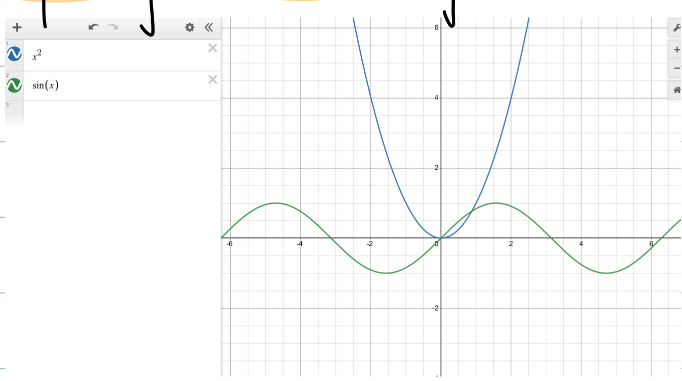


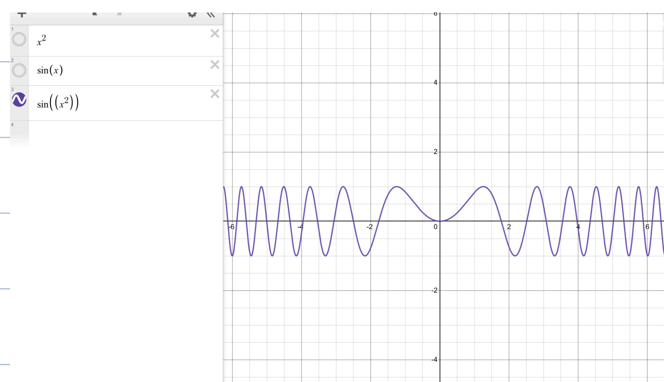
Multiplying functions $f(x)\,g(x)$





Composing functions $g(f(x))$





Example $\cos(x) = \sin\left(x+\frac{\pi}{2}\right) = \sin\left(x-\frac{3\pi}{2}\right)$

$$\left.\begin{aligned} \sin(x) &= \sin(x+2\pi) \\ \tan(x) &= \tan(x+\pi) \end{aligned}\right\} \text{periodicity}$$

Theorem There is only "one" parabola.
Say $f(x) = ax^2+bx+c$. Then the graph of f is a stretched an shifted version of $f(x) = x^2$.

proof:

$$\begin{aligned} ax^2+bx+c &= a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right) \\ &= a\left(x^2+\frac{2b}{2a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}\right) \\ &= a\left(\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}\right) \\ &= a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}\right) \end{aligned}$$

So the graph of ax^2+bx+c is x^2
shifted by $\frac{b}{2a}$ to the left,
shifted by $\frac{b^2-4ac}{4a^2}$ down, and
stretched by along the y-axis.