• Find $(\sqrt{x})'$ and $(x^3-3x)'$ using the limit definition of the derivative.

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$
$$= \lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$
$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

$$\lim_{h\to 0}\frac{((x+h)^3-3(x+h))-(x^3-3x)}{h} = \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-3h}{h}$$
$$= 3x^2-3$$

• Find the following limits:

$$\lim_{x\to\infty} 2^{-x}+1 = 1$$
$$\lim_{x\to 0}\frac{2^x-1}{x} = \lim_{x\to 0}\frac{(\ln 2)\,2^x}{1} = \ln 2$$
$$\lim_{x\to\infty}\frac{2^x}{x} = \infty$$
$$\lim_{x\to 0}\frac{\sin(x)}{x}+\frac{x}{\sin(x)} = 1+1 = 2$$
$$\lim_{x\to 1}\frac{x}{x^2+1} = \frac{1}{2}$$
$$\lim_{x\to\infty}\frac{x}{x^2+1} = 0$$
$$\lim_{x\to -\infty}\frac{3x^2}{4x^2+2} = 3/4$$
$$\lim_{x\to\infty}\frac{\sqrt{x}}{x+\sin(x)} = 0$$

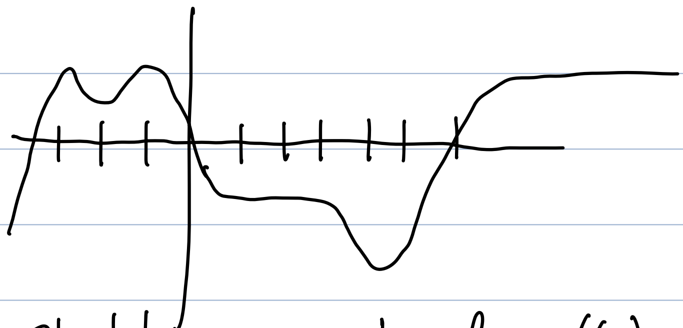
Consider the function
$$f(x) = \begin{cases} -x-2 & x\le -1 \\ \operatorname{sgn}(x) & -1<x<1 \\ 0 & x=1 \\ (x-1)^3+1 & x>1 \end{cases}$$

Sketch a graph of $y=f(x)$, label the two points

of discontinuity stating if they are poles, holes, jumps, or essential discontinuities, label the one point of failure of differentiability stating if it's a corner, cusp, or vertical tangent.
Sketch a graph of $y=f'(x)$.
Find the equation of the tangent line to f at $x=2$.
Find the following limits $\lim_{x\to 1} f(x)$, $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0^-} f(x)$.

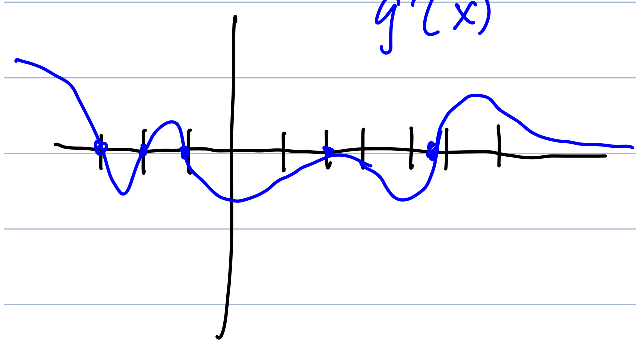
(did during office hours)

Suppose the graph of $y=g(x)$ is

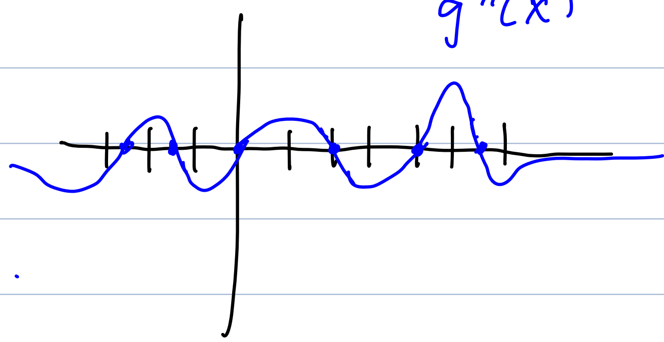


Sketch a graph of $g'(x)$ and $g''(x)$.

$g'(x)$



$g''(x)$



Find $\frac{dy}{dx}$ in the following equations

$\tan(xy)=x$ $\qquad \frac{dy}{dx} = \frac{1-y\sec^2(xy)}{x\sec^2(xy)} = \frac{\cos^2(xy)-y}{x}$

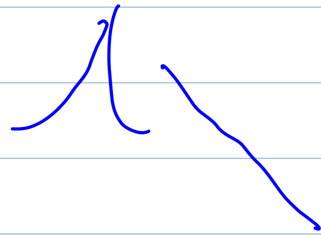
$\frac{x}{y} - \frac{\cos(x)}{x} = y$ $\qquad \frac{dy}{dx} = \left(\frac{\sin(x)x+\cos(x)}{x^2} + \frac{1}{y}\right) \Big/ \left(1+\frac{x}{y^2}\right)$

$$= \frac{\sin(x)xy^2 + \cos(x)x^2y^2 + x^2y}{x^2y^2 + x^3}$$

$x^2+y^3=1$ $\qquad \frac{dy}{dx} = -\frac{2x}{3y^2}$

$1+x = 1+y+xy \qquad \frac{dy}{dx} = \frac{1-y}{1+x}$

Suppose $f'(x) = \begin{cases} \frac{1}{x^2} & x \le 3 \\ 5-x & x>3 \end{cases}$



Find $\lim_{x\to 3^-} f'(x)$, $\lim_{x\to 0} f'(x)$.
Where is $f(x)$ increasing?
Where is $f(x)$ negatively curved?
Find the x value of the local extremum of $f(x)$, is it a maximum or minimum?

$\lim_{x\to 3^-} f'(x) = \frac{1}{9} \qquad \lim_{x\to 0} f'(x) = \infty$
$f(x)$ is increasing when $x<5$ except $x=0$
$f(x)$ is negatively curved when $x>0$ except $x=3$
$x=5$ is a local extremum, it's a maximum

Find the following derivatives and higher order derivatives

$(1-x^{-1}+x^{-2}-x^{-3}+x^{-4})' \qquad \frac{1}{x^2}-\frac{2}{x^3}+\frac{3}{x^4}-\frac{4}{x^5}$

$(x^5)'''' \qquad 120x$

$\sec(t^2)' \qquad 2t\sec(t^2)\tan(t^2)$

$\sqrt{\cot(x)-\csc(x)}\,' \qquad \frac{-\csc^2(x)-\cot(x)\csc(x)}{2\sqrt{\cot(x)-\csc(x)}}$

$\frac{d}{dx}\left(cx-\frac{c}{x}\right)$ (c is a constant) $\qquad c+\frac{c}{x^2}$

$\frac{d^2}{(dx)^2}x2^x \qquad 2\ln(2)2^x + x\ln(2)^2 2^x$

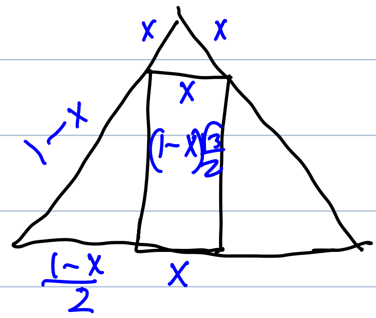
$\frac{d}{dx}\left(\frac{\sin(x)}{1+x}\right) \qquad \frac{\cos x(1+x)-\sin(x)}{(1+x)^2}$

$\frac{d^2}{(dx)^2}(e^x+1)^{1001} \qquad 1{,}001{,}000\, e^{2x}(e^x+1)^{999}$

Suppose a ball is falling, with height equal to
$h(t) = 25.2 - 2.1t - 4.9t^2$ meters, t is in seconds
Find the velocity and the acceleration as a function of time.

$v(t) = -2.1 - 9.8t$ m/s
$a(t) = -9.8$ m/s^2

Inscribe a rectangle in an equilateral triangle of length 1.
What is the maximum area?



$\text{Area} = x(1-x)\frac{\sqrt{3}}{2}$ $\qquad \frac{d(\text{Area})}{dx} = \frac{\sqrt{3}}{2}(1-2x) = 0$
So $x = \frac{1}{2}$ is a potential extreme value.
$\text{Area}\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{8}}$.

Suppose f is a differentiable function such that $f(1)=1$ and $f'(1)=2$.
Approximate $f(0.9)$ and $f(1.2)$
Let $g(x) = f(x)^3$. Find $g'(1)$.
Let $h(x) = f(f(x))$. Find $h'(1)$.

$f(0.9) \approx f(1) - 0.1 f'(1) = 0.8$
$f(1.2) \approx f(1) + 0.2 f'(1) = 1.4$
$g'(1) = 3f(1)^2 f'(1) = 6$
$h'(1) = f'(f(1)) f'(1) = 4$

Find the absolute maximum and absolute minimum of
$f(x) = x^4 - 2x^2 + 3$ on $[-2, 3]$
$f'(x) = 4x^3 - 4x = 4x(x-1)(x+1)$
$f(-2) = 11$, $f(-1) = 2$, $f(0) = 3$, $f(1) = 2$, $f(3) = 66$

Absolute min $= 2$

Absolute max $= 66$