Problem Set 6 (Total Points: 205), Due July 27th

Work is required on most of these problems to receive full credit.

Review Questions

Problem (1). (25 points) The force of gravity on Earth is $F = -mg$ where m is the mass of a constant and g is the gravitational constant. Using Newton's 2nd law we can find that the acceleration an object has due to gravity is $-g$. The gravitational constant g is about 9.8 meters per second squared but it varies across the earth depending on the geology of the ground below you. For example on top of a mountain gravity is slightly weaker.

(a) Suppose you throw a ball in the air in Mexico City and find it's height over time as $h_{MC}(t) = 2.12+0.30t-4.89t^2$. Find the velocity of the ball over time and the acceleration over time. Use this to find the gravitational constant in Mexico City g_{MC}.

(b) Now suppose you throw a ball in the air in Helsinki, Finland and find it's height over time as $h_H(t) = 1.23 - 0.53t - 4.91t^2$. Find the gravitation constant in Helsinki g_H.

(c) How much stronger is gravity in Helsinki than in Mexico City. Express your answer as a percent.

Problem (2). (10 points) Write out the first 5 terms of the Maclaurin series for $\tan^{-1}(x) = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1}$.

Problem (3). (20 points) Find the Maclaurin series for $e^x - 1 - x$. Express your answer with sigma notation.

Problem (4). (20 points) Find the Maclaurin series for $\frac{1}{1-x^2}$ Express your answer with sigma notation.

Taylor Series

Problem (5). (15 points, 10 point bonus) Find the first 3 terms of the Taylor series for x^3 centered at $x = 1$. Bonus: Find the 4th term (which is the last term). Then expand and add up all the terms of the Taylor series and show you get x^3 again.

Problem (6). (15 points) Look up in my notes the Maclaurin series for $\log(x + 1)$. Say the series is $\sum_{k=1}^{\infty} a(k)$ (the terms are $a(k)$). Using Desmos sketch a graph of $\log(x + 1)$ and the partial sums

$$\sum_{k=1}^{1} a(k), \sum_{k=1}^{2} a(k), \sum_{k=1}^{3} a(k), \sum_{k=1}^{4} a(k), \sum_{k=1}^{5} a(k).$$

Problem (7)**.** (25 points) Find the Taylor series of $\sin(x)$ centered at $x = \pi$.

Problem (8)**.** (25 points) Find the Taylor series of e^{2x-4} centered at $x = 2$.

Riemann Sums

Problem (9)**.** (10 points) The following sum is the right Riemann sum with 8 boxes to estimate the area under the $f(x) = x^2$ from 0 to 4

$$\sum_{n=1}^{8} \frac{1}{2}\left(\frac{n}{2}\right)^2.$$

Evaluate this sum.

Problem (10)**.** (20 points) Estimate the area under the curve $f(x) = x^2 - 10$ from $x = 3$ to $x = 11$ using right Riemann sums with 4 boxes.

Problem (11)**.** (20 points) Estimate the area under the curve $f(x) = sin(x) + 2$ from $x = -2\pi$ to $x = 0$ using left Riemann sums with 8 boxes. Your final answer will contain $\sqrt{2}$.

Problem (12)**.** (Bonus, 30 points) This problem is about find the exact area under a curve. Consider the function $f(x) = 1 + 2x$.

(a) Find the area under the curve from $x = 0$ to $x = 3$ using the area of a trapezoid. For your reference the area formula is $h\left(\frac{b_1+b_2}{2}\right)$ where h is the height and b_1 and b_2 are the two bases.

(b) Find the area under the curve from $x = 0$ to $x = 3$ by find a formula for the right Riemann sum with n boxes. Then take the limit as n goes to infinity. To evaluate this limit you will need to use the following sum.

$$\sum_{k=1}^{n} k = \frac{k(k+1)}{2}$$