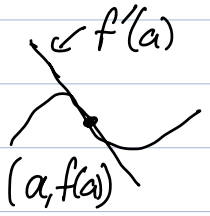
Definition

$$f'(x) = \lim_{y \to x} \frac{f(y) - f(x)}{y - x}$$
$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \frac{d}{dx} f(x)$$

Interpretations

- slope of tangent line



- linear approximation
  $f(x) \approx f(a) + f'(a)(x-a)$

- instantaneous rate of change / variability

Rules

sum rule	$(f(x) + g(x))' = f'(x) + g'(x)$
constant multiple rule	$(c\, f(x))' = c\, f'(x)$
power rule	$(x^n)' = n x^{n-1}$
	$(x^\alpha)' = \alpha x^{\alpha-1}$
product rule	$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
quotient rule	$(f(x)/g(x))' = (f'(x)g(x) - f(x)g'(x))/g(x)^2$
chain rule	$g(f(x))' = g'(f(x))\, f'(x)$
inverse function rule	$(f^{-1}(x))' = 1/f'(f^{-1}(x))$

Standard Derivatives

$(c)' = 0$ $\quad x' = 1$ $\quad (1/x)' = -1/x^2$

$\sin(x)' = \cos(x)$ $\quad \tan(x)' = \sec^2(x)$ $\quad \sec(x)' = \tan(x)\sec(x)$

$\cos(x)' = -\sin(x)$ $\quad \cot(x)' = -\csc^2(x)$ $\quad \csc(x)' = -\cot(x)\csc(x)$

$(e^x)' = e^x$ $\quad (a^x)' = \ln(a)\, a^x$ $\quad \ln(x)' = 1/x$

$(\sqrt{x})' = 1/2\sqrt{x}$ $\quad |x|' = \operatorname{sgn}(x)$

$\sin^{-1}(x)' = \frac{1}{\sqrt{1-x^2}}$ $\quad \cos^{-1}(x)' = -\frac{1}{\sqrt{1-x^2}}$ $\quad \tan^{-1}(x)' = \frac{1}{1+x^2}$