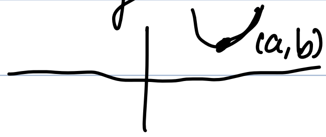
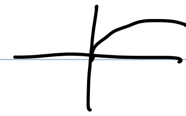
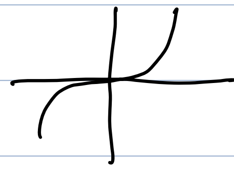
# Standard Graphs

Lines: $y = mx + b$, $x = a$, $y - y_1 = m(x - x_1)$

Parabolas: $y = k(x-a)^2 + b$, $y = \sqrt{x}$

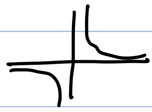


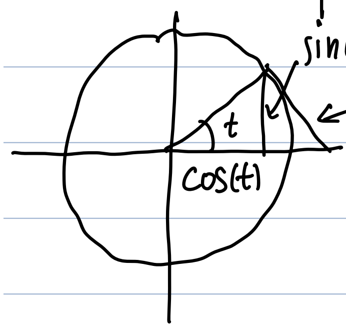


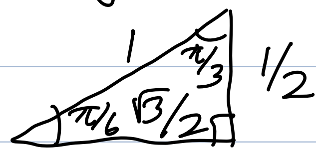
Cubics: $y = x^3$, etc

Hyperbolas: $y = 1/x$



Circles & Ellipses: $x^2 + y^2 = r^2$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

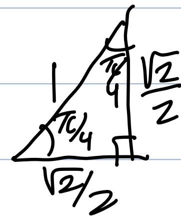



$\sin^2(t) + \cos^2(t) = 1$

$\sin^2(t) = \frac{1 - \cos(2t)}{2}$

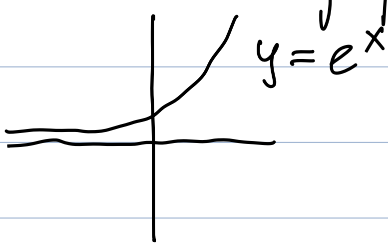
$\cos(a+b) = \cos a \cos b - \sin a \sin b$

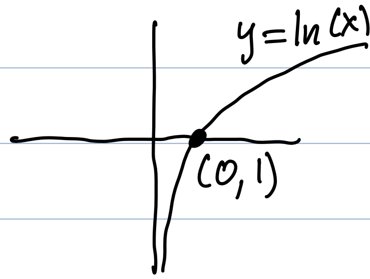


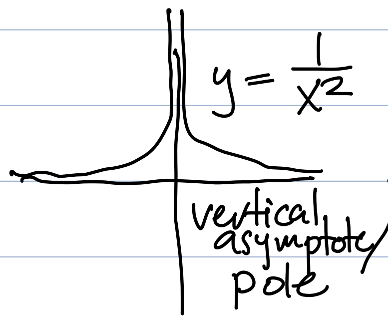

$\tan(t) = \frac{\sin(t)}{\cos(t)}$

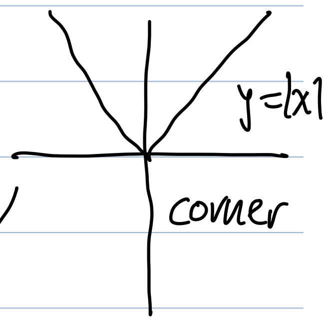
$\sec(t) = \frac{1}{\cos(t)}$

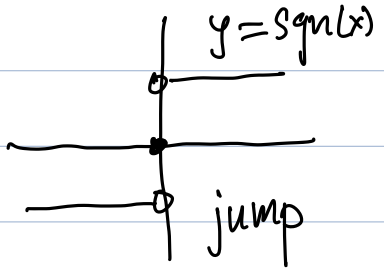
## Other useful graphs

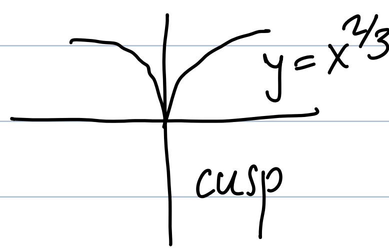


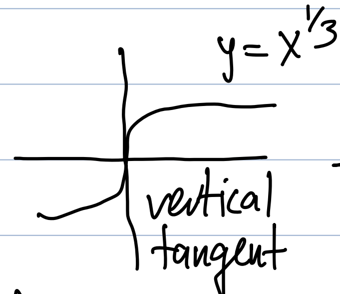



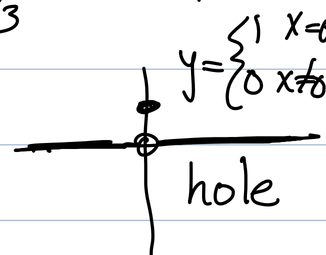



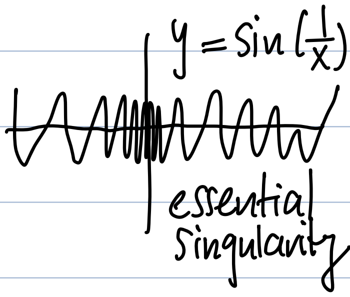



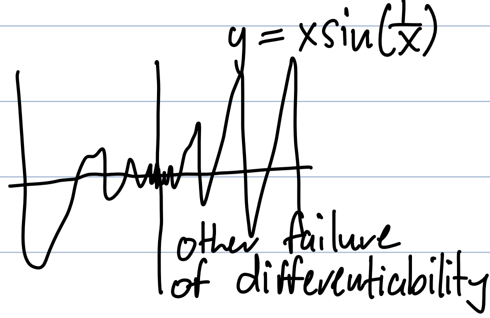



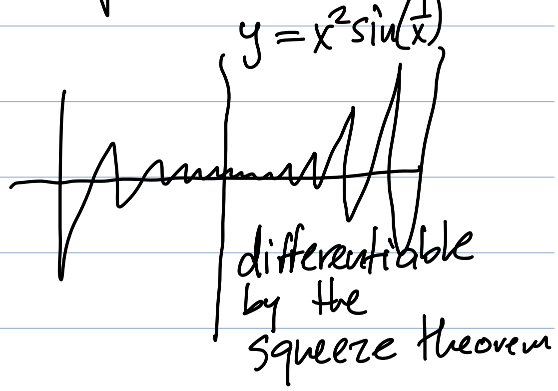



## Limits

Squeeze Theorem: If $f(x) \le g(x) \le h(x)$ for all $x$ near $a$ and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$ then $\lim_{x\to a} g(x) = L$

$\lim_{x\to\infty} e^{-x} = 0$, $\lim_{x\to\infty} \frac{1}{x} = 0$,

Limit from the right: $\lim_{x\to a^+}$, Limit from the left: $\lim_{x\to a^-}$

L'Hopitals Rule: If $f(a)=g(a)=0$ or $\infty$ $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(a)}{g'(a)}$

## Derivatives

$\frac{d}{dx} f = \frac{df}{dx} = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$(f+g)' = f' + g'$ $\qquad$ $(kf)' = kf'$

The power rule:

$(x^n)' = nx^{n-1}$ $\qquad$ (consequences: $1'=0$, $x'=1$, $\sqrt{x}' = \frac{1}{2\sqrt{x}}$, $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$)

The product rule: $(fg)' = f'g + fg'$

The quotient rule: $(f/g)' = \frac{f'g - fg'}{g^2}$

The chain rule: $f(g(x))' = f'(g(x))\, g'(x)$

Standard Derivatives: $(e^x)' = e^x$, $\ln(x)' = 1/x$

$\sin(x)' = \cos(x)$ $\qquad$ $\tan(x)' = \sec^2(x)$ $\qquad$ $\sec(x)' = \tan(x)\sec(x)$

$\cos(x)' = -\sin(x)$ $\qquad$ $\cot(x)' = -\csc^2(x)$ $\qquad$ $-\csc(x)' = -\cot(x)\csc(x)$

$\arctan(x)' = \frac{1}{1+x^2}$ $\qquad$ $\arcsin(x)' = \frac{1}{\sqrt{1-x^2}}$

Linear approximation: $f(x) \approx f(a) + (x-a) f'(a)$

$f$ is increasing $\Longleftrightarrow$ $f' > 0$

$f$ is positively curved (smiling) $\Longleftrightarrow$ $f'' > 0$

Extreme value theorem: If $(x, f(x))$ is a local minimum of maximum inside the domain of $f$ then $f'(x) = 0$

Mean value theorem: There is a point $c$ in $[a,b]$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Know implicit differentiation: $\frac{d}{dx} = \frac{dy}{dx} \frac{d}{dy}$

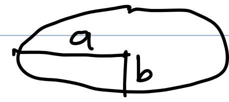
# Integrals

$\int_a^b f(x)\,dx$ = Signed area under the curve $y=f(x)$

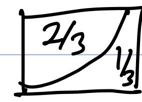
Triangle Area $= \frac{b \cdot h}{2}$

Circle Area $= \pi r^2$

Ellipse Area $= \pi ab$




Parabola Area $= \frac{2}{3}$ (rectangle's area)




$$\int_a^b f\,dx + \int_a^b g\,dx = \int_a^b f+g\,dx, \quad \int_a^b kf\,dx = k\int_a^b f\,dx$$

$$\int_a^b f\,dx + \int_b^c f\,dx = \int_a^c f\,dx, \quad \int_a^b f\,dx = -\int_b^a f\,dx$$

The fundamental theorem of calculus: If $F'(x) = f(x)$,

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Also $\frac{d}{dx}\int_0^x f(t)\,dt = f(t)$

The inverse power law: $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \quad (n \neq -1)$

Integration by parts: $\int u\,dv = uv - \int v\,du$

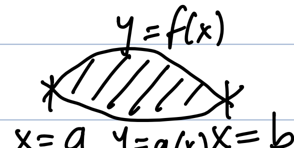
(if $u(x)$ is given $du = u'dx = \frac{du}{dx}dx$)

u-Substitution: $\int g(u(x))\,u'(x)\,dx = \int g(u)\,du$

$$\int_a^b g(u(x))\,u'(x)\,dx = \int_{u(a)}^{u(b)} g(u)\,du$$

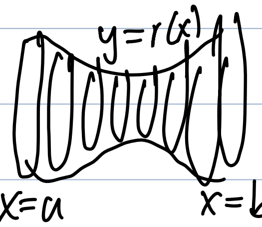
## Area and Volume:

$$\int_a^b f(x) - g(x)\,dx$$



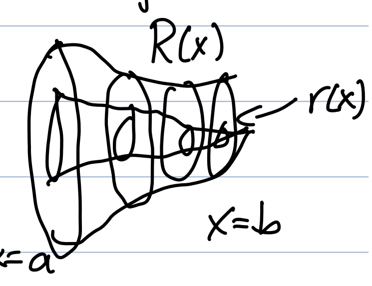

Disk Method:

$$\pi\int_a^b r(x)^2\,dx$$




Washer Method:

$$\pi\int_a^b R(x)^2 - r(x)^2\,dx$$




## Differential Equations:

Separable: $y' = f(x)g(y) \Rightarrow \int \frac{1}{g(y)}\,dy = \int f(x)\,dx$