Differential Equations

$y' = y$

$y' = y$ with $y(0) = 1$

$y' = 3y$, with $y(1) = 2$

$y' = y^2$

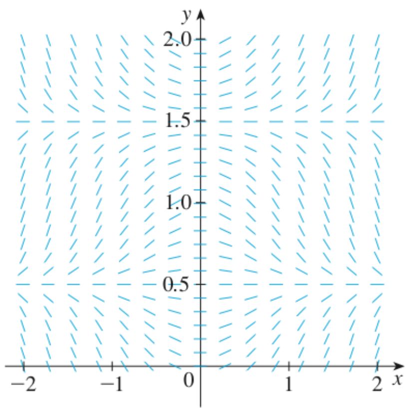
1. A direction field for the differential equation $y' = x \cos \pi y$ is shown.

(a) Sketch the graphs of the solutions that satisfy the given initial conditions.

(i) $y(0) = 0$ (ii) $y(0) = 0.5$

(iii) $y(0) = 1$ (iv) $y(0) = 1.6$

(b) Find all the equilibrium solutions.



3–6 Match the differential equation with its direction field (labeled I–IV). Give reasons for your answer.

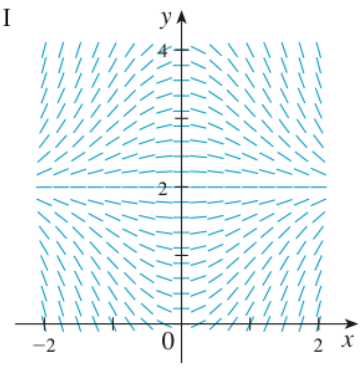
3. $y' = 2 - y$

4. $y' = x(2 - y)$

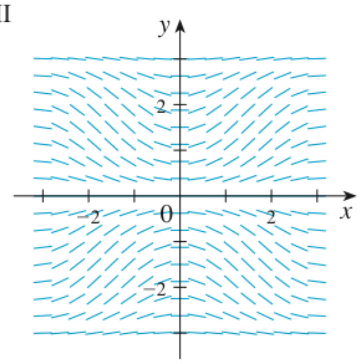
5. $y' = x + y - 1$

6. $y' = \sin x \sin y$

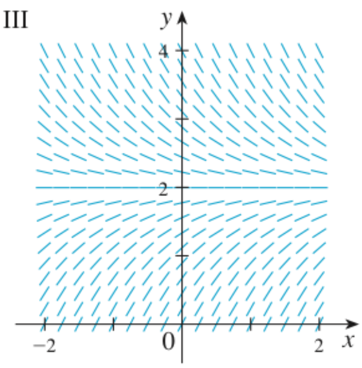
I



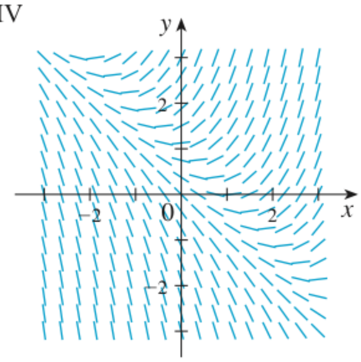
II



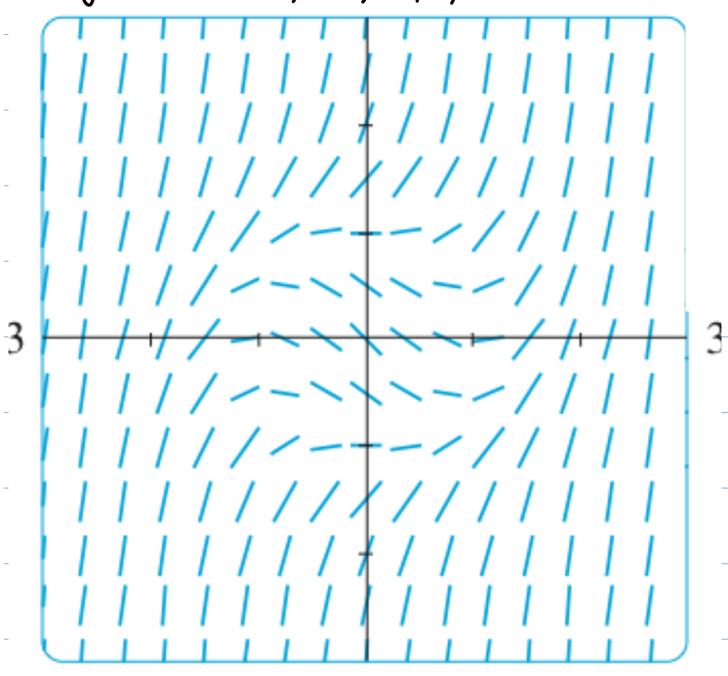
III



IV



$y(0) = -2, -1, 0, 1, 2$



1–10 Solve the differential equation.

1. $\dfrac{dy}{dx} = xy^2$

2. $\dfrac{dy}{dx} = xe^{-y}$

3. $(x^2 + 1)y' = xy$

4. $(y^2 + xy^2)y' = 1$

5. $(y + \sin y)y' = x + x^3$

6. $\dfrac{du}{dr} = \dfrac{1 + \sqrt{r}}{1 + \sqrt{u}}$

7. $\dfrac{dy}{dt} = \dfrac{te^t}{y\sqrt{1 + y^2}}$

8. $\dfrac{dy}{d\theta} = \dfrac{e^y \sin^2\theta}{y \sec \theta}$

9. $\dfrac{du}{dt} = 2 + 2u + t + tu$

10. $\dfrac{dz}{dt} + e^{t+z} = 0$

11–18 Find the solution of the differential equation that satisfies the given initial condition.

11. $\dfrac{dy}{dx} = \dfrac{x}{y}, \quad y(0) = -3$

12. $\dfrac{dy}{dx} = \dfrac{\ln x}{xy}, \quad y(1) = 2$

13. $\dfrac{du}{dt} = \dfrac{2t + \sec^2 t}{2u}, \quad u(0) = -5$

14. $y' = \dfrac{xy \sin x}{y + 1}, \quad y(0) = 1$

15. $x \ln x = y\left(1 + \sqrt{3 + y^2}\right)y', \quad y(1) = 1$

16. $\dfrac{dP}{dt} = \sqrt{Pt}, \quad P(1) = 2$

17. $y' \tan x = a + y, \; y(\pi/3) = a, \quad 0 < x < \pi/2$

18. $\dfrac{dL}{dt} = kL^2 \ln t, \; L(1) = -1$

Find the function f such that $f'(x) = f(x)(1 - f(x))$ and $f(0) = \frac{1}{2}$.

$y' = x + y$

$xy' = y + xe^{y/x}$

. $y(x) = 2 + \int_2^x [t - ty(t)]\, dt$

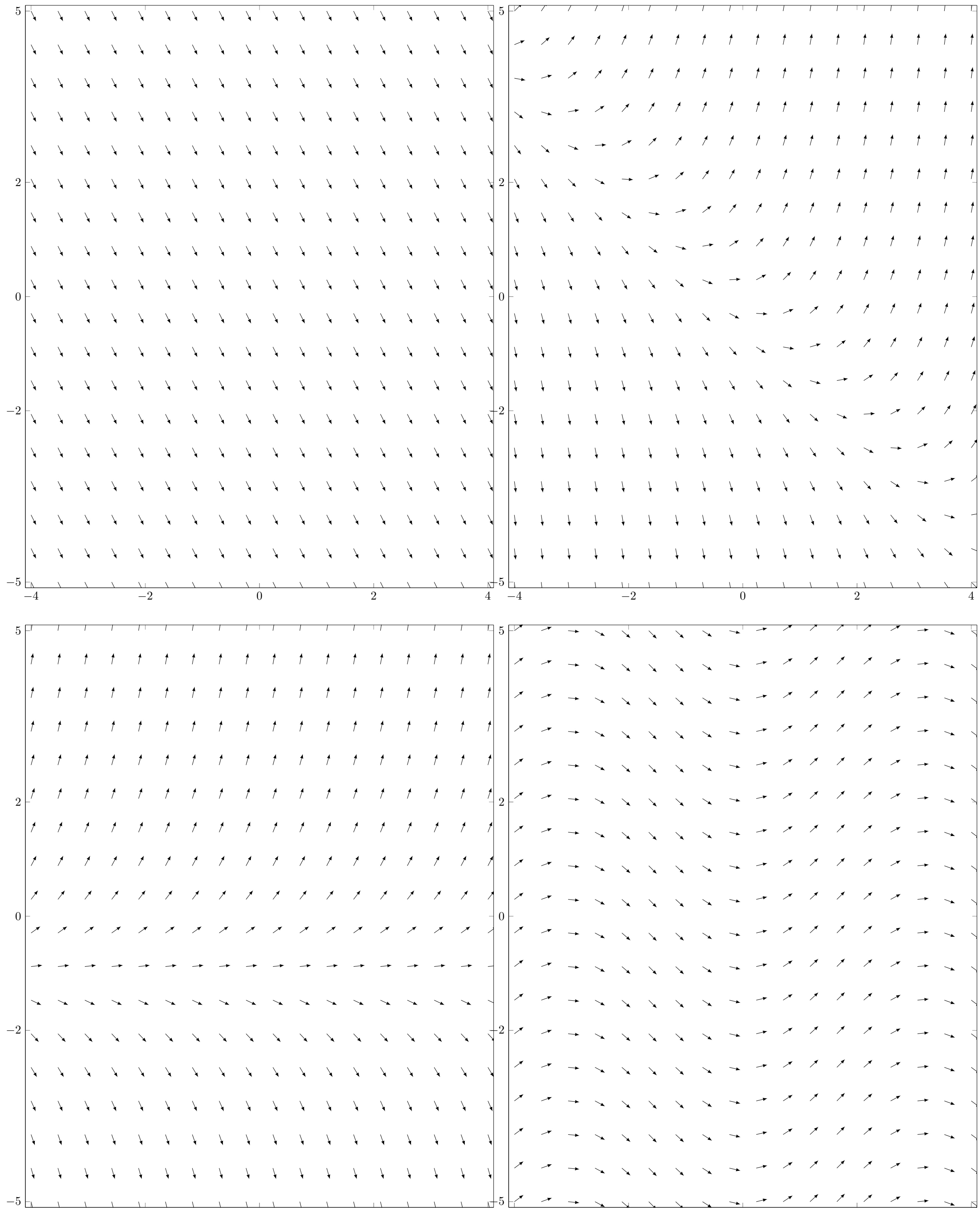
. $y(x) = 2 + \int_1^x \dfrac{dt}{ty(t)}, \quad x > 0$

Find a function f such that $f(3) = 2$ and

$$(t^2 + 1)f'(t) + [f(t)]^2 + 1 = 0 \qquad t \neq 1$$

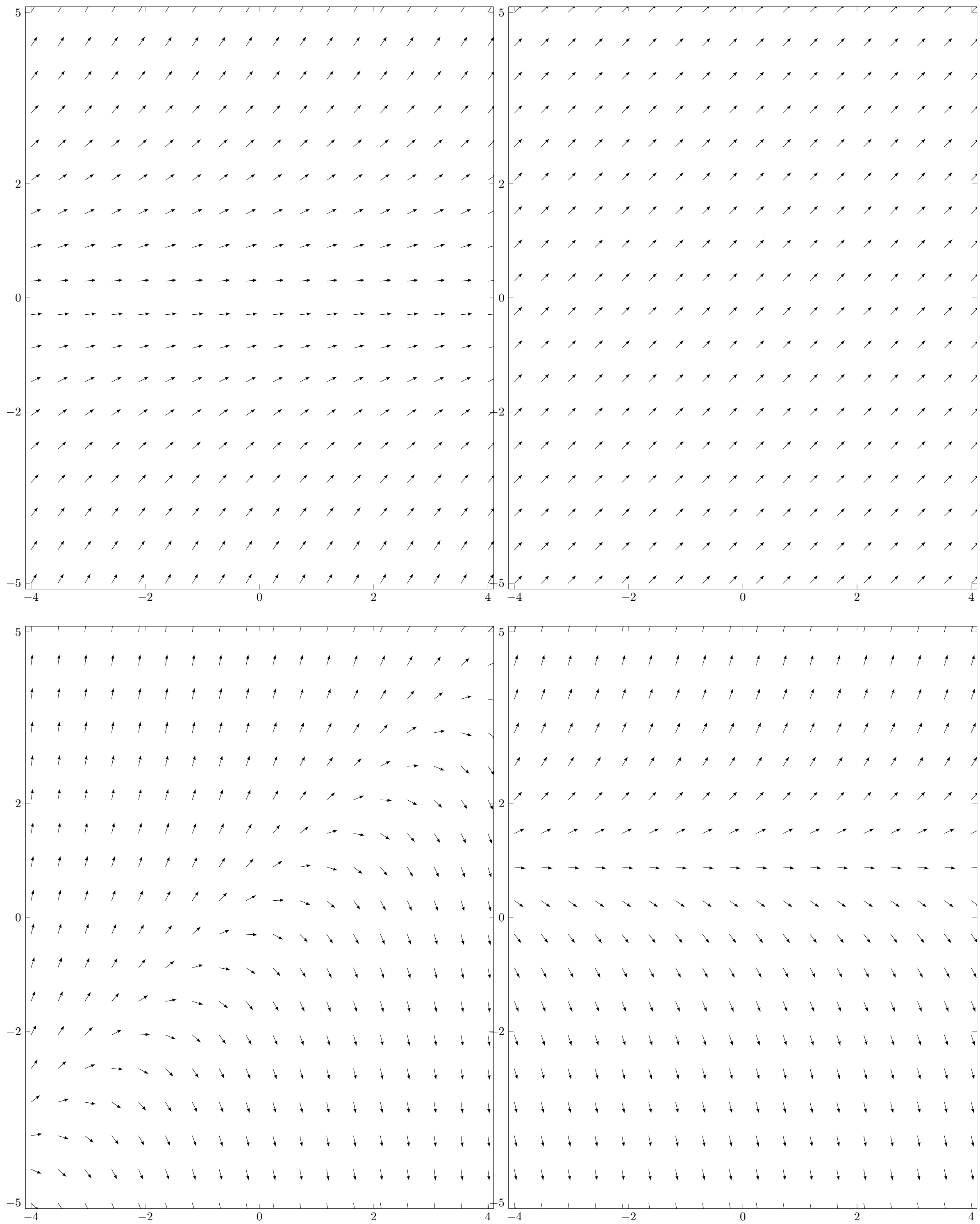
b) Match differential equations and direction fields:

$$y' = 3, \quad y' = -2, \quad y' = 1 + y \quad y' = x + y, \quad y' = \sin x$$



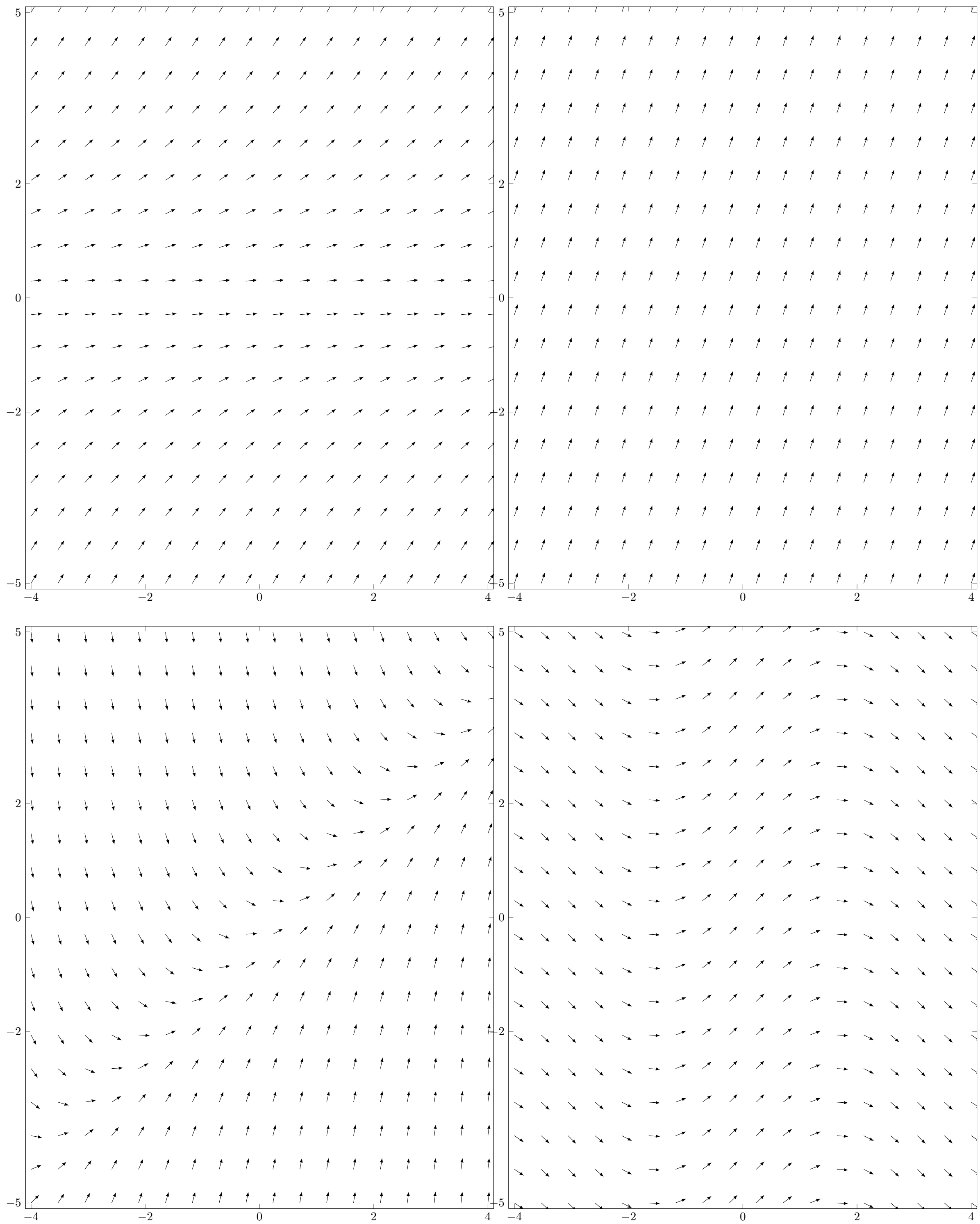
(b) Match differential equations and direction fields (one equation has no match):

$$y' = 1, \quad y' = -2, \quad y' = |y|/3 \quad y' = y - x, \quad y' = y - 1$$



b) Match differential equations and direction fields (one equation has no match):

$$y' = 3, \quad y' = -2, \quad y' = |y|/3 \quad y' = x - y, \quad y' = \cos x$$



(b) Match differential equations and direction fields (one equation has no match):

$$y' = 1, \quad y' = -2, \quad y' = |y|/3 \quad y' = y - x, \quad y' = y - 1$$