Final Review

Format
- 1 Hour 45 Minutes
- Half of the test like the midterm
- Half of the test AP Calculus AB questions
  - bonus Calc BC questions

Topics

New Topics (60%)
- Mclaurin & Taylor series
- Riemann sums
- the fundamental theorem of calculus, the definition of the integral
- finding area with integrals
- u-substitution
- integration by parts
- differential equations

From The Midterm (40%)
- graphing (shifts, features on graphs)
- limits (elementary and L'Hopital),
- limit definition of the derivate
- derivatives (polynomials, trigonometry, product rule, chain rule, quotient rule),
- linear approximation,
- implicit differentiation,
- related rates,
- extreme value problems,
- optimization problems

Review Questions

1) Review the review questions on the midterm review
2) State the formulas for
   a) Taylor series & Mclaurin series
   b) Left & right Riemann sums
   c) the integral (as a limit of Riemann sums)
   d) $\int x^n dx$, $\int \frac{1}{x} dx$, $\int e^x dx$,
   e) $\int \sin(x) dx$, $\int \cos(x) dx$, $\int \frac{1}{1+x^2} dx$, $\int \frac{1}{\sqrt{1-x^2}} dx$
   f) the fundamental theorem of calculus (both versions)
3) Make sure you can explain to yourself how to
   a) find a taylor series
   b) do u-substitution
   c) do integration by parts
   d) Solve a differential equation with an initial value
4) Fill in the blanks:

Say we compute $I = \int_a^b f(t)\,dt$ if f represents

velocity over time I is the distance traveled from time a to time b
acceleration over time I is __________ from time a to time b
inflation I is __________ from time a to time b
population growth rate I is __________ from time a to time b
power over time I. is __________ from time a to time b

AB Calculus Questions (One free response question, 15 multiple choice questions)

From the 2012 exam on the course webpage do the questions
1, 3 − 12, 14, 16, 18, 19, 21, 22, 24, 25, 28

From the 2023 exam

6. Consider the curve given by the equation $6xy = 2 + y^3$.

(a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.

(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

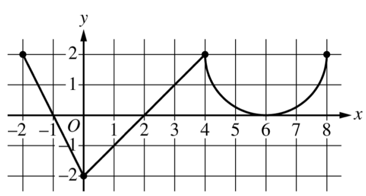
5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. ~~Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.~~ Find $k''(4)$

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

~~(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.~~



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

~~(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.~~

(c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Other Questions

Find the following definite integrals:

1) $\int_2^3 x^4 \, dx$

2) $\int_{\pi/4}^{\pi/2} \sin(x) + \cos(x) \, dx$

3) $\int_0^1 2xe^{x^2} \, dx$

Find the following indefinite integrals:

4) $\int x e^{-x} \, dx$

5) $\int \cos(x) \cos(\sin(x)) \, dx$

6) $\int \frac{12x^2}{x^3+1} \, dx$

7) $\int \ln(x) \, x^3 \, dx$ (Hint: $u = \ln(x)$ $dv = x^3 dx$)

Find the Taylor series for the following functions:

8) $f(x) = \frac{1}{1+x}$ at $x = 0$

9) $f(x) = e^x + e^{-x}$ at $x = 0$

10) $f(x) = 2\sin(x)$ at $x = 2\pi$

11) $f(x) = \ln(x)$ at $x = 2$

Find the following derivatives:

12) $\frac{d}{dx} \sqrt{x^2+1}$

13) $\frac{d}{dx} x^7 + x^6 - x^5$

14) $\frac{d}{dx} x^2 \sin(x)$

15) $\frac{d}{dx} \frac{1+x}{1+\ln(x)}$

Find the following limits:

16) $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ where $f(x) = x^2 - 1$

17) $\lim_{x \to 0^+} 2\,\mathrm{sgn}(-x)$

18) $\lim_{x \to 2\pi} \frac{\cos(x)}{x}$

19) $\lim_{x \to \infty} \frac{\ln(x)}{x^2}$

20) $\lim_{x \to \infty} \frac{3x^5 + x^3}{5x^5 - x^2 + 1}$

21) $\lim_{x \to 0} \frac{\cos(2x) - 1}{x^2}$ (Hint: L'Hopital twice)

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22) Solve the differential equation for y in terms of x:
$y' = x^3 y^4$, $y(0) = 0$

23) Approximate the area under the curve $f(x) = 1/x$ from 1 to 7 using 3 boxes and right Riemann sums

24) Suppose $x+y=6$ and $x \geq 0$, $y \geq 0$
What's the maximum value of

a) xy

b) xy^2

25) Find y' in terms of x and y given the equation $x^3+y^3 = xy$

26) Find the area bounded by the curves
$y = x^2+1$ and $y = 2x^2$

27) Find the linear approximation for $f(x) = e^x$ at $x=2$. (Express your answer as the equation of a line)

28) Choose the right multiple choice answer in each step to complete the calculation of $\cos(x)'$ with the limit definition of the derivative

i) a) $= \lim_{h\to 0} \cos(x+h) - \cos(x)$

b) $= \lim_{h\to 0} (\cos(x+h) - \cos(x))/h$

c) $= \lim_{h\to 0} \cos(x+h) + \cos(x)$

d) $= \lim_{h\to 0} (\cos(x+h) + \cos(x))/h$

ii) a) $= \lim_{h\to 0} (\sin(x)\sin(h) - \cos(x)\cos(h) - \cos(x))/h$

b) $= \lim_{h\to 0} (\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x))/h$

c) $= \lim_{h\to 0} (\sin(x)h + \cos(x) - \cos(x))/h$

d) $= \lim_{h\to 0} (\cos(x) - \sin(x)h - \cos(x))/h$

iii) a) $= \lim_{h\to 0} \sin(x)h/h$

b) $= \lim_{h\to 0} -\sin(x)h/h$

c) $= \sin(x)\lim_{h\to 0}\frac{\sin(h)}{h} - \cos(x)\lim_{h\to 0}\frac{\cos(h)+1}{h}$

d) $= -\sin(x)\lim_{h\to 0}\frac{\sin(h)}{h} + \cos(x)\lim_{h\to 0}\frac{\cos(h)-1}{h}$

iv) a) $= \sin(x)$

b) $= -\sin(x)$

c) $= \sin(x)\cdot(-1) - \cos(x)\cdot 0$

d) $= \sin(x)\cdot(1) - \cos(x)\cdot 0$

e) $= -\sin(x)\cdot(-1) - \cos(x)\cdot 0$

f) $= -\sin(x)\cdot(1) - \cos(x)\cdot 0$

v) a) Already done

b) $= \sin(x)$

c) $= -\sin(x)$

d) $= -\sin(-x)$

e) $= \sin(-x)$

vi) a) Already done

b) $= \sin(x)$ (sin is an odd function)

c) $= -\sin(x)$ (sin is an odd function)