Problem Set 8 (Total Points: 430), Due August 1st

Integration Questions

Find the following integrals (15 points each).

1. $\int x^3 \, \mathrm{d}x$

2. $\int x^4 + 2x - 1 \, \mathrm{d}x$

3. $\int_0^1 4x - x^{0.3} \, \mathrm{d}x$

4. $\int \sin(x) - 2e^x \, \mathrm{d}x$

5. $\displaystyle\int_{-1}^{1} \frac{4}{\sqrt{1 - x^2}} \, \mathrm{d}x$

6. $\int \sin(2x + 1) \, \mathrm{d}x$

7. $\int \frac{1}{3+4x} \, \mathrm{d}x$

8. $\int x^3 \sin(x^4 + 1) \, \mathrm{d}x$

9. $\int \sin(x) e^{\cos(x)} \, \mathrm{d}x$

10. $\displaystyle\int \frac{\cos(x)}{\sin(x) + 1} \, \mathrm{d}x$

11. $\int_1^2 2^x \, \mathrm{d}x$ (Hint: $2^x = e^{\ln(2)x}$)

12. $\displaystyle\int \frac{\sec^2(x) + \cos(x)}{\tan(x) + \sin(x)} \, \mathrm{d}x$

13. $\displaystyle\int \frac{x^2}{\sqrt{x^3 - 1}} \, \mathrm{d}x$

14. $\int 2x e^x \, \mathrm{d}x$

15. $\int x \ln(x) \, \mathrm{d}x$

Applications of Integrals

Work is required in this section to receive full credit.

Problem (16). (25 points) Suppose the company Connor's AI is currently having financial trouble. The rate their stock is changing is accurately modelled by $\frac{\mathrm{d}V}{\mathrm{d}t} = -30 + 8t - 3t^2$. Suppose their initial value is $V(0) = 400$

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dollars. Find the stocks value at the time $t = 10$.

Problem (17). (20 points) Find the area enclosed between the curve $y = \sin(\pi x) + 4x - \frac{1}{2}x^2$ and the x-axis.

Problem (18). (25 points) Find the area enclosed between the curves $y = 2x^2$, $y = 2$, and $x = 18$.

Problem (19). (25 points) Solve the following initial value problem: $y' = y^2(x + 1)$ and $y(0) = 2$.

Problem (20). (30 points) Radiocarbon dating is a method to determine the age of a material frequently used in archaeology. It uses the carbon-14 which is an isotope of carbon. The amount of carbon-14 found in an artifact is given by the differential equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N$$

The constant $\lambda = 0.1$ is a reasonable approximation with the unit being inverse millenniums. Suppose $N = 10^{13}$ at $t = 0$. Find $N(t)$.

     Bonus: Find the half life of carbon-14. That is the amount of time it takes for there to by half as many particles as there are at an initial time.

Challenge Questions (80 points)

Each of these problems are worth 40 points so you only need to solve 2 of them to get full credit. Solving any more will be counted as bonus points (they are only worth 20 points as bonus problems so you can get a maximum of 80 points points). Work is required to receive full credit.

21. $\displaystyle\int \frac{x^2 + x + 1}{x + 1}\,\mathrm{d}x$

22. $\displaystyle\int \sqrt{9 - \sqrt{x}}\,\mathrm{d}x$

23. $\displaystyle\int \frac{(2 + \ln(x))^2(1 - \ln(x))}{x}\,\mathrm{d}x$

24. $\int x^4 e^{-x}\,\mathrm{d}x$

25. $\int 2\cos(x)e^x\,\mathrm{d}x$

26. $\int \tan^{-1}(x)\,\mathrm{d}x$