Analysis of Non-Pulsar Variable Stars Using PRESTO

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ABSTRACT

We present a new method for using the pulsar analysis software suite, PRESTO (Ransom 2001), to analyze time series data for variable objects on much slower time scales than pulsars. This technique resamples time series data into evenly-spaced bins and prepares it for Fast Fourier Transform analysis. PRESTO's advanced Fourier techniques then enable detection and characterization of the strength and consistency of periodic signals in the data. Using this method, we have analyzed data from the Transiting Exoplanet Survey Satellite (Ricker et al. 2015), and have recovered several periodic signals present in light curves of HD 269582, as well as simulated data. We have confirmed the existence of a \sim 5 day periodicity in HD 269582 as reported in Dorn-Wallenstein et al. (2019) using a longer time series, and have measured a significant period derivative indicating the feature is quasi-periodic.

Keywords: PRESTO, TESS, EvryScope

1. INTRODUCTION

The goal of this project at its start was to measure the periods of several specific stars of interest—originally, these were several ultra-magnetic B stars suspected to possess rigidly rotating magnetospheres (Townsend & Owocki 2005). The strong magnetic fields of these stars are suspected to trap circumstellar material, which modulates the star's brightness as it rotates (Eikenberry et al. 2014). These stars were to be observed with EvryScope (Law et al. 2015), an all-sky telescope that images the entire night sky every two minutes. With these observations, EvryScope light curves for the ultramagnetic stars could be analyzed using Fourier techniques to identify and measure periodic signals. The strength of these periodic signals, and the frequencies at which they occur, would then indicate the rotational period of the ultra-magnetic star to a degree of certainty.

Advanced Fourier techniques have been used to analyze time series with great success, notably leading to the discovery of multitudes of new pulsars (Ransom 2011). However, until now these methods have remained specialized for pulsar-finding *only*. PRESTO, the software that neatly contains these advanced Fourier techniques, cannot read in a light curve directly from EvryScope. Instead, the data must be pre-processed into a format that PRESTO can read: a double-precision binary file, containing fluxes sampled at an even interval.

I have developed code to prepare a time series of any sampling rate for analysis in PRESTO. Using this code, we have analyzed several light curves from telescopes with a fairly regular 2 minute cadence. Sample EvryScope light curves were used during development of the core algorithms for data processing. The code (available at https://github.com/mccbc/TESS) has since been revised to work with TESS light curves as well.

There is high potential for non-pulsar, "slow" variables to be discovered and characterized using this method, as has been done with pulsars previously. As a consequence of the development of this code, slow variables can be analyzed with the same techniques that have been used to discover pulsars for the past decade. Among the most readily achieved results are a characterization of the variable star's period and period-derivative $(\dot{p} \text{ or } p\text{-}dot)$, as well as an automatic statistical analysis of the significance of the signal. Many other useful analytic results are reported by PRESTO upon finishing its analysis, which will be explored further in Section 4.

2. FOURIER BASICS

2.1. Overview

A large category of interesting astrophysical phenomena are objects that vary in brightness with time. The study of these objects, or "time-domain astronomy", usually involves making extended observations of these objects to characterize their long- or short-term behavior

Two fundamental tools are needed to study variable objects: a time series, and a way to extract periodic signals from it. A time series (or, light curve) is simply many measurements of brightness over some time inter-

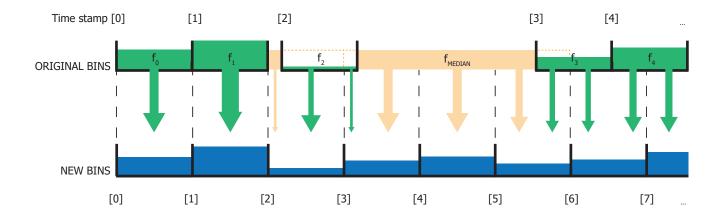


Figure 1: A diagram illustrating the resampling of flux in original bins into new, evenly-spaced bins. Original fluxes are shown in green, rebinned fluxes in blue, and the median flux replacement contribution is shown in orange. Where there are gaps in the data, the median replacement algorithm makes a contribution to the rebinned flux such that bins entirely contained within a gap end up with the median flux of the time series. This method ensures that the proper contribution is made to bins at the edge of gaps as well.

val. For this project, the time series data comes from TESS (Ricker et al. 2015). While examining this data, it is important to have an observation of sufficient length for the variable behavior to repeat itself many times—this helps determine how consistent the signal is, and it increases that behavior's statistical significance over other interfering signals. In general, the longer the time series, the easier one can extract periodic signals (the method of which is discussed in Section 2.2).

2.2. Fourier Analysis

Any star observed over some time interval will have a waveform. Let's call it F(t), for flux as a function of time. A Fourier Transform separates a waveform into a sum of sine waves of different frequencies—in theory, any real waveform can be broken down into a sum of sinusoids (thefouriertransform.com). Thus,

$$F(t) = A_1 \sin(2\pi f_1 t + \phi_1) + A_2 \sin(2\pi f_2 t + \phi_2) + \dots$$
 (1)

$$F(t) = \sum_{i=1,2,\dots}^{\infty} A_i \sin(2\pi f_i t + \phi_i)$$
 (2)

Then, one knows the amplitude (A_i) of each sinusoidal contribution to the waveform, the frequencies at which those contributions are made (f_i) , and the phase (ϕ_i) . This is all the theory this study requires, for the moment. To put it simply, the Fourier Transform allows us to examine how strongly certain frequencies are represented in a waveform. This is the method by which we

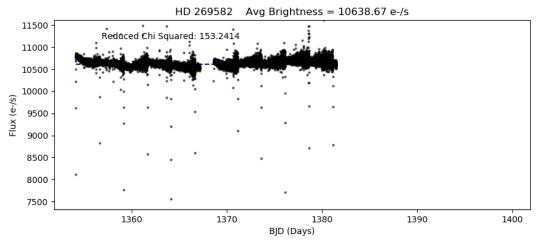
will extract periodic signals from time series data, allowing us to pinpoint which periodicities are the strongest and most consistent.

3. METHOD

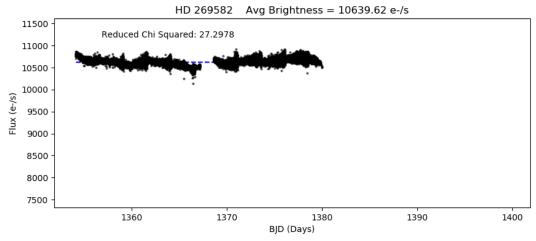
In order to perform advanced Fourier analysis in PRESTO, the time series must be loaded in the proper format for PRESTO's functions to operate on. Its primary tool is the Fast Fourier Transform—or FFT—which has a few computational and mathematical constraints.

First and foremost, time series data to be FFT'd requires evenly-spaced intervals between data points. Unfortunately, the raw light curves output by TESS are only quasi-evenly spaced. As TESS orbits Earth, it must adjust its pointing every once in awhile to ensure its targets remain in its field of view. These readjustment periods, labeled "Reaction wheel desaturation events" in the TESS documentation (Tenenbaum & Jenkins 2018) lead to considerable drops in flux and are excluded by removing all data flagged for quality issues before any analysis is done. This, as well as observational downtime, leaves gaps in the time series that must be dealt with, as shown in Fig. 2.

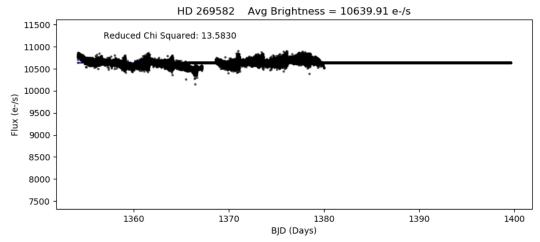
Additionally, an FFT necessitates that the total number of bins in the time series be a power of 2. This requirement is satisfied during the rebinning procedure; when the original time series is resampled, the new time series has a total number of data bins equal to the nearest power of 2 that contains all of the original flux data. The empty bins at the end of the time series are sim-



(a) A raw light curve of HD 269582 from TESS, showing data dropouts due to reaction wheel desaturation events. There is a 2 day downtime in the middle of the time series.



(b) A filtered light curve of HD 269582 from TESS. After removing data points flagged for bad quality, the dropouts no longer occur.



(c) A rebinned light curve of HD 269582. Here, the data has been resampled into evenly-spaced, 2 minute time bins, and all gaps in the data have been replaced with the median flux. As is made clear with this plot, the rebinned light curve is still faithful to the form of the original light curve, but contains no gaps or zeros which would be problematic for PRESTO's FFT algorithms.

Figure 2: Light curves at three different stages in the data pre-processing procedure.

ply filled with a constant value (the median of the time series).

3.1. The Pre-Processing Algorithm

Much of the heavy lifting is done by a single Python class, called Star, built as a generalized format for containing and performing functions on light curve data that has been loaded. Given FITS header names to look for, it will extract all relevant information and organize the data into a standard format that can be easily understood and operated on within Python. Thus, to generalize the code for use with any telescope, the class just needs input on which FITS headers correspond to the data it needs for analysis. The steps for loading an arbitrary light curve and producing processed data ready for analysis in PRESTO is as follows.

Reading the data. The light curve, consisting of data points of flux and the time stamps at which the exposures were taken, is read from a FITS file. Relevant bits of information from the FITS header, such as exposure time, target name, RA, and DEC, are recorded as attributes of the Star class.

Filtering. TESS's quality flags are loaded along with the flux and time stamps, and any data flagged as problematic is discarded. The data are then sorted by their error as reported by TESS, and the top 10 percent are also discarded. This ensures any outlying or erroneous data points that aren't accounted for by TESS's quality flags will not cause problems during analysis.

Rebinning. This is the most computationally expensive step, usually taking about 20 seconds to run on a standard laptop computer for a single light curve of 100,000 data points. First, an empty array of evenlyspaced data points is created: each point separated by the reported exposure time of each frame of the time series, and with a total number of points equal to the nearest power of 2 that can fit all of the original data. In this case, the exposure time of each frame of the time series is 2 minutes. The data in the original time series will be sorted into these newly-created bins. The algorithm goes through each data point in the original time series one by one, using the exposure time reported by the FITS header and the time stamps to determine where the original time bins overlap with the new, evenlyspaced bins. This is illustrated in Fig. 1. If two original bins overlap (as is the case for barycentered data, where the time stamps are relativistically corrected for the spacecraft's motion), the preceding overlapping bin is changed to end right where the next bin begins.

Median Replacement. It is helpful to replace the gaps in the data with a constant flux value other than zero, for compatibility with PRESTO's FFT algorithms. Since this flux value is constant, it doesn't contribute any periodicity to the Fourier transform, meaning it won't affect any of the Fourier analysis done in PRESTO. So, wherever there is a gap in the original time series, the code replaces it with a value equal to the median flux of the time series if the gap were exactly as long as a standard exposure time. It is probably better to understand this intuitively in graphical terms by looking at Fig. 1. In mathematical terms, wherever a gap is detected, the duration of the gap, t_{gap} , is measured. The replacement flux value inserted into the time series, f_{gap} , is then

$$f_{gap} = \frac{t_{gap}}{t_{exp}} \cdot f_{median} \tag{3}$$

where t_{exp} is the length of a standard exposure and f_{median} is the median flux of the time series. This calculated flux is then inserted into the time series and rebinned just like the rest of the data. Since the new, evenly-spaced bins are t_{exp} wide, when the replacement flux f_{gap} is split into each of these new bins, it ends up contributing exactly as much flux to each bin as would be recorded if one were observing at a 2 minute cadence and getting exactly the median flux for every data point. As a quick proof-of-concept: if t_{qap} is exactly the same length as a single exposure, t_{exp} , the first factor in Eq. 3 becomes 1 and the gap in the data is simply replaced with the median flux, as desired. If t_{qap} is zero, then no flux is contributed to any of the new bins, since there is no gap to fill in. The benefit of replacing gaps in the data this way is that it handles all cases of arrangements of original and new bins; no matter where the original bins overlap the new, or how large the gaps may be, this algorithm ensures that the final rebinned data has all the right flux in all the right places.

After the data is rebinned, it is exported as a .DAT file along with an accompanying .INF file; the former contains the raw 32x2 bit time series as a single data column, and the latter contains some of the

3.2. Understanding PRESTO

After a light curve has been rebinned and formatted properly, two output files are saved: a .DAT file, containing a single column vector of double-precision floats representing the flux at each time interval, and a .INF file, which contains information such as the time series' bin widths and the total number of bins in the time series. These two files are used together by PRESTO's functions realfft and prepfold.

realfft performs a Fast Fourier Transform, returning back a 32x2 bit complex array. The transform is best visualized using another PRESTO tool, explorefft,

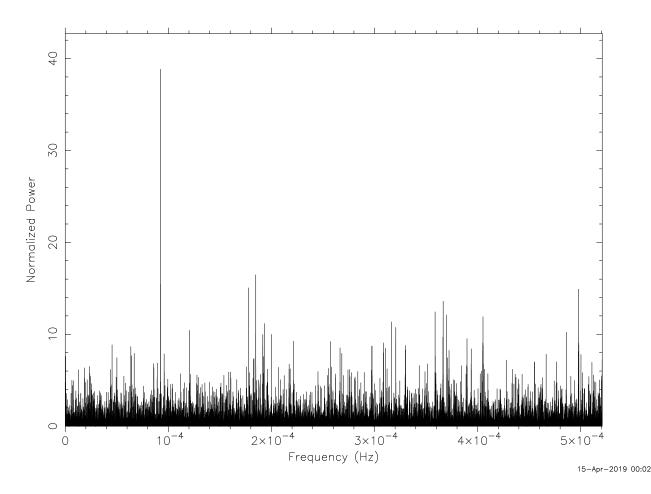


Figure 3: The FFT of a simulated sine wave with a window function, shown in PRESTO's explorefft tool. The corresponding light curve is shown in Fig. 4(a). The large peak near 10^{-4} Hz is the injected sine wave.

which displays the Fourier transform on axes of frequency and power. Fig. 3 shows an example using some simulated data.

The next step is to search for strong periodic signals. PRESTO has a built-in method to perform this procedure, called accelsearch, but so far it has been unable to reliably detect simulated signals at the right frequency with the level of significance we expect. However, we are able to search for signals by eye using explorefft, which has the ability to mark strong peaks and return precise frequencies at those peaks. Thus, until it is determined what causes accelsearch to malfunction, strong periodic signals will be detected by eye and their information input manually into the rest of the analysis functions. The pulse profile shown at the top left of PRESTO's output plot displays the program's best effort at reconstructing the shape of the signal it has detected.

Provided a certain period, prepfold folds the light curve at that period to see how well the signal lines up with itself over the course of the time series. First, the time series is separated into several chunks, the number of which can be specified using the optional argument -npart, for "number of partitions". Each of these partitions is searched independently for consistent periodic signals, and the results are stitched back together where the closest agreement of a final period is made. This is displayed on the left side of Fig. 7. The vertical alignment of bright spots in the figure indicates how well the periodic signal agrees in phase between subsequent folds. For a clean signal whose period has been accurately determined, there should be a straight vertical bright patch that goes all the way down at a single phase. The reduced χ^2 plot just to the right of this figure starts at 0 in the bottom right, and slowly increases over the course of the time series until reaching its maximum at 1.0 on the y-axis, "Fraction of Observation". A smooth, straight line here indicates consistent variability that makes a steady contribution to the reduced χ^2 of the data with each folded period. As we will see in Section 4, this is not always the case.

The right-hand side of the PRESTO prepfold output plot shows a Period Derivative v. Period plot, or "P-Pdot" plot. Like the name suggests, this figure displays the best constraints of a signal's period and how the period changes with time. For a steady, consistent periodicity that doesn't change with time, it is expected that a highly localized hotspot is visible in the center of the plot. This indicates that PRESTO has accurately determined the signal's period and period derivative, and has enough data to constrain them to a small region around zero with high certainty.

PRESTO also returns many numerical parameters, such as P_{topo} , P'_{topo} , and (σ) shown at the top right of the plot page. The period and period derivative parameters can be used to manually hone in on the signal if it doesn't show up centered in the P-Pdot plot, and the (σ) indicates the relative significance of the signal that has been detected. As expected, for the strong signals simulated in Fig. 7 and Fig. 8, the sigma is very high for both, but lower for the signal with window noise introduced.

3.3. Simulated Data

The intricacies of the rebinning algorithm leave lots of room for mathematical missteps. To put it into perspective: a small but consistent error in handling one of any of the possible arrangements of bins and gaps could introduce a false signal into the data that would be indistinguishable from the real periodicities we're trying to detect. To ensure our procedure does exactly what we intend, we tested it on simulated data we generated ourselves.

We started with a real light curve from TESS—the light curve of HD 269582. The reason for this is to preserve the unique, systematic function created by the placement of time stamps in the data. This is called the "window function", and can influence the forthcoming Fourier analysis substantially by introducing window noise, which is dependent on the pattern of when the data was taken. One of the goals of analyzing this simulated data is to confirm that window has a minimal effect on PRESTO's ability to recover strong, consistent signals even if there are gaps in the time series put into it

The time stamps from HD 269582 were preserved, but all of the corresponding flux data points were replaced with a flat, median value. On top of this flat light curve, we introduced a sine wave in each of two ways.

In one approach, a sine wave of the form $Asin(2\pi ft)$ with a period of 3 hours was added to each flux point, plugging in each of the original time stamps for t. This preserved the window function of the time series, leaving gaps in the sine wave that are replaced with the median flux during the rebinning procedure. This is shown graphically in Fig. 4(a).

In the other approach, the same sine wave was injected after the time series had been rebinned. This meant the sine wave was continuous, and was not affected by window noise. This is shown in Fig. 4(b). The noticeable difference is that, in this case, the gaps are completely filled in with the injected sine wave.

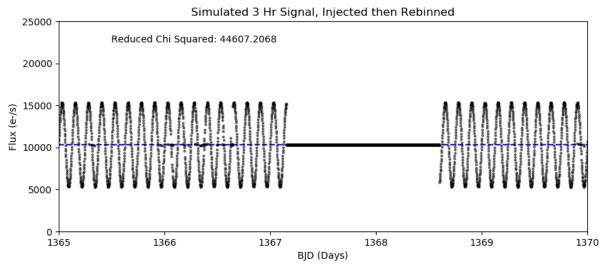
Exporting the data after each of these procedures have been performed separately, the light curve was folded about its known period of 3 hours and analyzed to quantify the strength and consistency of its periodicity. The resulting plots, for both approaches, are shown in Fig. 7 and Fig. 8.

Comparing these two output plots for simulated data, we paint a picture of what we expect to see from a strong periodic signal analyzed in PRESTO. What we're looking for are

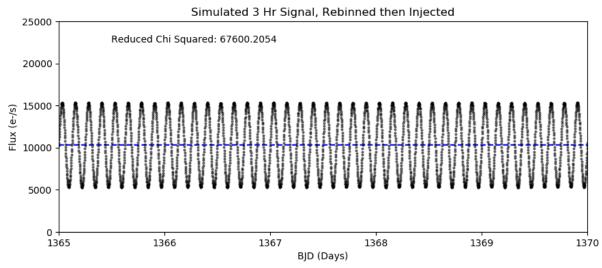
- 1. Vertically-aligned folded light curve. This means subsequent periods of the signal line up with each other in phase.
- 2. Horizontal bands in the folded light curve where data has been replaced with the median flux. The window function is inseparable from the real data, but it should be replaced with a constant and should not contribute anything to the reduced χ^2 .
- A centered, localized hotspot in the P-Pdot plot.
 The period and period derivative should be accurately determined and have high certainty.
- 4. A high σ , indicating the relative significance of this detection¹.

Indeed, looking at Fig. 7, all four of these criteria are met. The folded light curve has good alignment and is consistent for the entire duration of the time series. The horizontal bands in the folded light curve, representing median-replaced values, correspond with flat sections of the reduced χ^2 plot just to the right, meaning the gaps in the data aren't contributing to any variable behavior.

¹ The actual numbers in this field should be considered only at face value; they appear much larger than what one would typically expect even for poor signals.



(a) A 5-day section of a simulated light curve that preserves the original window function of the data. The time stamps for HD 269582 are assigned a new flux value from a sine function, and then the data are rebinned. As shown here, gaps in the original data are replaced with the median flux and don't contain the sine signal.



(b) A 5-day section of a simulated light curve where the injected sine wave is not affected by the window function of the time series. In this instance, the data are rebinned *before* being assigned a new flux value from the sine function, such that the gaps do contain the sine signal. Note that the axes for both of these plots are the same.

Figure 4: Two simulated light curves, one (Top) preserving the window function of the time series and another (Bottom) without gaps in the time series.

The red spot in the P-Pdot plot is close to the center, and the reduced χ^2 plots on either axis are strongly peaked about the center. And, finally, the sigma at the top right is large. After making these checks, it is seen from the P_{topo} field that PRESTO's recovered period is 10799967 ms with an uncertainty of 14 ms. Since the true injected signal had a period of 3 hr = 10,800,000 ms, the recovered period in PRESTO agrees

within $(10800000 \text{ ms} - 10799967 \text{ ms})/14 \text{ ms} = 2.3571\sigma.^2$ So, we have confirmed that simulated signals can be recovered in PRESTO with a reasonable amount of precision.

3.4. Real Data

 $^{^2}$ Note that this is different from the "sigma" reported by PRESTO.

For real data, most of these analysis steps remain the same except for the modifications made to the flux. A raw light curve is loaded from a FITS file, filtered using the Star class, and rebinned into an evenly sampled time series. The data is then exported in the .DAT and .INF formats for analysis in PRESTO. First, realfft is used to produce an FFT, which is then searched for signals by eye using explorefft. When a peak of interest is identified, explorefft's best guesses at its frequency, period, and period derivative are recorded. These quantities are then specified as keyword arguments to PRESTO's prepfold, which splits up the time series into parts and searches for signals near the specified period. It then folds the time series around the best detected period, and performs all the necessary analysis to produce the output plots. An example is Fig. 9.

4. RESULTS

Using the procedure described in Section 3, TESS data for the variable star HD 269582 were processed and analyzed using PRESTO.

Dorn-Wallenstein et al. (2019) reports a "strong peak" in periodicity at $0.201~\rm days^{-1}$, corresponding to a period of $\sim 4.97~\rm days$. In their analysis, they state TESS "...only observed HD 269582 for 5 full cycles" of the measured 5-day period. Since then, much more data on this target has been released. This longer time series (162 days total duration instead of 27 days) enables us to look at much lower frequencies with higher precision, since PRESTO works better with many full cycles of the periodic behavior.

However, even for the longer time series, the data is plagued with red noise—that is, random noise that is stronger at lower frequencies. Functionally, this means lower frequency signals are harder to extract reliably. Since the lower limit of useful information in Fourier space is determined by the total length of the time series, red noise is even more of an issue for short data sets.

In Fig. 5, the lower end of the FFT of HD 269582's light curve is plotted. Here, it appears that the signal reported in Dorn-Wallenstein et al. (2019) is the feature with the largest power in the spectrum. However, the red noise at the low frequency end of the spectrum may be contributing to this apparent power. For example, in Fig. 6, the FFT is shown again with normalized powers using a running median, to show the local significance of peaks in the Fourier transform. When shown this way, some of the visual effect of red noise is removed and it is easier to see that the peak corresponding to a 5 day period is not the most locally significant feature of the power spectrum.

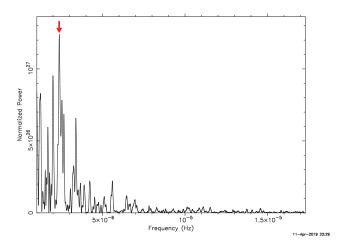


Figure 5: A section of the Fourier transform for HD 269582 displayed using raw powers, with the peak reported by Dorn-Wallenstein et al. (2019) highlighted by a red arrow.

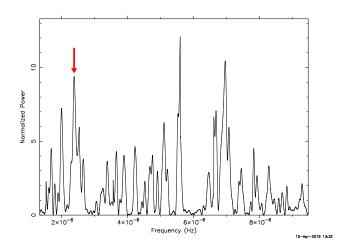


Figure 6: Another look at the FFT of HD 269582, except with powers normalized using a running median.

Putting this longer time series data through our own analysis procedure with a presumed 5 day period, we recover the information shown in Fig. 9. Interpreting the results plotted here, we see a rough pulse profile and a relatively inconsistent folded light curve. The strong variation in the pulse shape implies right away that this feature may be quasi-periodic, rather than periodic. By looking where the reduced χ^2 increases the most (between about 0.25 and 0.5 of the way through

the observation), we can see that this region of the time series is where most of the periodicity in the light curve comes from. Two strong pulses for each folded period line up well in this area of the plot, but this is not consistent throughout the whole time series. In fact, immediately after this strong, steady portion of the observation, the next folded section of the light curve actually detrends against the previous behavior, as evidenced by a slight decrease in the reduced χ^2 . Thus, the periodic behavior seems to come and go. As a result, this means the signal is not well constrained in period-derivative space, shown in the rightmost χ^2 box next to the P-Pdot plot. A semi-regular period of 4.2476×10^8 ms is detected (read from the P_{topo} field) with an uncertainty of 1.1×10^5 ms, corresponding to a period of $5.0208\pm$ 0.0013 days. This is approximately 40σ away from the period detected by Dorn-Wallenstein et al. (2019). Due to its semi-consistent folded light curve and notable period derivative of -0.001126 ± 0.000053 s/s (approximately 5σ away from 0), we conclude that, simply, the variable behavior of HD 269582 is changing over time. Additionally, while the 5 day peak in question does have substantial local power, it is by no means the only (or even the most powerful) periodic feature in HD 269582, since it is overpowered by at least two other signals in the Fourier transform, shown in Fig. 6.

5. SUMMARY

A quasi-regular periodicity of 5.0208 ± 0.0013 days has been measured for the star HD 269582. A substan-

tial period derivative, semi-consistent folded light curve, and unstable pulse profile indicate that this periodicity is changing with time. This could possibly confirm its suspected categorization as a Luminous Blue Variable, or LBV (Walborn et al. 2017). This star certainly has a complex light curve, and should continue to be observed by TESS in order to improve our ability to extract clean periodicities at low frequencies.

Now that the method using PRESTO to analyze slow variable stars has been established and tested, future studies can use the advanced Fourier techniques contained in PRESTO to analyze data from any extended observation, with any time resolution. The potential of discovering new variables is great—as soon as accelsearch can reliably find and sort the best candidate signals in a time series, the search procedure has the ability to be at least partially automated. When this becomes possible, searches for new variables with completely unknown periods can be made from any of the growing number of long-term sky surveys, such as TESS and EvryScope.

Several unanswered questions remain, but the method by which those questions will be answered has been established. Now, variable stars slower than pulsars can be analyzed comparably to what has been done in the field of pulsar astrophysics over the past decade. Future work on this project will necessitate a deeper understanding of all that PRESTO has to offer—now that it can be used, we should strive to use it to its fullest extent.

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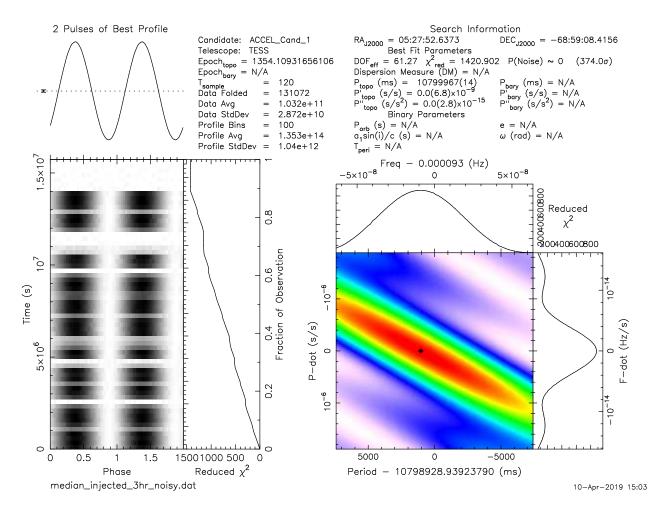


Figure 7: Simulated 3 hour sine wave with window noise. PRESTO's prepfold output plot for simulated data with the window function preserved, containing the folded light curve (left) and a Period Derivative v. Period plot, or "P-Pdot" plot (right). The horizontal white lines in the folded light curve indicate locations where the data was replaced with a constant flux. In general, the more localized the red region on the P-Pdot plot is, the more consistent and clean the periodic signal is.

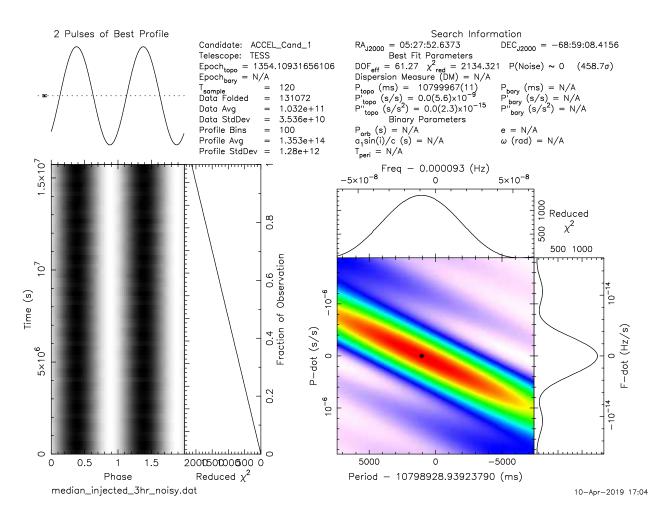


Figure 8: Simulated 3 hour sine wave without window noise. PRESTO's prepfold output plot for simulated data with no window function. In this plot, the sine source is consistent throughout the folded light curve on the left side of the image.

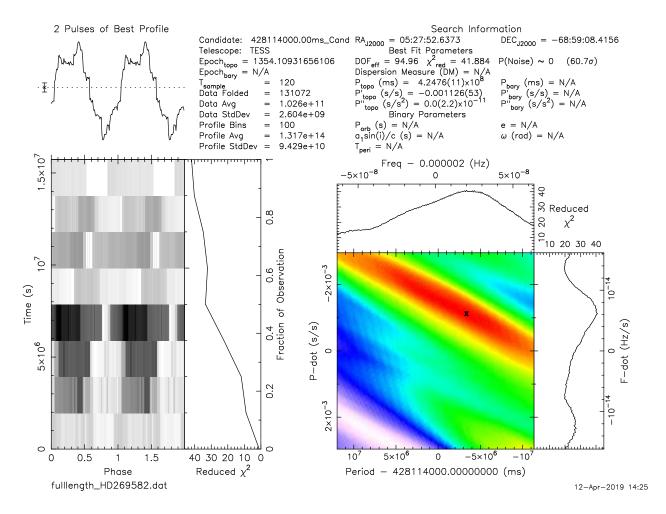


Figure 9: HD 269582, \sim 5 day period. A come-and-go signal with a period of roughly 5 days is detected, with modest significance. Most of the strength of this signal comes from only 1/3 of the total time series, indicating this feature is quasi-periodic.