MATHD022: Discrete Mathematics

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1. The Language of Mathematics

1.1 Variables

Definition A **variable** is a symbol that is used as a placeholder when:

- The quantity has one of more values, but is not known.
  - For example: $2x^2 - x = 7$
- The quantity represents **any element** from a given set.
  - For example: The reciporical of any non-zero integer n is $\frac{1}{n}$.

Writing Sentences using Variables We can rewrite the following sentences using variables:

- Is there an integer n that has a remainder of 2 when it is divided by 5?
  - Is there an integer n such that $n\%5 = 2$?
- The cube root of any negative real number is negative.
  - For any real number s, if $s < 0$, then $\sqrt[3]{s} < 0$.

Types of Statements

- A **universal statement** is a statement that is true always true.
  - For example: **All** positive numbers are greater than 0.
- A **conditional statement** is a statement that is true if a certain condition is met.
  - For example: **If** 378 is divisible by 18, **then** 378 is divisible by 6.
- A **universal conditional statement** is a statement that is both conditional and universal.
  - For example: **For all** animals a, if a is a dog, **then** a is a mammal.

- As a universal statement: **For all** dogs a, a is a mammal.
- As a conditional statement: **If** a is a dog, **then** a is a mammal.

- An **existential statement** gives a property that is true for at least one thing.
  - **There is** a prime number that is even.

- A **universal existential statement** is a statement where the first part is universal and the second part is existential.
  - **Every** real number **has** an additive inverse.
  - **For all** real numbers r, **there is** an additive inverse $-r$.
  - **For all** real numbers r, **there is** a real number s such that $r+s = 0$.

- An **existential universal statement** is a statement where the first part is existential and the second part is universal.
  - **There is** a positive integer that is less than or equal to **every** positive integer.
  - **There is** a positive intefer m such that **every** positive integer is greater than or equal to m.
  - **There is** a positive integer m with the property that **for all** positive integers n, $m \leq n$.

1.2 Sets

Definition A **set** is a collection of objects.

Notation

- $x \in S$: x is an element of S.
- $x \notin S$: x is not an element of S.
- $S = \{1, 2, 3, \dots\}$: is **set roster notation.**

Axion of Extension A set is determined by what its elements are. Orders of elements or repeated elements can't be determine the set.
For example: $\{1, 2, 3\} = \{3, 2, 2, 1, 2, 3, 1\}$. There are 3 elements in both sets.

Common Sets

- $\mathbb{R}$: the set of all real numbers.
- $\mathbb{Z}$: $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ the set of all integers.
- $\mathbb{N}$: $\{1, 2, 3, \dots\}$ the set of all natural numbers.
- $\mathbb{Q}$: the set of all rational numbers.
- $\emptyset = \{\}$: the empty set, or null set.

Set Builder Notation Let S denote a set and let $x \in S$ be and element in S. $P(x)$ is a property that some elements of S satisfy.

$$A = \{x \in S | P(x)\}$$

A constains elements in S such that (—) P(x) is true.

Subsets Let A and B be sets. A is a **subset** ($\subseteq$) of B if every element of A is also an element of B.

Proper Subsets Let A and B be sets. A is a **proper subset** ($\subset$) of B if every element of A is also an element of B, **and** there is at least one element in B that is not in A.

Example Let $A = \mathbb{Z}^+, B = \{n \in \mathbb{Z} | 0 \leq n \leq 100\}, and C = \{100, 200, 300, 400, 500\}$.

- $B \subseteq A$ is false.
- $C \subset A$ is true.
- $C \subseteq B$ is false.
- $C \subseteq C$ is true.

Cartesian Product of sets Let A and B be sets. The **Cartesian product** of A and B, denoted $A \times B$, is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A, b \in B\}$$

Example Let $A = \{1, 2, 3\}$ and $B = u, v$.

$$A \times B = \{(1, u), (1, v), (2, u), (2, v), (3, u), (3, v)\}$$
$$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$