PHYS4B Electromagnitism for Scientists and Engineers:

Circuits

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Contents

1 Current and Resistance

Current

Definition

Current is defined as the time rate of change of charge through an object. It is measured in amps.

$$I = \frac{dQ}{dt} \Rightarrow I = nqv_dA$$

Where I is the current in amps, n is the free electron density in electrons/m^3, v_d is the drift velocity in m/s, and A is the cross-sectional area of the conductor in m^2.

$$[Amps] = [\frac{Coulombs}{second}]$$

Drift Velocity

Electrons are bouncing around randomly. When an electric field is applied, the bouncing is directed in a direction, but it is still chaotic. This bouncing results in heat being generated. Heat is defined as the kinetic energy of a particle. The speed of the drift of the electrons is called the drift velocity (v_d).

$$v_d = \frac{I_{avg}}{nqA} = \frac{I}{nqA}$$

Where v_d is the drift current, n is the free electron density, q is the charge of the current carrier (usually an electron) and A is the cross-sectional area of the conductor.

Current Density

Current density is the current per unit area. It is defined as:

$$J = \frac{I}{A} = \sigma A = nqv_d$$

Where J is the current density in $Amps/m^2$, I is the current in amps, A is the cross-sectional area of the conductor in m^2, σ is the conductivity of the material, and n is the free electron density in electrons/m^3.

Voltage can be calculated as a function of current density and conductivity as follows:

$$\Delta V = E\ell = \frac{\ell J}{\sigma}$$

Resistance

Definition

Resistance is defined as the ratio of voltage to current, also known as Ohm's law. It is measured in ohms.

$$R = \frac{\Delta V}{I}$$

Where R is the resistance in ohms, ΔV is the voltage in volts, and I is the current in amps.

Resistivity

Resistivity is the fundimental property of a material that determines how much it resists the flow of current. It is measured in ohm-meters.

$$\rho = \frac{1}{\sigma} \Rightarrow R = \rho\frac{\ell}{A}$$

Where ρ is resistivity, σ is conductivity, R is resistance in Ω, I is current in amps, and ΔV is voltage in volts.

$$[\Omega] = [\frac{V}{A}]$$

Ohmic vs. Non-Ohmic devices

Ohmic devices are devices that have a Voltage vs Current slope of $\frac{1}{R}$. Non-ohmic devices have a slope that changes with voltage or current.

Resistance and Temperature

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

$$\alpha = \frac{\Delta\rho/\rho}{\Delta T}$$

Where ρ is resistivity and T is temperature.

Electrical Power

Definition

Power is defined as the rate at which energy is transferred or converted. It is measured in watts.

$$P = \frac{dU_e}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

$$\implies P = I^2R = \frac{(\Delta V)^2}{R}$$

Where P is power in watts, Q is total charge in Coulombs, ΔV is voltage in volts, and I is current in amps.