PHYS4B Electromagnitism for Scientists and Engineers

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Contents

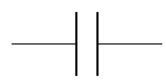
1 Introduction to Electrostatics

Charge

Much like inertial, charge is a fundimental property of matter. It represents the number the of excess electrons or protons in an object. It is measured in Coulombs [C].

2 Capacitors

Capacitors are devices that can store charge. They are made of one or more pairs of conductors seperated by an insulator. In circuit diagrams, they are denoted using the following symbol:



Capacitance

Definition

Capacitance is the ability of a capacitor to store charge. It is measured in farads.

$$C = \frac{Q}{\Delta V} = \epsilon_0 \frac{A}{d}$$

Where C is capacitance in farads, Q is charge in Coulombs, ΔV is voltage in volts, ϵ_0 is the permittivity of free space, A is the area of the plates in m^2, and d is the distance between the plates in meters.

Electric Field Within a Capacitor

The electric field within a capacitor is given by the equation:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Where E is the electric field in N/C, σ is the charge density in C/m^2, Q is the charge in Coulombs, ϵ_0 is the permittivity of free space, and A is the area of the plates in m^2.

Note that any 2 parallel objects with opposite charges can act as a capacitor, for example the sky and the ground during a stormy day.

Voltage Across a Capacitor

The voltage across a capacitor is given by the equation:

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Where ΔV is the voltage in volts, E is the electric field in N/C, d is the distance between the plates in meters, Q is the charge in Coulombs, ϵ_0 is the permittivity of free space, and A is the area of the plates in m^2.

Energy Stored in a Capacitor

The electric potential energy stored in a capacitor is given by the equation:

$$U_e = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Where U_e is the electric potential energy in Joules, Q is the charge in Coulombs, ΔV is the voltage in volts and C is the capacitance in farads

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Dielectrics

Up until this point, we have made the assumption that the conductors in the capacitor have been seperated by air, however this is not always the case. Dielectrics are insulating materials that are placed between the plates of a capacitor. They increase the capacitance of a capacitor by a factor of κ, or the dielectric constant of that material. κ is a direct property of a material that has been determined through experimentation, and thus this value will have to be given in the problem or retrieved from a table.

Capacitance and Electric Field with a Dielectric

When a dielectric is placed between the plates of a capacitor, the capacitance and electric field change based off of the dielectric constant.

$$C_\kappa \propto \kappa$$
$$E_\kappa \propto \tfrac{1}{\kappa}$$

This changes the equations for capacitance, electric field, and voltage across a capacitor to the following:

$$C_\kappa = \kappa C_0 = \tfrac{\kappa Q}{\Delta V}$$
$$E_\kappa = \tfrac{\sigma}{\kappa \epsilon_0} = \tfrac{Q}{\kappa \epsilon_0 A}$$
$$\Delta V_\kappa = E_\kappa d = \tfrac{Qd}{\kappa \epsilon_0 A}$$

Capacitors in Series and Parallel

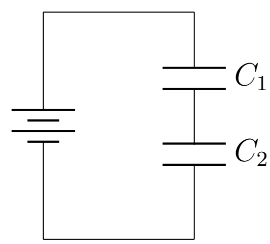

Figure 1: Wired in series.

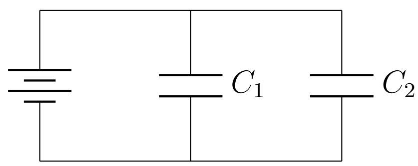

Figure 2: Wired in parallel.

Equivilant Capacitance

When capacitors are wired in parallel, the voltage drop across them are the same, and the equivilant capacitance is given by the equation:

$$C_{eq} = \sum_{i=1}^{n} C_i$$

When capacitors are wired in series, each one holds the same amount of charge, and the equivilant capacitance is given by the equation:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

3 Current and Resistance

Current

Definition

Current is defined as the time rate of change of charge through an object. It is measured in amps.

$$I = \frac{dQ}{dt} \Rightarrow I = nqv_dA$$

Where I is the current in amps, n is the free electron density in electrons/m^3, v_d is the drift velocity in m/s, and A is the cross-sectional area of the conductor in m^2.

$$[Amps] = [\frac{Coulombs}{second}]$$

Drift Velocity

Electrons are bouncing around randomly. When an electric field is applied, the bouncing is directed in a direction, but it is still chaotic. This bouncing results in heat being generated. Heat is defined as the kinetic energy of a particle. The speed of the drift of the electrons is called the drift velocity (v_d).

$$v_d = \frac{I_{avg}}{nqA} = \frac{I}{nqA}$$

Where v_d is the drift current, n is the free electron density in $\frac{g}{m^2}$, q is the charge of the current carrier (usually an electron) in C, and A is the cross-sectional area of the conductor in m^2.

Current Density

Current density is the current per unit area. It is defined as:

$$J = \frac{I}{A} = \sigma A = nqv_d$$

Where J is the current density in $Amps/m^2$, I is the current in amps, A is the cross-sectional area of the conductor in m^2, σ is the conductivity of the material, and n is the free electron density in electrons/m^3.

Voltage can be calculated as a function of current density and conductivity as follows:

$$\Delta V = E\ell = \frac{\ell J}{\sigma}$$

Resistance

Definition

Resistance is defined as the ratio of voltage to current, also known as Ohm's law. It is measured in ohms.

$$R = \frac{\Delta V}{I}$$

Where R is the resistance in Ω, ΔV is the voltage in volts, and I is the current in A.

Resistivity and Conductivity

Resistivity is the fundimental property of a material that determines how much it resists the flow of current. It is measured in ohm-meters [Ωm].

$$\rho = \frac{1}{\sigma} = \frac{RA}{\ell} \implies R = \rho\frac{\ell}{A}$$

Conductivity is the inverse of resistivity. It is measured in Siemens per meter $[\frac{S}{m}]$. $[S] = [\frac{1}{\Omega}]$

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA} \Longrightarrow R = \frac{\ell}{\sigma A}$$

Where ρ is resistivity, σ is conductivity, R is resistance in Ω, I is current in amps, and ΔV is voltage in volts.

Ohmic vs. Non-Ohmic devices

Ohmic devices are devices that have a Voltage vs Current slope of $\frac{1}{R}$. Non-ohmic devices have a slope that changes with voltage or current.

Resistivity and Temperature

The resistivity of a material changes with the temperature of the material. The equation for resistivity as a function of temperature is given by:

$$\rho_t = \rho_0[1 + \alpha\Delta T]$$

$$\alpha = \frac{\Delta\rho/\rho_0}{\Delta T}$$

Where ρ_t is the resistivity of the material at a given temperature in Ωm, ρ_0 is the resistivity of the material at a reference temperature in Ωm, α is the temperature coefficient in $^\circ C^{-1}$ (a material constant, much like κ), $\Delta\rho$ is the change in resistivity in Ωm, and ΔT is the change in temperature in $^\circ C$.

Electrical Power

Definition

Power is defined as the rate at which energy is transferred or converted. It is measured in watts.

$$P = \frac{dU_e}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

$$\implies P = I^2 R = \frac{(\Delta V)^2}{R}$$

Where P is power in watts, Q is total charge in Coulombs, ΔV is voltage in volts, and I is current in amps.