PHYS4B: Electromagnitism for Scientists and Engineers

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Contents

1. Electrostatics

Introduction to Electrostatics

Charge

Definition Much like inertia, charge is a fundamental property of matter that describes how an object interacts with electric fields. It is measured in Coulombs $[C]$. Practically, charge indicates if the object has an excess or deficiency of electrons.

Quantization of Charge Charge is quantized, meaning that it can only exist in discrete values. The smallest possible charge is the charge of an electron. Protons and electrons carry equal but opposite charges, represented by e.

$$q = \pm Ne \Longrightarrow e = 1.6 \times 10^{-19} C$$

Where N is any integer. This means that the charge of an object is always a multiple of the charge of an electron.

Conservation of Charge Much like energy and matter, charge is conserved in a closed system. This means that the total charge in a system will remain constant.

$$\sum q_i = \sum q_f$$

Conductors and Insulators Electrical conductors are materials in which some of the electrons are not bound to atoms and can move relatively freely through the material. This allows for the easy transfer of charge. Metals are the most common conductors. Electrical insulators are materials in which all of the electrons are bound to atoms and cannot move freely. This impededs the transfer of charge. Glass, rubber, and plastic are common insulators.

Coloumb's Law

Definition Coulomb's law describes the fundimental force between 2 charged objects. It is given by the equation:

$$F_e = k_e \frac{q_1 q_2}{r^2} \Longrightarrow \vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

Where F_e is the electrostatic force in Newtons, k_e is Coulomb's constant, q_1 and q_2 are the charges of the objects in Coulombs, r is the distance between the objects in meters, and $\hat{r}_{12}$ is a unit vector pointing from object 1 to object 2.

Notice the similarities between Coulomb's law and Newton's law of gravitation. These similarities are because both are fundamental forces of nature. Both are inverse square laws, meaning that the force between the objects decreases exponentially as the distance between them increases. Both forces are proportional to a property of matter (mass for gravity and charge for electrostatics) and a constant (G and k_e respectively).

The main difference is that electrostatic forces can be attractive or repulsive, while gravity is always attractive. This is because charge can be positive or negative, while it is (for our purposes) impossible to have negative mass.

Electric Fields

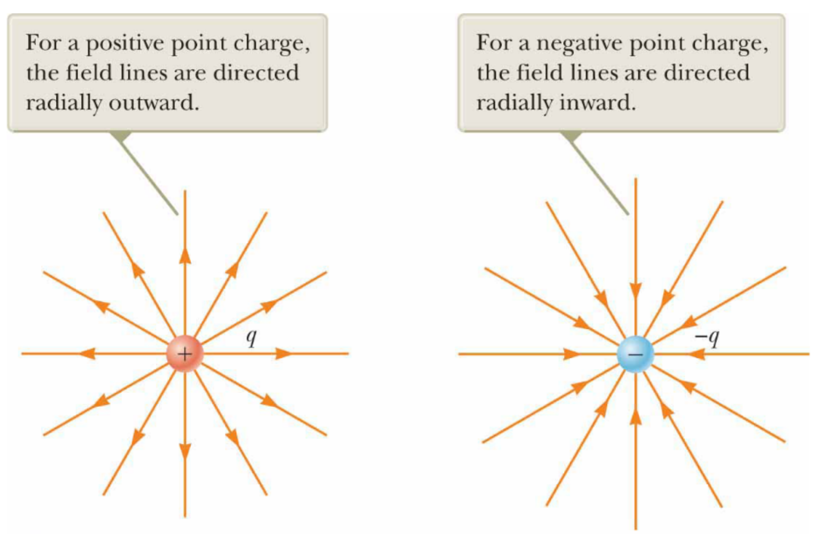
Definition The electric field is a vector field that describes the force experienced by a charge at any point in space. It is measured in Newtons per Coulomb $[\frac{N}{C}]$. It is given by the equation:

$$\vec{E} = \frac{\vec{F_e}}{q}$$

Where $\vec{E}$ is the electric field in Newtons per Coulomb, $\vec{F_e}$ is the electrostatic force experienced by the particle in Newtons, and q is the charge of the particle in Coulombs.

Electric Field Lines

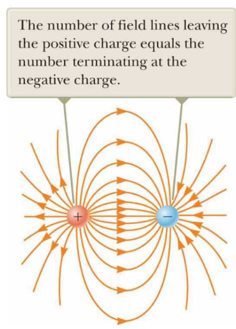
Definition Electric field lines are a visual representation of the electric field. They are drawn such that the electric field is tangent to the line at any point. The electric field lines are drawn such that they point away from positive charges and towards negative charges.



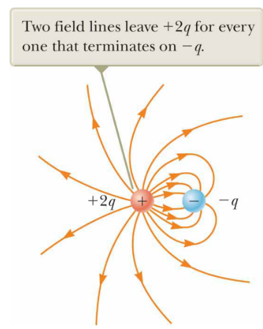
The density of the lines leaving or terminating at a particle is proportional to the charge of the particle.

$$\frac{N_2}{N_1} = |\frac{q_2}{q_1}|$$

Where N is the number of field lines coming from a charge, and q is the charge of that particle.



(a) Field lines between two equal and opposite charges



(b) Field lines between two unequal and opposite charges

Charge Density

Definition Charge density is the amount of charge per unit length (λ), area (σ), or volume (ρ) depending on the geometry of the object.

$$\text{Linear Charge Density} : \lambda = \frac{Q}{\ell} \implies dq = \lambda d\ell$$
$$\text{Surface Charge Density} : \sigma = \frac{Q}{A} \implies dq = \sigma dA$$
$$\text{Volume Charge Density} : \rho = \frac{Q}{V} \implies dq = \rho dV$$

Where Q is the total charge, ℓ is the length, A is the area, and V is the volume of the object.

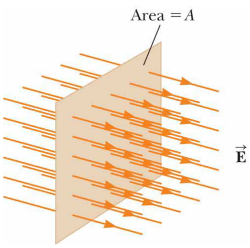
Electric Field Caused by Different Charged Geometry

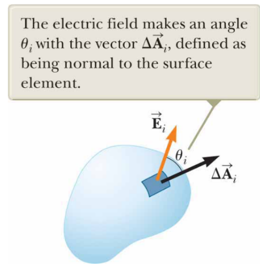
Charged objects can have different geometries resulting in different electric fields. Here are some common geometries and their electric fields.

$$
\begin{aligned}
\text{Infinite Line} &: E_{line} = \frac{\lambda}{2\pi\epsilon_0 r} \\
\text{Infinite Plane} &: E_{plane} = \frac{\sigma}{2\epsilon_0} \\
\text{Parallel Plates} &: E_{\|,plate} = \frac{\sigma}{\epsilon_0} \\
\text{Ring} &: E_{ring} = \frac{k_e Q x}{(x^2 + a^2)^{3/2}}
\end{aligned}
$$

Where λ is the linear charge density, σ is the surface charge density, k_e is Coulomb's constant, a is the radius of the ring, x is the distance from the ring, Q is the total charge of the object, and a is the radius of the ring.

Electric Flux





Definition Electric flux is defined as the number of electric field lines passing through a surface. It is measured in Volt meters $[Vm]$. It is given by the equation:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA\cos(\theta)$$

Where Φ_E is the electric flux in Newtons per Coulomb, $\vec{E}$ is the electric field in Newtons per Coulomb, $d\vec{A}$ is the differential area vector, and θ is the angle between $\vec{E}$ and $d\vec{A}$.

Gauss's Law

Definition Gauss's Law states that electric flux through a closed surface is equal to the charge enclosed by the surface divided by the permittivity of free space. This is to say that any flux generated by electric fields originating from charges outside the surface will cancel out, thus we only care about the enclosed charge.

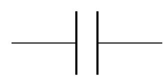
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Where Φ_E is the electric flux in Vm, $\vec{E}$ is the electric field in Newtons per Coulomb, $d\vec{A}$ is the differential area vector, Q_{enc} is the charge enclosed by the surface, and ϵ_0 is the permittivity of free space.

2. Circuits

Capacitors

Definition Capacitors are devices that can store charge. They are made of one or more pairs of conductors seperated by an insulator. In circuit diagrams, they are denoted using the following symbol:



Capacitance

Definition Capacitance is the ability of a capacitor to store charge. It is measured in farads.

$$C = \frac{Q}{\Delta V} = \epsilon_0 \frac{A}{d}$$

Where C is capacitance in farads, Q is charge in Coulombs, ΔV is voltage in volts, ϵ_0 is the permittivity of free space, A is the area of the plates in m^2, and d is the distance between the plates in meters.

Electric Field Within a Capacitor The electric field within a capacitor is given by the equation:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Where E is the electric field in N/C, σ is the charge density in C/m^2, Q is the charge in Coulombs, ϵ_0 is the permittivity of free space, and A is the area of the plates in m^2.

Note that any 2 parallel objects with opposite charges can act as a capacitor, for example the sky and the ground during a stormy day.

Voltage Across a Capacitor The voltage across a capacitor is given by the equation:

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Where ΔV is the voltage in volts, E is the electric field in N/C, d is the distance between the plates in meters, Q is the charge in Coulombs, ϵ_0 is the permittivity of free space, and A is the area of the plates in m^2.

Energy Stored in a Capacitor The electric potential energy stored in a capacitor is given by the equation:

$$U_e = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Where U_e is the electric potential energy in Joules, Q is the charge in Coulombs, ΔV is the voltage in volts and C is the capacitance in farads

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Dielectrics

Up until this point, we have made the assumption that the conductors in the capacitor have been seperated by air, however this is not always the case. Dielectrics are insulating materials that are placed between the plates of a capacitor. They increase the capacitance of a capacitor by a factor of κ, or the dielectric constant of that material. κ is a direct property of a material that has been determined through experimentation, and thus this value will have to be given in the problem or retrieved from a table.

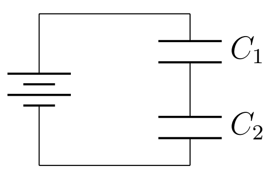
Capacitance and Electric Field with a Dielectric When a dielectric is placed between the plates of a capacitor, the capacitance and electric field change based off of the dielectric constant.

$$C_\kappa \propto \kappa$$
$$E_\kappa \propto \frac{1}{\kappa}$$

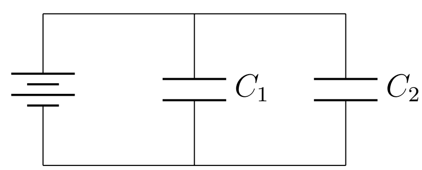
This changes the equations for capacitance, electric field, and voltage across a capacitor to the following:

$$C_\kappa = \kappa C_0 = \frac{\kappa Q}{\Delta V}$$
$$E_\kappa = \frac{\sigma}{\kappa \epsilon_0} = \frac{Q}{\kappa \epsilon_0 A}$$
$$\Delta V_\kappa = E_\kappa d = \frac{Qd}{\kappa \epsilon_0 A}$$

Capacitors in Series and Parallel



(a) Wired in series.



(b) Wired in parallel.

Differences Capacitors wired in series and parallel behave differently. Capacitors wired in series carry the same charge, but different voltages.

$$q_{eq(s)} = q_1 = q_2 = \cdots = q_n$$

Capacitors wired in parallel behave in the opposite manner, carrying the same voltage, but different charges.

$$\Delta V_{eq(\|)} = \Delta V_1 = \Delta V_2 = \cdots = \Delta V_n$$

Equivilant Capacitance Multiple capacitors can be represented by a single capacitor with an equivilant capacitance. The equivilant capacitance of capacitors in series and in parallel is given by the following equations:

$$\frac{1}{C_{eq(s)}} = \sum_{i=1}^{n} \frac{1}{C_i}$$
$$C_{eq(\|)} = \sum_{i=1}^{n} C_i$$

Current and Resistance

Current

Definition Current is defined as the time rate of change of charge through an object. It is measured in amps.

$$I = \frac{dQ}{dt} \Rightarrow I = nqv_dA$$

Where I is the current in amps $[A] = [\frac{C}{s}]$, n is the free electron density in electrons$/m^3$, v_d is the drift velocity in m/s, and A is the cross-sectional area of the conductor in m^2.

Drift Velocity Electrons are bouncing around randomly. When an electric field is applied, the bouncing is directed in a direction, but it is still chaotic. This bouncing results in heat being generated. Heat is defined as the kinetic energy of a particle. The speed of the drift of the electrons is called the drift velocity (v_d).

$$v_d = \frac{I_{avg}}{nqA} = \frac{I}{nqA}$$

Where v_d is the drift current, n is the free electron density in $\frac{g}{m^2}$, q is the charge of the current carrier (usually an electron) in C, and A is the cross-sectional area of the conductor in m^2.

Current Density Current density is the current per unit area. It can be calculated using the following formula:

$$J = \frac{I}{A} = \sigma A = nqv_d$$

Where J is the current density in $Amps/m^2$, I is the current in amps, A is the cross-sectional area of the conductor in m^2, σ is the conductivity of the material, and n is the free electron density in electrons$/m^3$.

Voltage can be calculated as a function of current density and conductivity as follows:

$$\Delta V = E\ell = \frac{\ell J}{\sigma}$$

Where ΔV is the voltage in volts, E is the electric field in volts per meter, ℓ is the length of the conductor in meters,

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Resistance

Definition Resistance is defined as the ratio of voltage to current, also known as Ohm's law. It is measured in ohms.

$$R = \frac{\Delta V}{I}$$

Where R is the resistance in Ω, ΔV is the voltage in volts, and I is the current in A.

Resistivity Resistivity is the fundimental property of a material that determines how much it resists the flow of current. It is measured in ohm-meters $[\Omega m]$.

$$\rho = \frac{1}{\sigma} = \frac{RA}{\ell} \Longrightarrow R = \rho\frac{\ell}{A}$$

Where ρ is resistivity in Ωm, σ is conductivity in S/m, R is resistance in Ω, A is the cross-sectional area of the conductor in m^2, and ℓ is the length of the conductor in meters.

Conductivity Conductivity is the inverse of resistivity. It is measured in Siemens per meter $[\frac{S}{m}]$. $[S] = [\frac{1}{\Omega}]$

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA} \Longrightarrow R = \frac{\ell}{\sigma A}$$

Where ρ is resistivity, σ is conductivity, R is resistance in Ω, I is current in amps, and ΔV is voltage in volts.

Ohmic vs. Non-Ohmic devices Ohmic devices are devices that have a Voltage vs Current slope of $\frac{1}{R}$. Non-ohmic devices have a slope that changes with voltage or current.

Resistivity and Temperature The resistivity of a material changes with the temperature of the material. The equation for resistivity as a function of temperature is given by:

$$\rho_t = \rho_0[1 + \alpha \Delta T]$$
$$\alpha = \frac{\Delta \rho / \rho_0}{\Delta T}$$

Where ρ_t is the resistivity of the material at a given temperature in Ωm, ρ_0 is the resistivity of the material at a reference temperature in Ωm, α is the temperature coefficient in $^\circ C^{-1}$ (a material constant, much like κ), $\Delta\rho$ is the change in resistivity in Ωm, and ΔT is the change in temperature in $^\circ C$.

Electrical Power

Definition Power is defined as the rate at which energy is transferred or converted. It is measured in watts.

$$P = \frac{dU_e}{dt} = I^2 R = \frac{(\Delta V)^2}{R}$$

Where P is power in watts, U_e is electric potential energy in Joules, ΔV is voltage in volts, and I is current in amps.

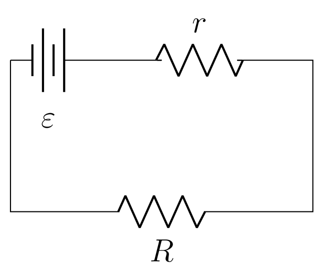
Direct Current Circuits

Electromotive Force

Definition Electromotive force (emf) is the voltage produced by a battery or generator. It is measured in volts. The emf of a circuit is the total voltage supplied by the battery or generator.

$$\Delta V = \varepsilon - Ir$$
$$\varepsilon = IR + Ir$$
$$I = \frac{\varepsilon}{R + r}$$

Where ΔV is the voltage in volts, ε is the emf in volts, I is the current in amps, and R and r are resistors in ohms.



Power :

$$P_R = I^2 R$$
$$P_r = I^2 r$$

Equivilant Resistance

Much like capacitors, multiple resistors can be represented by a single resistor with an equivilant resistance.

In Series

$$R_{eq(s)} = \sum_{i=1}^{n} R_i$$
$$I_{eq(s)} = I_1 = I_2 = \cdots = I_n$$
$$\Delta V_{eq(s)} = \sum_{i=1}^{n} \Delta V_i$$

In Parallel

$$\frac{1}{R_{eq(\parallel)}} = \sum_{i=1}^{n} \frac{1}{R_i}$$
$$I_{eq(\parallel)} = \sum_{i=1}^{n} I_i$$
$$\Delta V_{eq(\parallel)} = \Delta V_1 = \Delta V_2 = \cdots = \Delta V_n$$

Kirchhoff's Laws

Junction Rule At any junction in a circuit, the sum of the currents must equal zero.

$$\sum_{junction} I = 0 \quad \Longrightarrow \quad \sum_{junction} I_{in} = \sum_{junction} I_{out}$$

Loop Rule The sum of the voltages around any closed loop in a circuit must equal zero.

$$\sum_{closedloop} \Delta V = 0$$

Sign Conventions

- If the current enters the positive terminal of a resistor, it is positive.
- If the current enters the negative terminal of a resistor, it is negative.
- If the current enters the positive terminal of a battery, it is negative.

- If the current enters the negative terminal of a battery, it is positive.

Charging a Capacitor

$$
\begin{aligned}
Q_{max} &= C\varepsilon \\
I &= \frac{dq}{dt} = -\frac{q - C\varepsilon}{RC} \\
q(t) &= C\varepsilon(1 - e^{-\frac{t}{RC}}) = Q_{max}(1 - e^{-\frac{t}{RC}}) \\
i(t) &= \frac{dq}{dt} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}
\end{aligned}
$$

Discharging a Capacitor

$$
\begin{aligned}
q(t) &= Q_i e^{-\frac{t}{RC}} \\
i(t) &= -\frac{Q_i}{RC}e^{-\frac{t}{RC}}
\end{aligned}
$$

3. Magnitism

Magnetic Fields

Definition A magnetic field is a vector field that describes the magnetic influence on moving electric charges, electric currents, and magnetic materials. They are generated by moving charges, or current. It is represented by the symbol $\vec{B}$ and is measured in teslas (T).

Unlike electric fields, which can be generated by a single source charge, magnetic poles always come in pairs. There are no magnetic monopoles. The two types of magnetic poles are called north and south.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Where $\vec{F}_B$ is the magnetic force in newtons, q is the charge in coulombs, $\vec{v}$ is the velocity of the charge in meters per second, and $\vec{B}$ is the magnetic field in teslas. [Insert right hand rule stuff]