

# PHYS4B Electromagnetism for Scientists and Engineers

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# 1. Electrostatics

## Introduction to Electrostatics

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### Charge

**Definition** Much like inertia, charge is a fundamental property of matter that describes how an object interacts with electric fields. It is measured in Coulombs  $[C]$ . Practically, charge indicates if the object has an excess or deficiency of electrons.

**Quantization of Charge** Charge is quantized, meaning that it can only exist in discrete values. The smallest possible charge is the charge of an electron. Protons and electrons carry equal but opposite charges, represented by  $e$ .

$$q = \pm Ne \implies e = 1.6 \times 10^{-19} C$$

Where  $N$  is any integer. This means that the charge of an object is always a multiple of the charge of an electron.

**Conservation of Charge** Much like energy and matter, charge is conserved in a closed system. This means that the total charge in a system will remain constant.

$$\sum q_i = \sum q_f$$

**Conductors and Insulators** Electrical conductors are materials in which some of the electrons are not bound to atoms and can move relatively freely through the material. This allows for the easy transfer of charge. Metals are the most common conductors. Electrical insulators are materials in which all of the electrons are bound to atoms and cannot move freely. This impedes the transfer of charge. Glass, rubber, and plastic are common insulators.

## Coloumb's Law

**Definition** Coulomb's law describes the fundimantal force between 2 charged objects. It is given by the equation:

$$F_e = k_e \frac{q_1 q_2}{r^2} \implies \vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

Where  $F_e$  is the electrostatic force in Newtons,  $k_e$  is Coulomb's constant,  $q_1$  and  $q_2$  are the charges of the objects in Coulombs,  $r$  is the distance between the objects in meters, and  $\hat{r}_{12}$  is a unit vector pointing from object 1 to object 2.

Notice the similarities between Coulomb's law and Newton's law of gravitation. These similarities are because both are fundamental forces of nature. Both are inverse square laws, meaning that the force between the objects decreases exponentially as the distance between them increases. Both forces are proportional to a property of matter (mass for gravity and charge for electrostatics) and a constant ( $G$  and  $k_e$  respectively).

The main difference is that electrostatic forces can be attractive or repulsive, while gravity is always attractive. This is because charge can be positive or negative, while it is (for our purposes) impossible to have negative mass.

# Electric Fields

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**Definition** The electric field is a vector field that describes the force experienced by a charge at any point in space. It is measured in Newtons per Coulomb [ $\frac{N}{C}$ ]. It is given by the equation:

$$\vec{E} = \frac{\vec{F}_e}{q} \implies \vec{E} = k_e \frac{q}{r^2} \hat{r}$$

Where  $\vec{E}$  is the electric field in Newtons per Coulomb,  $\vec{F}_e$  is the electrostatic force experienced by the particle in Newtons, and  $q$  is the charge of the particle in Coulombs.

## 2. Circuits

### Capacitors

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**Definition** Capacitors are devices that can store charge. They are made of one or more pairs of conductors separated by an insulator. In circuit diagrams, they are denoted using the following symbol:



#### Capacitance

**Definition** Capacitance is the ability of a capacitor to store charge. It is measured in farads.

$$C = \frac{Q}{\Delta V} = \epsilon_0 \frac{A}{d}$$

Where  $C$  is capacitance in farads,  $Q$  is charge in Coulombs,  $\Delta V$  is voltage in volts,  $\epsilon_0$  is the permittivity of free space,  $A$  is the area of the plates in  $m^2$ , and  $d$  is the distance between the plates in meters.

**Electric Field Within a Capacitor** The electric field within a capacitor is given by the equation:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Where  $E$  is the electric field in  $N/C$ ,  $\sigma$  is the charge density in  $C/m^2$ ,  $Q$  is the charge in Coulombs,  $\epsilon_0$  is the permittivity of free space, and  $A$  is the area of the plates in  $m^2$ .

Note that any 2 parallel objects with opposite charges can act as a capacitor, for example the sky and the ground during a stormy day.

**Voltage Across a Capacitor** The voltage across a capacitor is given by the equation:

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Where  $\Delta V$  is the voltage in volts,  $E$  is the electric field in  $N/C$ ,  $d$  is the distance between the plates in meters,  $Q$  is the charge in Coulombs,  $\epsilon_0$  is the permittivity of free space, and  $A$  is the area of the plates in  $m^2$ .

**Energy Stored in a Capacitor** The electric potential energy stored in a capacitor is given by the equation:

$$U_e = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Where  $U_e$  is the electric potential energy in Joules,  $Q$  is the charge in Coulombs,  $\Delta V$  is the voltage in volts and  $C$  is the capacitance in farads

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## Dielectrics

Up until this point, we have made the assumption that the conductors in the capacitor have been separated by air, however this is not always the case. Dielectrics are insulating materials that are placed between the plates of a capacitor. They increase the capacitance of a capacitor by a factor of  $\kappa$ , or the dielectric constant of that material.  $\kappa$  is a direct property of a material that has been determined through experimentation, and thus this value will have to be given in the problem or retrieved from a table.

**Capacitance and Electric Field with a Dielectric** When a dielectric is placed between the plates of a capacitor, the capacitance and electric field change based off of the dielectric constant.

$$C_{\kappa} \propto \kappa$$

$$E_{\kappa} \propto \frac{1}{\kappa}$$

This changes the equations for capacitance, electric field, and voltage across a capacitor to the following:

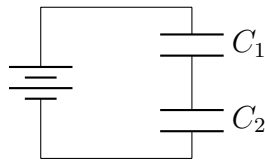
$$C_{\kappa} = \kappa C_0 = \frac{\kappa Q}{\Delta V}$$

$$E_{\kappa} = \frac{\sigma}{\kappa \epsilon_0} = \frac{Q}{\kappa \epsilon_0 A}$$

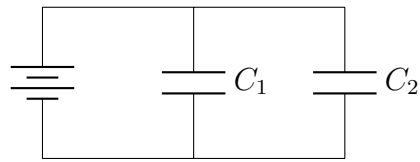
$$\Delta V_{\kappa} = E_{\kappa} d = \frac{Qd}{\kappa \epsilon_0 A}$$


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## Capacitors in Series and Parallel



(a) Wired in series.



(b) Wired in parallel.

**Differences** Capacitors wired in series and parallel behave differently. Capacitors wired in series carry the same charge, but different voltages.

$$q_{eq(s)} = q_1 = q_2 = \cdots = q_n$$

Capacitors wired in parallel behave in the opposite manner, carrying the same voltage, but different charges.

$$\Delta V_{eq(||)} = \Delta V_1 = \Delta V_2 = \cdots = \Delta V_n$$

**Equivilant Capacitance** Multiple capacitors can be represented by a single capacitor with an equivilant capacitance. The equivilant capacitance of capacitors in series and in parallel is given by the following equations:

$$C_{eq(s)} = \sum_{i=1}^n C_i$$
$$\frac{1}{C_{eq(||)}} = \sum_{i=1}^n \frac{1}{C_i}$$



# Current and Resistance

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## Current

**Definition** Current is defined as the time rate of change of charge through an object. It is measured in amps.

$$I = \frac{dQ}{dt} \Rightarrow I = nqv_d A$$

Where  $I$  is the current in amps  $[A] = [\frac{C}{s}]$ ,  $n$  is the free electron density in electrons/ $m^3$ ,  $v_d$  is the drift velocity in  $m/s$ , and  $A$  is the cross-sectional area of the conductor in  $m^2$ .

**Drift Velocity** Electrons are bouncing around randomly. When an electric field is applied, the bouncing is directed in a direction, but it is still chaotic. This bouncing results in heat being generated. Heat is defined as the kinetic energy of a particle. The speed of the drift of the electrons is called the drift velocity ( $v_d$ ).

$$v_d = \frac{I_{avg}}{nqA} = \frac{I}{nqA}$$

Where  $v_d$  is the drift current,  $n$  is the free electron density in  $\frac{g}{m^2}$ ,  $q$  is the charge of the current carrier (usually an electron) in  $C$ , and  $A$  is the cross-sectional area of the conductor in  $m^2$ .

**Current Density** Current density is the current per unit area. It can be calculated using the following formula:

$$J = \frac{I}{A} = \sigma A = nqv_d$$

Where  $J$  is the current density in  $Amps/m^2$ ,  $I$  is the current in amps,  $A$  is the cross-sectional area of the conductor in  $m^2$ ,  $\sigma$  is the conductivity of the material, and  $n$  is the free electron density in electrons/ $m^3$ .

Voltage can be calculated as a function of current density and conductivity as follows:

$$\Delta V = E\ell = \frac{\ell J}{\sigma}$$

Where  $\Delta V$  is the voltage in volts,  $E$  is the electric field in volts per meter,  $\ell$  is the length of the conductor in meters,

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## Resistance

**Definition** Resistance is defined as the ratio of voltage to current, also known as Ohm's law. It is measured in ohms.

$$R = \frac{\Delta V}{I}$$

Where  $R$  is the resistance in  $\Omega$ ,  $\Delta V$  is the voltage in volts, and  $I$  is the current in  $A$ .

**Resistivity** Resistivity is the fundamental property of a material that determines how much it resists the flow of current. It is measured in ohm-meters [ $\Omega m$ ].

$$\rho = \frac{1}{\sigma} = \frac{RA}{\ell} \implies R = \rho \frac{\ell}{A}$$

Where  $\rho$  is resistivity in  $\Omega m$ ,  $\sigma$  is conductivity in  $S/m$ ,  $R$  is resistance in  $\Omega$ ,  $A$  is the cross-sectional area of the conductor in  $m^2$ , and  $\ell$  is the length of the conductor in meters.

**Conductivity** Conductivity is the inverse of resistivity. It is measured in Siemens per meter [ $\frac{S}{m}$ ].  $[S] = [\frac{1}{\Omega}]$

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA} \implies R = \frac{\ell}{\sigma A}$$

Where  $\rho$  is resistivity,  $\sigma$  is conductivity,  $R$  is resistance in  $\Omega$ ,  $I$  is current in amps, and  $\Delta V$  is voltage in volts.

**Ohmic vs. Non-Ohmic devices** Ohmic devices are devices that have a Voltage vs Current slope of  $\frac{1}{R}$ . Non-ohmic devices have a slope that changes with voltage or current.

**Resistivity and Temperature** The resistivity of a material changes with the temperature of the material. The equation for resistivity as a function of temperature is given by:

$$\rho_t = \rho_0[1 + \alpha\Delta T]$$
$$\alpha = \frac{\Delta\rho/\rho_0}{\Delta T}$$

Where  $\rho_t$  is the resistivity of the material at a given temperature in  $\Omega m$ ,  $\rho_0$  is the resistivity of the material at a reference temperature in  $\Omega m$ ,  $\alpha$  is the temperature coefficient in  $^{\circ}C^{-1}$  (a material constant, much like  $\kappa$ ),  $\Delta\rho$  is the change in resistivity in  $\Omega m$ , and  $\Delta T$  is the change in temperature in  $^{\circ}C$ .

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## Electrical Power

**Definition** Power is defined as the rate at which energy is transferred or converted. It is measured in watts.

$$P = \frac{dU_e}{dt} = I^2 R = \frac{(\Delta V)^2}{R}$$

Where  $P$  is power in watts,  $U_e$  is electric potential energy in Joules,  $\Delta V$  is voltage in volts, and  $I$  is current in amps.