Econ 204 // Spring 2024

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**Quantitative Variables in Finite Populations and Samples:**

**Notation, Definitions, and Facts**

**I. QUANTITATIVE VARIABLES IN POPULATIONS**

Notation

A ***finite population*** consists of \(N\) elements, labelled \(I = 1,...,N\), where \(N\) is a real number.

\(X\) is the name of a quantitative variable (QVAR).

For \(I = 1,...,N\), \({X\_I}\) is the value of \(X\) for elements \(I\).

Definition: The population mean of a QVAR

The ***population mean of a QVAR*** \(X\) is represented by the symbol \({\mu \_X}\) , and defined as

\({\mu \_X} = \frac{{\sum\limits\_{I = 1}^N {{X\_I}} }}{N}\)

Definition: A deviation from the population mean of a QVAR for a given element

A ***deviation from the population*** ***mean of a QVAR*** \(X\) ***for a given element*** is the difference between the value of \(X\) for the given element and the population mean of \(X.\) The deviation from the population mean of \(X\) for element \(I\)is represented by the symbol \({D\_{{X\_I}}}\):

\({D\_{{X\_I}}} = {X\_I} - {\mu \_X}\)

Fact

For any finite population and any QVAR \(X\), if you find the deviation from the population mean of \(X\) for every element in the population, and then add up all those deviations, you will find that the sum is zero.

To put that more succinctly: for any finite population and any quantitative variable \(X\), the sum of the deviations from the means is always equal to 0.

This is true no matter how many elements there are in the population and no matter what the values of \(X\) are for the elements.

More formally: Given a finite population consisting of any number \(N\) of elements, and given any values \({X\_1},\,{X\_2},\,...,\,{X\_N}\) of a QVAR \(X\),

\(\sum\limits\_{I = 1}^N {{D\_{{X\_I}}}} = 0\)

or equivalently,

\(\sum\limits\_{I = 1}^N {\left( {{X\_I} - {\mu \_X}} \right)} = 0\)

Proof of this fact

We can prove this equality through several steps of algebra. Every step is just a rearrangement of terms.

\[\begin{array}{l}\sum\limits\_{I = 1}^N {{D\_{{X\_I}}}} = \sum\limits\_{I = 1}^N {\left( {{X\_I} - {\mu \_X}} \right)} \\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = \sum\limits\_{I = 1}^N {{X\_I}} - \sum\limits\_{I = 1}^N {{\mu \_X}} \\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = \sum\limits\_{I = 1}^N {{X\_I}} - N{\mu \_X}\\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = N\left( {\frac{{\sum\limits\_{I = 1}^N {{X\_I}} }}{N}} \right) - N{\mu \_X}\\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = N{\mu \_X} - N{\mu \_X}\\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = 0\end{array}\]

Definition: The population mean absolute deviation of a QVAR

The ***population mean absolute deviation*** ***of a QVAR*** \(X\) is represented by the symbol \[MA{D\_X}\], and defined as

\[MA{D\_X} = \frac{{\sum\limits\_{I = 1}^N {\left| {{X\_I} - {\mu \_X}} \right|} }}{N}\]

Definition: The population variance of a QVAR

The ***population variance*** ***of a QVAR*** \(X\) is represented by the symbol \[\sigma \_X^2\], and defined as

\[\sigma \_X^2 = \frac{{\sum\limits\_{I = 1}^N {{{\left( {{X\_I} - {\mu \_X}} \right)}^2}} }}{N}\]

Definition: The population standard deviation of a QVAR

The ***population standard deviation*** ***of a QVAR*** \(X\) is represented by the symbol \[{\sigma \_X}\], and defined as

\[{\sigma \_X} = \sqrt[ + ]{{\sigma \_X^2}}\]

**II. QUANTITATIVE VARIABLES IN SAMPLES**

The notation we use for samples is different from the notation we use for populations.

For some concepts, the definitions for samples are identical to the definitions for populations, and the only differences are in the notation.

For other concepts, the differences are not only in the notation—the definitions for samples are similar, but not identical, to the definitions for populations.

Notation

A ***sample*** consists of \(n\) observations, labelled \(i = 1,...,n\).

\(X\) is the name of a quantitative variable.

For \(i = 1,...,n\), \({x\_i}\) is the value of \(X\) for observation \(i\).

Definition: The sample mean of a QVAR

The ***sample mean of a QVAR*** \(X\) is represented by the symbol \(\bar x\) , and defined as

\(\bar x = \frac{{\sum\limits\_{i = 1}^n {{x\_i}} }}{n}\)

Definition: A deviation from the sample mean of a QVAR for a given observation

A ***deviation from the sample mean of a QVAR*** \(X\) ***for a given observation*** is the difference between the value of \(X\) for the given observation and the population mean of \(X\). The deviation from the sample mean of \(Q\) for observation \(i\) is represented by the symbol \({d\_{{X\_i}}}\):

\({d\_{{X\_i}}} = {x\_i} - \bar x\)

Fact

For every sample and any QVAR \(X\), if you find the deviation from the sample mean of \(X\) for every observation in the sample, and then add up all those deviations, you will find that the sum is zero.

To put that more succinctly: for any sample and any quantitative variable \(X\), the sum of the deviations from the means is always equal to 0.

This is true no matter how many observations there are in the sample and no matter what the values of \(X\) are for the observations.

More formally: Given a sample consisting of any number \(n\) of observations, and given any values \({x\_1},\,{x\_2},\,...,\,{x\_n}\) of a QVAR \(X\),

\(\sum\limits\_{i = 1}^n {{d\_{{X\_i}}}} = 0\)

Proof of this fact

We can prove this equality through several steps of algebra. Every step is just a rearrangement of terms.

\[\begin{array}{l}\sum\limits\_{i = 1}^n {{d\_{{X\_i}}}} = \sum\limits\_{i = 1}^n {\left( {{x\_i} - \bar x} \right)} \\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = \sum\limits\_{i = 1}^n {{x\_i}} - \sum\limits\_{i = 1}^n {\bar x} \\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = \sum\limits\_{i = 1}^n {{x\_i}} - n\bar x\\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = n\left( {\frac{{\sum\limits\_{i = 1}^n {{x\_i}} }}{n}} \right) - n\bar x\\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = n\bar x - n\bar x\\\,\,\,\,\,\,\,\,\,\,\,\,\,\,\, = 0\end{array}\]

Definition: The sample mean absolute deviation of a QVAR

The ***sample mean absolute deviation*** ***of a QVAR*** \(X\) is represented by the symbol \[ma{d\_X}\], and defined as

\[ma{d\_X} = \frac{{\sum\limits\_{i = 1}^n {\left| {{x\_i} - \bar x} \right|} }}{{n - 1}}\]

Definition: The sample variance of a QVAR

The ***sample variance*** ***of a QVAR*** \(X\) is represented by the symbol \[s\_X^2\], and defined as

\[s\_X^2 = \frac{{\sum\limits\_{i = 1}^n {{{\left( {{x\_i} - \bar x} \right)}^2}} }}{{n - 1}}\]

Definition: The sample standard deviation of a QVAR

The ***sample standard deviation*** ***of a QVAR*** \(X\) is represented by the symbol \[{s\_X}\], and defined as

\[{s\_X} = \sqrt[ + ]{{s\_X^2}}\]