Assignment #1

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ECE 651

09/14/2025

Question 2.5

5. The sinusoidal signal $\cos(\omega_0 n + \theta_0)$ is periodic in n if the normalized frequency $f_0 \triangleq \frac{\omega_0}{2\pi}$ is a rational number, that is, $f_0 = \frac{M}{N}$, where M and N are integers.
   (a) Prove the above result.
   (b) Generate and plot (use the `stem` function) $\cos(0.1n - \pi/5)$, $-20 \leq n \leq 20$. Is this sequence periodic? Can you conclude periodicity from the plot?
   (c) Generate and plot (use the `stem` function) $\cos(0.1\pi n - \pi/5)$, $-10 \leq n \leq 20$. Is this sequence periodic? If it is, what is the fundamental period. What interpretation can you give to the integers M and N?

a)

A function $x(t)$ is periodic if $x(t) = x(t+T)\ \forall\ t$

For the signal $cos(\omega_0 n + \theta_0)$ to be periodic, the argument of the cosine must be off by $\pm 2\pi$one time period later. Neglecting θ_0, the argument is:

$$2\pi \frac{M}{N} n$$

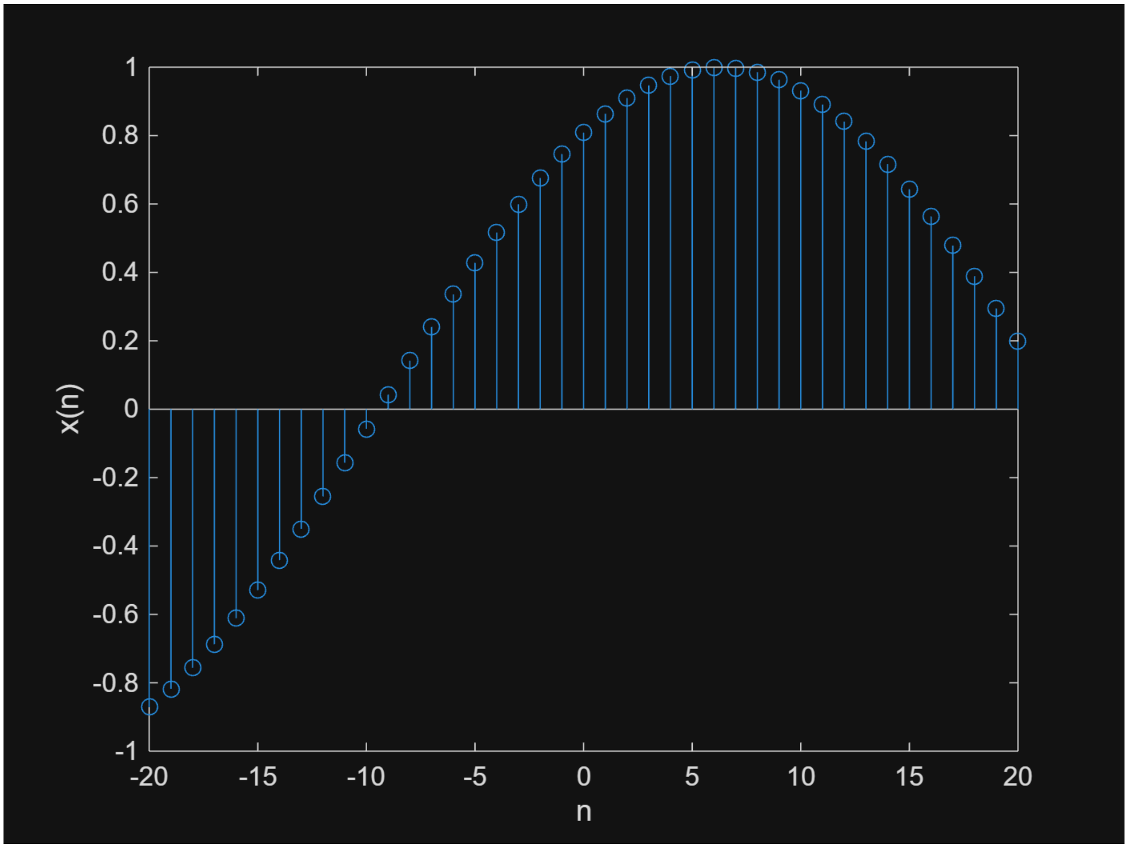
The time period N_0 can be defined as the inverse of f_0 which here is $\frac{N}{M}$. Replacing n with $(n+N_0)$, we get:

$$2\pi \frac{M}{N}(n+N_0) = 2\pi \frac{M}{N}\left(n+\frac{N}{M}\right)...$$
$$= 2\pi \frac{M}{N} n + 2\pi$$

An irrational number does not have this relationship, therefore the normalized frequency must be rational.

b)

```
n   = -20:20;
x_n = cos(0.1*n - pi/5);
stem(n, x_n)
xlabel('n');
ylabel('x(n)');
```



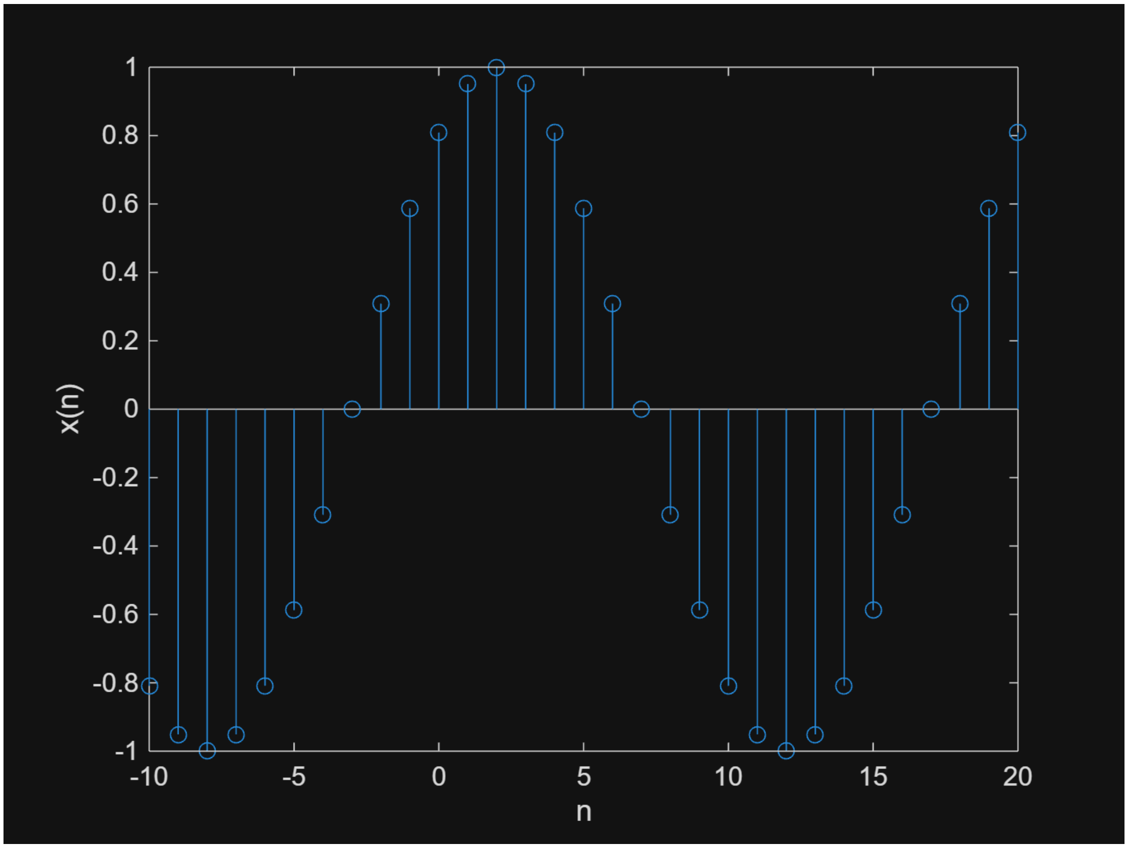
This sequence is **NOT** periodic. We can conclude that this signal is not periodic by considering ω_0:

$$\omega_0 = 0.1$$
$$2\pi f_0 = 0.1$$
$$f_0 = \frac{0.1}{2\pi}, \text{ Not Ratic}$$

However, we are not able to conclude from the graph alone due to the chart not capturing the entire period.

c)

```
n   = -10:20;
x_n = cos(0.1*pi*n-pi/5);
stem(n, x_n);
xlabel('n')
ylabel('x(n)');
```



This sequence **IS** periodic and the fundamental period is 20. We can determine this by considering ω_0:

$$
\begin{aligned}
&\omega_0 = 0.1\pi \\
&2\pi f_0 = 0.1\pi \\
&f_0 = \frac{0.1}{2} \rightarrow \frac{1}{20}, \text{ Ratior} \\
&N_0 = \frac{1}{f_0} = 20
\end{aligned}
$$

Question 2.6

6. This problem uses the sound file "handel" available in MATLAB. This sound is sampled at $F_s = 8192$ samples per second using 8-bits per sample.
   (a) Load the sound waveform "handel" in an array `x` and listen to it using the `sound` function at the full sampling rate.
   (b) Select every other sample in `x` which reduces the sampling rate by a factor of two. Now listen to the new sound array using the `sound` function at half the sampling rate.
   (c) Select every fourth sample in `x` which reduces the sampling rate by a factor of four. Listen to the resulting sound array using the `sound` function at quarter the sampling rate.
   (d) Save the generated sound in part (c) using the `wavwrite` function.

Uncomment the desired sound command or the audiowrite command to verify results.

a)

Load file and play sound.

```
x = load("handel.mat");
%sound(x.y, x.Fs)
```

b)

Reduce the sample by a factor of two.

```
y_halfsample  = x.y(1:2:end);
Fs_halfsample = x.Fs / 2;
%sound(y_halfsample, Fs_halfsample)
```

c)

Reduce the sample by factor of four.

```
y_quartersample = x.y(1:4:end);
Fs_quartersample = x.Fs / 4;
%sound(y_quartersample, Fs_quartersample);
```

d)

Save the audio file.

```
% wavwrite has been deprecated, using MATLAB available function instead
%audiowrite("handel_quarter_sample.flac", y_quartersample, Fs_quartersample)
```

Question 2.11

11. A system is implemented by the statements

```
y1=conv(ones(1,5),x);
y2=conv([1 -1 -1 -1 1],x);
y=conv(ones(1,3),y1+y2);
```

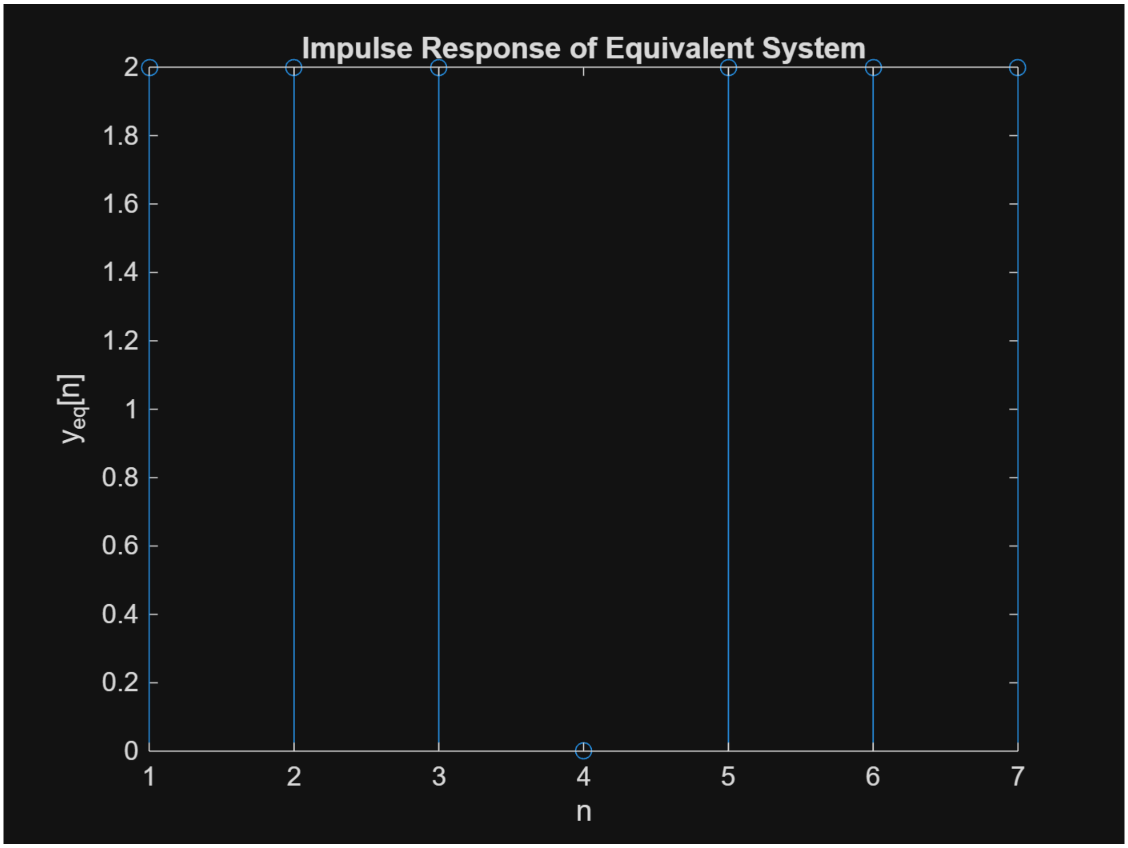
(a) Determine the impulse response of the equivalent system `y=conv(h,x)`.
(b) Compute and compare the step responses of the two equivalent system representations.

a)

The equivalent system can be determined by add the "h" of the first two subsystems and convolving it with the third subsystem:

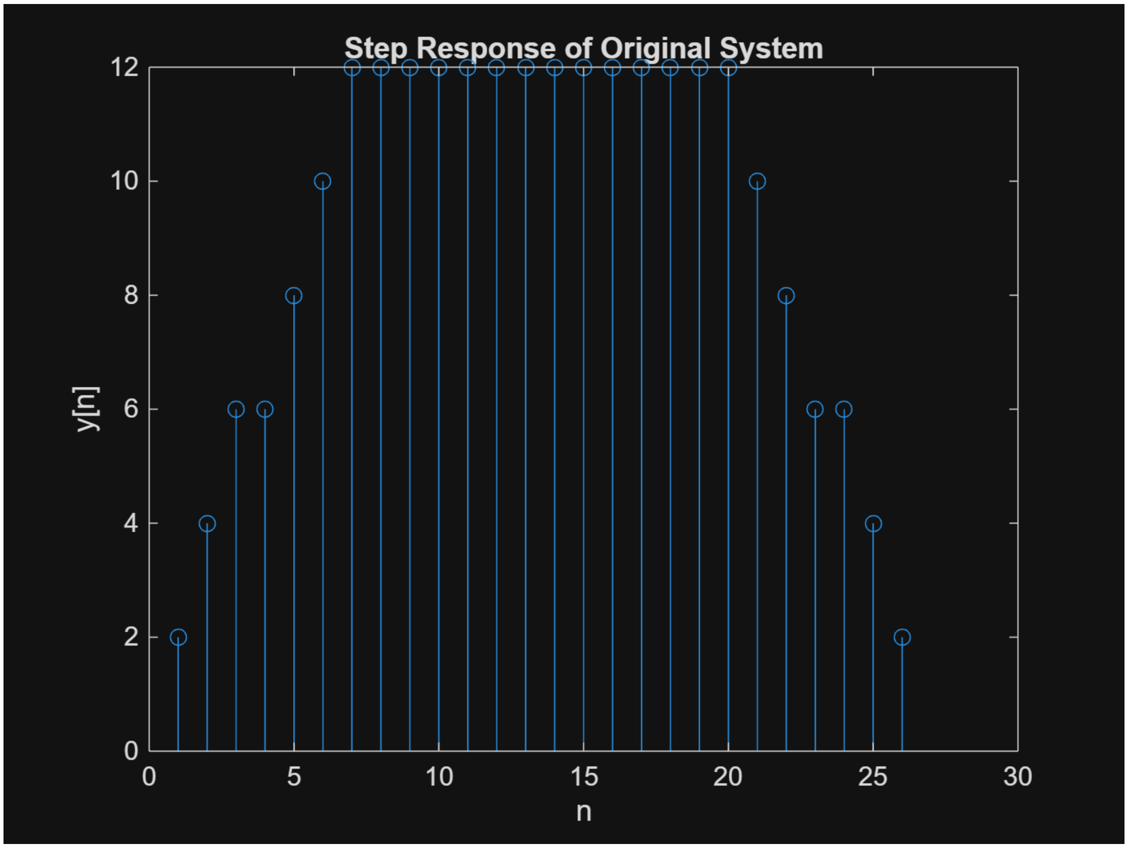
```
% Equivalent system
x = 1;
h1 = ones(1,5);
h2 = [1, -1, -1, -1, 1];
h  = conv(ones(1,3), h1+h2);
y_eq = conv(h, x);
figure;
stem(y_eq)
xlabel("n")
```

```
ylabel("y_{eq}[n]")
title("Impulse Response of Equivalent System")
```

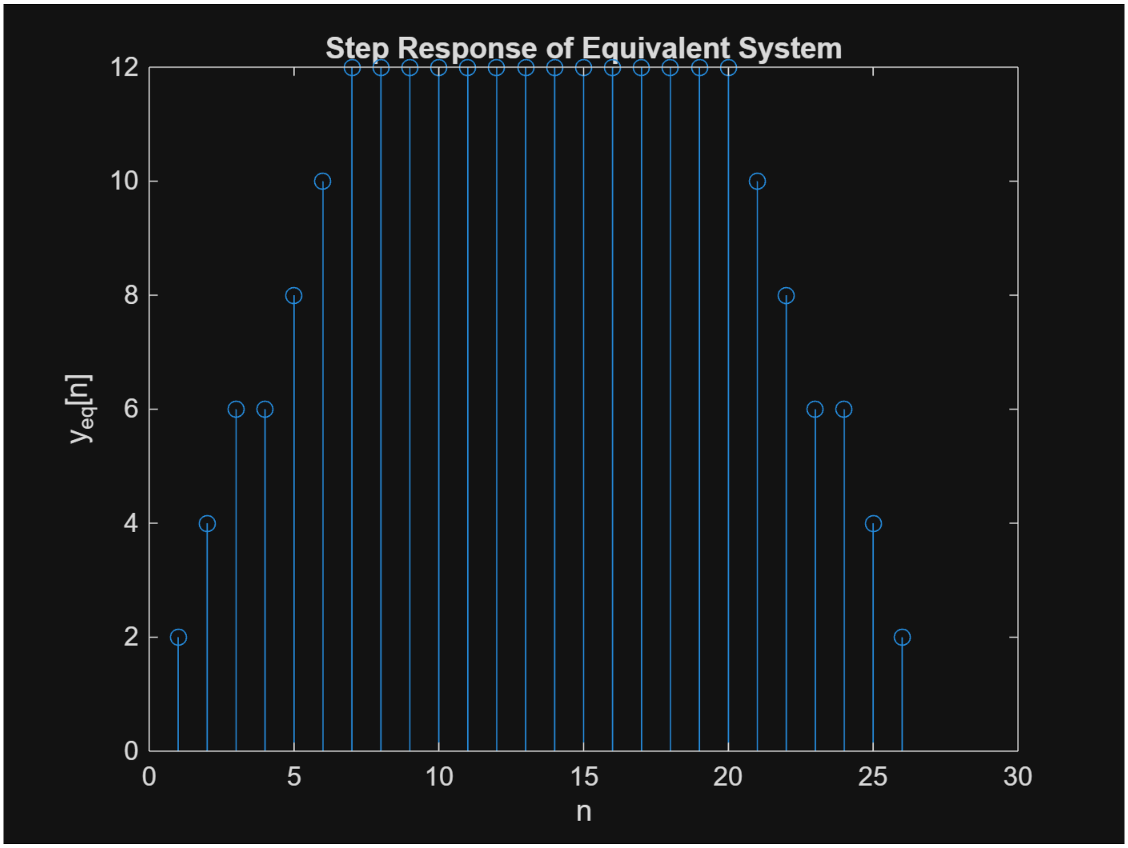


b)

```
% Original system
N = 20; x = ones(1,N); % Limit step response to finite value
y1 = conv(ones(1,5), x);
y2 = conv([1 -1 -1 -1 1], x);
y  = conv(ones(1,3), y1+y2);
figure;
stem(y)
xlabel("n")
ylabel("y[n]")
title("Step Response of Original System")
```




```
% Equivalent System
y_eq  = conv(h, x);
figure;
stem(y_eq)
xlabel("n")
ylabel("y_{eq}[n]")
title("Step Response of Equivalent System")
```



Question 2.19

19. A recursive implementation of reverberation is given by (2.103) which is given below

$$y[n] = x[n] + ay[n - D],$$

where $D = \tau F_s$ is the delay in sampling interval given the delay τ in seconds and sampling rate F_s and a is an attenuation factor. To generate digital reverberation we will use the sound file `handel` which is recorded at $F_s = 8192$ samples per second. (See Problem 6 for using this file.)

(a) For $\tau = 50$ ms and $a = 0.7$, obtain a difference equation for the digital reverberation and process the sound in `handel`. Comment on its audio quality.

(b) Repeat (a) for $\tau = 100$ ms.

(c) Repeat (a) for $\tau = 500$ ms.

(d) Which implementation sounds natural?

This system can be described as a filter by taking the z-transform of the difference equation:

$$y[n] = x[n] + ay[n-D]$$
$$y[n] - ay[n-D] = x[n]$$
$$(1 - az^{-D})Y(z) = X(z)$$
$$H(z) = \frac{1}{1 - az^{-D}}$$

a)

```
% Load handel
x = load("handel.mat");
```

```
% Define Parameters
a   = 0.7;
tau = 50e-3;
D   = ceil(tau*x.Fs);

% Apply filter
num   = 1;
den   = [1, zeros(1,D-1), -a];
y_rev = filter(num, den, x.y);
%sound(y_rev, x.Fs)
```

b)

```
% tau = 100 ms
tau = 100e-3;
D   = ceil(tau*x.Fs);

% Apply filter
num   = 1;
den   = [1, zeros(1,D-1), -a];
y_rev = filter(num, den, x.y);
%sound(y_rev, x.Fs)
```

c)

```
% tau = 500 ms
tau = 500e-3;
D   = ceil(tau*x.Fs);

% Apply filter
num   = 1;
den   = [1, zeros(1,D-1), -a];
y_rev = filter(num, den, x.y);
%sound(y_rev, x.Fs)
```

d)

When the tau is 50 ms, that is when the audio sounds most natural.

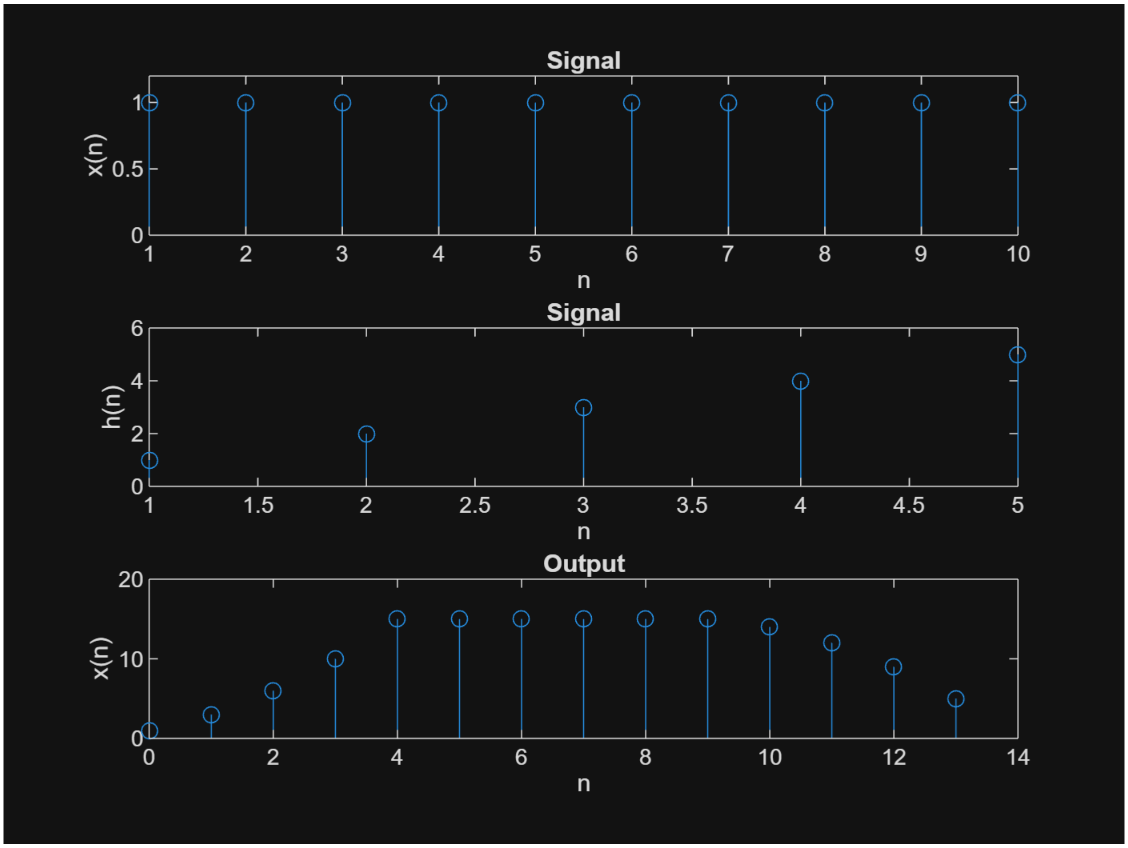
Question 2.25

25. Consider the finite duration sequences $x[n] = u[n] - u[n-N]$ and $h[n] = n(u[n] - u[n-M])$, $M \leq N$.
    (a) Find an analytical expression for the sequence $y[n] = h[n] * x[n]$.
    (b) Verify the result in (a) for $N = 10$ and $M = 5$ using function `y=conv(h,x)`.

b)

```
N = 10; M = 5;
x = ones([1,N]);
```

```
h = 1:M;
y = conv(h,x);

% Plot the convolution result
n_conv = 0:length(y)-1;
figure;

% Signal plot
subplot(3,1,1)
stem(x);
ylim([0,1.2]);
xlabel('n');
ylabel('x(n)');
title('Signal');

% impulse response plot
subplot(3,1,2)
stem(h);
xlabel('n');
ylabel('h(n)');
title('Signal');

% Signal plot
subplot(3,1,3)
stem(n_conv, y);
ylim([0,20]);
xlabel('n');
ylabel('x(n)');
title('Output');
```



Question 2.34

34. A system is described by the difference equation

$$y[n] = x[n] - 0.9y[n-1] + 0.81y[n-2]. \qquad (2.121)$$

Using MATLAB determine and plot

(a) Impulse response of the system.

(b) Step response of the system.

(c) Identify the transient response and the steady-state response in (b).

An effecient method of determing the impulse and step response using matlab is to identify the transfer function of the system in the z-domain:

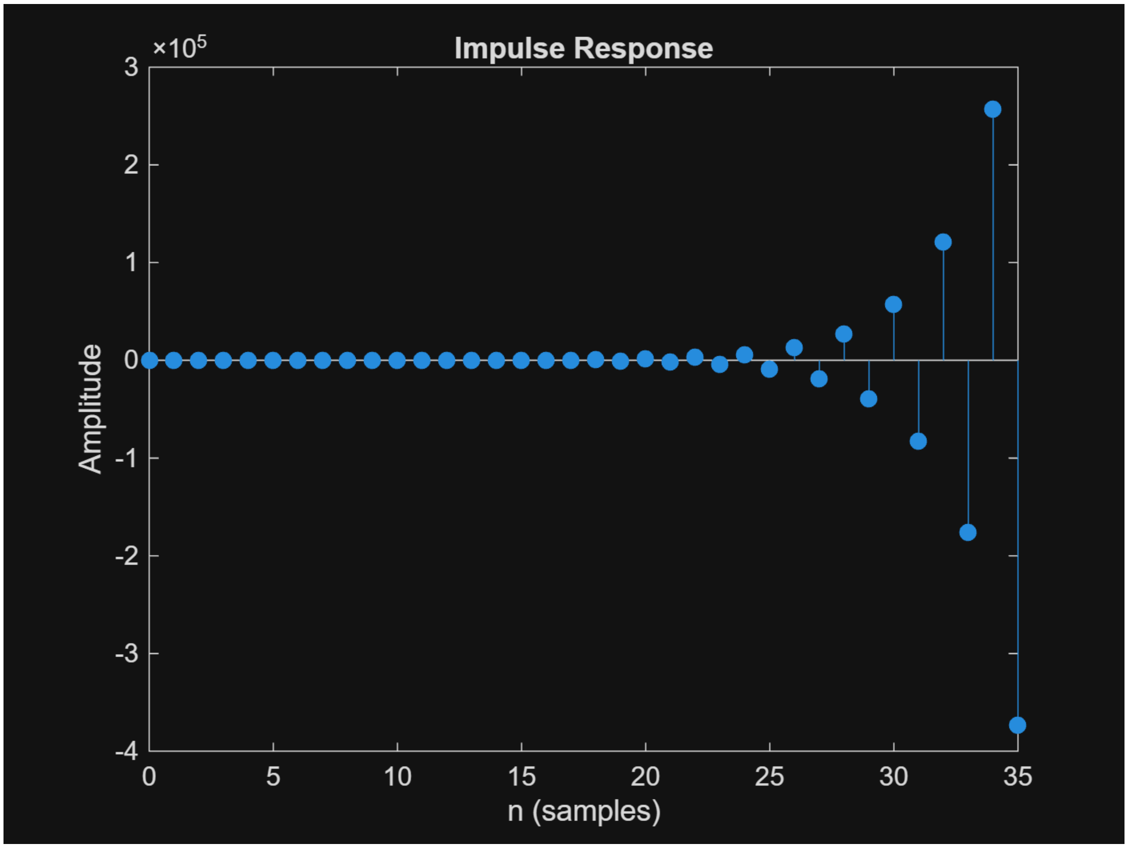
$$\begin{aligned}
&y[n] = x[n] - 0.9y[n-1] + 0.81y[n-2] \\
&y[n] + 0.9y[n-1] - 0.81y[n-2] = x[n] \\
&(1 + 0.9z^{-1} - 0.81z^{-2})Y(z) = X(z) \\
&\frac{Y(z)}{X(z)} = \frac{1}{1 + 0.9z^{-1} - 0.81z^{-2}}
\end{aligned}$$

The system can now be designed in matlab by using the coeffecients of the transfer function.

a)

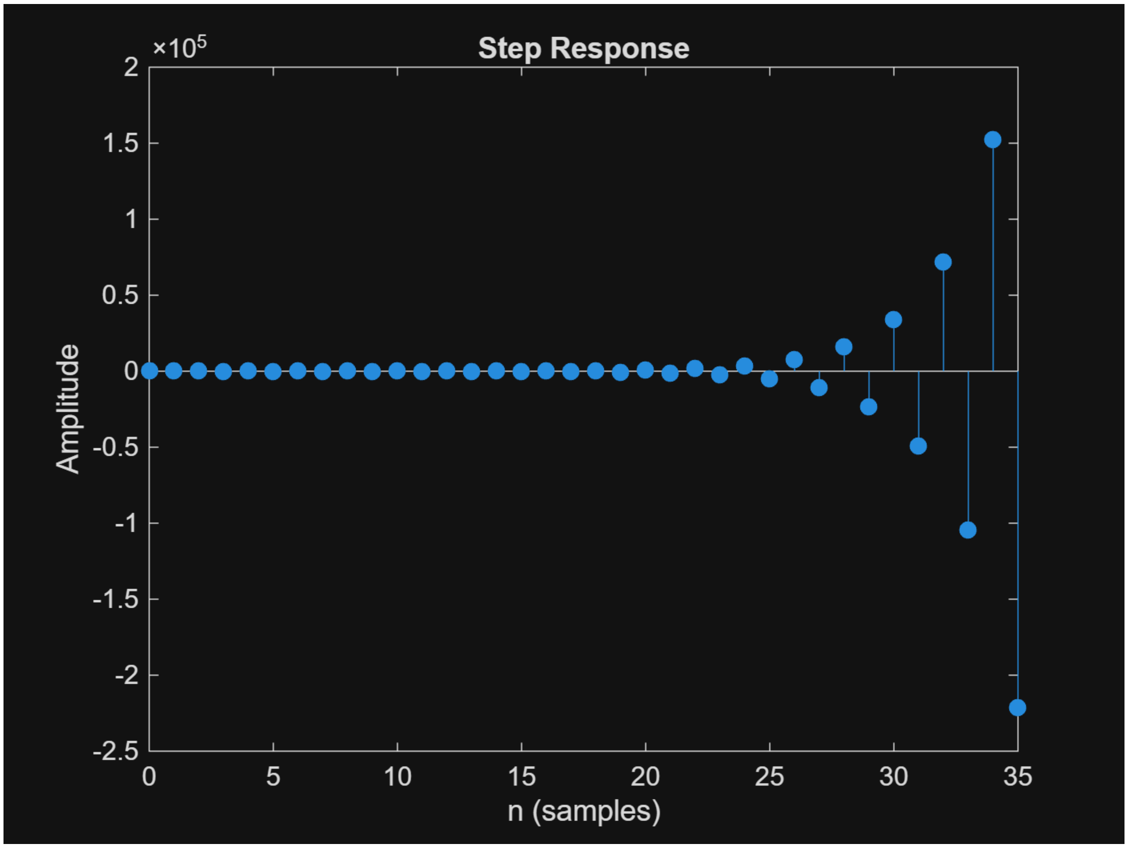
```
% Define system
b = 1;                  % numerator coeffecients
a = [1, 0.9, -0.81]; % denominator coeffecients

% Impulse Response
```

```
figure;
impz(b,a)
```



b)

```
% Step response
figure;
stepz(b,a)
```



c)

The system defined is unstable, therefore the entire response is transient and there is no steady state.

Question 2.41

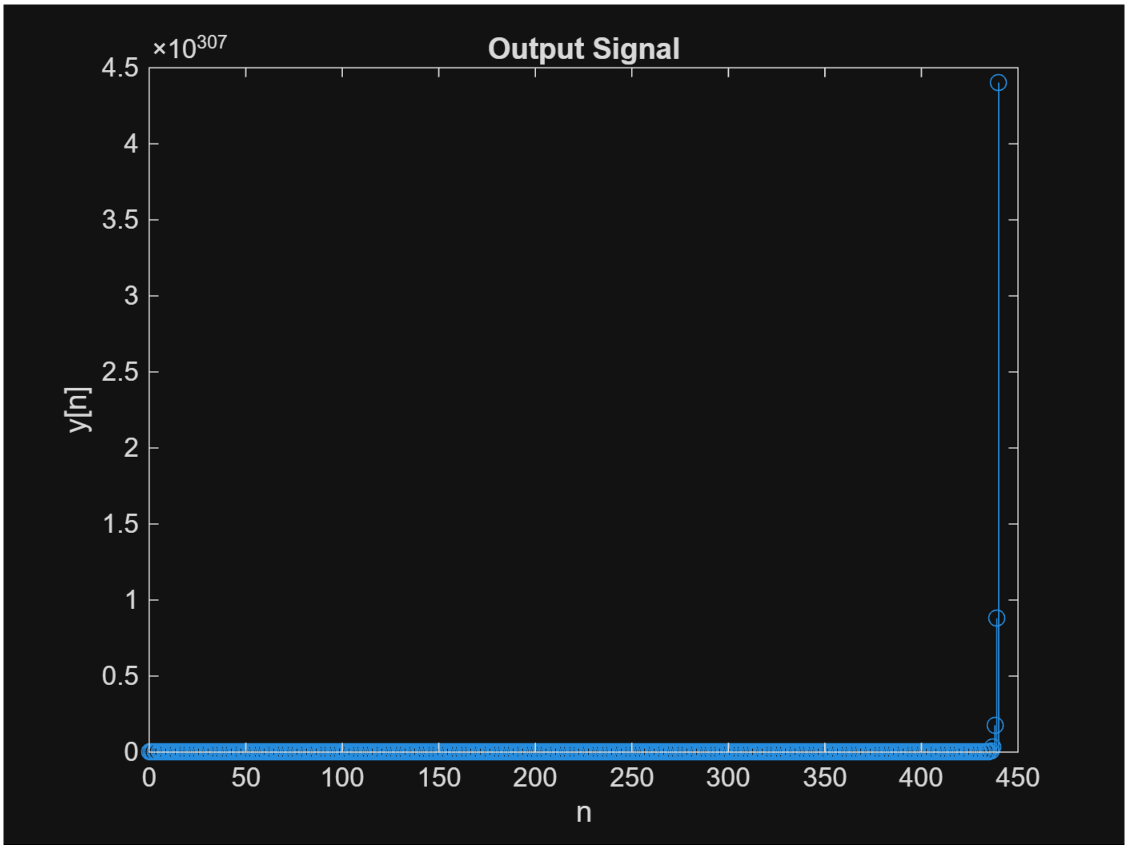
41. Write a MATLAB function to compute and plot the output of the discrete-time system

$$y[n] = 5y[n-1] + x[n], \quad y[-1] = 0$$

for $x[n] = u[n]$, $0 \leq n \leq 1000$. Based on these results can you make a statement regarding the stability of the system? Hint: Check the value $y[600]$.

```
%% Using recursion to solve
n = 0:1e3;

% Input signal
x = ones([1,length(n)]); % x[n] = u[n]

% Set up output signal
y = zeros([1,length(n)]);

% Initial condition
y_0  = 0;
y(1) = 5*y_0 + x(1);
```

```
% recursively solve system
for i=2:length(n)
   y(i) = 5*y(i-1) + x(i);
end

% Plot output signal
figure;
stem(n,y)
xlabel("n")
ylabel("y[n]")
title("Output Signal")
```



As can be seen from the resulting plot, the system is **UNSTABLE**. This can be clear when we display the value at n = 600.

```
fprintf("y[600] = %f",y(600))
```

```
y[600] = Inf
```