Assignment #2

Connor Cavarretta

ECE 651

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Question 3.2

2. The `filter` function in MATLAB can be used to verify the z-transform expression of a causal sequence. Let $x[n]$ be a causal sequence with a rational $X(z) \triangleq B(z)/A(z)$ expression.
   (a) Show that the fragment

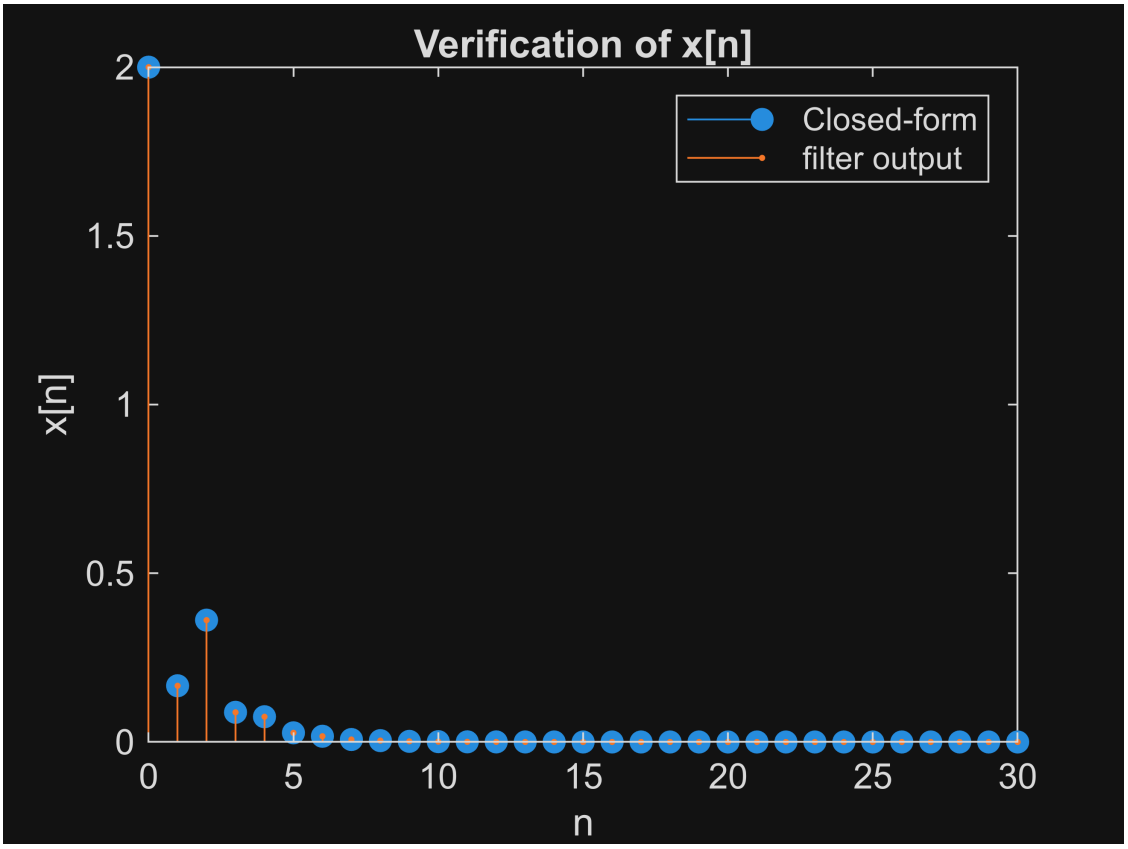
   ```
   x=filter(b,a,[1,zeros(1,N)]);
   ```

   will generate the first `N+1` samples of $x[n]$ where `b` and `a` contain polynomial coefficients of $B(z)$ and $A(z)$, respectively.
   (b) Let $x[n] = \left[\left(\frac{1}{2}\right)^n + \left(-\frac{1}{3}\right)^n\right] u[n]$. Determine $X(z)$.
   (c) Verify your expression in (b) using MATLAB by comparing output of the `filter` function with the given sequence.

```
a1 = [1 -0.5];
a2 = [1  1/3];
A  = conv(a1, a2);      % denominator
B  = a1 + a2;           % numerator

%% Generate samples with filter
N = 30;
imp = [1 zeros(1,N)];
x_filter = filter(B, A, imp);

%% Compare with closed-form sequence
n = 0:N;
x_true = (1/2).^n + (-1/3).^n;

disp(['Max abs error = ', num2str(max(abs(x_filter - x_true)))])
```

```
Max abs error = 3.4694e-18
```

```
figure;
stem(n, x_true, 'filled'); hold on
stem(n, x_filter, '.')
xlabel('n'); ylabel('x[n]')
legend('Closed-form','filter output')
title('Verification of x[n]')
```



Question 3.27

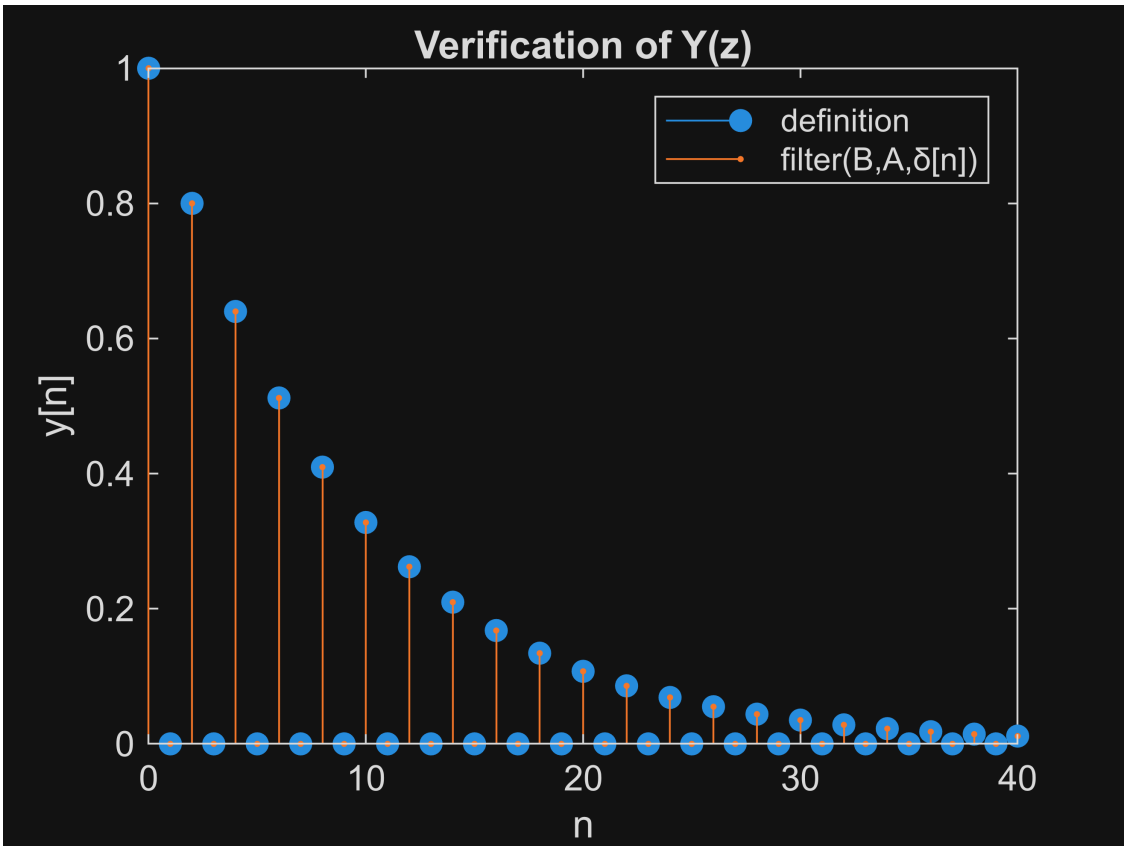
27. Let $x[n] = 0.8^n u[n]$ and let

$$y[n] = \begin{cases} x[n/2], & n = 0, \pm 2, \pm 4, \ldots \\ 0. & \text{otherwise} \end{cases}$$

(a) Show that $Y(z) = X(z^2)$.
(b) Determine $Y(z)$.
(c) Using MATLAB verify that $y[n]$ has the z-transform $Y(z)$. (Hint: See Problem 2.)

```
%% Determine Y(z) = B(z)/A(z)
B = 1;                % numerator
A = [1 0 -0.8];       % denominator

%% Verify
N = 40;
imp = [1 zeros(1,N)];
y_filter = filter(B, A, imp);

n = 0:N;
x = (0.8).^n;
y_def = zeros(1,N+1);
y_def(1:2:end) = x(1:floor(N/2)+1);
```

```
figure;
stem(n, y_def, 'filled'); hold on
stem(n, y_filter, '.')
xlabel('n'); ylabel('y[n]')
legend('definition','filter(B,A,δ[n])'); title('Verification of Y(z)')
```



Question 3.29

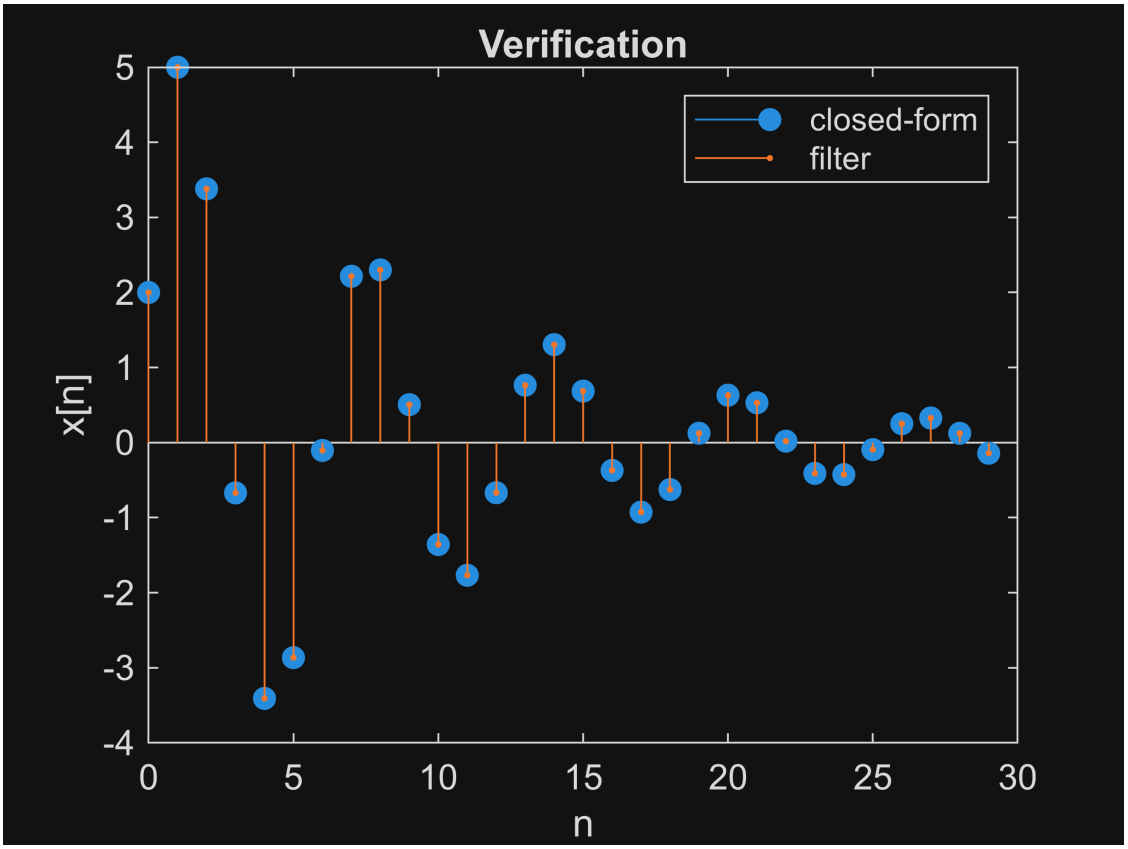
29. The z-transform of a signal $x[n]$ is given by

$$X(z) = \frac{2z^2 + 3z}{z^2 - z + 0.81}, \quad |z| > 0.9$$

(a) Express $x[n]$ as a real-valued signal.
(b) Using MATLAB, determine the first 30 samples of $x[n]$ and compare them with your answer in (a). (See Problem 2.)

```
%% Real-time signal x[n] from X(z)
% X(z) = (2 + 3 z^{-1}) / (1 - z^{-1} + 0.81 z^{-2}),  ROC: |z|>0.9 (causal)
rho = 0.9;                                  % pole radius
theta = acos(5/9);                          % angle; cos(theta)=0.5/0.9
k = 2.672612419;                            % = 10/sqrt(14)
% x[n] = 2*rho^n*(cos(n*theta) + k*sin(n*theta)) u[n]
```

```
%% First 30 samples via filter and compare to (a)
b = [2 3]; a = [1 -1 0.81];
N = 29; n = 0:N;

x_filter = filter(b, a, [1 zeros(1,N)]);
x_closed = 2 * (rho.^n) .* (cos(n*theta) + k*sin(n*theta));

figure;
stem(n, x_closed, 'filled'); hold on
stem(n, x_filter, '.'); xlabel n; ylabel('x[n]')
legend('closed-form','filter'); title('Verification')
```



Question 3.43

43. A difference equation is given by

$$y[n] = x[n] - x[n-1] + 0.81y[n-2],\ n \geq 0$$

with initial conditions $y[-1] = y[-2] = 2$ and excited by $x[n] = (0.7)^n u[n+1]$.

(a) Determine the solution $y[n]$, $n \geq 0$.

(b) Generate the first 50 samples of $y[n]$ using MATLAB and compare these samples with those in (a) above.

```
%% Generate samples
b = [1 -1 0];                % x[n] - x[n-1]
a = [1 0 -0.81];             % y[n] - 0.81 y[n-2]
```

```
N = 49; n = 0:N;

x = (0.7).^n;
x_m1 = 1/0.7; x_m2 = 0;
zi = filtic(b, a, [2 2], [x_m1 x_m2]);
y_filter = filter(b, a, x, zi);

C1 = 261/350; C2 = -1179/5600; A = 21/32;
y_closed = C1*(0.9).^n + C2*(-0.9).^n + A*(0.7).^n;

figure;
stem(n,y_closed,'filled'); hold on, stem(n,y_filter,'.')
legend('closed-form','filter'); xlabel n; ylabel('y[n]');
```