Bayes Theorem states that the discrimination function used for classification between distributions is:

$$g_i(\mathbf{x}) = \ln\, p(\mathbf{x}|\omega_i) + \ln\, P(\omega_i).$$

The general formula for a multivariate normal distribution is:

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\mathbf{\Sigma}|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^t\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right],$$

Plugging in the equation for a multivariate normal distribution into Bayes discrimination function gives:

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^T\Sigma_i^{-1}(\mathbf{x}-\boldsymbol{\mu}_i) - \frac{1}{2}\ln|\Sigma_i| + \ln P(\omega_i)$$

This equation is the discrimination function for a multivariate normal distribution.

In my code, I use this discrimination function as a way of classifying between points belonging to two different normal distributions.

For both of the distributions (described by a mean and covariance) , the value of the discrimination function is calculated for each point. The point is assigned to the distribution whose discriminant function yields a larger value.

In order to plot the decision boundary between points, I set the discriminant function for both distributions to each other. By finding the intersection, I was able to find the equation of the decision boundary and plot it over the data points.

I run this function on two cases:

Case 1- The two distributions have different means but both have an identity covariance matrix

Case 2- The two distributions have different means and the same, non-identity covariance matrix.

The discrimination function is the same for both, by the way I solve for the decision boundary is different.