Video source: https://www.youtube.com/watch?v=iMjpZwISIlA

Gates introduced here: CNOT, Bell state generator, 2 qubit swap, and toffoli gate

Classical gate vs quantum gate
- Classical gates have only one output regardless of outputs
- Quantum computers have as many outputs as there are bits.
- Quantum gates change the state of a physical quantum system, while a classical gate is just true or false.
- The number of qubits is conserved during the computation, so the number of outputs should equal the number of inputs
- Classical gates are generally not reversible
- While a not gate can be reversible since it’s just a single gate, XOR gates and NAND gates are not reversible due to them having multiple inputs.
- Quantum gates, because they have the same number of outputs as inputs, are reversible.

Combining two Hilbert Spaces
- Qubit and quantum gates are mathematically represented by matrices.
- Individual qubits are vectors living in a 2D hilbert space and quantum gates are linear transformations
- When considering a compositie system comprising of two qubits, whose state vectors belong to the single qubit hilbert spaces Ha and Hb, the state vector of the composite syetsm is then given by the product of the state vectors of the constituent system.

Tensor product takes dimensionality from one 1x2 to 1x4 in example shown.

Because the state vector of composite two qubit system is a 4D vector, there are four possible basis vectors.

00 =	1	01 =	0	10 =	0	11 =	0
	0		1		0		0
	0		0		1		0
	0		0		0		1

General two qubit composite state vector can be written as the superposition of these basis state vectors.

When we have an n qubit system, the most general state will be a superposition of all the 2^n basis states with 2^n normalized complex coefficients

Such operators, like the gate operator, are represented by 2^n by 2^n square matrices.

Two qubit CNOT gate

The quantum CNOT gate is known as the controlled not gate, which is analogous to a standard XOR gate with binary. Here, it does the addition modulo 2 operator (so operator is true when the variables match and false otherwise.

The CNOT gate takes two inputs and two outputs. The two input qubit are designated as the control and target qubit.
Control qubit q1 remains unchanged during gate operation, but target qubit q2 becomes q1 XOR q2. The truth table of this XOR is the same as the XOR seen on the standard XOR gate in binary.

The matrix of the CNOT gate is as follows

1 0 0 0
0 1 0 0
0 0 0 1
0 0 1 0

00 corresponds to the leftmost column, 01 is the next column, and so forth.

Swapping qubits with CNOT gates

2 qubit swap - swaps the states of qubits 1 and 2.
Swap circuit has 3 CNOT gates in the row,
The logic table is the same as XOR

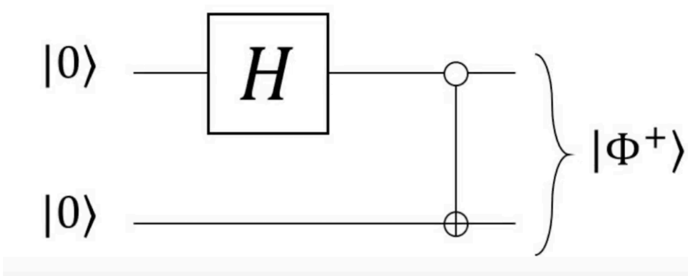
On top of what's already obtained, do A + (A + B) - the A's cancel each other out and what's left is B.

So when having 3 CNOT gates in a row, we actually go from (q1, q2) to (q2, q1)

- T0 (before any gates) = (q1, q2)
- T1 = q1, q1 + q2
- T2: q1, q1 + q2 = q1 + (q1 + q2), q1 + q2), simplifying to q2, q1 + q2
- T3: q2, q1 + q2 = q2(q1 + q2) + q2 = q2, q1

Circuit for Bell states generation

The bell states are two qubit states with maximum entanglement
Property is useful for some applications
Generate this with single qubit gates and CNOT gates



(Image shown around 10:10 in video)
This gate produces (|00> + |11>) / sqrt(2)

Using a combination of hadamard gate and cnot gate, initialized at the beginning.
In the next step, hadamard gate is applied to first qubit, no action on second qubit. The first qubit turns into |+>, represented as

1/sqrt(2) [1]
[1]

Working out the tensor yields a four column value. Apply the CNOT gate to |+> X |0> by doing matrix multiplication to get the new product.
The new vector is a superposition of two qubits

When a Z gate is added on the second qubit) (after the CNOT gate), you'll make the conjugate version. Z gate is a phase change gate and applies a relative phase, which causes the phi minus state.

Replacing that z gate with an x gate is the psi+ gate. This results in (|01> + |10>)/sqrt(2)

Then there's the conjugate, where a z gate is added on top of the present x gate. This gives the conjugate version.

CNOT gate is important because it allows the entanglement of 2 qubits. Represents a new source of computation, enabling quantum systems to process information in ways that would otherwise be impossible.

Toffoli gate, the 3 qubits gate

Toffoli gate is like a controlled CNOT gate, the 3 qubit extension on the CNOT gate covered before. Instead of having one control qubits, there are two. Target operator is given by the direct product of q3 with q1*q2.

Truth table of the top gate - the first two, control qubit never change their state. But the target qubit flips from 0 to 1 and vice versa (boolean operator - XOR?)

Basis for a 3 qubit system has 8 different states.
When the first two qubits are not 11, the third qubit stays the same. But when it is, the third qubit flips. This is consistent with the boolean operation of the toffoli gate. So, the first 6 basis are unchanged while the last 2 are flipped.