Name: **Date:** **Exam 4** **Precalculus - Hargus**

Instructions: Please **show all work** on the test paper (partial credit may be awarded for correct work, even if your answer is wrong).

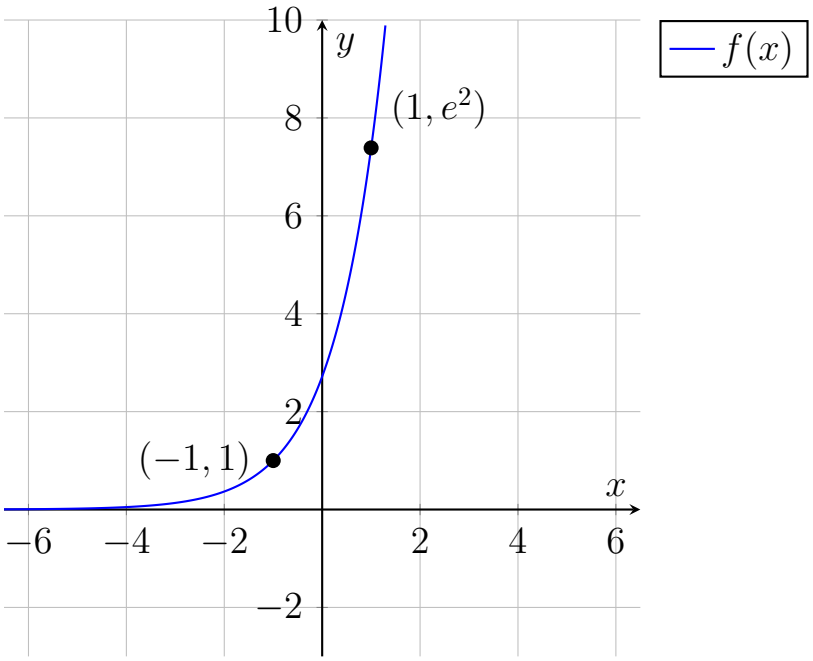
1. (4 points) Convert from radians to degrees or from degrees to radians.

   (a) $\frac{5\pi}{3}$ (radians) = (b) 120° =

2. (4 points) Evaluate the expression.

   (a) $\log_5 125$ = (b) $\tan 180^\circ$ =

3. (4 points) Write an equation $f(x)$ for the following exponential graph.



$f(x) =$ _____

4. (6 points) Solve for x.

(a) $\dfrac{\ln(x)}{e} = e^3$

(b) $\sin(x) = \dfrac{\sqrt{2}}{2} \qquad \left(\dfrac{\pi}{2} \leq x \leq \pi\right)$

5. (8 points) Consider the function $f(x) = \frac{1}{x-3} + 2$. True or false?

(a) ____ $f(x)$ has a horizontal asymptote at $y = 3$.

(b) ____ $f(x)$ has a vertical asymptote at $x = 3$.

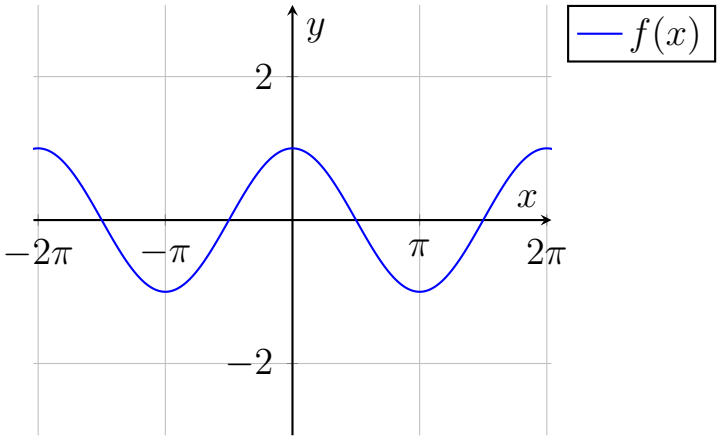
(c) ____ $\lim\limits_{x \to \infty} f(x) = \infty$

(d) ____ $\lim\limits_{x \to 3^+} f(x) = \infty$

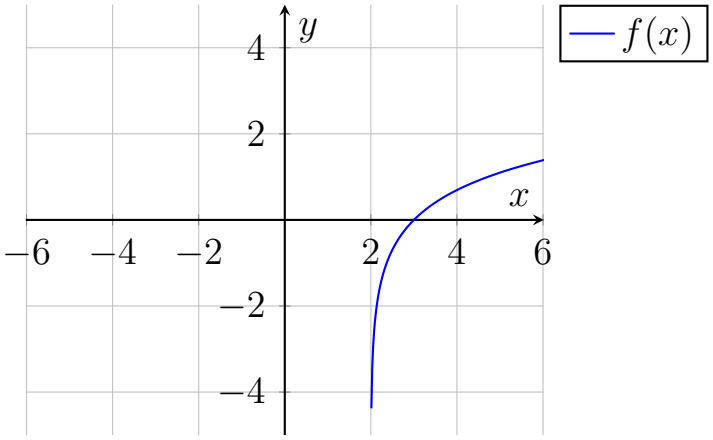
6. (4 points) Use synthetic division to divide the following (Write your answer in **fraction form**).

$$\frac{x^3 - 3x^2 - 2x + 25}{x - 3}$$

7. (4 points) The graph of $f(x) = \cos(x)$ is below. Draw $g(x) = 2\cos(\frac{1}{2}x)$.



8. (4 points) The graph of $f(x)$ is drawn below. Draw the graph of $f^{-1}(x)$.



9. (4 points) Simplify the expression to either 1 or -1.

a) $\sec(-x)\cos(x)$

b) $\dfrac{1}{2}(-2\cos^2(x) - 2\sin^2(x))$

10. (6 points) Prove the identity.

a) $\cos(x) = \dfrac{\cot(x)}{\csc(x)}$

b) $(\sin(x))(\cos(x)\tan(x) + \cot(x)) = \cos(x) + \sin^2(x)$

11. (4 points) Find an **explicit** rule for the nth term of the sequence.

a) 4, 8, 12, 16, ...

$a_n =$ _____

b) $a_1 = 5$, $a_{n+1} = 3a_n$

$a_n =$ _____

12. (6 points) You do not need to simplify your answers for these questions: answers with powers, products, and factorials are okay.

a) How many ways are there to select a group of 4 students from a class of 8 students? (order does not matter)

Ways: _____

b) How many unqiue ways are there to rearrange the letters in the name CONNOR? (for instance, CRONON is one way)

Ways: _____

c) If I roll a six-sided die 5 times, what is the probability that we get the sequence 5, 4, 3, 2, 1? (order matters here)

Probability: _____

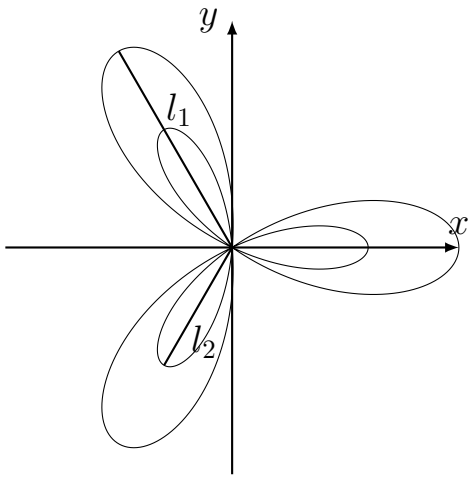
13. (4 points) Eliminate the parameter t from the following parametric equations. For your answer, write y in terms of x.

$$x = 3 - 3t$$

$$y = 2 + t$$

$y =$ _____

14. (4 points) The graph of $r = 4\cos(2\theta) + 1$ is shown below. What are the lengths of the petals l_1 and l_2?



l_1= _____

l_2= _____

15. (4 points) Below are the matrices A and B. Find the product AB. **Show how you got each element in the answer matrix without a calculator**:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

16. (5 points) Solve the following system of equations **using an inverse matrix** (you can use a calculator to find the inverse, but show other work):

$$4x + 7y + 1z = 5$$

$$2x + 3y + 2z = 4$$

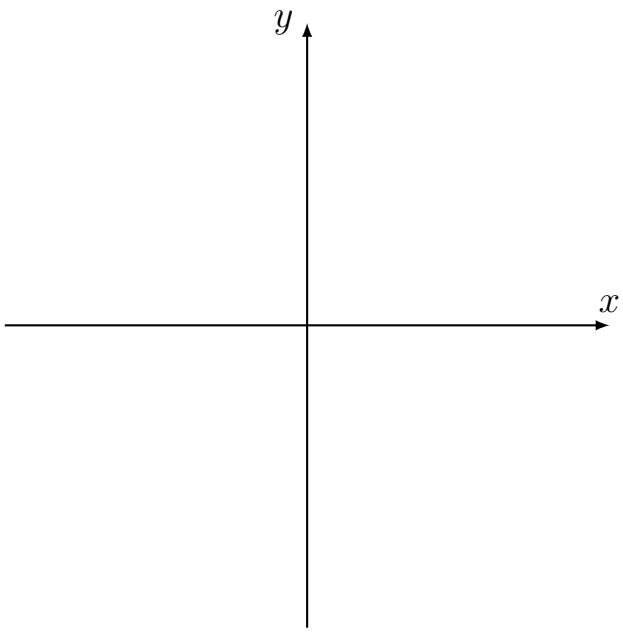
$$1x + 7z = 3$$

$x =$ _____

$y =$ _____

$z =$ _____

17. (5 points) Draw the graph of the ellipse with equation $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{16} = 1$.



18. (4 points) Let $A = (3, 4, 5)$, $B = (1, 2, 3)$, and $C = (1, 1, 1)$.

a) What is the midpoint between A and C?

Midpoint: _____

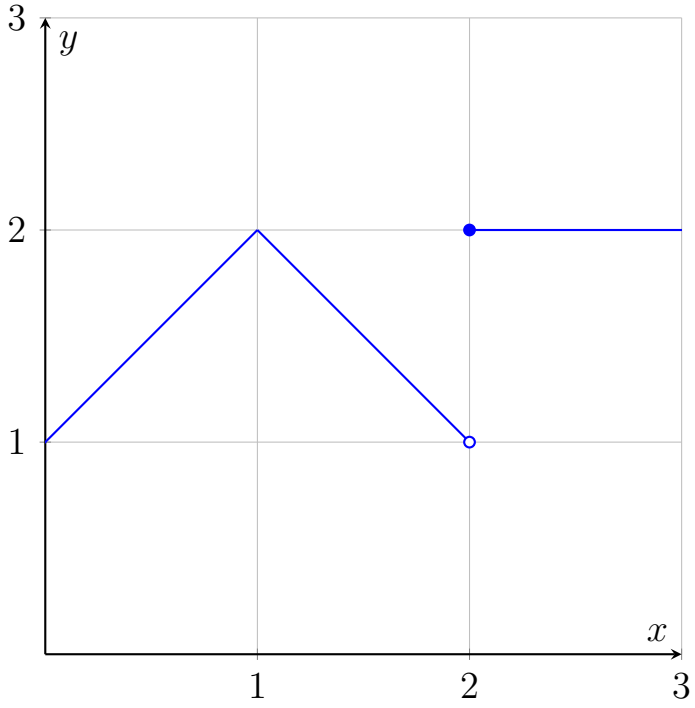
b) Find the dot product $\overrightarrow{AB} \cdot \overrightarrow{BC}$.

$\overrightarrow{AB} \cdot \overrightarrow{BC} =$ _____

19. (4 points) Researchers ask 13 students how many cups of coffee they drink each week and get the data below. Draw a box-and-whisker plot to display this data.

$$0, 0, 0, 1, 5, 5, 7, 7, 8, 10, 11, 15, 21$$

20. (12 points) Use the graph of $f(x)$ below to answer the following questions.



a) Does $\lim_{x\to 1} f(x)$ exist? If yes, what does it equal?

$\lim_{x\to 1} f(x) =$ _____

b) Does $\lim_{x\to 2^-} f(x)$ exist? If yes, what does it equal?

$\lim_{x\to 2^-} f(x) =$ _____

c) Does $\lim_{x\to 2} f(x)$ exist? If yes, what does it equal?

$\lim_{x\to 2} f(x) =$ _____

d) Does $f'(0)$ exist? If yes, what does it equal?

$f'(0) =$ _____

e) Does $f'(1)$ exist? If yes, what does it equal?

$f'(1) =$ _____

f) Find $\int_0^3 f(x)dx$.

$\int_0^3 f(x)dx =$ _____