Instructions: Please **show all work** (partial credit may be awarded for correct work, even if your answer is wrong).

1. (15 points) Evaluate the **indefinite** integral using u-substitution. **You must show the steps of using u-substitution to get credit**.

   (a) $\displaystyle\int \cos(2\theta + 4)d\theta$

   (b) $\displaystyle\int \frac{x}{1 - 25x^2}dx$

   (c) $\displaystyle\int x \cdot e^{x^2} dx$

2. (10 points) Evaluate the **definite** integral using u-substitution. **You must show the steps of using u-substitution to get credit**.

   (a) $\displaystyle\int_1^2 x^2(x^3 - 1)^4 dx$

   (b) $\displaystyle\int_0^1 \ln(2) \cdot 2^{x+3} dx$

3. (10 points) Find the average value of $f(x) = x^3$ between $x = 0$ and $x = 2$

4. (10 points) Let $f(x)$ be a differentiable function for which $f'(x) = 3f(x)$.

   (a) $f(x)$ is which kind of function? (choose one)
       i. Quadratic
       ii. Exponential
       iii. Cosine
   (b) Write an equation for $f(x)$, if $f(0) = 5$.

5. (15 points) Write an expression for the gray area below between $y = \frac{1}{x}$ and $y = \frac{1}{2}$ from $x = 1$ to $x = 3$. **Write your answer using integrals, you do not need to solve the integrals**.
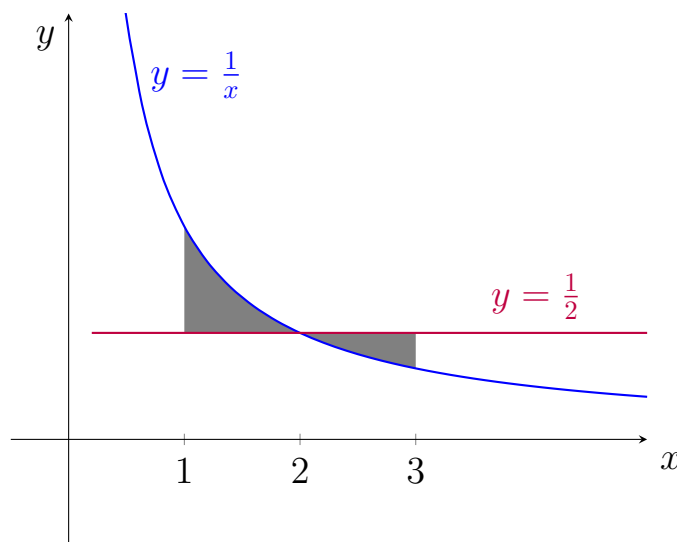

Area= _____

6. (20 points) Answer each of the following questions by writing the correct integral. You **do** need to find the endpoints a and b for each integral $\int_a^b$. You **do not** need to solve the integrals.

(a) What is the area of the gray region between the functions below?

(b) What is the volume of the object made by rotating the gray area around the x-axis?

(c) What is the volume of the object made by rotating the gray area around the line $x = 1$?

$$y = -x^2 + 4x + 3$$

$$y = x^2 - 2x + 3$$
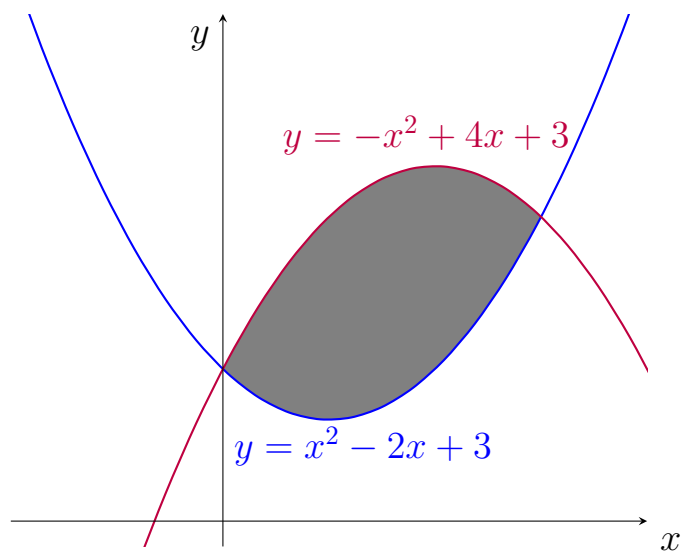
7. (**Extra Credit: 10 points**) A solid has a base inside the circle $x^2 + y^2 = 16$. The cross sections perpendicular to the x-axis are triangles with height equal to 3 times the base. Write an integral for the volume of the solid and solve to find this volume.