

Instructions: Please **show all work** (partial credit will be given for correct work, even if your answer is wrong). You may use a calculator.

1. (10 points) True or false? (circle your answer)
 - a) Each term in a geometric sequence is the previous term plus some constant. **T or F**
 - b) A proof by induction requires an “anchor” and an “inductive step”. **T or F**
 - c) The sequence with $a_1 = 5$ and $a_{k+1} = a_k - 1$ converges. **T or F**
 - d) The series $\sum_{k=1}^{\infty} 2^k$ converges. **T or F**
 - e) The series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ converges. **T or F**
2. (4 points) Compute the sum $\sum_{k=1}^{200} 6k + 4$. (Hint: note that this is an arithmetic series)

$$\sum_{k=1}^{50} 6k + 4 = \underline{\hspace{2cm}}$$

3. (8 points) Simplify the expression to either 1 or -1.

a) $\cos(-x) \sec(x)$

b) $\frac{\sin^2(x) - 1}{\cos^2 x}$

4. (8 points) Prove the identity.

a) $(\sin x)(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

b) $(1 - 2 \cos^2 x + \cos^4 x)(\sin x) = \sin^5 x$

5. (6 points) Use a half-angle identity to find the exact value of $\sin(75^\circ)$. Show your work.

$$\sin(75^\circ) = \underline{\hspace{2cm}}$$

6. (6 points) Find all solutions to the equation $\cos(2x) = \cos(x)$ in the interval $[0, 2\pi)$.

$$x = \underline{\hspace{2cm}}$$

7. (12 points) Find an **explicit** rule for the n th term of the sequence.

a) $4, 2, 0, -2, \dots$

$$a_n = \underline{\hspace{2cm}}$$

b) $3, 1, \frac{1}{3}, \dots$

$$a_n = \underline{\hspace{2cm}}$$

c) $a_1 = 5, a_n = 6 \cdot a_{n-1}$

$$a_n = \underline{\hspace{2cm}}$$

8. (15 points)

- a) Connor has 12 pairs of socks and 3 pairs of shoes. How many ways are there for him to choose a set of socks and shoes to wear?

Ways: _____

- b) If the classroom has 9 chairs and there are 9 students, how many ways are there to choose who sits in each chair?

Ways: _____

- c) If the classroom has 9 chairs and there are 9 students, how many ways are there to choose who sits in each chair if Chester needs to be in the back row?

Ways: _____

- d) If we roll two six-sided dice, what is the probability that the sum of the dice is 11?

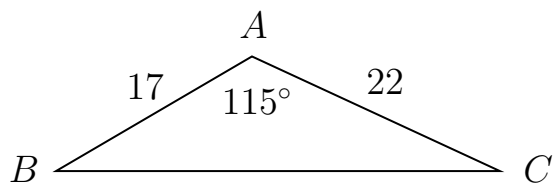
Probability: _____

- e) When it rains, Steven never plays basketball. If it does not rain, Steven plays basketball 50% of the time. If there is a 60% chance of rain tomorrow, what is the probability that Steven will play basketball?

Probability: _____

9. (15 points) Solve for missing sides and angles using the Law of Cosines and Law of Sines.

(a)

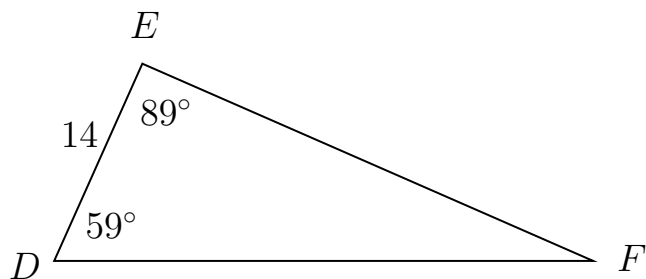


$$a = \underline{\hspace{2cm}}$$

$$\angle B = \underline{\hspace{2cm}}$$

$$\angle C = \underline{\hspace{2cm}}$$

(b)

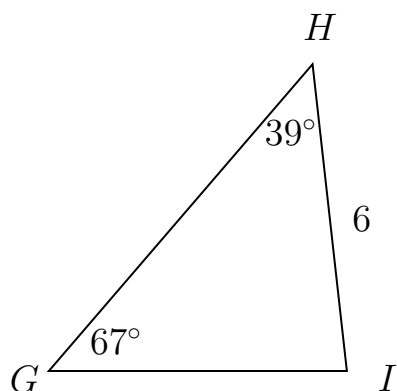


$$d = \underline{\hspace{2cm}}$$

$$e = \underline{\hspace{2cm}}$$

$$\angle F = \underline{\hspace{2cm}}$$

(c)



$$h = \underline{\hspace{2cm}}$$

$$i = \underline{\hspace{2cm}}$$

$$\angle I = \underline{\hspace{2cm}}$$

10. (4 points) How many triangles can be made with the measurements $B = 30^\circ$, $b = 5$, $c = 8$? (with angle B opposite from side b)

Number of triangles: $\underline{\hspace{2cm}}$

11. (4 points) What is the area of the triangle with measurements $A = 50^\circ$, $b = 10$, $c = 6$?

Area = _____

12. (**Extra Credit:** 5 points) Prove the following statement for all positive integers n using induction.

$$1 + 2 + 4 + \dots + 2^{n-1} = 2^n - 1$$