

Part 3

Question 1

(a)

Relation schema R has attributes ABCDEFGHIJK

For BCNF to hold, the LHS of a FD must be a super key.

$S = \{ I \rightarrow DGF, H \rightarrow CEA, BI \rightarrow J, B \rightarrow H, CI \rightarrow K \}$

LHS of FD	Closure	Superkey?	Does this FD fulfill BCNF?
I	$I^+ = IDGF$	×	×
H	$H^+ = HCEA$	×	×
BI	$BI^+ =$ BIJHDGFCEAK = ABCDEFGHIJK	✓	✓
B	$B^+ = BHCEA$	×	×
CI	$CI^+ = CIKDGF$	×	×

Out of the given FDs, only $BI \rightarrow J$ fulfills BCNF, all other FDs violate BCNF.

(b)

Decomposing R using FD $I \rightarrow DGF$.

$I^+ = IDGF$, so this yields two relations:

$R_1 = DFGI$ with FDs $S_{R_1} = \{ I \rightarrow DGF \}$

$R_2 = R - I^+ + I = ABCEHIJK$

with FDs $S_{R_2} = \{ H \rightarrow ACE, BI \rightarrow J, B \rightarrow H, CI \rightarrow K \}$

Project the FDs onto $R_1 = DFGI$

D	F	G	I	Closure	FDs
✓				$D^+ = D$	Nothing
	✓			$F^+ = F$	Nothing
		✓		$G^+ = G$	Nothing
			✓	$I^+ = IDGF$	$I \rightarrow DGF$; I is a superkey of R_1

Supersets of I				Irrelevant	Can only generate weaker FDs
✓	✓			$DF^+ = DF$	Nothing
✓		✓		$DG^+ = DG$	Nothing
	✓	✓		$FG^+ = FG$	Nothing
✓	✓	✓		$DFG^+ = DFG$	Nothing

This relation satisfies BCNF

Project the FDs onto $R_2 = ABCEHIJK$

A	B	C	E	H	I	J	K	Closure	FDs
✓								$A^+ = A$	Nothing
	✓							$B^+ = BHCEA$	$B \rightarrow ACEH$ violates BCNF, stop projection

Decompose R_2 further.

Decompose R_2 using FD $B \rightarrow ACEH$. $B^+ = BHCEA$, so this yields two relations:

$R_3 = ABCEH$ with FDs $S_{R_3} = \{ H \rightarrow ACE, B \rightarrow H \}$

$R_4 = R_2 - B^+ + B = BIKJ$ with FDs $S_{R_4} = \{ BI \rightarrow J \}$

Project the FDs onto $R_3 = ABCEH$

A	B	C	E	H	Closure	FDs
	✓				$B^+ = BHCEA$	$B \rightarrow ACEH$; B is a superkey of R_3
				✓	$H^+ = HCEA$	$H \rightarrow ACE$ violates BCNF, stop projection

Decompose R_3 using FD $H \rightarrow ACE$. $H^+ = HCEA$, so this yields two relations:

$R_5 = ACEH$

$R_6 = R_3 - H^+ + H = BH$

Project the FDs onto R5 = ACEH

A	C	E	H	Closure	FDs
✓				$A^+ = A$	Nothing
	✓			$B^+ = B$	Nothing
		✓		$E^+ = E$	Nothing
			✓	$H^+ = HCEA$	$H \rightarrow ACE$; H is a superkey of R5
Supersets of H				Irrelevant	Can only generate weaker FDs
✓	✓			$AC^+ = AC$	Nothing
✓		✓		$AE^+ = AE$	Nothing
	✓	✓		$CE^+ = CE$	Nothing
✓	✓	✓		$ACE^+ = ACE$	Nothing

This relation satisfies BCNF

Project the FDs onto R6 = BH

B	H	Closure	FDs
✓		$B^+ = BHCEA$	$B \rightarrow H$; B is a superkey of R6
	✓	$H^+ = HCEA$	Nothing

This relation satisfies BCNF

Return to R4 = BIJK, project the FDs.

B	I	J	K	Closure	FDs
✓				$B^+ = BHCEA$	Nothing
	✓			$I^+ = IDGF$	Nothing
		✓		$J^+ = J$	Nothing
			✓	$K^+ = K$	Nothing
✓	✓			$BI^+ = BIHJCEADGFK$	$BI \rightarrow JK$; BI is a superkey of R4
Supersets of H				Irrelevant	Can only generate weaker FDs
✓			✓	$BK^+ = BKHCEA$	Nothing
✓		✓		$BJ^+ = BJHCEA$	Nothing
	✓	✓		$IJ^+ = IJDGF$	Nothing
	✓		✓	$IK^+ = IKDGF$	Nothing

		✓	✓	$JK^+ = JK$	Nothing
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This relation satisfies BCNF

The final decomposition is: (in alphabetical order)

1. R5 = ACEH with FD $H \rightarrow ACE$
2. R6 = BH with FD $B \rightarrow H$
3. R4 = BIJK with FD $BI \rightarrow JK$
4. R1 = DFGI with FD $I \rightarrow DFG$

Question 2:

(a)

Relation schema P has attributes ABCDEFGH

$T = \{ ACDE \rightarrow B, BF \rightarrow AD, B \rightarrow CF, CD \rightarrow AF, ABF \rightarrow CDH \}$

Simplify to singleton RHS to set S1

1. $ACDE \rightarrow B$
2. $BF \rightarrow A$
3. $BF \rightarrow D$
4. $B \rightarrow C$
5. $B \rightarrow F$
6. $CD \rightarrow A$
7. $CD \rightarrow F$
8. $ABF \rightarrow C$
9. $ABF \rightarrow D$
10. $ABF \rightarrow H$

$S1 = \{ ACDE \rightarrow B, BF \rightarrow A, BF \rightarrow D, B \rightarrow C, B \rightarrow F, CD \rightarrow A, CD \rightarrow F, ABF \rightarrow C, ABF \rightarrow D, ABF \rightarrow H \}$

Looking for redundant FDs

FD	Exclude from S1 when computing closure	Closure	Decision
1	1	No way to get B without this FD	Keep

2	2	$BF^+ = BFDCA\dots$	Discard
3	2, 3	$BF^+ = BCF$, can't get D	Keep
4	2, 4	$B^+ = BFD$, can't get C	Keep
5	2, 5	$B^+ = BC$, can't get F	Keep
6	2, 6	$CD^+ = CDF$, can't get A	Keep
7	2, 7	$CD^+ = CDA$, can't get F	Keep
8	2, 8	$ABF^+ = ABFCDH$	Discard
9	2, 8, 9	$ABF^+ = ABFCDH$	Discard
10	2, 8, 9, 10	No way to get H without this FD	Keep

Call the remaining FDs S2:

- 1 $ACDE \rightarrow B$
- 3 $BF \rightarrow D$
- 4 $B \rightarrow C$
- 5 $B \rightarrow F$
- 6 $CD \rightarrow A$
- 7 $CD \rightarrow F$
- 10 $ABF \rightarrow H$

$S2 = \{ACDE \rightarrow B, BF \rightarrow D, B \rightarrow C, B \rightarrow F, CD \rightarrow A, CD \rightarrow F, ABF \rightarrow H\}$

We will try reducing the LHS of any FDs with multiple attributes on LHS. We will close over the full set S2, including the FD being considered for simplification.

- 1 $ACDE \rightarrow B$
 - $A^+ = A$, can't reduce LHS to A
 - $C^+ = C$, can't reduce LHS to C
 - $D^+ = D$, can't reduce LHS to D
 - $E^+ = E$, can't reduce LHS to E
 - $AC^+ = AC$, can't reduce LHS to AC
 - $AD^+ = AD$, can't reduce LHS to AD
 - $AE^+ = AE$, can't reduce LHS to AE
 - $ACD^+ = ACDF$, can't reduce LHS to ACD
 - $ACE^+ = ACE$, can't reduce LHS to ACE
 - $CDE^+ = CDEAFB$, we can reduce the LHS to CDE

3 $BF \rightarrow D$

$B^+ = BCFD$, we can reduce LHS to B

6 $CD \rightarrow A$

$C^+ = C$, can't reduce LHS to C

$D^+ = D$, can't reduce LHS to D

This FD remains unchanged.

7 $CD \rightarrow F$

$C^+ = C$, can't reduce LHS to C

$D^+ = D$, can't reduce LHS to D

This FD remains unchanged.

10 $ABF \rightarrow H$

$A^+ = A$, can't reduce LHS to A

$B^+ = BCFDAH$, we can reduce LHS to B

Call the FDs with reduced LHS S3:

1' $CDE \rightarrow B$

3' $B \rightarrow D$

4 $B \rightarrow C$

5 $B \rightarrow F$

6 $CD \rightarrow A$

7 $CD \rightarrow F$

10' $B \rightarrow H$

$S3 = \{ CDE \rightarrow B, B \rightarrow D, B \rightarrow C, B \rightarrow F, CD \rightarrow A, CD \rightarrow F, B \rightarrow H \}$

We now have new FDs (1', 3' and 10') so we attempt to simplify further.

FD	Exclude from S1 when computing closure	Closure	Decision
1'	1'	No way to get B without this FD	Keep
3'	3'	No way to get D without this FD	Keep
4	4	No way to get C without this FD	Keep
5	5	$B^+ = BDCAFH$	Discard
6	5, 6	No way to get A without this FD	Keep

7	5, 7	No way to get F without this FD	Keep
10'	5, 10'	No way to get H without this FD	Keep

Thus, we can now remove FD #5 from the set of relations.

This set, let's call it S4, is minimal basis:

4 $B \rightarrow C$
 3' $B \rightarrow D$
 10' $B \rightarrow H$
 6 $CD \rightarrow A$
 7 $CD \rightarrow F$
 1' $CDE \rightarrow B$

Impossible to simplify further.

Minimal basis for T, $S4 = \{ B \rightarrow C, B \rightarrow D, B \rightarrow H, CD \rightarrow A, CD \rightarrow F, CDE \rightarrow B \}$

(b)

From S4,

Attribute	Appears on		Conclusion
	LHS	RHS	
G	-	-	Must be in every key
E	✓	-	Must be in every key
F, H, A	-	✓	Are not in any key
B, C, D	✓	✓	We should check

We will consider all combinations of attributes B, C, and D, and we must add is G and E to each combination, since they must be in every key.

- $EGB^+ = EGBCDHAF = ABCDEFGH$, SO $EGB = BEG$ is a key
- $EGC^+ = EGC$, this is not a key
- $EGD^+ = EGD$, this is not a key
- $EGCD^+ = EGCDAFBH = ABCDEFGH$, SO $EGCD = CDEG$ is a key

All other possible combinations have either EGB or EGCD and therefore must be non-minimal superkeys.

So, there are two keys for P: (rearranged alphabetically)

- i. BEG
- ii. CDEG

(c)

We will merge the RHSs of S4, and call these FDs S5:

$CDE \rightarrow B$

$B \rightarrow CDH$

$CD \rightarrow AF$

Resulting set of relations, with attributes:

R1(B,C,D,E), R2(B, C, D, H), R3(A, C, D, F)

Since none of the keys is a subset of R1, R2, or R3, we need to add one of them, say R4(B,E,G) because BEG is a key as per part B above.

The final set of relations (decomposition of relation P in 3NF) is:

R1(B, C, D, E),

R2(B, C, D, H),

R3(A, C, D, F),

R4(B, E, G)

(d)

Each relation was formed from an FD in S5, so the LHS for each FD is a superkey for that relation.

However, we can look for other FDs because of whom BCNF is violated and then this schema is known to allow redundancy.

To check, we project the FDs from S5 onto these relations.

Projecting FDs onto R1 = ABCDE

B	C	D	E	Closure	FDs
✓				$B^+ =$ BCDHAF $= ABCDFH$	$B \rightarrow CDHAF$; this is not a superkey of R1 because it is missing E

Thus, because not all LHS of each FD form superkeys in their relations, redundancy based on functional dependency is present. Therefore BCNF is violated and thus this schema allows redundancy. This is expected as 3NF does not necessarily guarantee no redundancy.