Part 3

Question 1

(a)

Relation schema *R* has attributes ABCDEFGHIJK

For BCNF to hold, the LHS of a FD must be a super key.

S = { I → DGF, H → CEA, BI → J, B → H, CI → K }

LHS of FD	Closure	Superkey?	Does this FD fulfill BCNF?
I	I^+ = IDGF	×	×
H	H^+ = HCEA	×	×
BI	BI^+ = BIJHDGFCEAK = ABCDEFGHIJK	✓	✓
B	B^+ = BHCEA	×	×
CI	CI^+ = CIKDGF	×	×

Out of the given FDs, only BI → J fulfills BCNF, all other FDs violate BCNF.

(b)

Decomposing *R* using FD I → DGF.

I^+ = IDGF, so this yields two relations:
R1 = DFGI
R2 = R – I^+ + I = ABCEHIJK

Project the FDs onto R1 = DFGI

D	F	G	I	Closure	FDs
✓				D^+ = D	Nothing
	✓			F^+ = F	Nothing
		✓		G^+ = G	Nothing
			✓	I^+ = IDGF	I → DGF; I is a superkey of R1
Supersets of I				Irrelevant	Can only generate weaker FDs

✓	✓			$DF^+ = DF$	Nothing
✓		✓		$DG^+ = DG$	Nothing
	✓	✓		$FG^+ = FG$	Nothing
✓	✓	✓		$DFG^+ = DFG$	Nothing

This relation satisfies BCNF

Project the FDs onto R2 = ABCEHIJK

A	B	C	E	H	I	J	K	Closure	FDs
✓								$A^+ = A$	Nothing
	✓							$B^+ = BHCEA$	B → ACEH violates BCNF, stop projection

Decompose R2 further.

Decompose R2 using FD B → ACEH. $B^+ = BHCEA$, so this yields two relations:
R3 = ABCEH
R4 = R2 – B^+ + B = BIKJ

Project the FDs onto R3 = ABCEH

A	B	C	E	H	Closure	FDs
	✓				$B^+ = BHCEA$	B → ACEH; B is a superkey of R3
				✓	$H^+ = HCEA$	H → ACE violates BCNF, stop projection

Decompose R3 using FD H → ACE. $H^+ = HCEA$, so this yields two relations:
R5 = ACEH
R6 = R3 – H^+ + H = BH

Project the FDs onto R5 = ACEH

A	C	E	H	Closure	FDs
√				$A^+ = A$	Nothing
	√			$B^+ = B$	Nothing
		√		$E^+ = E$	Nothing
			√	$H^+ = HCEA$	H → ACE; H is a superkey of R5
Supersets of H				Irrelevant	Can only generate weaker FDs
√	√			$AC^+ = AC$	Nothing
√		√		$AE^+ = AE$	Nothing
	√	√		$CE^+ = CE$	Nothing
√	√	√		$ACE^+ = ACE$	Nothing

This relation satisfies BCNF

Project the FDs onto R6 = BH

B	H	Closure	FDs
√		$B^+ = BHCEA$	B → H; B is a superkey of R6
	√	$H^+ = HCEA$	Nothing

This relation satisfies BCNF

Return to R4 = BIJK, project the FDs.

B	I	J	K	Closure	FDs
√				$B^+ = BHCEA$	Nothing
	√			$I^+ = IDGF$	Nothing
		√		$J^+ = J$	Nothing
			√	$K^+ = K$	Nothing
√	√			$BI^+ = BIHJCEADGFK$	BI → JK; BI is a superkey of R4
Supersets of H				Irrelevant	Can only generate weaker FDs
√			√	$BK^+ = BKHCEA$	Nothing
√		√		$BJ^+ = BJHCEA$	Nothing
	√	√		$IJ^+ = IJDGF$	Nothing
	√		√	$IK^+ = IKDGF$	Nothing
		√	√	$JK^+ = JK$	Nothing

This relation satisfies BCNF

The final decomposition is: (in alphabetical order)

1. R5 = ACEH with FD H → ACE
2. R6 = BH with FD B → H
3. R4 = BIJK with FD BI → JK
4. R1 = DFGI with FD I → DFG

Question 2:

(a)

Relation schema *P* has attributes ABCDEFGH

T = { ACDE → B, BF → AD, B → CF, CD → AF, ABF → CDH }

Simplify to singleton RHS to set S1

1. ACDE → B
2. BF → A
3. BF → D
4. B → C
5. B → F
6. CD → A
7. CD → F
8. ABF → C
9. ABF → D
10. ABF → H

S1 = { ACDE → B, BF → A, BF → D, B → C, B → F, CD → A, CD → F, ABF → C, ABF → D, ABF → H }

Looking for redundant FDs

FD	Exclude from S1 when computing closure	Closure	Decision
1	1	No way to get B without this FD	Keep
2	2	BF^+ = BFDCA…	Discard
3	2, 3	BF^+ = BCF, can't get D	Keep
4	2, 4	B^+ = BFD, can't get C	Keep

5	2, 5	B^+ = BC, can’t get F	Keep
6	2, 6	CD^+ = CDF, can’t get A	Keep
7	2, 7	CD^+ = CDA, can’t get F	Keep
8	2, 8	ABF^+ = ABFCDH	Discard
9	2, 8, 9	ABF^+ = ABFCDH	Discard
10	2, 8, 9, 10	No way to get H without this FD	Keep

Call the remaining FDs S2:

1 ACDE → B
3 BF → D
4 B → C
5 B → F
6 CD → A
7 CD → F
10 ABF → H

S2 = {ACDE → B, BF → D, B → C, B → F, CD → A, CD → F, ABF → H}

We will try reducing the LHS of any FDs with multiple attributes on LHS. We will close over the full set S2, including the FD being considered for simplification.

1 ACDE → B
A^+ = A, can’t reduce LHS to A
C^+ = C, can’t reduce LHS to C
D^+ = D, can’t reduce LHS to D
E^+ = E, can’t reduce LHS to E
AC^+ = AC, can’t reduce LHS to AC
AD^+ = AD, can’t reduce LHS to AD
AE^+ = AE, can’t reduce LHS to AE
ACD^+ = ACDF, can’t reduce LHS to ACD
ACE^+ = ACE, can’t reduce LHS to ACD
CDE^+ = CDEAFB, we can reduce the LHS to CDE

3 BF → D
B^+ = BCFD, we can reduce LHS to B

6 $CD \rightarrow A$

$C^+ = C$, can't reduce LHS to C

$D^+ = D$, can't reduce LHS to D

This FD remains unchanged.

7 $CD \rightarrow F$

$C^+ = C$, can't reduce LHS to C

$D^+ = D$, can't reduce LHS to D

This FD remains unchanged.

10 $ABF \rightarrow H$

$A^+ = A$, can't reduce LHS to A

$B^+ = BCFDAH$, we can reduce LHS to B

Call the FDs with reduced LHS S3:

1' $CDE \rightarrow B$

3' $B \rightarrow D$

4 $B \rightarrow C$

5 $B \rightarrow F$

6 $CD \rightarrow A$

7 $CD \rightarrow F$

10' $B \rightarrow H$

S3 = { $CDE \rightarrow B$, $B \rightarrow D$, $B \rightarrow C$, $B \rightarrow F$, $CD \rightarrow A$, $CD \rightarrow F$, $B \rightarrow H$ }

We now have new FDs (1', 3' and 10') so we attempt to simplify further.

FD	Exclude from S1 when computing closure	Closure	Decision
1'	1'	No way to get B without this FD	Keep
3'	3'	No way to get D without this FD	Keep
4	4	No way to get C without this FD	Keep
5	5	$B^+ = BDCAFH$	Discard
6	5, 6	No way to get A without this FD	Keep
7	5, 7	No way to get F without this FD	Keep
10'	5, 10'	No way to get H without this FD	Keep

Thus, we can now remove FD #5 from the set of relations.

This set, let's call it S4, is minimal basis:

4 $B \rightarrow C$
3' $B \rightarrow D$
10' $B \rightarrow H$
6 $CD \rightarrow A$
7 $CD \rightarrow F$
1' $CDE \rightarrow B$

Impossible to simplify further.

Minimal basis for T, S4 = { $B \rightarrow C$, $B \rightarrow D$, $B \rightarrow H$, $CD \rightarrow A$, $CD \rightarrow F$, $CDE \rightarrow B$}

(b)

From S4,

Attribute	Appears on		Conclusion
	LHS	RHS	
G	-	-	Must be in every key
E	✓	-	Must be in every key
F, H, A	-	✓	Are not in any key
B, C, D	✓	✓	We should check

We will consider all combinations of attributes B, C, and D, and we must add is G and E to each combination, since they must be in every key.

- EGB^+ = EGBCDHAF = ABCDEFGH, SO EGB = BEG is a key
- EGC^+ = EGC, this is not a key
- EGD^+ = EGD, this is not a key
- $EGCD^+$ = EGCDAFBH = ABCDEFGH, SO EGCD = CDEG is a key

All other possible combinations have either EGB or EGCD.

So, there are two keys for P: (rearranged alphabetically)

i. BEG
ii. CDEG

(c)

We will merge the RHSs of S4, and call these FDs S5:

CDE → B
B → CDH
CD → AF

Resulting set of relations, with attributes:

R1(B,C,D,E), R2(B, C, D, H), R3(A, C, D, F)

Since none of the keys is a subset of R1, R2, or R3, we need to add one of them, say R4(B,E,G)

The final set of relations (decomposition of relation P in 3NF) is:

R1(B,C,D,E),
R2(B, C, D, H),
R3(A, C, D, F),
R4(B,E,G)

(d)

Each relation was formed from an FD in S5, so the LHS for each FD is a superkey for that relation.

However, we can look for other FDs because of whom BCNF is violated and then this schema is known to allow redundancy.

To check, we project the FDs from S5 onto these relations.

Projecting FDs onto R1 = ABCDE

B	C	D	E	Closure	FDs
✓				B^+= BCDHAF	B → CDHAF; this is not a superkey of R1 because it is missing E

Thus, due to the LHS of each FD not necessarily being a superkey, redundancy based on functional dependency is present, BCNF is violated and thus this schema allows redundancy.