`pyop3`: A Domain-Specific Language for Expressing Iterations over Mesh-like Data Structures

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Abstract

Chapter 1

Introduction

The remainder of this paper is laid out as follows...

Chapter 2

Background

2.0.1 An example of a complicated stencil function: solving the Stokes equations using the finite element method

For a moderately complex stencil operation that we will refer to throughout this thesis we consider solving the Stokes equations using the finite element method (FEM) larsonFiniteElementMethod2013. The Stokes equations are a linearisation of the Navier-Stokes equations and are used to describe fluid flow for laminar (slow and calm) media. For domain Ω they are given by
- $\nu\Delta$u + ∇p = f in Ω,
$\nabla\cdot$u = 0 in Ω,

where u is the fluid velocity, p the pressure, ν the viscosity and f is a known forcing term. We also prescribe Dirichlet boundary conditions for the velocity across the entire boundary

$$u = g \quad \text{on } \Gamma. \tag{2.1}$$

For the finite element method we seek the solution to the *variational*, or *weak*, formulation of these equations. These are obtained by multiplying each equation by a suitable *test function* and integrating over the domain. For 2.0.1, with v as the test function and integrating by parts, this gives

$$\int \nu\nabla u : \nabla v \mathrm{d}\Omega - \int p\nabla \cdot v \mathrm{d}\Omega = \int f \cdot v \mathrm{d}\Omega \tag{2.2}$$

Note that the surface terms from the integration by parts can be dropped since v is defined to be zero at Dirichlet nodes.

For the second equation we simply get

$$\int q \, \nabla \cdot u \mathrm{d}\Omega = 0. \tag{2.3}$$

In order for these equations to be well-posed we require that the functions u, v, p and q be drawn from appropriate function spaces...

Chapter 3

Mesh-like data layouts

Chapter 4

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4.1 Examples

4.1.1 Residual assembly

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