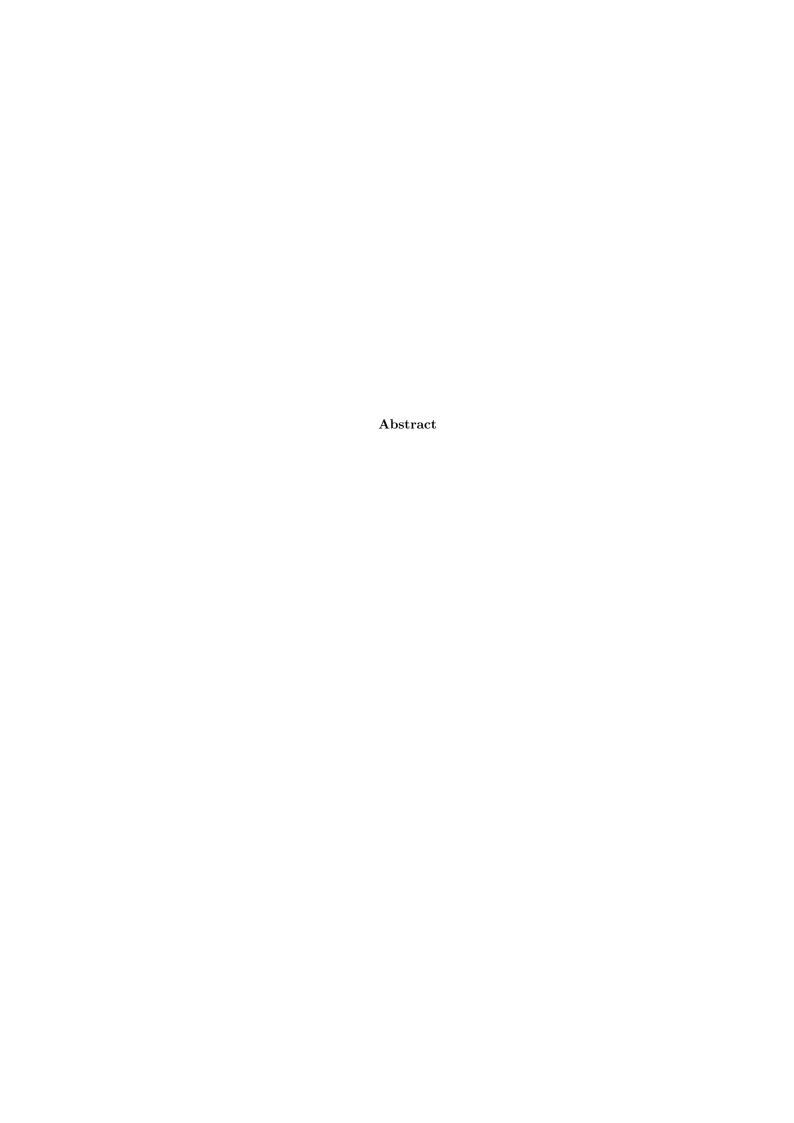
pyop3: A Domain-Specific Language for Expressing Iterations over Mesh-like Data Structures

Connor J. Ward and David A. Ham

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Introduction

The remainder of this paper is laid out as follows... $\,$

Background

2.0.1 An example of a complicated stencil function: solving the Stokes equations using the finite element method

For a moderately complex stencil operation that we will refer to throughout this thesis we consider solving the Stokes equations using the finite element method (FEM) larson FiniteElementMethod2013. The Stokes equations are a linearisation of the Navier-Stokes equations and are used to describe fluid flow for laminar (slow and calm) media. For domain Ω they are given by

$$-\nu\Delta u + \nabla p = f \quad \text{in } \Omega,$$

$$\nabla \cdot u = 0 \quad \text{in } \Omega,$$

where u is the fluid velocity, p the pressure, ν the viscosity and f is a known forcing term. We also prescribe Dirichlet boundary conditions for the velocity across the entire boundary

$$u = g \quad \text{on } \Gamma.$$
 (2.1)

For the finite element method we seek the solution to the *variational*, or weak, formulation of these equations. These are obtained by multiplying each equation by a suitable *test function* and integrating over the domain. For 2.0.1, with v as the test function and integrating by parts, this gives

$$\int \nu \nabla u : \nabla v d\Omega - \int p \nabla \cdot v d\Omega = \int f \cdot v d\Omega \tag{2.2}$$

Note that the surface terms from the integration by parts can be dropped since v is defined to be zero at Dirichlet nodes.

For the second equation we simply get

$$\int q \, \nabla \cdot u \mathrm{d}\Omega = 0. \tag{2.3}$$

In order for these equations to be well-posed we require that the functions $u,\,v,\,p$ and q be drawn from appropriate function spaces...

Mesh-like data layouts

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