Chapter 1

Background

1.0.1 Inspector-executor model

[**knepleyExascaleComputingThreads2015**]
[**stroutSparsePolyhedralFramework2018**] [**mirchandaneyPrinciplesRuntimeSupport1988**] [**arenazInspectorExecutorAlgorithmIrregular2004**]

1.1 Domain-specific languages

1.1.1 An example of a complicated stencil function: solving the Stokes equations using the finite element method

For a moderately complex stencil operation that we will refer to throughout this thesis we consider solving the Stokes equations using the finite element method (FEM) [**larsonFiniteElementMethod2013**]. The Stokes equations are a linearisation of the Navier-Stokes equations and are used to describe fluid flow for laminar (slow and calm) media. For domain Ω they are given by

$$-\nu \Delta u + \nabla p = f \quad \text{in } \Omega, \tag{1.1}$$

$$\nabla \cdot u = 0 \quad \text{in } \Omega, \tag{1.2}$$

$$\tag{1.3}$$

where u is the fluid velocity, p the pressure, ν the viscosity and f is a known forcing term. We also prescribe Dirichlet boundary conditions for the velocity across the entire boundary

$$u = g \quad \text{on } \Gamma. \tag{1.4}$$

For the finite element method we seek the solution to the *variational*, or *weak*, formulation of these equations. These are obtained by multiplying each equation by a suitable *test function* and integrating over the domain. For 1.1, with v as the test function and integrating by parts, this gives

$$\int \nu \nabla u : \nabla v \mathrm{d}\Omega - \int p \nabla \cdot v \mathrm{d}\Omega = \int f \cdot v \mathrm{d}\Omega \tag{1.5}$$

Note that the surface terms from the integration by parts can be dropped since v is defined to be zero at Dirichlet nodes.

For the second equation we simply get

$$\int q \, \nabla \cdot u \mathrm{d}\Omega = 0. \tag{1.6}$$

In order for these equations to be well-posed we require that the functions u, v, p and q be drawn from appropriate function spaces…

1.2 Related work