MATH 228: Project 2

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We want to design an artificial pancreas that has a success rate of *93%* with an average score per trial of below *20*. Let's first try and understand what variables we are looking at to better design the pancreas. Our model includes I(t), X(t), G(t), $v_1(t)$, $v_2(t)$, x(t), and u(t). I(t) is the blood insulin level in the body at time *t*, measured as the concentration of insulin in the bloodstream that will diffuse into the interstitial fluid within the cells. This value should remain close to our desired value I_b, or 10μU/mL. X(t) is the intercellular insulin level in the body at time *t*, measured as the concentration of the intracellular insulin which is responsible for clearing glucose from the bloodstream. This value should remain close to 0. G(t) is the blood glucose level in the body at time *t*. It is measured as the concentration of glucose in the bloodstream. We want this value to be close to G_b, or 80mg/dL. The equations for I'(t), X'(t), G'(t) are shown below.

$$I'(t) = -nI(t) + p_4 v_1(t) = -8.52I(t) + 0.098\, v_1(t)$$

$$X'(t) = -p_2 X(t) + p_3\left(I(t) - I_b\right) = -3X(t) + 0.1008\left(I(t) - 10\right)$$

$$G'(t) = -X(t)G(t) - p_1\left(G(t) - G_b\right) + \frac{1}{V} v_2(t) = -X(t)G(t) - 2.1\left(G(t) - 80\right) + \frac{v_2(t)}{117}$$

Our artificial pancreas wants to keep our patient alive. It does that by measuring the amount of glucose in the bloodstream, and then adds the right amount of insulin back into the bloodstream in order to combat the glucose. $v_1(t)$ measures the rate at which we are adding insulin back into the bloodstream. Our feedback control method will be denoted as u(t). This is what I will choose in order to get the patient to survive with the limitations imposed on me. Throughout the day our patient will get hungry, as humans tend to do, and they will consume meals that will have a random amount of glucose in them. $v_2(t)$ is the rate at which this glucose is added into the bloodstream.

Throughout the explanation of the model, I have talked about where we want our levels, such as blood insulin, intercellular insulin, and blood glucose, to be at. x(t) is the 3x1 matrix that contains the limitations, such as keeping the blood insulin level close to I_b, the intercellular insulin level close to 0, and the blood glucose level close to G_b. Shown below are the three matrices that make up these parameters.

$$A = \begin{bmatrix} -n & 0 & 0 \\ P_3 & -P_2 & 0 \\ 0 & -G_b & -P_1 \end{bmatrix} = \begin{bmatrix} -8.52 & 0 & 0 \\ 0.1008 & -3 & 0 \\ 0 & -80 & -2.1 \end{bmatrix} \qquad B = \begin{bmatrix} P_4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.098 \\ 0 \\ 0 \end{bmatrix}$$

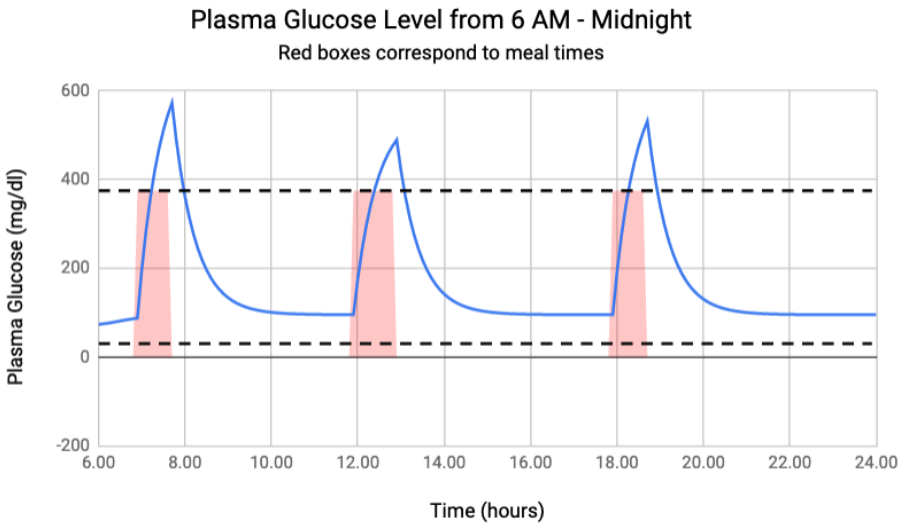
$$K = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix}$$

If we want to control u(t) we have to choose *k* values that will have a large enough effect on the artificial pancreas such that the patient stays within its limits. If we want to control the *k* values we have to control the eigenvalues. First we will solve for the *k* values in order to pick eigenvalues at will. We do this by solving det(A-BK-λI) and then comparing it to a cubic λ function with solutions at λ_1, λ_2, and λ_3**.** The final expressions for the three *k* values are shown below (all full calculations will be in appendix):

$$K_1 = \frac{-\lambda_1 - \lambda_2 - \lambda_3 - 13.62}{0.098} \qquad K_2 = \frac{\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 + 5.1\lambda_1 + 5.1\lambda_2 + 5.1\lambda_3 + 19.71}{0.0098784}$$

$$K_3 = \frac{6.3(-\lambda_1 - \lambda_2 - \lambda_3) + 2.1(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) + 10.71(\lambda_1 + \lambda_2 + \lambda_3) + \lambda_1\lambda_2\lambda_3 + 9.261}{0.790272}$$

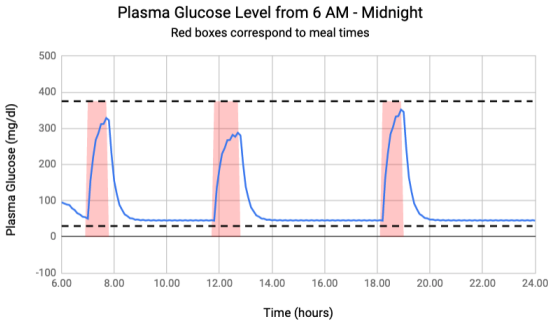
Let's first run a test to see how the patient fairs when the artificial pancreas is turned off. As you can see below, after the patient has had their first meal, plasma glucose levels go above the limit. In this case, my goal for the success rate was not met but I did meet my score goal.

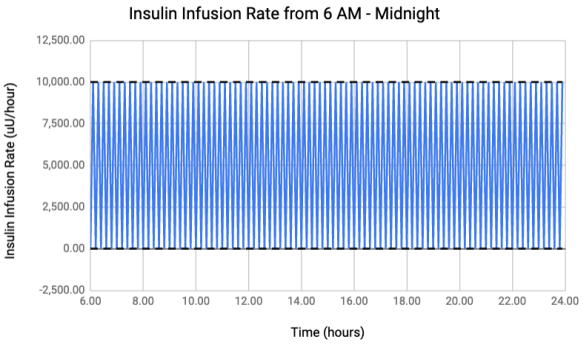


Picking eigenvalues values turned out to be a 2D needle in a 4D haystack. Once I thought I found a decent set of numbers, trying to improve them by moving even just one caused the success rate to plummet. Fortunately, I found some patterns in the numbers. The simulation runs 100 trials, but I wanted to get more than just 1 number for each set of lambda values. I ran 100 trials 10 times, and took the average from those trials for both the score and the success rate. This generally gets rid of any outlier scores and shows me how the values behave as time approaches infinity. The first set of numbers I chose was (-10,-100,-100), for (λ_1, λ_2, λ_3) respectively. This had an average score of 14.64 and an average success rate of 86.9%.

I noticed that having a smaller negative number, such as anything below -100 and above -10, and then two larger numbers below -1000 and above -100 resulted in a decent score in the low 90's everytime. The next set of numbers was (-70,-400,-500), for (λ_1, λ_2, λ_3) respectively. This had an average score (over 10 runs of 100 simulations) of 15.56 and an average success rate of 91.1%.

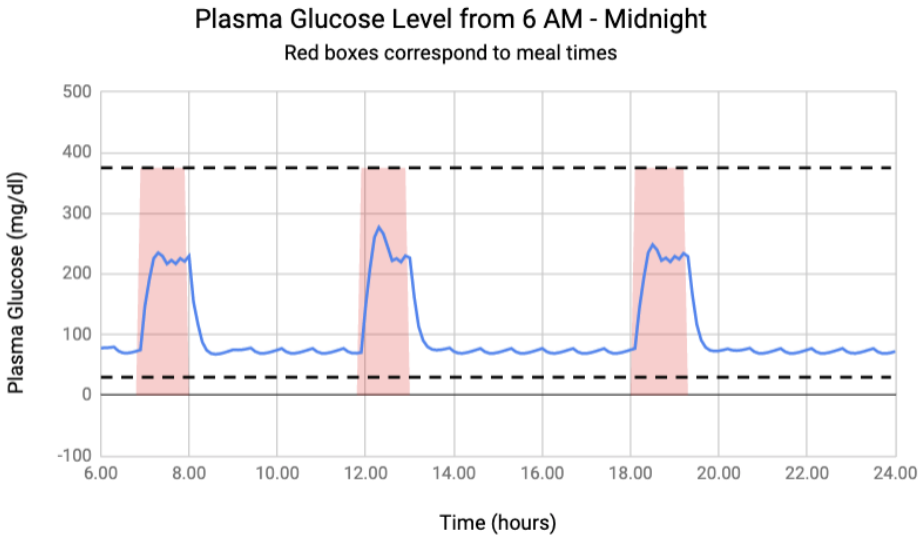
Playing around with the numbers more I finally found what I was looking for, the glorious 100% success rate. The first set of numbers in which this occurred was (-100,-2,-2), for (λ_1, λ_2, λ_3) respectively. Upon further analysis, as long as λ_2 and λ_3 stayed at -2, λ_1 could be anything from -84.9 (number found testing values closer and closer to final value until error was negligible) to negative infinity and gives a success rate on average of 99.8%. For example, choosing (-12345678996543700000, -2, -2), for (λ_1, λ_2, λ_3) respectively, gives a success rate of 100%. The plots for the Plasma Glucose Level and the Insulin Infusion Rate are shown below.

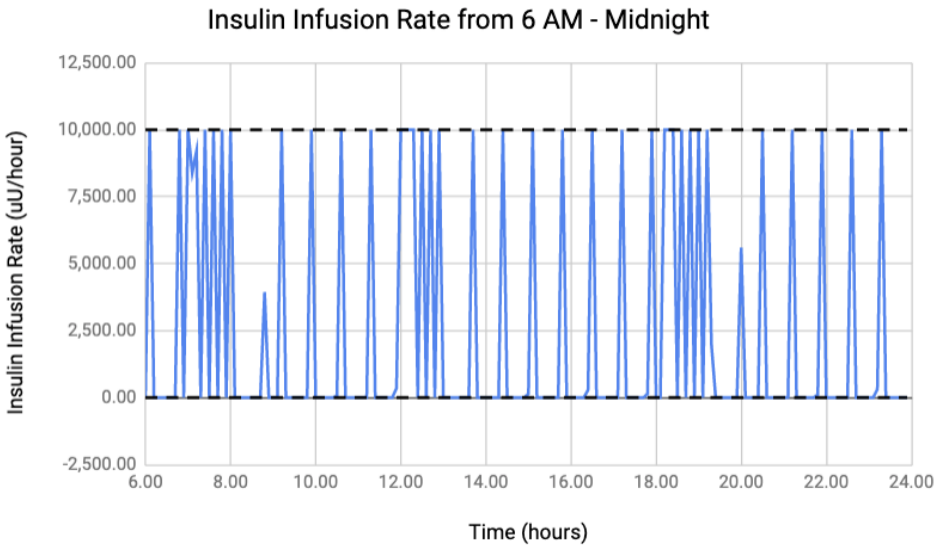




As you can see, the insulin infusion rate changes drastically throughout the day. In fact, after I checked the plotting tab and looked at the column titled *Insulin Infusion Rate*, every 6 minutes it changed from releasing 10,000μU/h to 0μU/h. I don't know much about artificial pancreas' but I suspect that this is not possible.

My last set of eigenvalues produced a graph that makes more sense when speaking about the real world. The set of numbers was (-200,-4,-7), for (λ_1, λ_2, λ_3) respectively. These eigenvalues produced k values of (2014.081633, 118603.2151, -2331.41121), for (k_1, k_2, k_3) respectively. After running 10 sets of 100 trials each we got an average score of 12.94, and an average success rate of 93.5%. These numbers met both of my goals at the start of the project, even though technically I achieved that with the previous set of eigenvalues I felt it was a bit of a cheat to make those the main/favorite set.





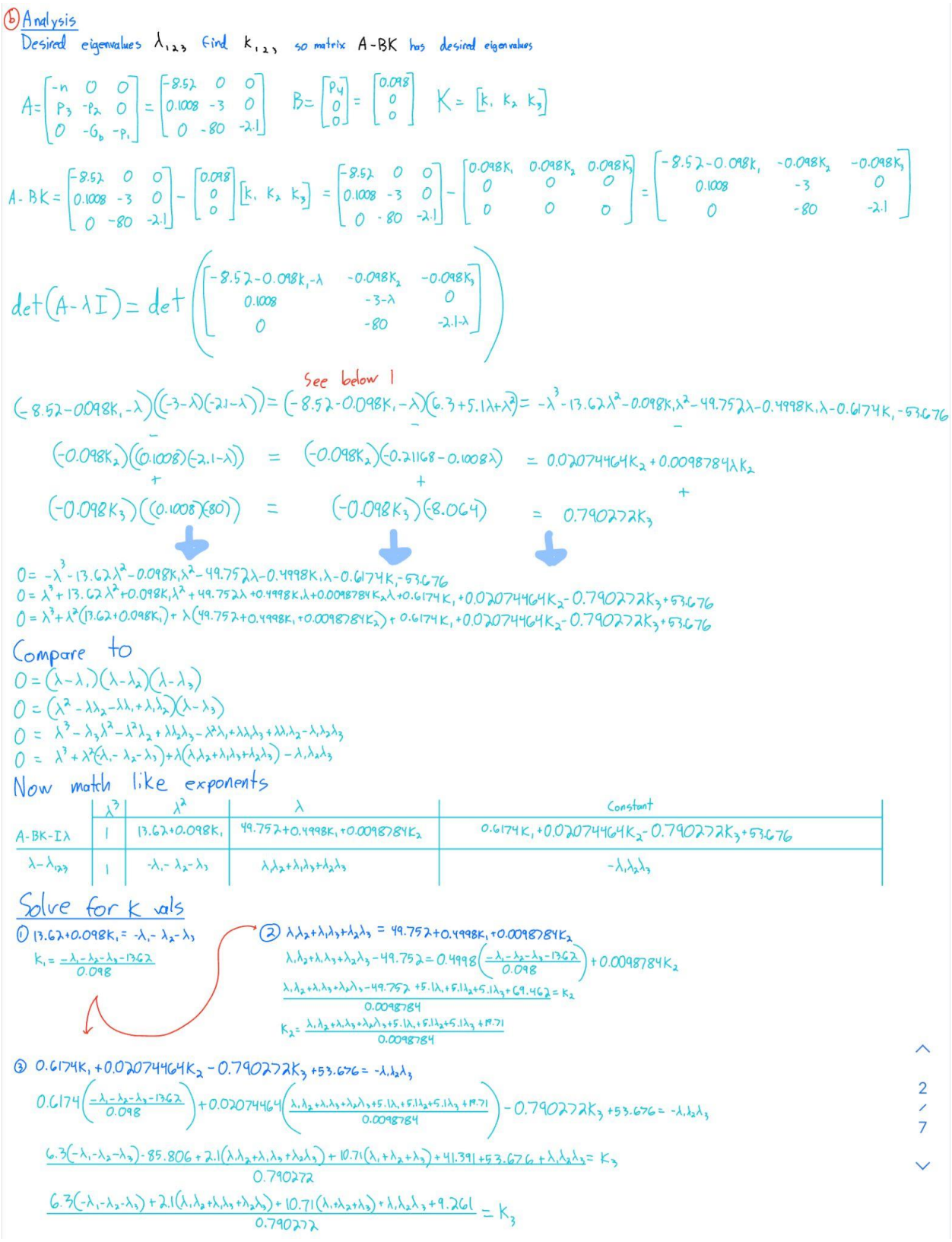
What happens if our initial values are the lowest we can go? Or the highest? How do my chosen eigenvalues hold up? My favorite set of eigenvalues was (-200,-4,-7), for (λ_1, λ_2, λ_3) respectively. Running the simulation with the plasma insulin, remote insulin, and plasma glucose with values of (5, 0, 60) respectively, yields a .2% increase in score, and a 1% increase in the success rate.

When we start off with the maximum plasma insulin (15) in the system along with the minimum plasma glucose level (60), our score jumps up 2% while our success rate goes down by 1%. This shows that starting off with more insulin in your system actually has a negative effect on the goal you're trying to achieve, which is minimizing glucose levels.

When we start off with the maximum plasma insulin(15) in the system, along with a huge initial plasma glucose level of 300mg/dL, we saw a 6.8% increase in the score along with a 1% decrease in success rate. The average score going way up makes sense, because if you add around half a meals' worth of glucose into the system, the system will release more insulin, forcing our score to go up.

Appendix:

Calculating *k* expressions



Calculating Average Score and Success Rate for Various Lambda Sets

Set 1

$\lambda_1 = -10$
$\lambda_2 = -100$
$\lambda_3 = -100$

⇒

K values

$K_1 = 2003.87755102$
$K_2 = 1,108,348.51798$
$K_3 = -95,811.1118703$

⇒

Result after running 10 simulations w/ 100 trials
Average Score: 14.64
Average Success Rate: 86.9

Trial	1	2	3	4	5	6	7	8	9	10	Average
Score	14.65	14.63	14.62	14.66	14.62	14.68	14.63	14.62	14.62	14.63	14.64
Success Rate(%)	89	89	88	83	87	86	89	85	92	81	86.9

Set 2

$\lambda_1 = -70$
$\lambda_2 = -400$
$\lambda_3 = -500$

⇒

K values

$K_1 = 9758.98$
$K_2 = 26,249,50.4$
$K_3 = -17021947.4$

⇒

Result after running 10 simulations w/ 100 trials
Average Score: 15.56
Average Success Rate: 91.1

Trial	1	2	3	4	5	6	7	8	9	10	Average
Score	15.56	15.62	15.53	15.51	15.57	15.58	15.60	15.51	15.56	15.56	15.56
Success Rate(%)	95	95	89	92	90	87	91	90	89	93	91.1

Set 3

$\lambda_1 = -100$
$\lambda_2 = -2$
$\lambda_3 = -2$

⇒

K values

$K_1 = 922.24$
$K_2 = -10,800.33$
$K_3 = -1.2388$

⇒

Result after running 10 simulations w/ 100 trials
Average Score: 20.59
Average Success Rate: 99.8

Trial	1	2	3	4	5	6	7	8	9	10	Average
Score	20.62	20.57	20.62	20.57	20.56	20.60	20.58	20.63	20.60	20.64	20.59
Success Rate(%)	100	100	100	100	100	99	100	99	100	100	99.8

Set 4

$\lambda_1 = -200$
$\lambda_2 = -4$
$\lambda_3 = -7$

⇒

K values

$K_1 = 2014.08$
$K_2 = 118,603.21$
$K_3 = -2331.41$

⇒

Result after running 10 simulations w/ 100 trials
Average Score: 12.94
Average Success Rate: 93.5

Trial	1	2	3	4	5	6	7	8	9	10	Average
Score	12.97	12.91	12.97	12.91	12.97	12.90	12.94	12.97	12.93	12.92	12.94
Success Rate(%)	94	91	99	93	93	95	93	92	93	92	93.5

Set 4.b

$\lambda_1 = -200$
$\lambda_2 = -4$
$\lambda_3 = -7$

⇒

K values

$K_1 = 2014.08$
$K_2 = 118,603.21$
$K_3 = -2331.41$

⇒

Result after running 10 simulations w/ 100 trials
Average Score: 12.97
Average Success Rate: 94.5

Trial	1	2	3	4	5	6	7	8	9	10	Average
Score											
Success Rate(%)											

Note:
Plasma insulin = min

Set 4.c

$\lambda_1 = -200$
$\lambda_2 = -4$
$\lambda_3 = -7$

$\Rightarrow$ K values

$K_1 = 2014.08$
$K_2 = 118{,}603.21$
$K_3 = -2331.41$

$\Rightarrow$ Result after running 10 simulations w/ 100 trials

Average Score: 13.2

Average Success Rate: 92.6

Trial	1	2	3	4	5	6	7	8	9	10	Average
Score	12.98	13.04	12.99	13.04	13.02	13.05	13.04	13.03	13.02	13.03	13.2
Success Rate (%)	95	91	94	94	95	91	87	93	94	92	92.6

Note:
Plasma insulin = max
Plasma glucose = min

Set 4.d

$\lambda_1 = -200$
$\lambda_2 = -4$
$\lambda_3 = -7$

$\Rightarrow$ K values

$K_1 = 2014.08$
$K_2 = 118{,}603.21$
$K_3 = -2331.41$

$\Rightarrow$ Result after running 10 simulations w/ 100 trials

Average Score: 13.82

Average Success Rate: 92.5

Trial	1	2	3	4	5	6	7	8	9	10	Average
Score	13.79	13.91	13.85	13.85	13.81	13.77	13.74	13.80	13.89	13.83	13.82
Success Rate (%)	92	92	87	93	93	95	96	91	90	96	92.5

See Below Section

See below section

$$1 = (-8.52 - 0.098K_1 - \lambda)(6.3 + 5.1\lambda + \lambda^2)$$

$$= -\lambda^3 - 13.62\lambda^2 - 0.098K_1\lambda^2 - 49.752\lambda - 0.4998K_1\lambda - 0.6174K_1 - 53.676$$

Useful Links:

Google Sheets Simulation

Desmos Calculator