In-Class

2D Linear Systems: $\vec{x}' = Ax$ $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Case 1: real distinct eigenvalues $\Rightarrow \lambda_1 \neq \lambda_2$
w/ independent eigenvectors $\Rightarrow \vec{v}_1, \vec{v}_2$

Solution: $\vec{x}(t) = c_1 e^{\lambda_1 t}\vec{v}_1 + c_2 e^{\lambda_2 t}\vec{v}_2$
Find c_1, c_2 using $x(0)$

We can have:
- Saddles $\lambda_1 < 0 < \lambda_2$
- Nodal source $\lambda_1 > \lambda_2 > 0$
- Nodal sink $\lambda_1 < \lambda_2 < 0$

Case 2: Complex eigenvalues $\Rightarrow \lambda_{1,2} = a \pm ib$
w/ complex eigenvectors $\Rightarrow \vec{v}_1, \vec{v}_2$

Solution: $x(t) = c_1 x_r + c_2 x_I$
Where: $e^{\lambda_1 t} v_1 = x_r + i x_I$

Eulers Formula
$e^{(a+bi)t} = e^{at}(\cos(bt) + i\sin(bt))$

We can have:
- Centers $a = 0$
- Spiral Source $a > 0$
- Spiral Sink $a < 0$

Case 3: Repeated Eigenvalues $\Rightarrow \lambda_1 = \lambda_2$

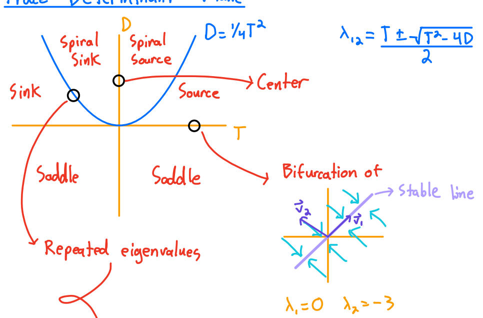
Case 1) Find 2 independent eigenvectors $\vec{v}_1, \vec{v}_2 \Rightarrow$ Case 1

Case 2) Find 1 independent eigenvector $\vec{v}_1$

Solution: $x(t) = c_1 e^{\lambda_1 t}\vec{v}_1 + c_2 e^{\lambda_2 t}\vec{v}_2(v_2 + v_1 t)$
Where: $(A - \lambda I)\vec{v}_2 = \vec{v}_1$
To find $\vec{v}_2$: Choose $\vec{w}$ (not multiple of $\vec{v}_1$)
Compute: $(A - \lambda I)\vec{w} = \alpha \vec{v}_1$
$\vec{v}_2 = \frac{\vec{w}}{\alpha}$

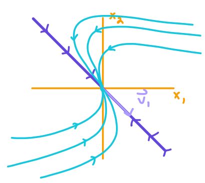
Trace - Determinant Plane

$\lambda_{1,2} = \frac{T \pm \sqrt{T^2 - 4D}}{2}$



If A depends on a parameter α we can plot T, D in terms of α to see bifurcations that occur

Case 2



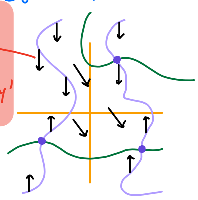
Non Linear Systems

$x' = f(x, y)$
$y' = g(x, y)$

Typically Isolated points

Equilibrium points: $f(x,y) = g(x,y) = 0$

To find vectors plug points into x' and y'



fixed points

x-nullcline: $f(x,y) = 0 \rightarrow$ typically a curve
y-nullcline: $g(x,y) = 0 \nearrow$

Partially Decoupled Systems

$x' = f(x)$
$y' = g(x, y)$

solve for $x(t)$ using 1D methods then substitute into $g(x,y)$ and solve

$\boxed{\text{HW 5} \rightarrow \text{Present On Test}}$