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2a)

C and G matrices are shown below

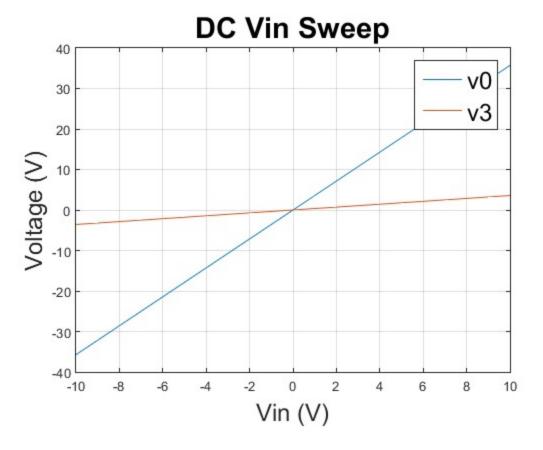
```
G1=1/1;
C=0.25;
G2=1/2;
L=0.2;
G3=1/10;
alpha=100;
G4=1/0.1;
G0=1/1000;
G = [1 0 0 0 0 0 0;
    -G2 G1+G2 -1 0 0 0 0;
    0 1 0 -1 0 0 0;
    0 0 -1 G3 0 0 0;
    0 0 0 0 -alpha 1 0;
    0 0 0 G3 -1 0 0;
    0 0 0 0 0 -G4 G4+G0];
C = [0 \ 0 \ 0 \ 0 \ 0 \ 0;
    -C C 0 0 0 0 0;
    0 0 -L 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0];
% V = [V1
       V2
        IL
        V3
9
        I3
용
        V4
```

```
% V0]
F = [0; 0; 0; 0; 0; 0];
```

2b)

DC SWEEP

```
vinvec = zeros(1,21); % vin vector
v0vec = zeros(1,21); % v0 vector
v3vec = zeros(1,21); % v3 vector
for i=1:21 % sweeping vin from -10 to 10
   F = [i-11; 0; 0; 0; 0; 0; 0];
    V = G \setminus F; % DC solution
    vinvec(i) = i-11;
    v0vec(i) = V(7);
    v3vec(i) = V(4);
end
figure (1)
plot(vinvec, v0vec)
hold on
grid on
plot(vinvec, v3vec)
title('\fontsize{22}DC Vin Sweep')
xlabel('\fontsize{18}Vin (V)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}v0','\fontsize{18}v3')
```

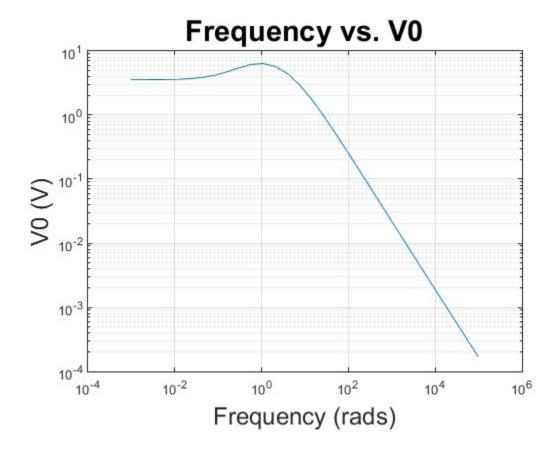


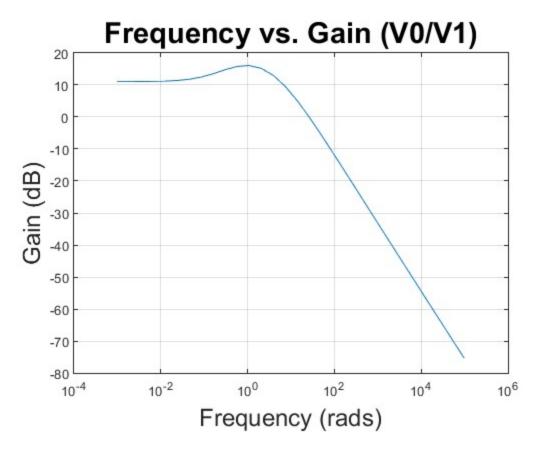
2c)

AC SWEEPS

```
omegavec = logspace(-3,5,30); % 30 log spaced values from 10^-3 to 10^5
v0vec = zeros(1,30);
F = [1; 0; 0; 0; 0; 0; 0];
for i=1:30
    V = (G+i*omegavec(i)*C) \F; % AC solution
    v0vec(i) = V(7);
end
figure (2)
loglog(omegavec, v0vec)
hold on
grid on
title('\fontsize{22}Frequency vs. V0')
xlabel('\fontsize{18}Frequency (rads)')
ylabel('\fontsize{18}V0 (V)')
figure (3)
semilogx(omegavec,20*log10(v0vec))
hold on
grid on
title('\fontsize{22}Frequency vs. Gain (V0/V1)')
```

xlabel('\fontsize{18}Frequency (rads)')
ylabel('\fontsize{18}Gain (dB)')





2d and e)

Numerical solution in time domain, with Fourier Transforms

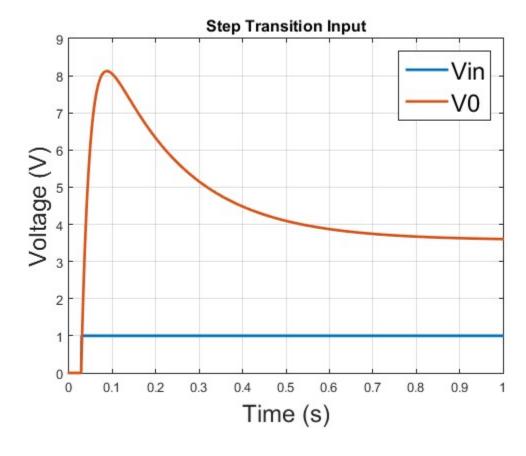
```
dt = 1e-3;
T = 1;
fs = T/dt;
time = 0:(1/fs):(1-1/fs);
n=fs;
f = (0:n-1);
V = zeros(7,T/dt);
F = zeros(7,T/dt);
Ap = inv(C/dt + G);
% Vin defined as 0 until 0.03 when source turns on
F(1,0.03/dt:T/dt) = ones([1,T/dt - 0.03/dt+1]);
for i=2:T/dt
    V(:,i) = Ap*(C*V(:,i-1)/dt + F(:,i));
end
figure (4)
plot(time,F(1,:)','LineWidth',2)
hold on
plot(time, V(7,:)', 'LineWidth',2)
title('\fontsize{12}Step Transition Input')
xlabel('\fontsize{18}Time (s)')
```

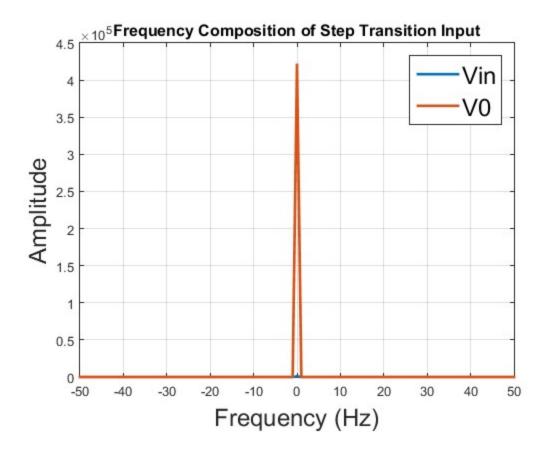
```
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}V0')
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftF1 = abs(YF1).^2/n; % zero-centered power
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
figure (5)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Step Transition Input')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}V0')
xlim([-50 50])
% Vin defined as a sinusoid
f = 1/0.03;
F1 = zeros(7,T/dt);
F2 = zeros(7,T/dt);
V = zeros(7,T/dt);
V1 = zeros(7,T/dt);
V2 = zeros(7,T/dt);
for i=dt/dt:T/dt
    F(1,i) = \sin(2*pi*f*dt*i);
    F1(1,i) = sin(2*pi*(f*10)*dt*i); % 10 times the frequency
    F2(1,i) = \sin(2*pi*(f*0.1)*dt*i); % 1/10 of the frequency
end
for i=2:T/dt
    V(:,i) = Ap*(C*V(:,i-1)/dt + F(:,i));
   V1(:,i) = Ap*(C*V1(:,i-1)/dt + F1(:,i));
    V2(:,i) = Ap*(C*V2(:,i-1)/dt + F2(:,i));
end
figure (6)
plot(time, F(1,:)', 'LineWidth', 2)
hold on
grid on
plot(time, V(7,:)', 'LineWidth', 2)
title('\fontsize{12}Sinusoid Input, f=1/0.03')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}V0')
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
```

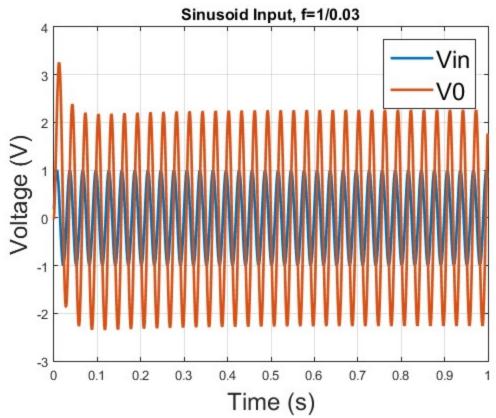
```
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftV7 = abs(YV7).^2/n; % zero-centered power
figure (7)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Sinusoid Input, f=1/0.03')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}V0')
xlim([-50 50])
figure (8)
plot(time,F1(1,:)','LineWidth',2)
hold on
grid on
plot(time, V1(7,:)', 'LineWidth', 2)
title('\fontsize{12}Sinusoid Input, f=1/0.003')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}V0')
axis([0 0.2 -1 1]);
XF1 = (fft(F1(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
XV7 = (fft(V1(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftV7 = abs(YV7).^2/n; % zero-centered power
figure (9)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift, powershiftV7, 'LineWidth', 2)
title('\fontsize{12}Frequency Composition of Sinusoid Input, f=1/0.003')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}V0')
xlim([-500 500])
figure (10)
plot(time,F2(1,:)','LineWidth',2)
hold on
grid on
plot(time, V2(7,:)', 'LineWidth',2)
```

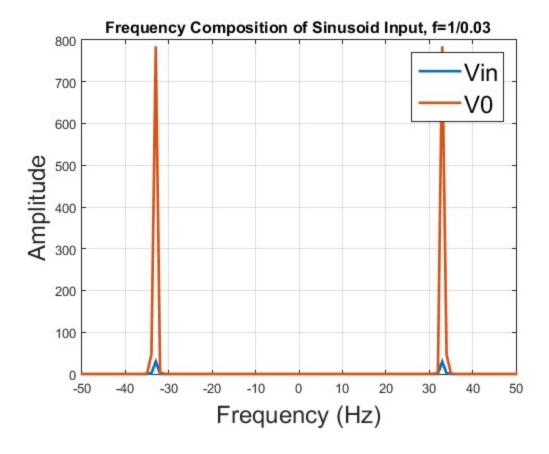
```
title('\fontsize{12}Sinusoid Input, f=1/0.3')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}V0')
XF1 = (fft(F2(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
XV7 = (fft(V2(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftV7 = abs(YV7).^2/n; % zero-centered power
figure (11)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Sinusoid Input, f=1/0.3')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}VO')
xlim([-50 50])
% Vin defined as a gaussian
V = zeros(7,T/dt);
F = zeros(7,T/dt);
F(1,:) = normpdf(time, 0.06, 0.03)*max(normpdf(time, 0.06, 0.03))^(-1);
for i=2:T/dt
   V(:,i) = Ap*(C*V(:,i-1)/dt + F(:,i));
end
figure (12)
plot(time,F(1,:)','LineWidth',2)
hold on
grid on
plot(time, V(7,:)', 'LineWidth', 2)
title('\fontsize{12}Gaussian Pulse')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}VO')
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftF1 = abs(YF1).^2/n; % zero-centered power
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftV7 = abs(YV7).^2/n; % zero-centered power
figure (13)
```

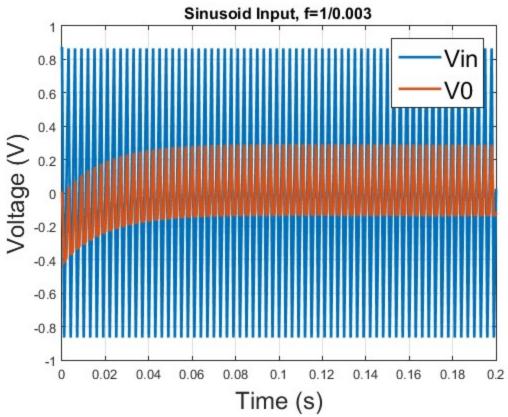
```
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Gaussian Pulse')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}V0')
xlim([-50 50])
```

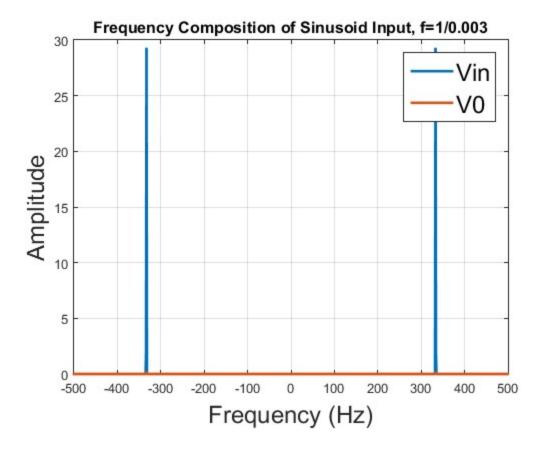


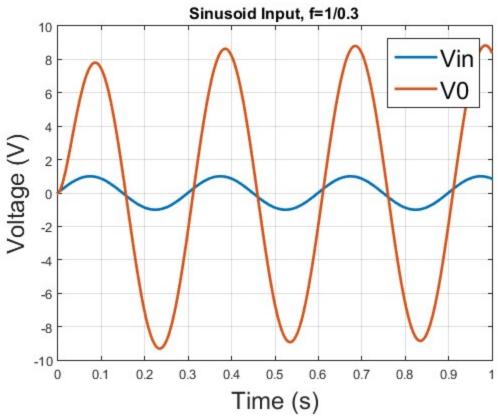


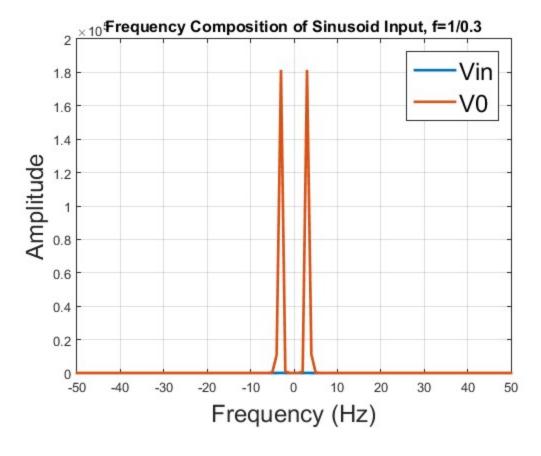


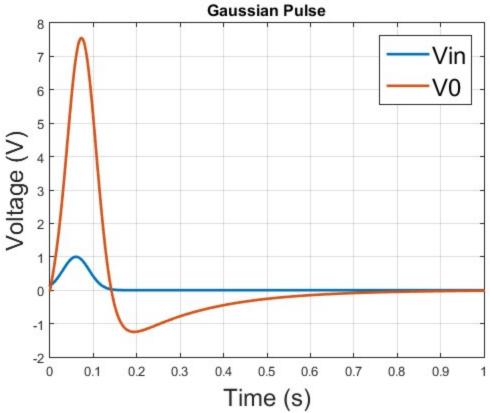


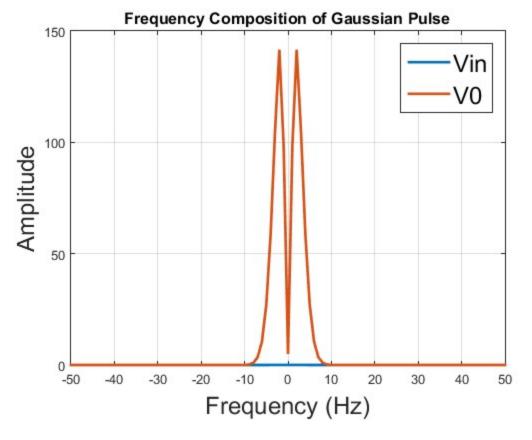












2) Comments

Increasing the time step by small amounts does not drastically reduce the accuracy of the simulation, but at around dt=30e-3 the simulation loses consistency with previous results. There is no significant improvement in accuracy when reducing dt past 1e-3

3a)

Updated C matrix

```
G1=1/1;
C=0.25;
G2=1/2;
L=0.2;
G3=1/10;
alpha=100;
G4=1/0.1;
G0=1/1000;
Cn = 0.00001;
% V = [V1]
         V2
        IL
        V3
         IЗ
        V4
        V0]
```

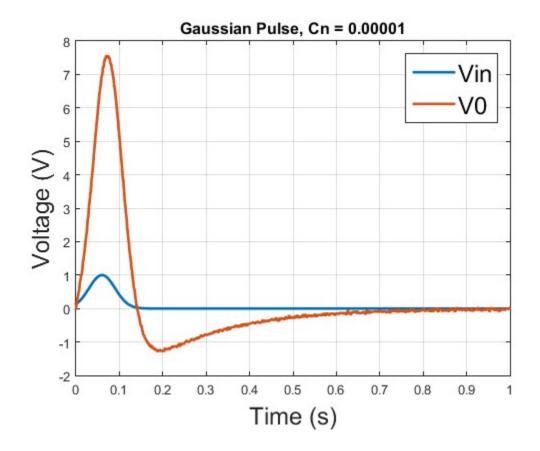
```
% F = [Vin
양
        0
용
        0
        In
용
        0
        0
용
        0]
G = [1 0 0 0 0 0 0;
    -G2 G1+G2 -1 0 0 0 0;
    0 1 0 -1 0 0 0;
    0 0 -1 G3 0 0 0;
    0 0 0 0 -alpha 1 0;
    0 0 0 G3 -1 0 0;
    0 0 0 0 0 -G4 G4+G0];
C = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0;
    -C C O O O O O;
    0 0 -L 0 0 0 0;
    0 0 0 Cn 0 0 0; % Cn added to C matrix
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0];
```

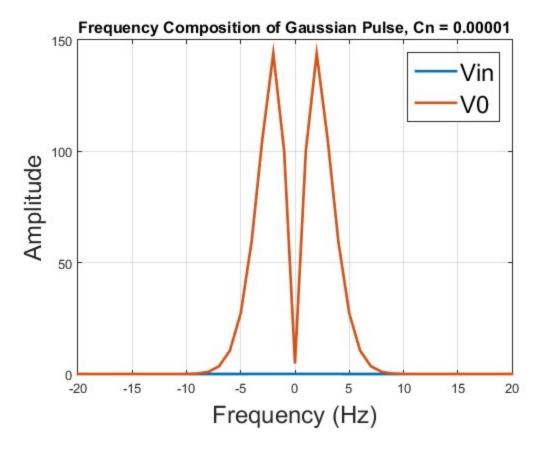
3b and c)

Plots of Vout with noise, and the Fourier transform

```
dt = 1e-3;
T = 1;
fs = T/dt;
time = 0:(1/fs):(1-1/fs);
n=fs;
f = (0:n-1);
V = zeros(7,T/dt);
F = zeros(7,T/dt);
Ap = inv(C/dt + G);
% Vin defined as a gaussian
F(1,:) = normpdf(time, 0.06, 0.03) *max(normpdf(time, 0.06, 0.03))^(-1);
F(4,:) = normrnd(0.001, 0.0003, 1, T/dt); % In randomly picked from normal dist
for i=2:T/dt
    V(:,i) = Ap*(C*V(:,i-1)/dt + F(:,i));
end
figure (14)
plot(time,F(1,:)','LineWidth',2)
hold on
grid on
plot(time, V(7,:)','LineWidth',2)
title('\fontsize{12}Gaussian Pulse, Cn = 0.00001')
xlabel('\fontsize{18}Time (s)')
```

```
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}V0')
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
figure (15)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Gaussian Pulse, Cn = 0.00001')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}V0')
xlim([-20 20])
```





3e)

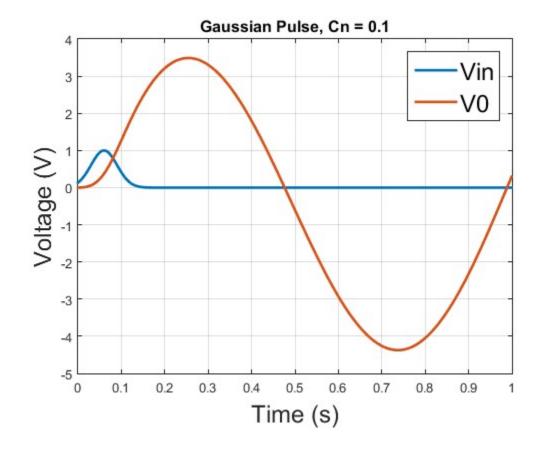
Plots as above, but with different Cn values (2 in this section, plus 1 in previous, 3 total)

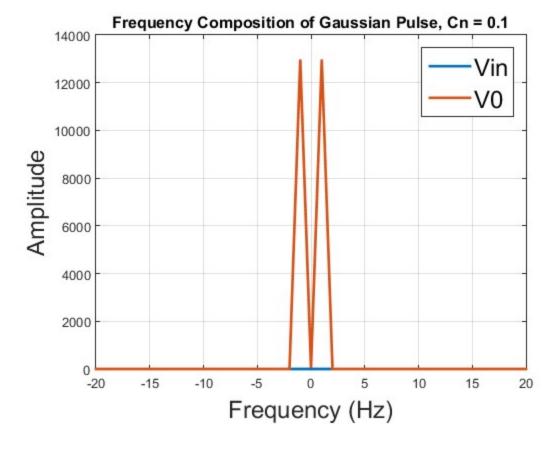
```
% Cn = 0.1
Cn = 0.1;
C(4,4) = Cn; % updating matrix
V = zeros(7,T/dt);
F = zeros(7,T/dt);
Ap = inv(C/dt + G);
% Vin defined as a gaussian
F(1,:) = normpdf(time, 0.06, 0.03)*max(normpdf(time, 0.06, 0.03))^(-1);
F(4,:) = normrnd(0.001, 0.0003, 1, T/dt); % In randomly picked from normal dist
for i=2:T/dt
    V(:,i) = Ap*(C*V(:,i-1)/dt + F(:,i));
end
figure (16)
plot(time,F(1,:)','LineWidth',2)
hold on
grid on
plot(time, V(7,:)','LineWidth',2)
title('\fontsize{12}Gaussian Pulse, Cn = 0.1')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}VO')
```

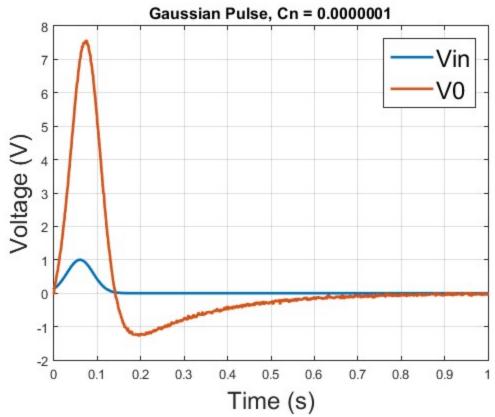
```
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftF1 = abs(YF1).^2/n; % zero-centered power
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
figure (17)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Gaussian Pulse, Cn = 0.1')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}V0')
xlim([-20 20])
% Cn = 0.0000001
Cn = 0.0000001;
C(4,4) = Cn; % updating matrix
V = zeros(7,T/dt);
F = zeros(7,T/dt);
Ap = inv(C/dt + G);
% Vin defined as a gaussian
F(1,:) = normpdf(time, 0.06, 0.03) *max(normpdf(time, 0.06, 0.03))^(-1);
F(4,:) = normrnd(0.001, 0.0003, 1, T/dt); % In randomly picked from normal dist
for i=2:T/dt
   V(:,i) = Ap^*(C^*V(:,i-1)/dt + F(:,i));
end
figure (18)
plot(time, F(1,:)', 'LineWidth', 2)
hold on
grid on
plot(time, V(7,:)', 'LineWidth',2)
title('\fontsize{12}Gaussian Pulse, Cn = 0.0000001')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}V0')
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftF1 = abs(YF1).^2/n; % zero-centered power
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
```

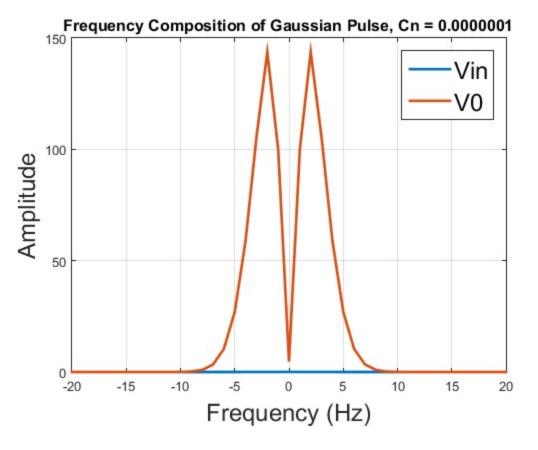
```
powershiftV7 = abs(YV7).^2/n;  % zero-centered power

figure (19)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Gaussian Pulse, Cn = 0.0000001')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}VO')
xlim([-20 20])
```







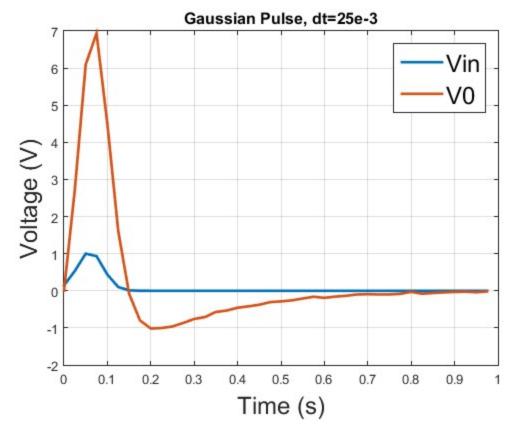


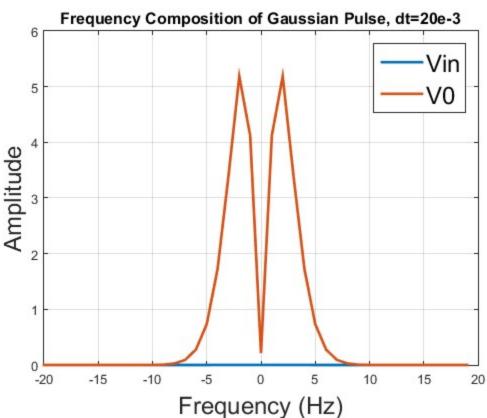
3f)

Plots of Vout with dt = 25e-3 (1 plot in this section, plus one in previous, total of 2)

```
% dt = 25e-3
Cn = 0.0001;
C(4,4) = Cn; % updating matrix
dt = 25e-3;
T = 1;
fs = T/dt;
time = 0:(1/fs):(1-1/fs);
n=fs;
f = (0:n-1);
V = zeros(7,T/dt);
F = zeros(7,T/dt);
Ap = inv(C/dt + G);
% Vin defined as a gaussian
F(1,:) = normpdf(time, 0.06, 0.03) *max(normpdf(time, 0.06, 0.03))^(-1);
F(4,:) = normrnd(0.001, 0.0003,1,T/dt); % In randomly picked from normal dist
for i=2:T/dt
    V(:,i) = Ap*(C*V(:,i-1)/dt + F(:,i));
end
figure (20)
plot(time,F(1,:)','LineWidth',2)
```

```
hold on
grid on
plot(time, V(7,:)', 'LineWidth', 2)
title('\fontsize{12}Gaussian Pulse, dt=25e-3')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}VO')
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftF1 = abs(YF1).^2/n; % zero-centered power
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftV7 = abs(YV7).^2/n; % zero-centered power
figure (21)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
grid on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Gaussian Pulse, dt=20e-3')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}VO')
xlim([-20 20])
```





3) Comments

The changes to the time stepping are consistent with the results discussed in part 2. Increasing Cn to around 0.1 causes the voltage output to swing much larger than with the starting Cn value of 0.00001, and also causes the bandwidth to decrease significantly. Decreasing the capacitor value past 0.00001 does not have a visible impact on the bandwidth of the signal.

4a)

The equation used to iterate through time was $V(:,i)=inv(C/dt+G)^*(C^*V(:,i-1)/dt+F(:,i))$. With the addition of a non-linear component, the equation would need to change to $V(:,i)=inv(C/dt+G)^*(C^*V(:,i-1)/dt+F(:,i)-B(V(5,i-1)))$, where B is the non-linear vector that is a function of the current I3, which is V(5,i-1). The vector B would be:

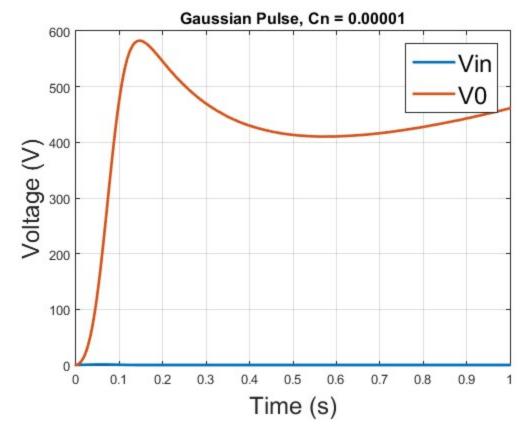
B = [0 0 0 0 0 alpha*I3 + beta*I3^2 + gamma*I3^3 0]

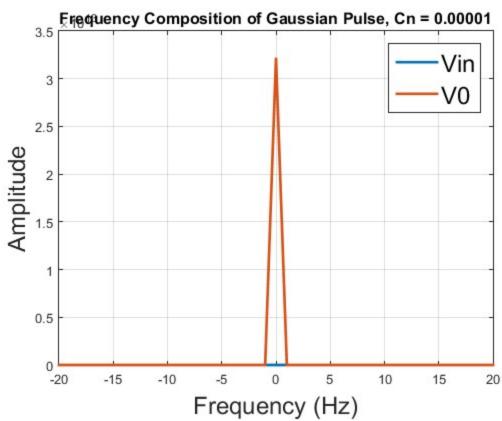
4b)

Non-linear implimentation

```
G1=1/1;
C=0.25;
G2=1/2;
L=0.2;
G3=1/10;
alpha=100;
G4=1/0.1;
G0=1/1000;
%Cn = 0.00001;
Cn = 0.00001;
% V = [V1
        V2
        IL
        V3
용
        Ι3
        V4
        V0]
% F = [Vin
        0
        0
        In
        0
        0
용
        0]
G = [1 0 0 0 0 0 0;
    -G2 G1+G2 -1 0 0 0 0;
    0 1 0 -1 0 0 0;
    0 0 -1 G3 0 0 0;
    0 0 0 0 -alpha 1 0;
    0 0 0 G3 -1 0 0;
    0 0 0 0 0 -G4 G4+G0];
C = [0 \ 0 \ 0 \ 0 \ 0 \ 0;
    -C C 0 0 0 0 0;
    0 0 -L 0 0 0 0;
    0 0 0 Cn 0 0 0; % Cn added to C matrix
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0;
    0 0 0 0 0 0 0];
```

```
dt = 1e-3;
T = 1;
fs = T/dt;
time = 0:(1/fs):(1-1/fs);
n=fs;
f = (0:n-1);
V = zeros(7,T/dt);
F = zeros(7,T/dt);
Ap = inv(C/dt + G);
B = zeros(7,1);
alpha = 1;
beta = 0.0001;
gamma = 0.000001;
% Vin defined as a gaussian
F(1,:) = \text{normpdf}(\text{time}, 0.06, 0.03) * \text{max}(\text{normpdf}(\text{time}, 0.06, 0.03))^{(-1)};
F(4,:) = normrnd(0.001, 0.0003, 1, T/dt); % In randomly picked from normal dist
for i=2:T/dt
    V(:,i) = Ap^*(C^*V(:,i-1)/dt + F(:,i)-B);
    B(6) = alpha*V(5,i) + beta*V(5,i)^2 + gamma*V(5,i)^3;
end
figure (22)
plot(time, F(1,:)', 'LineWidth', 2)
hold on
grid on
plot(time, V(7,:)','LineWidth',2)
title('\fontsize{12}Gaussian Pulse, Cn = 0.00001')
xlabel('\fontsize{18}Time (s)')
ylabel('\fontsize{18}Voltage (V)')
legend('\fontsize{18}Vin','\fontsize{18}VO')
XF1 = (fft(F(1,:))).^2/n;
YF1 = fftshift(XF1);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftF1 = abs(YF1).^2/n; % zero-centered power
XV7 = (fft(V(7,:))).^2/n;
YV7 = fftshift(XV7);
fshift = (-n/2:n/2-1)*(fs/n); % zero-centered frequency range
powershiftV7 = abs(YV7).^2/n; % zero-centered power
figure (23)
plot(fshift,powershiftF1,'LineWidth',2)
hold on
plot(fshift,powershiftV7,'LineWidth',2)
title('\fontsize{12}Frequency Composition of Gaussian Pulse, Cn = 0.00001')
xlabel('\fontsize{18}Frequency (Hz)')
ylabel('\fontsize{18}Amplitude')
legend('\fontsize{18}Vin','\fontsize{18}VO')
xlim([-20 20])
```





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