1. Suppose $X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(\mu)$, and that X and Y are independent. Let $Z = X + Y$. Prove that $Z \sim \text{Poisson}(\lambda + \mu)$ via both of the following approaches.
   (a) Using MGF method.
   (b) By finding $\mathbb{P}(Z = z)$ directly.

2. Two random variables X and Y independently follow a distribution with density function given by
$$f(y) = \begin{cases} \frac{1}{4} y e^{-y/2}, & y > 0 \\ 0, & \text{otherwise} \end{cases}.$$
Find density function of the average: $Z = (X + Y)/2$.

3. Let measurements from a machine be $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, 1)$, and denote the sample mean computed from n samples as $\bar{X}_n$: i.e.,
$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i.$$
   (a) Using above notation, $\bar{X}_9$ is sample mean of $X_1, \ldots, X_9$. Suppose we know that
$$\mathbb{P}\left(|\bar{X}_9 - \mu| \leq 0.3\right) = 0.6318.$$
   Interpret the probability statement in plain words.
   (b) Determine $\mathbb{P}\left(|\bar{X}_n - \mu| \leq 0.3\right)$ for increasing n. In particular, compute the probabilities for $\bar{X}_{16}$, $\bar{X}_{25}$, $\bar{X}_{36}$, $\bar{X}_{49}$, and $\bar{X}_{64}$.
   (c) What is the pattern of $\bar{X}_n$ as n increases?

4. Suppose now that a different machine's measurements are $V_1, \ldots, V_n \overset{iid}{\sim} N(\mu, 2)$, where definition of $\bar{V}_n$ is identical.
   (a) Again, determine $\mathbb{P}\left(|\bar{V}_n - \mu| \leq 0.3\right)$ for increasing n. In particular, compute the probabilities for $\bar{V}_{16}$, $\bar{V}_{25}$, $\bar{V}_{36}$, $\bar{V}_{49}$, and $\bar{V}_{64}$.
   (b) What is the pattern as n increases?
   (c) How do these probabilities $\bar{V}_n$ for each n compare with probabilities $\bar{X}_n$ in problem 3?

5. A scale measures weight in pounds, that are distributed following a normal distribution with variance of 4 square pounds.
   (a) Based on $n = 9$ measurements, find the probability that sample mean will be within 2 pounds of the true weight?
   (b) We would like to measure the sample mean within 1 pound with probability 0.9. How many measurements are needed to ensure this degree of accuracy?

6. Suppose that $V \sim f(v)$ where
$$f(v) = \begin{cases} e^{-(v-\alpha)}, & v > \alpha \\ 0, & \text{otherwise} \end{cases}.$$
where $\alpha > 0$. Let $V_1, \ldots, V_n$ denote random sample from this distribution.

(a) Find density function of $Y_1 = \min\{V_1, \ldots, V_n\}$

(b) Compute the expected value of Y_1.

7. Let $X_1, \ldots, X_n \overset{iid}{\sim}$ Uniform$(0, 1)$. Find the probability density function for $V = Y_n - Y_1$, where $Y_1 = \min\{X_1, \ldots, X_n\}$ and $Y_n = \max\{X_1, \ldots, X_n\}$.

8. A scale measures weight in pounds, that are distributed following a normal distribution with an unknown variance. Suppose we estimate the variance to be S^2 from $n = 9$ random sample of measurements. Find a and b such that

$$\mathbb{P}(a \leq \bar{Y} - \mu \leq b) = 0.90.$$

Is there an ambiguity in choosing a and b? What would be a reasonable decision to make? *Note: Your answer will be a function of S.*

9. (a) Let $V \sim \chi^2$ with ν degrees of freedom. Use the moment generating function to find $\mathbb{E}(V)$ and $\mathbb{V}(V)$.

   (b) We know $(n-1)S^2/\sigma^2 \sim \chi^2$ with $n-1$ degrees of freedom. Using the result from previous part to compute $\mathbb{E}(S^2)$ and $\mathbb{V}(S^2)$.

10. A fair coin is tossed 20 times and all tosses are independent.

   (a) What is the true probability of obtaining exactly 10 heads? Compute and compare the probability when approximated by an appropriate normal distribution.

   (b) Compute approximate probability of obtaining more than 7 heads *both with and without* continuity correction. Compare these approximations to the exact probability found using pmf of binomial distribution.