prof. balzano mentioned that anti-aligned vectors also cause problems with image reconstruction. trying to look into why that is.

suppose we have highly clustered sets of rows $\beta_1, ..., \beta_k$, where by "highly clustered" we mean that the vectors in each β_i are very close to one another w.r.t. orientation. Let's reorder U such that $U = \begin{bmatrix} U_{\beta_1} \\ ... \\ U_{\beta_k} \end{bmatrix}$. This shuffles the rows of UW, which should not change completability, so we're fine here.

the idea is that we can reshuffle the columns of UW by their alignment with these groups.

so basically we have $UW = \begin{bmatrix} 1\ 0...\ 0 \\ 0\ 1...\ 0 \\ .... \\ 0\ 0...\ 1 \end{bmatrix}$

and if we solve those subproblems (which should be easy), it's just a matter of fitting them back together. basically, how do we align these smaller subspaces correctly?

Let's consider k=2. So we have $U = \begin{bmatrix} U_{\beta_1} \\ U_{\beta_2} \end{bmatrix}$ and have found valid subspaces for U_{β_1} and U_{β_2}. Because this is a matter of alignment, WLOG we can set U_{β_2} to a basis for the subspace and orient U_{β_1} accordingly. Consider some orthogonal basis Γ for the subspace of U_{β_1}.

Let's consider our matrix $\begin{bmatrix} U_{\beta_1} \\ U_{\beta_2} \end{bmatrix} \begin{bmatrix} W_1 & W_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$. A and D have been used to recover the subspaces.

Given an orthogonal basis for a subspace, we can translate it into another basis via a linear combination: $\Gamma V = U_{\beta_1}$. When we want the result to be orthogonal as well, V is necesasrily orthogonal. So we are solving for an r by r orthogonal matrix.

We can solve for W_2 pretty easily by using

so we basically have $U_{\beta_1} V w = y$ for known U_{β_1}, w, y, and we're trying to solve for a linear combination of the columns of U (a valid basis) which satisfies the $U_\beta V w = y$.

V must be orthogonal because UV is a linear combination of the columns, but is still orthogonal, and $(UV)'(UV) = V'U'UV = V'V = I$.