Introduction to Game Theory

with Connor T. Wiegand

Intro

What are examples of games?

- Ticket to Ride
- League
- Football
- Baldur’s Gate
- Duopoly competition
- Job Market

Game theory Games

- Typically smaller-scale, isloated scenarios
  → Analyze a piece of a bigger game
  → In a vacuum (remember: modeling discipline)

- Prisoner’s Dilema
- Matching Pennies
- Chicken
- Beer/quiche
- RPS
- How much college? (job market signaling)
- BoS

Who Cares?

- Amazon
- Verizon
- Coke/Pepsi
- Valve
- Everyday Uses:
  → How should you fight family over the holidays?
  → How should you approach returning an item to a store?
  → When should you email your professor?

What’s in a Game?

From last time…

- There are three things that are essential to specifying a game:

1. Players
2. Strategies
3. Payoffs

Players

- Agents, users, etc.
- To be a proper game, there should be at least 2 players
- Ex Sukhjit is taking a penalty kick in soccer. There is a 50% chance the goalie defends left, and 20% chance they defend the middle, and a 30% chance they defend right.
- This is something we might think of as a "game", but is really just a decision with some exogeneity[1] from the environment
- Having two or more *real* players is sometimes known as "strategic interaction"

1. exogenous = 'falls from the sky'

Strategies

- AKA
  - → Moves
  - → Actions
- What choices the player has
- For now, the above terms can be treated as identical
- Later, when we get to sequential games, we will start using these terms with a little more care
  - → Namely, a strategy is an *an information-contingent plan of action*
  - → An action is a *component* of a strategy
- Typically finite, can be infinite
  - → Ultimatum game, 2/3 avg, price-setting

Payoffs

- Crucial to economics
- The payoff/utility[1] obtained from a specific outcome in a game
  - → (outcome = result of each player playing a pre-determined strategy)
- Can be specified via a function
  - → Ex. $\pi(P, Q) = (100 + q_1 + q_2) - 5q_1$
- Or a list
  - → Ex. Outcomes (A, B, C, D) correspond to payoffs $(20, 40, 60, 80)$
  - → Ex. $u(s_1, s_2) =$ <value> for each strategy s_i that player i could play
- Basically: a map from outcomes to numbers

1. We may use either of $u(\cdot)$ ("utility") or $\pi(\cdot)$ ("payoff")

Assumptions in Game Theory

Assumptions

- Suppose that you and I represent the players in a game
- One mild assumption we would like to impose is that you and I are on the same page regarding the details of the game
  → e.g., the players, strategies, and outcomes
- We refer to these details as the *rules of the game*
- But how do we formalize “on the same page”?

The Rules of the Game

1. I know the rules of the game
   → I know that you know the rules of the game
   → I know that you know that I know the rules of the game
   → I know that you know that I know that you know the rules of the game
   → …
2. You know the rules of the game
   → You know that I know the rules of the game
   → You know that I know that you know the rules of the game
   → You know that I know that you know that I know the rules of the game
   → …

Common Knowledge of RotG

Let Γ be a game with players $P_1, \ldots, P_n$

<u>Assumption 1:</u> Common Knowledge of the Rules of the Game

- *Every statement of the form "player i knows that player j knows that … player k knows the rules of the game" is true*

- Blue Eyed Islander Problem – backup

Assumptions, cont.

1. Common Knowledge of the Rules of the Game
2. Common Knowledge of Rationality
    a. I am rational
        → I know that you are rational
        → I know that you know that I am rational
        → …
    b. You are rational
        → You know that I am rational
        → You know that I know that you are rational
        → …

… but what is rationality?

Rationality

What is it?

- ~~Players maximize their expected payoffs~~

Technically speaking, rational agents are those who exhibit preferences which satisfy both of:

1. Completeness
2. Transitivity

Wait, what?

Recall:

- In microeconomics, agents are said to have *preferences* over goods (or *bundles* of goods)
- We represent these preference relations with the $\succsim$ symbol, as follows
- For (bundles of) goods and , consider Taylor's preferences:
  - → reads *"Taylor weakly prefers to "*
    - – or *"Taylor wants as least as much as "*
  - → reads *"Taylor strictly prefers to "*
- This allows us to define two more notions:
  - → When <u>and</u> , we say
    - – : *"Taylor is indifferent between and "*
  - → When but , we say
    - – : *"Taylor strictly prefers to "*

Rational Preferences

1. Completeness:
    - → Def Given , an agent's preferences are complete provided at least one of the following is true:
        - →
        - →
    - → Ex.: Jump to appendix
2. Transitivity:
    - → Def Given , suppose that and . Under these conditions, an agent's preferences are transitive provided that
    - → Ex.
      Jake roots for the Lions when they play the Bengals, and the Bengals when they play the Bears. But, Jake roots for the Bears when they play the Lions

Assumptions

…so…?

- We assume agents' preferences follow the classical assumptions of microeconomics…

  → (Players maximize their expected payoffs)

- Given this, our two assumptions moving forward are

  1. Common Knowledge of the Rules of the Game
  2. Common Knowledge of Rationality

- and that's it!

Review

Symbol Cheat-Sheets

Some selected tables of symbols which may be useful

- Greek
- Set Theory
- Logic

Some Math Resources

- As-needed basis:
  - → Paul's online math notes is great
    - – I have attached the link to the algebra section, but make sure to brief yourself with whatever you need (graphing, etc.).

- Overkill, but will be helpful to skim through before next class:
  - → Set theory video 1
  - → Set theory video 2

- *Please* reach out if you need further (specific) resources

Appendix

Incomplete Preferences

Via `economics.stackexchange`:

> Let be the number of possible baskets of goods that one can buy from a Walmart Superstore. Even if there were only 1,000 distinct items and we could only buy at most one of each item, that'd be possible baskets. (Note that "any estimate of the number of particles in the universe".)
>
> Even as a normative matter, it is debatable whether a perfectly rational being "should" have a complete preference ordering over these baskets.
>
> But as a positive matter, many (if not all) human beings will not have a complete preference ordering over these baskets.

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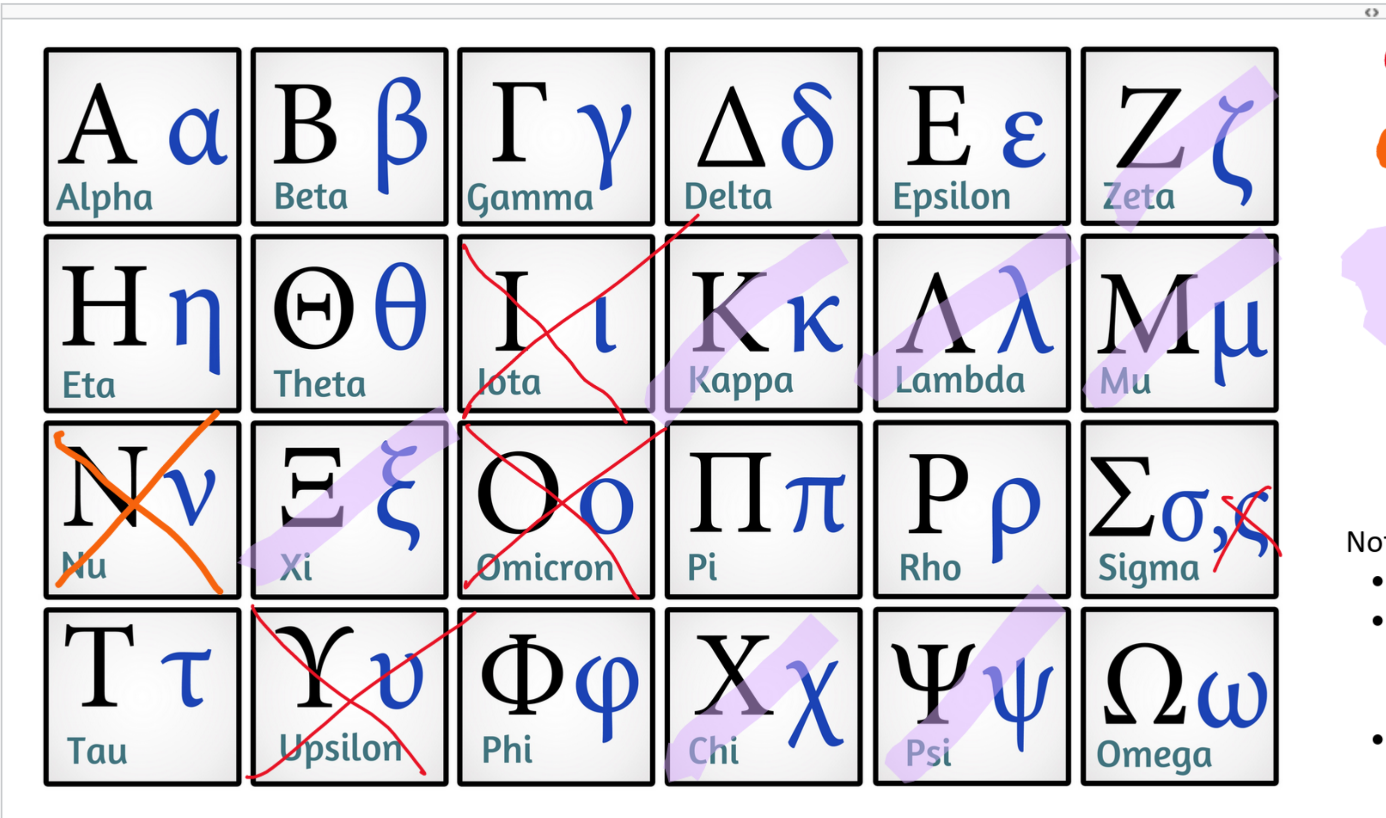
Select Greek



= don't need

= Probably don't need

= common in GT, but less likely to use

Notes:
- Capital Pi usually reserved for products
- Capital Sigma often refers to sums, but will also be our symbol for the (mixed) strategy spaces
- Capital letters in Greek that look like Latin (Engligh) letters will almost never be used (i.e. A will mean English A, not alpha)
- Capital Delta means "change" or "change in"
- Epsilon refers to a very small arbitrary value (usually positive)

$\forall x$	for all x
$\exists\, x$	there exists x

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Some Set Theory Symbols

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Symbol	Symbol Name	Meaning / definition	Example
$\{\ \}$	set	a collection of elements	$A=\{3,7,9,14\}, B=\{9,14,28\}$
$A \cap B$	intersection	objects that belong to set A and set B	$A \cap B = \{9,14\}$
$A \cup B$	union	objects that belong to set A or set B	$A \cup B = \{3,7,9,14,28\}$
$A \subseteq B$	subset	subset has less elements or equal to the set	$\{9,14,28\} \subseteq \{9,14,28\}$
$A \subset B$	proper subset / strict subset	subset has less elements than the set	$\{9,14\} \subset \{9,14,28\}$
$A \not\subset B$	not subset	left set not a subset of right set	$\{9,66\} \not\subset \{9,14,28\}$
$\cancel{A \supseteq B}$	superset	set A has more elements or equal to the set B	$\{9,14,28\} \supseteq \{9,14,28\}$
$\cancel{A \supset B}$	proper superset / strict superset	set A has more elements than set B	$\{9,14,28\} \supset \{9,14\}$
$\cancel{A \not\supset B}$	not superset	set A is not a superset of set B	$\{9,14,28\} \not\supset \{9,66\}$
$\cancel{2^A}$	~~power set~~	~~all subsets of A~~	
$\cancel{\mathcal{P}(A)}$	~~power set~~	~~all subsets of A~~	
$A = B$	equality	both sets have the same members	$A=\{3,9,14\}, B=\{3,9,14\}, A=B$
$\cancel{A^c}$	complement	all the objects that do not belong to set A	
$\cancel{A \setminus B}$	relative complement	objects that belong to A and not to B	$A=\{3,9,14\},\quad B=\{1,2,3\},$ $A-B=\{9,14\}$
$\cancel{A - B}$	~~relative complement~~	~~objects that belong to A and not to B~~	$\cancel{A=\{3,9,14\},\quad B=\{1,2,3\},}$ $\cancel{A-B=\{9,14\}}$
$\cancel{A \Delta B}$	~~symmetric difference~~	~~objects that belong to A or B but not to their intersection~~	$\cancel{A=\{3,9,14\},\quad B=\{1,2,3\}, A \Delta}$ $B=\{1,2,9,14\}$
$\cancel{A \ominus B}$	~~symmetric difference~~	~~objects that belong to A or B but not to their intersection~~	$\cancel{A=\{3,9,14\},\quad B=\{1,2,3\}, A \ominus}$ $B=\{1,2,9,14\}$
$a \in A$	element of	set membership	$A=\{3,9,14\}, 3 \in A$
$x \notin A$	not element of	no set membership	$A=\{3,9,14\}, 1 \notin A$
(a,b)	ordered pair	collection of 2 elements	
$A \times B$	cartesian product	set of all ordered pairs from A and B	
$\lvert A \rvert$	cardinality	the number of elements of set A	$A=\{3,9,14\}, \lvert A \rvert=3$
$\cancel{\#A}$	~~cardinality~~	~~the number of elements of set A~~	$\cancel{A=\{3,9,14\}, \#A=3}$
$\cancel{\aleph}$	~~aleph~~	~~infinite cardinality~~	
$\emptyset$	empty set	$\emptyset = \{\ \}$	$C = \{\emptyset\}$
$\cancel{U}$	~~universal set~~	~~set of all possible values~~	
$\mathbb{N}_m$	natural numbers set (with zero)	$\mathbb{N}_0 = \{0,1,2,3,4,...\}$	$0 \in \mathbb{N}_0$
$\cancel{\mathbb{N}_1}$	~~natural numbers set (without zero)~~	$\cancel{\mathbb{N}_1 = \{1,2,3,4,5,...\}}$	$\cancel{6 \in \mathbb{N}_1}$
$\mathbb{Z}$	integer numbers set	$\mathbb{Z} = \{...-3,-2,-1,0,1,2,3,...\}$	$-6 \in \mathbb{Z}$
$\mathbb{Q}$	rational numbers set	$\mathbb{Q} = \{x \mid x=a/b, a,b \in \mathbb{N}\}$	$2/6 \in \mathbb{Q}$
$\mathbb{R}$	real numbers set	$\mathbb{R} = \{x \mid -\infty < x < \infty\}$	$6.343434 \in \mathbb{R}$

Some Logical Symbols

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$\forall x$	for all x
$\exists\, x$	there exists x
$\therefore P$	therefore P
$\because P$	because of P
$a \Rightarrow b$	a implies b, if a then b
$\cancel{a \rightarrow b}$	~~a implies b~~
$\cancel{a \supset b}$	~~a implies b, a contains b~~ see below
$a \Leftarrow b$	a is implied by b, if b then a
$\cancel{a \leftarrow b}$	~~a is implied by b~~
$\cancel{a \subset b}$	~~a is implied by b, a is contained in b~~
a if b	a is implied by b
$a \Leftrightarrow b$	a is equivalent to b, a if and only if b
$a \leftrightarrow b$	a is equivalent to b
$a \equiv b$	a is equivalent to b
a iff b	a is equivalent to b
$\sim P \leftrightarrow \cancel{\neg P}$	not P
$\cancel{\bar{P}}$	~~not P~~
$a \cap b$	~~a and b,~~ intersection of a and b
$\cancel{a \bullet b}$	~~a and b~~
$\cancel{a \wedge b}$	~~a and b~~
$a \cup b$	~~a or b,~~ union of a and b
$\cancel{a + b}$	~~a or b~~
$\cancel{a \vee b}$	~~a or b~~
$a \in S$	a is an element of the set S
$a \notin S$	a is not an element of the set S
$\cancel{x \ni P}$	~~x such that P~~

Extra

We introduced some set-theory notation on the chalkboard, including

- Sets
  - → e.g. reads “S is the set of all pairs such that is even and is odd”
  - → I will use this for the examples below as well.
- Elements of sets
  - → e.g.
  - → Importantly, is the same as
- The cartesian product of sets
  - → If and , then

To introduce these concepts, we began discussing strategy spaces, strategy profiles, and the set of strategy profiles, but this will be covered formally at a later date (e.g. next week)