Game Theoryst

Connor T. Wiegand

Contents

Overview

Full Example

```
#nfg(
  players: ([A\ Joe], [Bas Pro]),
  s1: ([$x$], [a]),
  s2: ("x", "aaaa", [$a$]),
  pad: ("x": 12pt, "y": 10pt),
  eliminations: ("s11", "s21", "s22"),
  ejust: (
    s11: (x: (0pt, 36pt), y: (-3pt, -3.5pt)),
    s22: (x: (-10pt, -12pt), y: (-10pt, 10pt)),
    s21: (x: (-3pt, -9pt), y: (-10pt, 10pt)),
  ),
  mixings: (hmix: ($p$, $1-p$), vmix: ($q$, $r$, $1-q-r$)),
  custom-fills: (hp: maroon, vp: navy, hm: purple, vm: fuchsia, he: gray, ve:
gray),
  [$0,vul(100000000)$], [$0,1$], [$0,0$],
  [$hul(1),1$], [$0, -1$], table.cell(fill: yellow.lighten(30%),
[$hful(0),vful(0)$])
)
```

		Bas Pro		
		$[q]$ x	$[r]$ aaaa	$[1-q-r]$ a
A Joe	$[p]$ x	~~$0, \underline{100000000}$~~	~~$0, 1$~~	~~$0, 0$~~
	$[1-p]$ a	$\underline{\mathbf{1}}, 1$	$0, -1$	$\underline{0}, \underline{0}$

Importing

~~Simply insert the following into your Typst code:~~ (*Coming soon*)

```
#import "@preview/game-theoryst:0.1.0": *
```

This imports the `nfg()` function as well as the underlining methods. If you want to tweak the helper functions for generating an `nfg`, import them explicitly through the `utils/` directory. The package's repository is located at https://github.com/connortwiegand/game-theoryst.

Example

The main function to make strategic (or **normal**) form games is `nfg`. For a basic 2x2 game, you can do

```
#nfg(
  players: ("Jack", "Diane"),
  s1: ($C$, $D$),
  s2: ($C$, $D$),
  [$10,10$], [$2,20$],
  [$20,2$], [$5,5$],
)
```

		Diane	
		C	D
Jack	C	$10, 10$	$2, 20$
	D	$20, 2$	$5, 5$

All of `<players>`, `<s1>`, and `<s2>` have defaults for convenience sake. Payoffs (table entries) are provided via unamed arguments after all other payoff matrix options have been set.

Color

By default, player names, mixed-strategy parameters (called *mixings*), and elimination lines are shown in color. These colors can be turned off at the method-level by passing `bw: true`, or at the document level by running the state helper-function `#colorless()`.

`nfg` accepts custom colors for all of the aforementioned parameters by passing a `dictionary` of colors to the `custom-fills` arg. The keys for this dictionary are as follows (`<defaults>`):

- `hp` – "horizontal player" (red)
- `vp` – "vertical player" (blue)
- `hm` – "hor. mixing" (`#e64173`)
- `vm` – "ver. mixing" (eastern)
- `he` – "hor. elimination" line (orange)
- `ve` – "ver. elimination" line (olive)

Cell Customization

Since the payoffs are implemented as argument sinks (`..args`) which are passed directly to Typst's `#table()`, underlining of non-math can be accomplished via the standard `#underline()` command. Similarly, any of the payoff cells can be customized by using `table.cell()` directly. For instance, `table.cell(fill: yellow.lighten(30%), [$1,1$])` can be used to highlight a specific cell.

Padding

There are edge cases where the default padding may be off. These can be mended by passing the optional `pad` argument to `nfg()`. This should represent how much ***additional*** padding you want. The `pad` arg. is interpreted as follows:

- If a `length` is provided, it assumes you want that much length added to all cell walls
- If an array of the form `(L1, L2)` is provided, it assumes you want padding a horizontal (`x`) padding of `L1` and a vertical padding (`y`) of `L2`

- If a `dictionary` is provided, it operates identically to that of the array, but you must specify the `x` / `y` keys yourself

Automatic Cell Sizing

Cell are automatically sized to equal heights/widths according to the longest/tallest content. If you want to avoid this behavior, pass `lazy-cells: true` to `nfg`. This behavior can be combined with the custom `padding` argument.

Semantic Game Styling

Underlining

The package imports a small set of underlining utility functions.

The primary functions for underlining are

- `hul()` – *Horizontal Underline*
- `vul()` – *Vertical Underline*
- `bul()` – *Black Underline*

These can be wrapped around values in math-mode (`$..$`) within the payoff matrix. The underlines for `hul` and `vul` are colored by default according to the default colors for names, but they accept an optional `col` parameter for changing the color of the underline. `bul()` produces a black underline.

```
#nfg(
  players: ("Jack", "Diane"),
  s2: ($x$, $y$, $z$),
  s1: ($a$, $b$),
  [$hul(0),vul(0)$], [$1,1$], [$2,2$],
  [$3,3$], [$4,4$], [$5,5$],
)
```

Diane

Jack		x	y	z
	a	$\underline{\mathbf{0}}, \underline{0}$	$1, 1$	$2, 2$
	b	$3, 3$	$4, 4$	$5, 5$

By default, these commands leave the numbers themselves black, but boldfaces them. *Full Color* versions of `hul` and `vul`, which color the numbers and under-lines identically, are available via `hful()` and `vful()`. Like their counterparts, they accept an optional `col` command for the color.

Both of the colors can be modified individually via the general `cul()` command, which takes in content (`cont`), an underline color (`ucol`), and the color for the text value (`tcol`). For instance,

```
#let new-ul(cont, col: olive, tcol: fuchsia) = { cul(cont, col, tcol) }
```

will define a new command which underlines in olive and sets the text (math) color to fuchsia.

Mixed Strategies

You can optionally mark mixed strategies that a player will in a `nfg` using the `mixing` argument. This can be a `dictionary` with `hmix` and `vmix` keys, or an `array`, interpreted as a dictionary with the aforementioned keys in the `(hmix, vmix)` order. The values/entries here should be arrays which mimic `s1` and `s2` in size, with some parameter denoting the proportion of time the relevant player uses that strategy. If you would like to omit a strategy from this markup, pass `[]` in it's place.

For example, in a 2x3 game, the following dictionary would add mixing parameters to both of player 1's strategies and player 2's first and third strategies:

```
(hmix: ($p$, $1-p$), vmix: ($q$, [], $1-q$))
```

Example

```
#nfg(
  players: ("Chet", "North"),
  s1: ([$F$], [$G$], [$H$]),
  s2: ([$X$], [$Y$]),
  mixings: (
    hmix: ($p$, $1-p$),
    vmix: ($q$, [], $1-q$)),
  [$7,3$], [$2,4$],
  [$5,2$], [$6,1$],
  [$6,1$], [$5,4$]
)
```

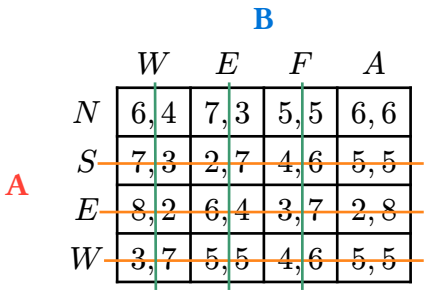
		North	
		$[q]$ X	$[1-q]$ Y
Chet	$[p]$ F	$7, 3$	$2, 4$
	G	$5, 2$	$6, 1$
	$[1-p]$ H	$6, 1$	$5, 4$

Iterated Deletion (Elimination) of Dominated Strategies

You can use the `pinit` package to cross out lines, semantically eliminating strategies. `pinit` comes pre-imported with `gametheoryst` by default.

```
#let directions = ([$N$], [$S$], [$E$], [$W$])
#let elements =  ([$W$], [$E$], [$F$], [$A$])
#let domd = ("s12", "s13", "s14", "s21", "s22", "s23")
```

```
#nfg(
    players: ("A", "B"),
    s1: directions,
    s2: elements,
    eliminations: domd,
    ejust:(
      "s12": (x: (0pt, 10pt), y: (-3pt, -3pt)),
      "s13": (x: (0pt, 10pt), y: (-3pt, -3pt)),
      "s14": (x: (0pt, 10pt), y: (-3pt, -3pt)),
      "s21": (x: (-6pt, -8pt), y: (3pt, 8pt)),
      "s22": (x: (-4pt, -8pt), y: (3pt, 8pt)),
      "s23": (x: (-4pt, -8pt), y: (3pt, 8pt)),
    ),
    [$6,4$], [$7,3$], [$5,5$], [$6,6$],
    [$7,3$], [$2,7$], [$4,6$], [$5,5$],
    [$8,2$], [$6,4$], [$3,7$], [$2,8$],
    [$3,7$], [$5,5$], [$4,6$], [$5,5$],
)
```

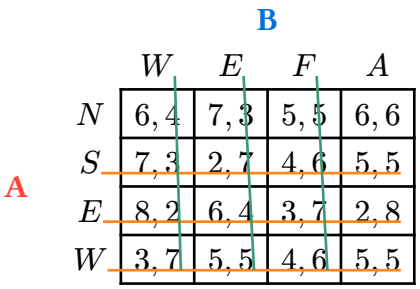


You can tell `nfg` which strategies to eliminate with the `eliminations` argument and the corresponding `ejust` helper-argument. The `eliminations` argument is simply an `array` of `strings` of the form `"s<i><j>"`, where `<i>` is the player – 1 or 2 – and `<j>` is player `i`'s `<j>` th strategy (in left-to-right / top-to-bottom order *starting from 1*). These strategy strings represent the rows/columns which you want to eliminate. For instance, `("s12", "s21")` denotes an elimination of player 1's second strategy as well as player 2's first strategy.

Due to `context` dependence, the lines typically need manual adjustments, which can be done via the `ejust` arg. `ejust` needs to be a dictionary with keys of matching those strings present in `eliminations` (`s11`, `s21`, etc.). The values of one of these dictionary entries is itself a dictionary: one with `x` and `y` keys. Each of these keys needs an array consisting of 2 lengths, corresponding to the starting/ending `dx/dy` adjustments from `pinit-line`.

Example

Here is the previous game with no `ejust` options included. As you can see, even similar lines may need different adjustments.



For example, one such `ejust` argument could be `("s12": (x: (5pt, -5pt), y: (-10pt, 3pt)))`. This says to adjust the "s12" elimination line by `5pt` in the x direction and `-10pt` in the y direction for the starting (strategy-) side of the line, and adjust by `-5pt` in x and `3pt` in y on the ending (payoff-) side of the line.

Pin modifications

If you would like to modify the pins and/or lines in any way, there are easy-to-follow naming conventions for the pins. The name of every pin is `<prefix-id>-s<i><j>--<start/end suffix>`.

Prefix ID

To prevent confusion of `pinit-lines` across games within a document, there is a counter called `_nfg-counter` which steps every time `nfg` is called. The counter begins from 0. By default, this counter is used to define the start of every pin; if the value of `_nfg-counter` is `<c>`, then the start of the pin label is "nfg-".

You may change the starting prefix of the elims within a game by passing a `string` to the `gid` argument of `nfg`. *No additional* `-` *will be added.* Note that the `_nfg-counter` will still incrememnt for every game in the document.

Strategy

The strategy portion of the pin name is identical to the strategy being crossed out; see above for explanation.

Start/End Suffix

For the suffix, the rule is as follows:

- The starting pin – e.g., strategy-side pin – is labelled with the suffix `"--start"`
- The ending pin – e.g., payoff-side pin – is labelled with the suffix `"--end"`

For example, consider an elimination line in the 1st game within a document which eliminates player 2's 1st strategy. This `pinit-line` would connect the pins `nfg0-s21--start` with `nfg0-s22--end`.

Debugging

If you want to see all of the lines for the table, including the ones for a players, strategies, and mixings, set the following at the top of your document.

```
#set table.cell(stroke: (thickness: auto))
```

Note that cells are always present for mixings, they just have 0 width/height when no mixings of a specific variety are provided.