ST3009: Week 4 Assignment

Conor McCauley - 17323203

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Question 1

(a) The only way that two dice rolls can sum to 2 is if both of the rolls are 1.

$$\{(1,1)\}$$

(b) The only ways that two dice rolls can sum to 3 is if one roll is 2 and the other is 1 (these rolls can be in either order).

$$\{(2,1),(1,2)\}$$

(c) The only ways that two dice rolls can sum to 4 is if one roll is 3 and the other is 1 (in either order), or if both rolls are 2.

$$\{(3,1),(1,3),(2,2)\}$$

(d) The given event contains 3 elements and there are 6^2 elements in the sample space.

$$P(X=1) = \frac{3}{6^2} = 0.08\overline{3}$$

Question 2

(a) The following distinct amounts of heads and tails are possible:

$$(3H, 0T) \Rightarrow 3 - 0 = +3$$
$$(2H, 1T) \Rightarrow 2 - 1 = +1$$

$$(1H, 2T) \Rightarrow 1 - 2 = -1$$
$$(0H, 3T) \Rightarrow 0 - 3 = -3$$

(b) There are $\binom{3}{0}$ ways to order 0 heads and 3 tails and 2^3 total outcomes.

$$\frac{\binom{3}{0}}{2^3} = 0.125$$

(c) There are $\binom{3}{1}$ ways to order 1 head and 2 tails and 2^3 total outcomes.

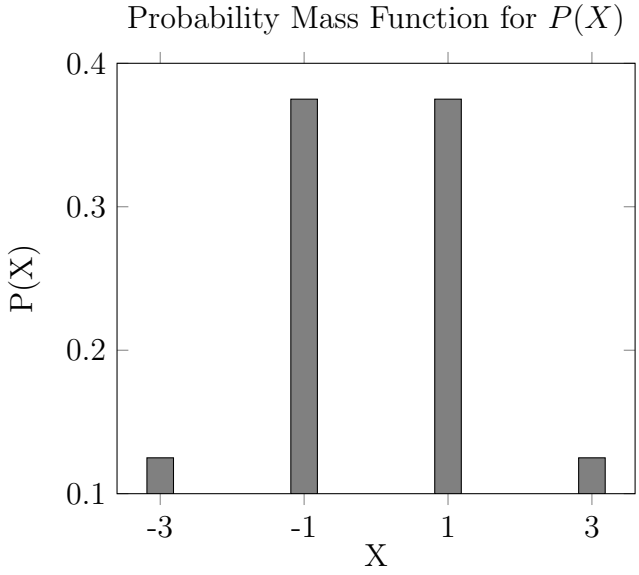
$$\frac{\binom{3}{1}}{2^3} = 0.375$$

(d) The probability of getting 3 heads and 0 tails is the same as the probability of getting 0 heads and 3 tails. Likewise, the probability of getting 2 heads and 1 tail is the same as the probability of getting 1 head and 2 tails.

The probabilities are as follows:

X	$P(X)$
-3	0.125
-1	0.375
1	0.375
3	0.125

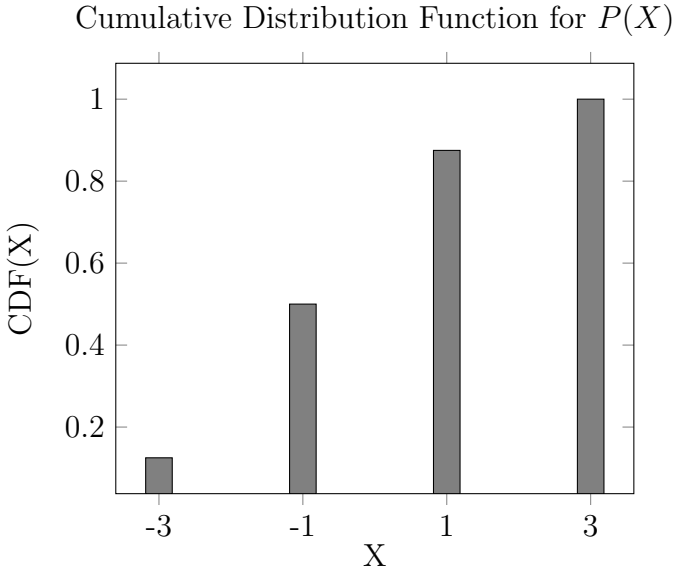
They can be graphed like so:



The cumulative probabilities are as follows:

X	$CDF(X)$
-3	0.125
-1	0.5
1	0.875
3	1.0

They can be graphed like so:



Question 3

(a) It's not possible for any die roll to be less than 1.

$$P(X \geq 1) = 1.0$$

(b) The probability that every die rolls is greater than or equal to 2 is the probability that none of the dice rolls are 1. Thus, for each roll there are 5 valid results.

$$P(X \geq 2) = \left(\frac{5}{6}\right)^4 = 0.48225$$

(c) The probability that $P(X \leq 1)$ is the inverse of the probability that none of the dice rolls are 1.

$$P(X \leq 1) = 1 - \left(\frac{5}{6}\right)^4 = 0.51774$$

The probability that $P(X \leq 2)$ is the inverse of the probability that none of the dice rolls are 1 or 2 - this is the cumulative probability of $P(X \leq 1) + P(X \leq 2)$.

$$P(X \leq 2) = 1 - \left(\frac{4}{6}\right)^4 = 0.80246$$

There is an obvious formula here:

$$P(X \leq k) = 1 - \left(\frac{6-k}{6}\right)^4, 1 \leq k \leq 6$$

Replacing k with the remaining values from 3 to 6 gives us the following probabilities:

$$P(X \leq 3) = 1 - \left(\frac{3}{6}\right)^4 = 0.9375$$

$$P(X \leq 4) = 1 - \left(\frac{2}{6}\right)^4 = 0.98765$$

$$P(X \leq 5) = 1 - \left(\frac{1}{6}\right)^4 = 0.99922$$

$$P(X \leq 6) = 1 - \left(\frac{0}{6}\right)^4 = 1.0$$

This can be graphed like so:

Cumulative Distribution Function for $P(X \leq k)$