CS4062: Probability Exercises

Conor McCauley - 17323203

October 13, 2020

Question 1

We know from lectures that

$$P(A \mid B) = \frac{P(A \land B)}{P(B)}$$

(i) If we are told that $P(A \land B) = P(A) \cdot P(B)$ then we can rewrite our initial equation like so:

$$P(A \mid B) = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

(ii) Similarly, if we are told that $P(A \mid B) = P(A)$ we can replace the left side of our initial equation with $P(A)$ like so:

$$P(A) = \frac{P(A \land B)}{P(B)}$$

$$P(A) \cdot P(B) = P(A \land B)$$

Question 2

(a) We can rewrite $P(gw \mid ps)$ like so:

$$P(gw \mid ps) = \frac{P(gw \land ps)}{P(ps)} = \frac{28}{28 + 2} = \frac{28}{30} = 0.9\dot{3}$$

(b) Again, we can rewrite $P(ps \mid gw)$ like so:

$$P(ps \mid gw) = \frac{P(ps \wedge gw)}{P(gw)} = \frac{28}{28 + 140} = \frac{28}{168} = 0.1\dot{6}$$

Question 3

(a) Given that $p(vmel) = 0.01$ we know that $p(\neg vmel) = 0.99$. We can calculate the following:

$$p(dbi \mid vmel) \cdot p(vmel) = 0.95 \cdot 0.01 = 0.0095$$
$$p(dbi \mid \neg vmel) \cdot p(\neg vmel) = 0.01 \cdot 0.99 = 0.0099$$

This tells us that $\neg vmel$ is the best guess since $0.0099 > 0.0095$.
(b) Given that $p(vmel) = 0.15$ we know that $p(\neg vmel) = 0.85$. We can calculate the following:

$$p(dbi \mid vmel) \cdot p(vmel) = 0.95 \cdot 0.15 = 0.1425$$
$$p(dbi \mid \neg vmel) \cdot p(\neg vmel) = 0.01 \cdot 0.85 = 0.0085$$

This tells us that $vmel$ is the best guess since $0.1425 > 0.0085$.
(c) Given that $p(vmel) = 0.01$ we know that $p(\neg vmel) = 0.99$. We can calculate the following:

$$p(dbi \mid vmel) \cdot p(vmel) = 0.95 \cdot 0.01 = 0.0095$$
$$p(dbi \mid \neg vmel) \cdot p(\neg vmel) = 0.001 \cdot 0.99 = 0.00099$$

This tells us that $vmel$ is the best guess since $0.0095 > 0.00099$.

Question 4

First we must find the total number of days: $62 + 108 + 38 + 292 = 500$. We can then calculate $p(cool : +)$ like so:

$$p(cool : +) = \frac{62 + 108}{500} = \frac{170}{500} = 0.34$$

We can calculate $p(cool : + \mid noisy : +)$ like so:

$$p(cool:+ \mid noisy:+) = \frac{p(cool:+ \land noisy:+)}{p(noisy:+)} = \frac{62}{62+30} = \frac{62}{92} \approx 0.674$$

It is clear from these results that, since $p(cool:+ \mid noisy:+) \neq p(cool:+)$, $cool:+$ is **not** independent of $noisy:+$.

Question 5

(i) First we must find the total number of $open:+$ days: $54+36+6+4 = 100$. Using the first table we can calculate $p(cool:+)$ like so:

$$p(cool:+) = \frac{54+36}{100} = \frac{90}{100} = 0.9$$

We can also calculate $p(cool:+ \mid noisy:+)$ like so:

$$p(cool:+ \mid noisy:+) = \frac{p(cool:+ \land noisy:+)}{p(noisy:+)} = \frac{54}{54+6} = \frac{54}{60} = 0.9$$

Since $p(cool:+ \mid noisy:+) = p(cool:+)$ we can conclude that $cool:+$ **is** conditionally independent of $noisy:+$ (all given $open:+$).

(ii) Again, we must first find the total number of $open:-$ days: $8+72+32+288 = 400$. Using the first table we can calculate $p(cool:+)$ like so:

$$p(cool:+) = \frac{8+72}{400} = \frac{80}{400} = 0.2$$

We can also calculate $p(cool:+ \mid noisy:+)$ like so:

$$p(cool:+ \mid noisy:+) = \frac{p(cool:+ \land noisy:+)}{p(noisy:+)} = \frac{8}{8+32} = \frac{8}{40} = 0.2$$

Since $p(cool:+ \mid noisy:+) = p(cool:+)$ we can conclude that $cool:+$ **is** conditionally independent of $noisy:+$ (all given $open:-$).