CS3081: Assignment 1

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Answers

Question 2.31

a. (ii) 13
b. (i) 0

Question 3.2

a. (ii) 0.8125
b. (ii) 0.85261
c. (iv) 0.85261

Question 4.24

a. (i)

$$\begin{pmatrix} -0.7143 & 0 & 1.4286 \\ 0.2571 & 0.1 & 0.2857 \\ -0.2286 & -0.2 & 0.8571 \end{pmatrix}$$

b. (i)

$$\begin{pmatrix} 1.6667 & 2.8889 & -2.2222 & 1 \\ 0 & 0.3333 & -0.3333 & 0 \\ -0.3333 & -0.4444 & 0.1111 & 0 \\ 1.5 & 2 & -1.5 & 0.5 \end{pmatrix}$$

Solutions

Question 2.31

a.

$$\det\begin{pmatrix} 1 & 5 & 4 \\ 2 & 3 & 6 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned}
&= 1 \cdot \begin{vmatrix} 3 & 6 \\ 1 & 1 \end{vmatrix} - 5 \cdot \begin{vmatrix} 2 & 6 \\ 1 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} \\
&= 1 \cdot (3 \cdot 1 - 6 \cdot 1) - 5 \cdot (2 \cdot 1 - 6 \cdot 1) + 4 \cdot (2 \cdot 1 - 3 \cdot 1) \\
&= 1 \cdot (3 - 6) - 5 \cdot (2 - 6) + 4 \cdot (2 - 3) \\
&= 1 \cdot (-3) - 5 \cdot (-4) + 4 \cdot (-1) \\
&= -3 + 20 - 4 = 13
\end{aligned}$$

b.

$$\det \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

$$= 1 \cdot \begin{vmatrix} 6 & 7 & 8 \\ 10 & 11 & 12 \\ 14 & 15 & 16 \end{vmatrix} - 2 \cdot \begin{vmatrix} 5 & 7 & 8 \\ 9 & 11 & 12 \\ 13 & 15 & 16 \end{vmatrix} + 3 \cdot \begin{vmatrix} 5 & 6 & 8 \\ 9 & 10 & 12 \\ 13 & 14 & 16 \end{vmatrix} - 4 \cdot \begin{vmatrix} 5 & 6 & 7 \\ 9 & 10 & 11 \\ 13 & 14 & 15 \end{vmatrix}$$

$$= 1 \cdot \left(6 \cdot \begin{vmatrix} 11 & 12 \\ 15 & 16 \end{vmatrix} - 7 \cdot \begin{vmatrix} 10 & 12 \\ 14 & 16 \end{vmatrix} + 8 \cdot \begin{vmatrix} 10 & 11 \\ 14 & 15 \end{vmatrix} \right)$$

$$-2 \cdot \left(5 \cdot \begin{vmatrix} 11 & 12 \\ 15 & 16 \end{vmatrix} - 7 \cdot \begin{vmatrix} 9 & 12 \\ 13 & 16 \end{vmatrix} + 8 \cdot \begin{vmatrix} 9 & 11 \\ 13 & 15 \end{vmatrix} \right)$$

$$+3 \cdot \left(5 \cdot \begin{vmatrix} 10 & 12 \\ 14 & 16 \end{vmatrix} - 6 \cdot \begin{vmatrix} 9 & 12 \\ 13 & 16 \end{vmatrix} + 8 \cdot \begin{vmatrix} 9 & 10 \\ 13 & 14 \end{vmatrix} \right)$$

$$-4 \cdot \left(5 \cdot \begin{vmatrix} 10 & 11 \\ 14 & 15 \end{vmatrix} - 6 \cdot \begin{vmatrix} 9 & 11 \\ 13 & 15 \end{vmatrix} + 7 \cdot \begin{vmatrix} 9 & 10 \\ 13 & 14 \end{vmatrix} \right)$$

$$= 1 \cdot (6 \cdot (11 \cdot 16 - 12 \cdot 15) - 7 \cdot (10 \cdot 16 - 12 \cdot 14) + 8 \cdot (10 \cdot 15 - 11 \cdot 14))$$

$$-2 \cdot (5 \cdot (11 \cdot 16 - 12 \cdot 15) - 7 \cdot (9 \cdot 16 - 12 \cdot 13) + 8 \cdot (9 \cdot 15 - 11 \cdot 13))$$

$$+3 \cdot (5 \cdot (10 \cdot 16 - 12 \cdot 14) - 6 \cdot (9 \cdot 16 - 12 \cdot 13) + 8 \cdot (9 \cdot 14 - 10 \cdot 13))$$

$$-4 \cdot (5 \cdot (10 \cdot 15 - 11 \cdot 14) - 6 \cdot (9 \cdot 15 - 11 \cdot 13) + 7 \cdot (9 \cdot 14 - 10 \cdot 13))$$

$$= 1 \cdot (6 \cdot (176 - 180) - 7 \cdot (160 - 168) + 8 \cdot (150 - 154))$$

$$-2 \cdot (5 \cdot (176 - 180) - 7 \cdot (144 - 156) + 8 \cdot (135 - 143))$$

$$+3 \cdot (5 \cdot (160 - 168) - 6 \cdot (144 - 156) + 8 \cdot (126 - 130))$$

$$-4 \cdot (5 \cdot (150 - 154) - 6 \cdot (135 - 143) + 7 \cdot (126 - 130))$$

$$= 1 \cdot (6 \cdot (-4) - 7 \cdot (-8) + 8 \cdot (-4))$$

$$-2 \cdot (5 \cdot (-4) - 7 \cdot (-12) + 8 \cdot (-8))$$

$$+3 \cdot (5 \cdot (-8) - 6 \cdot (-12) + 8 \cdot (-4))$$

$$-4 \cdot (5 \cdot (-4) - 6 \cdot (-8) + 7 \cdot (-4))$$

$$= 1 \cdot (-24 + 56 - 32) - 2 \cdot (-20 + 84 - 64)$$

$$+3 \cdot (-40 + 72 - 32) - 4 \cdot (-20 + 48 - 28)$$

$$= 1 \cdot 0 - 2 \cdot 0 + 3 \cdot 0 - 4 \cdot 0 = 0$$

Question 3.1

a.

	a	b	c	$f(a)$	$f(c)$	Comment
0	0	1	$\frac{0+1}{2} = 0.5$	-2	-0.713	$f(a)$ and $f(c)$ have same sign so set $a = c$
1	0.5	1	$\frac{0.5+1}{2} = 0.75$	-0.713	-0.1947	$f(a)$ and $f(c)$ have same sign so set $a = c$
2	0.75	1	$\frac{0.75+1}{2} = 0.875$	-0.1947	0.0412	$f(a)$ and $f(c)$ have different sign so set $b = c$
3	0.75	0.875	$\frac{0.75+0.875}{2} = 0.8125$	-0.1947	-0.0749	$root = 0.8125$

b.

n	x_{n-1}	x_n	x_{n+1}
2	$x_1 = 0$	$x_2 = 1$	$x_3 = 1 - \frac{f(1)\cdot(0-1)}{f(0)-f(1)} = 1 - \frac{0.264\cdot -1}{-2-0.264} = 0.883$
3	$x_2 = 1$	$x_3 = 0.883$	$x_4 = 0.883 - \frac{f(0.883)\cdot(1-0.883)}{f(1)-f(0.883)} = 0.883 - \frac{0.056\cdot 0.117}{0.264-0.056} = 0.8515$
4	$x_3 = 0.883$	$x_4 = 0.8515$	$x_5 = 0.8515 - \frac{f(0.8515)\cdot(0.883-0.8515)}{f(0.883)-f(0.8515)} = 0.8515 - \frac{-0.002\cdot 0.0315}{0.056-(-0.002)}$ $= 0.852 \approx 0.85261$

c. In order to use Newton's method we must first find the derivative of $f(x)$:

$$\frac{d}{dx}(x - 2e^{-x})$$
$$= 1 - \frac{d}{dx}(2e^{-x})$$
$$= 1 - (-1 \cdot 2e^{-x})$$
$$= 1 + 2e^{-x}$$

n	x_n	x_{n+1}
1	1	$1 - \frac{1-2e^{-1}}{1+2e^{-1}} = 0.84776$
2	0.84776	$0.84776 - \frac{0.84776-2e^{-0.84776}}{1+2e^{-0.84776}} = 0.8526$
3	0.8526	$0.8526 - \frac{0.8526-2e^{-0.8526}}{1+2e^{-0.8526}} = 0.852606 \approx 0.85261$

Question 4.24

The following Matlab code can be used to calculate the inverse of the matrices given in the question:

```
% calculate the inverse of a matrix A
function [Ainv] = Inverse(A)

    [N, ~] = size(A);    % get the size of the matrix
    Ainv = Identity(N); % construct the identity matrix

    % for each row in the (augmented) matrix
    for i = 1:N

        % make diagonal column 1
        u = A(i, i);
        for j = 1:N
            A(i, j) = A(i, j) / u;
            Ainv(i, j) = Ainv(i, j) / u;
        end

        % make non-diagonal columns 0
        for j = 1:N
            if j ~= i
                v = A(j, i);
                for k = 1:N
                    A(j, k) = A(j, k) - (A(i, k) * v);
                    Ainv(j, k) = Ainv(j, k) - (Ainv(i, k) * v);
                end
            end
```

```
            end

        end

    end

    % return the identity matrix of size N
    function [Imat] = Identity(N)
        Imat = zeros(N, N);
        for i = 1:N
            Imat(i, i) = 1;
        end
    end
```

For the matrix in question (a) the code gives the following answer:

$$\begin{pmatrix} -0.7143 & 0 & 1.4286 \\ 0.2571 & 0.1 & 0.2857 \\ -0.2286 & -0.2 & 0.8571 \end{pmatrix}$$

For the matrix in question (b) the code gives the following answer:

$$\begin{pmatrix} 1.6667 & 2.8889 & -2.2222 & 1 \\ 0 & 0.3333 & -0.3333 & 0 \\ -0.3333 & -0.4444 & 0.1111 & 0 \\ 1.5 & 2 & -1.5 & 0.5 \end{pmatrix}$$

These results both match the answers given in (i).