ST3009: Week 2 Assignment

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Question 1

(a) Each of the 3 rolls has 6 possible outcomes.

$$6^3 = 216$$

(b) For outcomes containing exactly one 2 there are 5 options each for the other rolls and $\binom{3}{2}$ ways of ordering the rolls.

$$5 \cdot 5 \cdot \binom{3}{2} = 75$$

For outcomes containing exactly two 2's there are 5 options for the other roll and $\binom{3}{1}$ ways of ordering the rolls.

$$5 \cdot \binom{3}{1} = 15$$

There is only one outcome where all of the rolls result in a 2.

$$\frac{75 + 15 + 1}{216} = 0.42129$$

(c) The following code confirms the result from the previous question:

```
times = 1000000;
twos = 0;

for i = 1:times
    r1 = randi([1 6], 1, 1);
    r2 = randi([1 6], 1, 1);
```

```
    r3 = randi([1 6], 1, 1);
    if r1 == 2 || r2 == 2 || r3 == 2
        twos = twos + 1;
    end
end

disp(twos / times);
```

(d) The only outcomes that could sum to 17 are some ordering of 6, 6 and 5. There are $\binom{3}{2}$ orderings of these rolls.

$$\left(\frac{1}{6}\right)^3 \cdot \binom{3}{2} = 0.013\overline{8}$$

(e) Given that the first roll was a 1 the remaining two rolls must sum to 11. This would require some ordering of 6 and 5. There are $\binom{2}{1}$ orderings of these rolls.

$$\left(\frac{1}{6}\right)^2 \cdot \binom{2}{1} = 0.0\overline{5}$$

Question 2

(a) The probability that the second throw is a 5 is $\frac{1}{6}$ if a six-sided die is rolled and $\frac{1}{20}$ if a 20-sided die is rolled - there is a $\frac{1}{6}$ chance that we roll the six-sided die and a $\frac{5}{6}$ chance that we roll the 20-sided die.

$$\frac{1}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{20} = 0.069\overline{4}$$

(b) The second throw can only result in a 15 if the first throw is not a 1. There is a $\frac{5}{6}$ chance that the 20-sided die is rolled and a $\frac{1}{20}$ chance that the second throw is a 15.

$$\frac{5}{6} \cdot \frac{1}{20} = 0.041\overline{6}$$

Question 3

I will refer to the event that a person is guilty as G and the event that a person has a certain characteristic as C.

- Likelihood - the probability that the criminal has a certain characteristic given that they are guilty:

$$P(C \mid G) = 1.0$$

- Prior - the probability of guilt prior to receiving the new evidence:

$$P(G) = 0.6$$

- Evidence - calculated using marginalisation:

$$\begin{aligned} P(C) &= P(C \mid G) \cdot P(G) + P(C \mid \overline{G}) \cdot P(\overline{G}) \\ &= 1.0 \cdot 0.6 + 0.2 \cdot (1 - 0.6) = 0.68 \end{aligned}$$

Therefore, the probability that the suspect is guilty given that they have a certain characteristic is

$$P(G \mid C) = \frac{P(C \mid G) \cdot P(G)}{P(C)} = \frac{1.0 \cdot 0.6}{0.68} = 0.88235$$

Question 4

I will refer to the event that a person is in a particular location as L and the event that a person is observed to be in a particular location as O.

For each cell in the grid the probability that the phone is in the location given the two observed bars is

$$P(L \mid O) = \frac{P(O \mid L) \cdot P(L)}{P(O \mid L) \cdot P(L) + P(O \mid \overline{L}) \cdot P(\overline{L}))}$$

The following code calculates the updated probability for each cell in the grid:

```
pLoc = [
    0.05 0.10 0.05 0.05;
    0.05 0.10 0.05 0.05;
    0.05 0.05 0.10 0.05;
    0.05 0.05 0.10 0.05
```

```
];

pObsLoc = [
    0.75 0.95 0.75 0.05;
    0.05 0.75 0.95 0.75;
    0.01 0.05 0.75 0.95;
    0.01 0.01 0.05 0.75
];

% init an array of zeros
pLocObs = zeros(4, 4);

% for each grid cell calculate the value of P(L|O)
for i = 1:numel(pLocObs)
    num = pObsLoc(i) * pLoc(i);
    den_a = (pObsLoc(i) * pLoc(i));
    den_b = ((1 - pObsLoc(i)) * (1 - pLoc(i)));
    pLocObs(i) = num / (den_a + den_b);
end

disp(pLocObs);
```

It produced the following results:

$$\begin{bmatrix} 0.1364 & 0.6786 & 0.1364 & 0.0028 \\ 0.0028 & 0.2500 & 0.5000 & 0.1364 \\ 0.0005 & 0.0028 & 0.2500 & 0.5000 \\ 0.0005 & 0.0005 & 0.0058 & 0.1364 \end{bmatrix}$$

The code stores the known values for $P(L)$ and $P(O \mid L)$ in 4x4 arrays and initialises another array containing only zeros. Then, for each of the sixteen values, it calculates the value of $P(L \mid O)$ using the formula given at the start of my answer for this question.