ST3009: Week 1 Assignment

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Question 1

(a) There are 10 choices for the first letter, 9 for the second, etc.

$$10! = 3628800$$

(b) There are 2 orderings of the letters 'E' and 'F', as well as 8 remaining letters. We have 9 choices for 'EF', 8 for the first remaining letter, etc.

$$2 \cdot 9! = 725760$$

(c) There are 6 letters in total, but there are 3 'A's, 2 'N's and 1 'B'.

$$\frac{6!}{1! \cdot 2! \cdot 3!} = 60$$

(d) There are 5 distinct options and we must choose 3.

$$\binom{5}{3} = 10$$

Question 2

(a) Each roll has 6 possible outcomes.

$$6^4 = 1296$$

(b) There are $\binom{4}{2}$ different placements of the two 3s and 5 options for each of the remaining rolls.

$$\binom{4}{2} \cdot 5 \cdot 5 = 150$$

(c) We know there are 150 ways to get exactly two 3s from the previous question. There are $\binom{4}{3}$ different placements of the three 3s and 5 options for the remaining roll. There is one scenario where only 3s are rolled.

$$150 + \left(\binom{4}{3} \cdot 2\right) + 1 = 171$$

Question 3

(a) There 8 possible position for the first card, 7 for the second, etc. However, there are 2 of each of the 4 suits.

$$\frac{8!}{2! \cdot 2! \cdot 2! \cdot 2!} = 2520$$

(b) There are 4 distinct cards and we must choose 2.

$$\binom{4}{2} = 6$$

(c) There are 2 hearts and 2 diamonds in the deck which means there are $\binom{4}{2}$ ways to get only "good" cards. However, since order doesn't matter only half of the ways are distinct.

$$\frac{\binom{4}{2}}{2} = 3$$