1 Overview - Spatio-temporal threshold

The mechanism linking gas extraction to induced seismicity is highly complex and not well understood. We aim to contribute to the understanding of this relationship. In particular, we aim to build upon the approach of Varty et al. (2021) by incorporating spatial variability into the threshold in order to improve modelling approaches and forecasts of hazards under future extraction scenarios. This spatial variability will be incorporated by utilising a relationship between the distance from a geophone and the likelihood of an earthquake being recorded in the catalogue.

Earthquakes are recorded if their locations and magnitudes can be inferred from ground vibrations at three geophone locations. An earthquake is detected or missed depending on its magnitude and location relative to the sensor network. This leads to the problem of missing observations across the gas field from areas/times where the network is too sparse or insensitive to detect low magnitude events. Data concerning the development of the earthquake detection network was provided by Shell.

Spatio-temporal variation in the density and sensitivity of this network leads to variability in the probability of an earthquake being recorded. For now, we treat sensors as having constant sensitivity across space and time and we rely on the relationship between the earthquake detection probability and the distance of an earthquake from the nearest three geophones.

Let $y_{i,t}$ denote the i^{th} earthquake magnitude to occur in some year t with corresponding location $x_{i,t}$, for $i = 1, \ldots, n_t$ and $t = 1995, \ldots, 2022$, where n_t is the total number of earthquakes to occur in some year t. Then, we denote as $V(x_{i,t}, t)$ the distance from the third-nearest geophone available in year t to $x_{i,t}$. To adjust this function $V(x, t)$ to be appropriate on the earthquake magnitude scale, we scale it by a parameter, $\theta > 0$, such that we have a threshold given by $u(x, t) = \theta V(x, t)$ below which observations are likely to be missed. With this framework, we can adjust our threshold selection method to estimate the scaling parameter θ given $V(x, t)$.

Let S be our spatial region and T be the set of years for which we have data. We make the assumption that there is a constant threshold, $u < \min\limits_{x \in S, t \in T}(u(x, t))$, above which the underlying true earthquake magnitudes follow a $\text{GPD}(\sigma, \xi)$, irrespective of the time or location of the earthquake. We also assume that the partial censoring of the earthquake magnitudes, due to limitations of the geophone network, is the sole reason for deviation from this distribution. Relying on these assumptions, we can use the threshold stability property to develop a non-stationary GPD as follows:

For some $y > 0$,

$$\begin{aligned}
\mathbb{P}(Y - u(x,t) > y | Y > u(x,t)) &= \mathbb{P}(Y > u(x,t) + y | Y > u(x,t)) \\
&= \frac{\mathbb{P}(Y > u(x,t) + y)}{\mathbb{P}(Y > u(x,t))} \\
&= \frac{1 - F(u(x,t) + y)}{1 - F(u(x,t))} \\
&= \frac{\left[1 + \frac{\xi(\theta V(x,t) + y)}{\sigma}\right]^{-1/\xi}}{\left[1 + \frac{\xi(\theta V(x,t))}{\sigma}\right]^{-1/\xi}} \\
&= \left[\frac{\sigma + \xi\theta V(x,t) + \xi y}{\sigma + \xi\theta V(x,t)}\right]^{-1/\xi} \\
&= \left[1 + \frac{\xi y}{\tilde{\sigma}}\right]^{-1/\xi}
\end{aligned}$$

where $\tilde{\sigma}(x,t) = \sigma + \xi\theta V(x,t)$.

Thus, we can model the earthquake magnitudes using the conditional distribution:

$$(Y - \theta V(x,t)) | (Y > \theta V(x,t)) \sim \text{GPD}(\sigma + \xi\theta V(x,t), \xi).$$

and estimate the parameters (θ, σ, ξ) based on the following log-likelihood.

For $\xi \neq 0$, the log-likelihood is given by:

$$l(\theta, \sigma, \xi) = -\sum_{t=t_{min}}^{t_{max}} \sum_{i=1}^{n_t} \mathbb{I}(y_{i,t} > u(x_i, t)) l_{i,t}(\theta, \sigma, \xi)$$

where

$$l_{i,t}(\theta, \sigma, \xi) = \log(\tilde{\sigma}(x_{i,t}, t)) + (1 + 1/\xi) \log\left(1 + \frac{\xi(y_{i,t} - u(x_{i,t}, t))}{\tilde{\sigma}(x_{i,t}, t)}\right)$$

with $\tilde{\sigma}(x_{i,t}, t) = \sigma + \xi\theta V(x_{i,t}, t)$. This likelihood holds provided $(1 + \xi\tilde{y}_{i,t}/\tilde{\sigma}(x_{i,t}, t)) > 0$. Otherwise, $l(\theta, \sigma, \xi) = -\infty$.

For the case of $\xi = 0$, the log-likelihood is given by:

$$l(\theta, \sigma) = -\sum_{t=t_{min}}^{t_{max}} \sum_{i=1}^{n_t} \left[\log(\tilde{\sigma}(x_{i,t}, t)) + \frac{(y_{i,t} - u(x_{i,t}, t))}{\tilde{\sigma}(x_{i,t}, t)}\right] \mathbb{I}(y_{i,t} > u(x_i, t)).$$

In order to use our metric, we need to transform the magnitudes onto common margins. We will compare a few different transformations using simulated data. We can then provide a set of possible choices for θ and compare the overall metric values to select the appropriate value for this parameter.

References

Varty, Z., Tawn, J. A., Atkinson, P. M., and Bierman, S. (2021). Inference for extreme earthquake magnitudes accounting for a time-varying measurement process. *arXiv preprint arXiv:2102.00884*.