$$\lambda_0(\underset{\sim}{x},t) = \exp\Big(\underset{\gamma_0}{\underset{\downarrow}{\alpha_0}} + \underset{\gamma_1}{\underset{\downarrow}{\alpha_1}}\, s(\underset{\sim}{x},t) \Big)$$

$$\lambda_{u(\underset{\sim}{x},t)}(\underset{\sim}{x},t) = \lambda_0(\underset{\sim}{x},t) \left[1 + \xi \left(\frac{u(\underset{\sim}{x},t) - 0}{\sigma_0(s(\underset{\sim}{x},t))} \right) \right]^{-1/\xi}$$

Need to fit (α_0, α_1) as treat ξ, $u(\underset{\sim}{x},t)$, $\sigma_0(s(\underset{\sim}{x},t))$ as known.

So ~~add~~ data $\{(\underset{\sim}{x}_i, t_i) : i = 1, \ldots, n\}$ corresponding to exceedances of $u(\underset{\sim}{x},t)$ threshold, i.e., with associated magnitudes

$$m_i > u(\underset{\sim}{x}_i, t_i).$$

Likelihood for Poisson process with points $\{y_1, \ldots, y_n\}$ and intensity $\lambda(y)$ in region S is

$$L \propto \left[\prod_{i=1}^{n} \cancel{y_i}\, \lambda(y_i) \right] e^{-\int_S \lambda(y)\, dy}$$

So here you have

[0,T] the time window of data.

$$L(\alpha_0, \alpha_1) \propto \left[\prod_{i=1}^{n} \lambda_{u(\underset{\sim}{x}_i,t_i)}(\underset{\sim}{x}_i, t_i) \right] \exp\left(- \int_0^T \int_{y \in G} \lambda(y,t)\, dy\, dt \right)$$

$u(y,t)$

G is a set of Groningen field