Economics 8106
Macroeconomic Theory
Recitation 4

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Outline:

- Sustainable Plans
- New Keynesian Model

1 Sustainable Plans

I want to go over some of the results and intuition behind sustainable plans. This will for the most part be similar to the treatment given in Jordan's Class Notes, but I will try to elaborate on some points. I will also use a slightly different example, generally following the environment defined in the Fall 2015 Prelim.

1.1 Environment

Consider a production economy with a large number of identical, infinitely lived individuals. There are two goods: labor and consumption. The period utility function is $U(c_t, l_t)$. Consumers discount future utility at β. There exists a per capita government expenditure g_t in each period. Specifically, government expenditure is given by $g_H > 0$ in even periods and $g_L = 0$ in odd periods. The government can default on inherited debt in any period. The government can raise revenue by levying labor taxes.

If there is a commitment technology, a TDCE can be defined as normal. It is a sequence of household allocation rules $Z^H = \{c_t, l_t, b_{t+1}\}_{t=0}^{\infty}$, a firm production plan $Z^F = \{l_t^f\}_{t=0}^{\infty}$, a government policy $\{g_t, \tau_t, b_{t+1}^g\}_{t=0}^{\infty}$ and prices $\{R_t, w_t\}_{t=0}^{\infty}$ such that, given prices and the policy, Z^H solves

$$\max_{Z^H} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$
$$s.t. \quad c_t + b_{t+1} \leq (1-\tau_t) w_t l_t + R_t b_t$$
$$s.t. \quad b_0 = 0, \quad b_{t+1} \geq -B, \quad c_t \geq 0 \quad l_t \in [0,1]$$

Given prices and the policy, Z^F solves

$$\max_{Z^F} l_t^f - w_t l_t^f \quad \forall t, \forall$$

the government budget constraint holds:

$$g_t + R_t b_t = \tau_t w_t l_t + b_{t+1} \quad \forall t, \forall$$

and markets clear:

$$c_t + g_t = l_t \quad \text{and} \quad b_t^g = b_t \quad \text{and} \quad l_t^f = l_t \quad \forall t, \forall.$$

1.2 Defining a Sustainable Equilibrium

The Ramsey problem in this environment is relatively standard. There is no uncertainty and the fiscal policies are simple. So let's move directly to defining sustainable equilibria. A sustainable equilibria, as we are going to define it, is truly a concept of game theory. We are treating consumers and the government as strategic players of a game. The game is a sequential description of the economy, and you can think of the timing as follows: At the beginning of every period, the government chooses a fiscal policy strategy for the current period and every future period. The consumer then makes her consumption and labor decisions for the current period and every future period. At the next period, the game repeats. We are looking for equilibria in which neither the consumer nor the government find it optimal to deviate from the path of future consumption and policy strategies. In game theoretic terms, we are looking for a sub-game perfect equilibria.

In this example, I am going to ignore aggregates. I am not entirely confident that we can do this in the definition. Chari has a proof in one of his papers that under certain circumstances, the aggregate allocations are superfluous as long as we keep track of the history of policies. Now let's attempt a precise definition.

Definition 1.1. Let $\pi_t = (\delta_t, B_{t+1}, \tau_t)$ be government policy in any period t. For simplicity, let's assume that δ, the default probability, is either 1 or 0.

Let h_t denote the history of all policies $h_t = (\pi_t, \pi_{t-1}, \ldots, \pi_0)$.

Let $f = \{f_t\}_{t=0}^{\infty}$ be a sequence of household allocation rules, where f_t maps from the space of time t histories into time t allocations: $f_t(h_t) = (c_t(h_t), l_t(h_t), b_{t+1}(h_t))$. I will denote the continuation rule as $f^\tau = \{f_t\}_{t=\tau}^{\infty}$.

Let $\sigma = \{\sigma_t\}_{t=0}^{\infty}$ be a sequence of government policy rules, where σ_t maps from the space of time $t-1$ histories into time t policies: $\sigma_t(h_{t-1}) = (\pi_t)$. I will denote the continuation rule as $\sigma^\tau = \{\sigma_t\}_{t=\tau}^{\infty}$.

A sustainable equilibrium is the pair (σ, f), and prices $p = \{R_t\}_{t=0}^{\infty}$ such that

1. For all periods τ and all histories h_τ, f^τ solves, given p and σ,

$$\max_{c_t, l_t, b_{t+1}} \sum_{s=t}^{\infty} \beta^{t-\tau} u(c_t, l_t)$$
$$s.t. \quad c_t + b_{t+1} \leq (1-\tau_t) w_t l_t + \delta_t R_t b_t$$
$$c_t \geq 0 \quad b_{t+1} \geq -B$$
$$\text{where } \pi_{t+1} = \sigma_{t+1}(h_t) \quad h_{t+1} = (\sigma_{t+1}(h_t), h_t)$$

2. For all periods τ, σ^τ solves, given p, f, and $h_{\tau-1}$,

$$\max_{\pi_t} \sum_{s=t}^{\infty} \beta^{t-\tau} u(c_t(h_t), l_t(h_t))$$
$$s.t. \quad R_t B_t \delta_t + g_t \leq \tau_t w_t l_t + B_{t+1}$$
$$\text{where } h_t = (\pi_t, h_{t-1})$$

3. Markets Clear: $g_t + c_t = l_t$ and $b_{t+1} = B_{t+1}$.

1.3 Characterizing Some Discernable Equilibria

D) Without commitment, what is the best sustainable equilibrium if g_0 is positive and $g_t = 0$ for all $t \geq 1$?

Refer to part E for a generalized proof.

E) Without commitment, what is the best sustainable equilibrium if $g_t = g > 0$ for $t = 1, 2, ..., T$ and $g_t = 0$ for all $t \geq T+1$?

For the remainder of the question, we will assume that sequential equilibria are characterized by trigger mechanisms, or *revert-to* strategies. This is in line with Chari, Kehoe (1990) and Chari, Kehoe (1993). Here, assume that if the government deviates from its announced

policy in period t, then there is autarky for periods $t+1$ and onward. That is, there is no bond market and the government can only finance expenditures through its labor tax. Lastly, note that we have moved into a deterministic environment.
Claim 1.2. *If $g_t = 0 \quad \forall t \geq \tau$ for some τ, then any sustainable plan must default at τ.*

Proof. Given that the government does not need to finance expenditures after period $\tau - 1$, the government budget constraint becomes: $\delta_t R_t b_t \leq \tau_t l_t + b_{t+1}$. In particular, for the first period τ, we have

$$\delta_\tau R_\tau b_\tau \leq \tau_\tau l_\tau$$

with the government objective of maximizing

$$\sum_{t=\tau}^{\infty} \beta^{t-\tau} u(c_t, l_t)$$

subject to $c_t + b_{t+1} \leq (1-\tau_t)l_t + \delta_t R_t b_t \quad \forall t \geq \tau$.[1] If the government defaults in period τ, the household receives welfare $U_d + \frac{\beta}{1-\beta}V$ whereas if the government does not default, the household receives $U_{ND} + \frac{\beta}{1-\beta}V$. Thus, the government's decision is with respect to which period τ utility is higher. Consider the household's feasibility set $(c,l) \in \Gamma(\tau, w)$ such that

$$\Gamma(\tau, w) = \{(c,l) : c \geq 0, l \in [0,1], c = (1-\tau)l + w\}$$

where w corresponds to a bond repayment. Given that the government must finance period τ bond repayments with a distortionary tax $\tau l = w$, we note that $\Gamma(\tau, w) \subseteq \Gamma(0,0)$ for any $\tau, w > 0$. Thus, from a planner's perspective, facing resource constraint $c = l$, the maximum is attained at $\frac{u_l}{u_c} = 1$. Further, when faced with $(\tau, \delta R_\tau b_\tau) = (0,0)$, the household first order condition coincides with the planner's. Thus, any scheme $(\tau, w) > 0$ is *feasible* under the $(0,0)$ fiscal and it is this policy that obtains the maximum utility for the household. Thus, the government will choose to set $\delta_\tau = 0$. □

corollary 1. *If $g_t = 0 \quad \forall t \geq \tau$ for some τ, then any sustainable plan must have $b_\tau = 0$.*
Claim 1.3. *If $b_t = 0 \quad \forall t = \tau, \tau+1, ...,$ then any sustainable plan must have default on $\tau - 1$ bonds.*

Proof. Suppose that pairing (σ, f) defines a sustainable plan for the economy in which i) $b_t = 0$ for all $t = \tau, \tau+1, ...$ and ii) there exists positive lending for all previous periods; that is, $b_t > 0 \quad \forall t = 0, 1, ..., \tau - 1$. The government faces the trigger mechanism which specifies that $b_{t+1} = 0$ for all future periods, given a deviation from policy in period t. Now, consider

[1] This proof assumes that the government does not choose to roll over debt with $b_{\tau+1}$. Given a no-ponzi condition and episolon argument, this proof will follow through for the more general case in which the government is allowed to roll over some debt.

the government in period $\tau - 1$. It seeks to solve

$$\max_{\{\delta_t,\tau_t\}_{t=\tau-1}^{\infty}} \sum_{t=\tau-1}^{\infty} \beta^{t-(\tau-1)} u(c_t, l_t) \tag{1}$$
$$s.t. \quad \delta_{\tau-1} R_{\tau-1} b_{\tau-1} + g_\tau \leq \tau_{\tau-1} l_{\tau-1}$$
$$s.t. \quad g_t \leq \tau_t l_t \qquad \forall t = \tau, \tau+1, ...$$

If the government deviates in period $\tau - 1$, the trigger mechanism relegates the government to optimizing household welfare with respect to the government budget constraint

$$g_t \leq \tau_t l_t \qquad \forall t = \tau, \tau+1, ...$$

which is no different than the original problem (1). Thus, the trigger mechanism does not constrain the government from optimizing in period $\tau - 1$. Given this, and through the same distortionary tax logic of the former proof, the government finds it optimal to maximize household welfare by setting $(\delta_{\tau-1}, \tau_{\tau-1}) = (0, 0)$.

Given this, the household allocation rule f would optimally set $b_{\tau-1} = 0$ in period $\tau - 2$. This contradicts the plan with $b_t > 0$ for $t = 0, 1, ..., \tau - 1$ being a sustainable one. Thus, a sustainable plan will have default in period $\tau - 1$. □

Through the same proof technique and using induction, we arrive at the following:
corollary 2. *If If $b_t = 0 \quad \forall t = \tau, \tau+1, ...$, then any sustainable plan must also have default at period $t = 0$.*[2]

Thus, if the government faces a trigger mechanism of no borrowing autarky, and has a finite stream of expenditures (stopping at some date τ), then any sustainable plan must have date 0 default on any positive government debt.

[2]This follows through inductive reasoning.