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Active Fault Tolerant Decentralized Control Strategy for an Autonomous 2WS4WD Electrical Vehicle Path Tracking

A. HADDAD *'**, A. AITOUCHE*'***, V. COCQUEMPOT*'**

**CRIStAL Laboratory, UMR CNRS 9189, Building M3, Cité Scientifique, 59655, Villeneuve d'Ascq, France*
***Lille 1 University: Sciences and Technologies, Villeneuve d'Ascq Cedex 59655, France (e-mail: alain.haddad @ polytech-lille.fr or vincent.cocquempot@univ-lille1.fr)*
****Hautes Etudes d'Ingénieur, 13 rue de Toul, 59046, Lille, France (e-mail: abdel.aitouche@hei.fr)*

Abstract: This paper presents an active fault tolerant control (AFTC) strategy for preserving the path tracking for an autonomous 2 wheel-steering 4 wheel-driving (2WS4WD) electrical vehicle in the presence of an actuator fault. It is based on a decentralized fault tolerant control strategy for overactuated systems. The strategy consists of generating new references for redundant actuators, which are only used when faults are detected, and tracking these references. For a 2WS4WD vehicle, five actuators are used in normal situations to ensure the vehicle path tracking: the front-wheel steering actuator and the four traction actuators. However, in faulty situations, the vehicle's rear-wheel steering actuator is controlled in order to preserve the system's path tracking and to ensure the desired performance. The elaborated control law that is used to control the rear-wheels steering system is composed of 2 interconnected control loops: an outer loop and an inner loop. This decentralized control strategy tolerates the traction and the front-wheel steering actuator faults. The main advantage of the proposed fault tolerant strategy is that the faults can be compensated by computing online new references for the control loops without changing the initial controllers. This strategy provides the necessary time for a diagnosis system to precisely isolate the faulty actuator. This method is tested and validated on a realistic vehicle dynamic model co-simulated using CarSim and Matlab-Simulink softwares.

Keywords: Autonomous vehicles, trajectory path tracking, fault-tolerant systems, active control, dynamic reference generator.

INTRODUCTION

The control of autonomous 2WS4WD vehicles has been the subject of intensive studies in recent years (Casavola, A. et al 2008, Wang, R et al. 2011, Zhou, Q. F., et al., 2005). Works show that this type of overactuated vehicles is superior to the traditional ones in different scenarios (Song J. and al 2009, Potluri R. et al, 2012). This is due to the possibility they offer to combine front-wheel steering control with rear-wheel steering control, as well as active differential control, in order to ensure better performances.

Existing strategies that deal with the control of 2WS4WD vehicles are mostly based on centralized control. This type of control uses a single algorithm to compute system inputs. (Casavola, A. et al 2008, Moriwaki, K., 2005, Yang, H., et al. 2010, Zhou et al., 2005). The computed inputs are then distributed between redundant actuators using allocation strategies, which can be determined offline (Zhou, Q. F., et al., 2005, Moriwaki, K., 2005), or online (Casavola, A. et al 2008, Yang, H., et al. 2010). An offline allocation strategy is presented in (Zhou, Q. F., et al., 2005), where equal steering torques are computed for front-wheel and rear-wheel steering actuators. In (Casavola, A. et al 2008, Yang, H., et al. 2010), the authors use an online allocation strategy that is function of the state of the steering actuators, in order to tolerate actuator faults.

When applying an active fault tolerant centralized control strategy, the system is viewed as a whole. If a single component is faulty, the system's initial controller has to be redesigned in order to tolerate the fault. During this step, the system's path tracking and performance cannot be guaranteed. For dynamic systems as 2WS4WD vehicles, it is necessary to react as soon as a fault is detected in order to avoid accidents.

In this paper, we present an active fault tolerant decentralized control strategy that ensures the path tracking and performance of a 2WS4WD vehicle as soon as a fault is detected. It is designed using a decentralized control approach, mainly used in aeronautics (Härkegård, O. et al., 2005, Luo, Y. et al. 2004, Levine, W. S., 2010). Compared to existing centralized control strategies, faults are not tolerated by modifying the initial controllers. Instead, as soon as a fault is detected, new references are generated locally for the redundant actuators, which are only used in faulty scenarios. The tracking of these new local references ensures the compensation of the fault. The global system's path tracking and performance are then recovered in the presence of the fault. A fault isolation module can later be applied in order to

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identify precisely the fault, as in (Haddad, A., et al., 2013), which is necessary for the system's reconfiguration. This strategy, first presented in (Haddad, A., et al., 2014), is detailed in this article.

The paper is organized as follows: Section 2 describes the kinematic model of a 2WS4WD vehicle that is used for elaborating the presented control method. Section 3 describes the strategy used for controlling the rear-wheel steering system. Section 4 presents simulation results. Conclusions and future work end the paper.

2. NONLINEAR VEHICLE MODEL

In this section, the nonlinear model, which is used to elaborate the proposed decentralized control law, is described. This model is presented in ((Dumont, P. E. et al., 2006, Sotelo, M. A., 2003)). It is valid under the assumptions of planar motion, rigid body, non-slipping tires, and considering that the two front wheels (resp. the two rear wheels) are oriented with the same angle. These assumptions make it possible to determine the position of the rotation center using kinematic rules (as shown in Fig. 1).

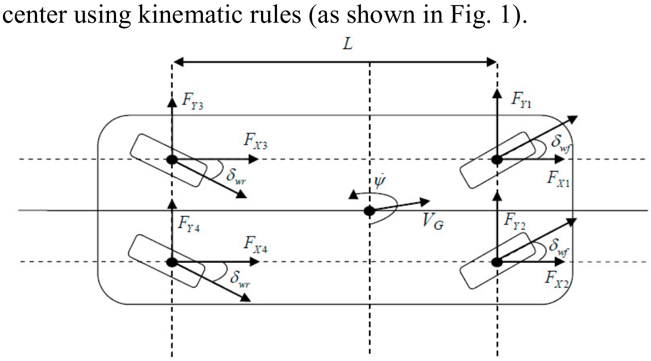

Fig. 1. 2WS4WD vehicle model

The state-space model of the nonlinear vehicle dynamics, in the frame OXYZ fixed to the ground, can be written as follows:

$$\dot{X} = f(X) + g(U) \tag{1}$$

$$X = \begin{bmatrix} x & y & \dot{x} & \dot{y} & \psi & \delta_{wf} & \dot{\delta}_{wf} & \delta_{wr} & \dot{\delta}_{wr} & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 & \dot{\theta}_4 \end{bmatrix}^T \tag{2}$$

$$U = \begin{bmatrix} u_f & u_r & u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T \tag{3}$$

x and y are respectively the vehicle's longitudinal and lateral positions, $\dot{x}$ and $\dot{y}$ are respectively the vehicle's longitudinal and lateral velocities, ψ is the vehicle's orientation, δ_{wf} and δ_{wr} are respectively the front and rear wheels steering angles, $\dot{\delta}_{wf}$ and $\dot{\delta}_{wr}$ are respectively the front and rear wheels steering velocities, $\dot{\theta}_i$ is the angular velocity of the wheel i with $i \in \{1,2,3,4\}$, u_i is the traction torque applied on the wheel i, and u_f and u_r are respectively the torques applied on the front and rear steering actuators.

$f(X)$ and $g(U)$ are expressed as

$$f(X) = \Bigg(\sqrt{\dot{x}^2+\dot{y}^2}\cos(\psi) \quad \sqrt{\dot{x}^2+\dot{y}^2}\sin(\psi) \quad -(\dot{x}^2+\dot{y}^2)\sin(\psi)\frac{tan(\delta_{wf})+tan(\delta_{wr})}{L}$$
$$(\dot{x}^2+\dot{y}^2)\cos(\psi)\frac{tan(\delta_{wf})+tan(\delta_{wr})}{L} \quad \frac{\sqrt{\dot{x}^2+\dot{y}^2}\,(tan(\delta_{wf})+tan(\delta_{wr}))}{L}$$
$$\dot{\delta}_{wf} \quad \dot{\delta}_{wr} \quad \frac{-B_f\dot{\delta}_{wf}}{J_f} \quad \frac{-B_r\dot{\delta}_{wr}}{J_r} \quad \frac{-f_1\dot{\theta}_1 + F_{x1}R\cos(\delta_{wf}) + F_{y1}R\sin(\delta_{wf})}{J_1}$$
$$\frac{-f_2\dot{\theta}_2 + F_{x2}R\cos(\delta_{wf}) + F_{y2}R\sin(\delta_{wf})}{J_2} \quad \frac{-f_1\dot{\theta}_1 + F_{x3}R\cos(\delta_{wr}) + F_{y3}R\sin(\delta_{wr})}{J_3}$$
$$\frac{-f_1\dot{\theta}_1 + F_{x4}R\cos(\delta_{wr}) + F_{y4}R\sin(\delta_{wr})}{J_4}\Bigg)^T$$

$$g(U) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{u_f}{J_f} & \frac{u_r}{J_r} & \frac{u_1}{J_1} & \frac{u_2}{J_2} & \frac{u_3}{J_3} & \frac{u_4}{J_4} \end{pmatrix}^T$$

In these equations, L is the wheelbase, B_f and B_r are respectively the front and rear tires rolling resistances, J_f and J_r are respectively the front and rear wheels steering inertia, R is the wheel radius, J_i and f_i are respectively the inertia and the friction of the wheel i, and F_{xi} and F_{yi} are respectively the longitudinal and lateral forces applied on the wheel i. Vehicle's longitudinal and lateral positions and velocities are obtained from a GPS, orientation and yaw rate are measured via an inertial measurement unit (IMU), vehicle's steering angles and velocities are obtained from absolute encoders, and wheels velocities are measured by incremental encoders.

3. FAULT TOLERANT CONTROL STRATEGY

The vehicle's lateral position y is monitored in order to detect undesirable deviations. A residual is generated based on the difference between the reference trajectory and the vehicle's lateral position measurement. This residual is then compared to a threshold that may be a static one or a dynamic one (Haddad, A., et al., 2013).

In normal situations, the path tracking is maintained by only using the front wheels steering system and the traction system. As soon as an abnormal behaviour (i.e. a lateral deviation from the vehicle's trajectory) is detected, the rear-wheel steering system is activated. The control of the rear-wheel steering system ensures that the trajectory tracking of the vehicle is preserved in the presence of the fault. The decentralized control strategy used to control the rear-wheels steering system is elaborated using two interconnected control loops: an outer loop and an inner loop. In the outer loop, a dynamic reference generator for rear-wheel steering is computed. This generator calculates the rear-wheel steering references that have to be followed in order to ensure the desired performances. In the inner loop, the control input is calculated in order to track the desired rear-wheel steering references, obtained in the outer loop (see Fig. 2).

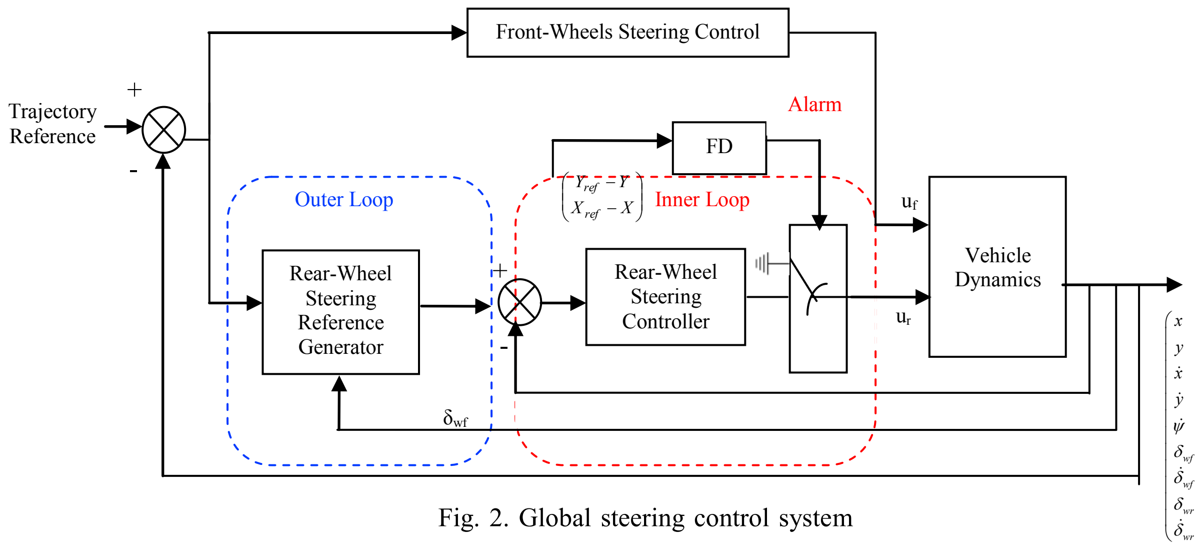

Fig. 2. Global steering control system

To design the interconnected control loops, the vehicle lateral behaviour model derived from (1) is rewritten as interconnected submodels. Then these submodels are used to elaborate independently each control loop. This decentralized control approach gives the possibility of modifying independently control loops without having to redesign the global control law. Initial controllers can then be maintained when faults are detected since these faults can be compensated by computing new local objectives in the outer loop to be followed in the inner loop.

Let us define

$$X_1=(y \quad \dot{y})^T \tag{4}$$

$$X_2=(x \quad \dot{x})^T \tag{5}$$

$$X_3=(\psi) \tag{6}$$

$$X_4=(\delta_{wf} \quad \dot{\delta}_{wf})^T \tag{7}$$

$$X_5=(\delta_{wr} \quad \dot{\delta}_{wr})^T \tag{8}$$

A submodel from (1) may then be written as

$$\dot{X}_1=f_1(X_1,X_2,X_3,X_4,X_5) \tag{9}$$

$$\dot{X}_2=f_2(X_1,X_2,X_3,X_4,X_5) \tag{10}$$

$$\dot{X}_3=f_3(X_1,X_2,X_4,X_5) \tag{11}$$

$$\dot{X}_4=f_4(X_4,u_f) \tag{12}$$

$$\dot{X}_5=f_5(X_5,u_r) \tag{13}$$

with

$$f_1(X_1,X_2,X_3,X_4,X_5)=\begin{pmatrix} \sqrt{(N_1X_1)^2+(N_1X_2)^2}\,sin(X_3) \\ ((N_1X_1)^2+(N_1X_2)^2)cos(X_3)\dfrac{(tan(N_2X_4)+tan(N_2X_5))}{L} \end{pmatrix}$$

$$f_2(X_1,X_2,X_3,X_4,X_5)=\begin{pmatrix} \sqrt{(N_1X_1)^2+(N_1X_2)^2}\,cos(X_3) \\ -((N_1X_1)^2+(N_1X_2)^2)sin(X_3)\dfrac{(tan(N_2X_4)+tan(N_2X_5))}{L} \end{pmatrix}$$

$$f_3(X_1,X_2,X_4,X_5)=\left(\frac{\sqrt{(N_1X_1)^2+(N_1X_2)^2}(tan(N_2X_4)+tan(N_2X_5))}{L}\right)$$

$$f_4(X_4,U_f)=\begin{pmatrix} N_1X_4 & \dfrac{-B_fN_1X_4+u_f}{J_f} \end{pmatrix}^T$$

$$f_5(X_5,U_r)=\begin{pmatrix} N_1X_5 & \dfrac{-N_1X_5+u_r}{J_r} \end{pmatrix}^T \tag{14}$$

with $N_1=\begin{pmatrix}0 & 1\end{pmatrix}$, and $N_2=\begin{pmatrix}1 & 0\end{pmatrix}$.

The initial control laws for the front-wheel steering system and the traction system are to be considered as designed previously. The design of the interconnected control loops is presented in the following subsections.

3.1 *Outer loop design*

In order to compute the desired rear-wheel steering angle δ_{wrdes}, the condition that the vehicle's yaw rate $\dot{\psi}$ has to satisfy in order to obtain the desired vehicle tracking performances is determined. Consider the following candidate Lyapunov function:

$$V_1(\tilde{X}_1)=V_1\left(X_{1ref}-X_1\right)=V_1(y_{ref}-y,\dot{y}_{ref}-\dot{y}) \tag{15}$$

If this function verifies the following conditions:

C_1: $V_1(y_{ref}-y,\dot{y}_{ref}-\dot{y})>0$

for $(y_{ref}-y,\dot{y}_{ref}-\dot{y})\neq 0$, and $V_1(0,0)=0$.

C_2: $\dfrac{dV_1(y_{ref}-y,\dot{y}_{ref}-\dot{y})}{dt}<0$ for $(y_{ref}-y,\dot{y}_{ref}-\dot{y})\neq 0$.

$(y_{ref}-y,\dot{y}_{ref}-\dot{y})=(0,0)$ is then a globally asymptotically stable equilibrium point. This guarantees that $(y,\dot{y})$ will converge asymptotically to $(y_{ref},\dot{y}_{ref})$ for any initial condition, thus ensuring the vehicle path tracking. The condition that the vehicle's yaw rate $\dot{\psi}_{des}$ has to satisfy for (15) to verify C_1 and C_2 is then determined. Next, the rear-wheel steering angle, which is needed to obtain the vehicle's desired yaw rate $\dot{\psi}_{des}$, is computed by using the vehicle's submodel (12).

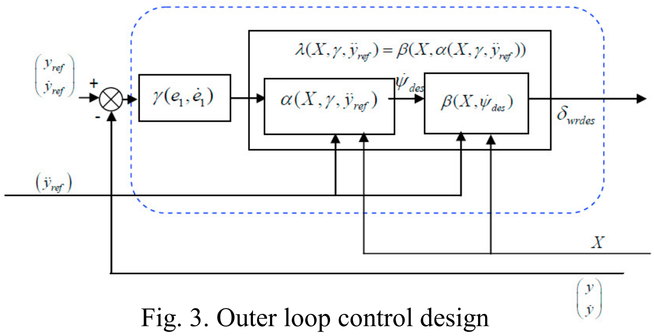

Fig. 3. Outer loop control design

Let us choose a positive definite function $V_1(e_1,\dot{e}_1)$ as

$$V_1(e_1,\dot{e}_1)=\frac{K_0(e_1)^2+(\dot{e}_1+e_1)^2}{2}>0 \tag{16}$$

with $e_1=y_{ref}-y$, $\dot{e}_1=\dot{y}_{ref}-\dot{y}$ and $K_0>0$.

If $V_1(e_1,\dot{e}_1)$ verifies C_1 and C_2, then $V_1(e_1,\dot{e}_1)$ is a Lyapunov function and $(e_1,\dot{e}_1)=(0,0)$ is a globally asymptotically stable equilibrium point for the system. Equation (15) shows that condition C_1 is satisfied. In order to satisfy C_2, there must be

$$\frac{dV_1(e_1,\dot{e}_1)}{dt}=K_0e_1\dot{e}_1+(e_1+\dot{e}_1)(\dot{e}_1+\ddot{e}_1)<0 \tag{17}$$

for $(e_1,\dot{e}_1)\neq 0$

One possible solution to obtain $\dot{V}_1(e_1,\dot{e}_1)<0$ is to impose

$$\dot{e}_1+\ddot{e}_1=-K_0e_1-K_1(e_1+\dot{e}_1) \tag{18}$$

with $\ddot{e}_1=\ddot{y}_{ref}-\ddot{y}$ and $K_1>0$.

From equation (18), $\dot{V}(e_1,\dot{e}_1)$ in equation (17) can be written as

$$\begin{aligned}\dot{V}_1(e_1,\dot{e}_1) &= K_0e_1\dot{e}_1+(e_1+\dot{e}_1)(-K_0e_1-K_1(e_1+\dot{e}_1)) \\ &= -K_0e_1^2-K_1(e_1+\dot{e}_1)^2<0\end{aligned} \tag{19}$$

From equation (18), the following error equation can be obtained

$$\ddot{e}_1+(1+K_1)\dot{e}_1+(K_0+K_1)e_1=0 \tag{20}$$

This error equation describes the free response of a 2nd order linear system. The system's overshoot M_p and settling time T_s within 2% can then be expressed as

$$M_p=\exp\left(-\frac{\pi(K_1+1)}{2\sqrt{(K_0+K_1)}\sqrt{1-\frac{(K_1+1)^2}{4(K_0+K_1)}}}\right) \tag{21}$$

$$T_s=-\frac{2\ln(0.02)}{(K_1+1)} \tag{22}$$

K_0 and K_1 in equation (20) can then be chosen such as to obtain desired tracking dynamic performances.

In the following, the condition that the vehicle's yaw rate $\dot{\psi}$ has to satisfy in order to obtain equation (20) is presented. In order to obtain this condition, equation (20) is firstly rewritten in function of $\dot{\psi}$. Then, the desired yaw rate $\dot{\psi}_{des}$ that has to be followed in order to obtain (20) is calculated.

Since $\ddot{e}_1=\ddot{y}_{ref}-\ddot{y}$, equation (20) can be rewritten as

$$(\ddot{y}_{ref}-\ddot{y})+(1+K_1)\dot{e}_1+(K_0+K_1)e_1=0 \tag{21}$$

$\ddot{y}$ in (21) can be expressed as in (9):

$$\begin{aligned}\ddot{y}&=((N_1X_1)^2+(N_1X_2)^2)\cos(X_3)\frac{(tan(N_2X_4)+tan(N_2X_5))}{L} \\ &=\sqrt{\dot{x}^2+\dot{y}^2}\cos(\psi)\dot{\psi}\end{aligned} \tag{22}$$

Introducing (22) into (21), we can write

$$(\ddot{y}_{ref}-\sqrt{\dot{x}^2+\dot{y}^2}\cos(\psi)\dot{\psi})+(1+K_1)\dot{e}_1+(K_0+K_1)e_1=0 \tag{23}$$

We then calculate the desired yaw rate $\dot{\psi}_{des}$ that has to be followed in order to verify (23).

Let us define $\gamma(e_1,\dot{e}_1)$ as

$$\gamma(e_1,\dot{e}_1)=-(1+K_1)\dot{e}_1-(K_0+K_1)e_1 \tag{24}$$

From (20) and (24) we have $\gamma(e_1,\dot{e}_1)=\ddot{e}_1$. If the vehicle's yaw rate $\dot{\psi}$ verifies the following equation:

$$\ddot{e}_1=\gamma(e_1,\dot{e}_1)=\ddot{y}_{ref}-\sqrt{\dot{x}^2+\dot{y}^2}\,cos(\psi)\dot{\psi} \tag{25}$$

equation (19) is then satisfied and the trajectory tracking of the system is in this case guaranteed. The yaw rate $\dot{\psi}_{des}$ verifying equation (25) can be expressed as

$$\begin{aligned}\dot{\psi}_{des}&=\frac{\ddot{y}_{ref}-\gamma(e_1,\dot{e}_1)}{\sqrt{(\dot{x}^2+\dot{y}^2)}\cos(\psi)} \\ &=\alpha(X,\gamma(e_1,\dot{e}_1),\ddot{y}_{ref})\end{aligned} \tag{26}$$

The desired rear-wheel steering angle δ_{wrdes}, which has to be followed in order to obtain $\dot{\psi}_{des}$, is computed in the following.

From (11), vehicle's yaw $\dot{\psi}$ is expressed as

$$\begin{aligned}\dot{\psi}&=\left(\frac{\sqrt{(N_1X_1)^2+(N_1X_2)^2}\,(tan(N_2X_4)+tan(N_2X_5))}{L}\right) \\ &=\frac{\sqrt{\dot{x}^2+\dot{y}^2}\,(tan(\delta_{wf})+tan(\delta_{wr}))}{L}\end{aligned} \tag{27}$$

Based on (27), the vehicle's desired yaw rate $\dot{\psi}_{des}$ is expressed as

$$\dot{\psi}_{des}=\frac{\sqrt{\dot{x}^2+\dot{y}^2}\,(tan(\delta_{wf})+tan(\delta_{wrdes}))}{L} \tag{28}$$

From (28), the desired rear-wheel steering angle δ_{wrdes} can be computed as

$$\begin{aligned}\delta_{wrdes}&=arctan\left(\frac{L\dot{\psi}_{des}}{\sqrt{(\dot{x}^2+\dot{y}^2)}}-tan(\delta_{wf})\right) \\ &=\beta(X,\dot{\psi}_{des})\end{aligned} \tag{29}$$

Finally, equations (26) and (29) are used to rewrite the desired rear-wheel steering angle as

$$\begin{aligned}\delta_{wrdes}&=arctan\left(\frac{L(\ddot{y}_{ref}-\gamma(e_1,\dot{e}_1))}{(\dot{x}^2+\dot{y}^2)\cos(\psi)}-tan(\delta_{wf})\right) \\ &=\beta(X,\alpha(X,\gamma(e_1,\dot{e}_1),\ddot{y}_{ref})) \\ &=\lambda(X,\gamma(e_1,\dot{e}_1),\ddot{y}_{ref})\end{aligned} \tag{30}$$

Knowing that $\psi=arctan(\frac{\dot{y}}{\dot{x}})$, (29) can be expressed as

$$\delta_{wrdes} = arctan(\frac{L(K_0 e_1 + K_1 \dot{e}_1 + \ddot{y}_{ref})}{(\dot{x}^2 + \dot{y}^2)\cos(arctan(\frac{\dot{y}}{\dot{x}}))} - tan(\delta_{wf})) \quad (31)$$

Remark 1: Singularities exist in equation (31) for $arctan(\frac{\dot{Y}}{\dot{X}}) = \pm\frac{\pi}{2}$. These singularities can be avoided by changing the vehicle's frame, as in (Rajamani et al., 2003), when vehicle rotation angle does not verify C_3: $-\frac{\pi}{4} \le arctan(\frac{\dot{Y}}{\dot{X}}) \le \frac{\pi}{4}$ or C_4: $\frac{3\pi}{4} \le arctan(\frac{\dot{Y}}{\dot{X}}) \le \frac{5\pi}{4}$.

The transition matrix can be expressed as

$$T = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (32)$$

By applying this transformation, the vehicle is controlled in the frame OYXZ instead of controlling it in the frame OXYZ. In that case, the rear-wheel steering reference can be rewritten as follows:

$$\delta_{wrdes} = arctan(-\frac{L}{(\dot{X}^2 + \dot{Y}^2)\sin(arctan(\frac{\dot{Y}}{\dot{X}}))}(K_0(X_{ref} - X) + K_1(\dot{X}_{ref} - \dot{X}) + \ddot{X}_{ref}) - tan(\delta_{wf})) \quad (33)$$

Remark 2: It is clear that singularities exist also for $arctan(\frac{\dot{Y}}{\dot{X}}) = 0$ and $arctan(\frac{\dot{Y}}{\dot{X}}) = \pi$. A transformation is also applied when vehicle rotation angle does not verify C_5: $\frac{\pi}{4} < arctan(\frac{\dot{Y}}{\dot{X}}) < \frac{3\pi}{4}$ or C_6: $\frac{5\pi}{4} < arctan(\frac{\dot{Y}}{\dot{X}}) < \frac{7\pi}{4}$. In this case, the transition matrix can be expressed as

$$T^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (34)$$

In other words, when conditions C_5 or C_6 are not verified, the vehicle is controlled in the initial frame OXYZ .

3.2 *Inner Loop design*

After computing the desired rear-wheel steering position δ_{wrdes} in the outer loop, the control input u_r needed to track this reference is calculated. For this purpose, the backstepping technique, which is a recursive control method, is used. First, a subsystem from the considered vehicle's lateral behavior model is used, for which a virtual control law is constructed. The design is then extended in several steps by adding subsystems to the considered model until a control law for the entire system is obtained. Along with the control law, Lyapunov functions are successively constructed in each step.

From (9) and (13), the following two subsystems are extracted

$$\dot{X}_1 = f_1(X_1, X_2, X_3, X_4, \xi) \quad (35)$$

$$\dot{\xi} = f_5(\xi, u_r) \quad (36)$$

with $\xi = (\delta_{wr} \quad \dot{\delta}_{wr})^T$ being the virtual input of the subsystem presented in (35).

As demonstrated in the previous section, ξ that stabilizes $\dot{\tilde{X}}_1 = (\dot{X}_{1ref} - \dot{X}_1)$ has to be equal to $(\delta_{wrdes} \quad \dot{\delta}_{wrdes})^T$. The subsystem (35) is then extended by adding (36) in order to calculate the input that ensures a global lateral trajectory tracking for the system. The following equation is obtained

$$\begin{pmatrix} \dot{X}_1 \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} f_1(X_1, X_2, X_3, X_4, \xi) \\ f_5(\xi, u_r) \end{pmatrix} \quad (37)$$

For the extended subsystem (37), a positive definite function $V_2(e_1, \dot{e}_1, \Delta\xi)$ is defined as

$$\begin{aligned} V_2(e_1, \dot{e}_1, \Delta\xi) &= V_1(e_1, \dot{e}_1) + \Delta\xi^T W \Delta\xi \\ &= V_1(e_1, \dot{e}_1) + \frac{K_2 \Delta\delta_{wr}{}^2}{2} + \frac{(\Delta\delta_{wr} + \Delta\dot{\delta}_{wr})^2}{2} \end{aligned} \quad (38)$$

with

$$\begin{aligned} \Delta\xi &= \xi - (\delta_{wrdes} \quad \dot{\delta}_{wrdes})^T \\ &= \begin{pmatrix} \delta_{wr}(t) - \delta_{wrdes}(t) \\ \dot{\delta}_{wr}(t) - \dot{\delta}_{wrdes}(t) \end{pmatrix} \\ &= \begin{pmatrix} \Delta\delta_{wr}(t) \\ \Delta\dot{\delta}_{wr}(t) \end{pmatrix} \end{aligned} \quad (39)$$

and $W = \frac{1}{2}\begin{pmatrix} K_2 + 1 & 1 \\ 1 & 1 \end{pmatrix} > 0$ with $K_2 > 0$.

If $V_2(e_1, \dot{e}_1, \Delta\xi)$ verifies the following conditions:

C_7: $V_2(e_1, \dot{e}_1, \Delta\xi) > 0$
for $(e_1, \dot{e}_1, \Delta\xi) \ne 0$, and $V_2(0, 0, (0,0)) = 0$.

C_8: $\frac{dV_2(e_1, \dot{e}_1, \Delta\xi)}{dt} < 0$ for $(e_1, \dot{e}_1, \Delta\xi) \ne 0$.

$(e_1, \dot{e}_1, \Delta\delta_{wr}, \Delta\dot{\delta}_{wr}) = (0,0,0,0)$ is then a globally asymptotically stable equilibrium point. This guarantees that $(\Delta\delta_{wr}, \Delta\dot{\delta}_{wr})$ converges asymptotically to (0,0), meaning that $(\delta_{wr}(t), \dot{\delta}_{wr}(t))$ converges to $(\delta_{wrdes}(t), \dot{\delta}_{wrdes}(t))$.

Equation (38) shows that $V_2(e_1, \dot{e}_1, \Delta\xi)$ satisfies condition C_7.

In the following, the rear-wheel steering control input u_r necessary for satisfying condition C_8 is computed.

$\frac{dV_2(e_1, \dot{e}_1, \Delta\xi)}{dt}$ is expressed as

$$\begin{aligned} \frac{dV_2(e_1, \dot{e}_1, \Delta\xi)}{dt} &= \dot{V}_1(e_1, \dot{e}_1) + \Delta\dot{\xi}^T W \Delta\xi + \Delta\xi^T W \Delta\dot{\xi} \\ &= \dot{V}_1(e_1, \dot{e}_1) + K_2 \Delta\delta_{wr}\Delta\dot{\delta}_{wr} + \\ &\quad (\Delta\delta_{wr} + \Delta\dot{\delta}_{wr})(\Delta\dot{\delta}_{wr} - \Delta\ddot{\delta}_{wr}) \end{aligned} \quad (40)$$

for $(e_1, \dot{e}_1, \Delta\delta_{wr}, \Delta\dot{\delta}_{wr}) \ne 0$, with $\Delta\ddot{\delta}_{wr} = \ddot{\delta}_{wr} - \ddot{\delta}_{wrdes}$.

Based on (40), C_8 is satisfied if

$$\begin{aligned} \frac{dV_2(e_1, \dot{e}_1, \Delta\xi)}{dt} &= \dot{V}_1(e_1, \dot{e}_1) + K_2 \Delta\delta_{wr}\Delta\dot{\delta}_{wr} \\ &\quad + (\Delta\delta_{wr} + \Delta\dot{\delta}_{wr})(\Delta\dot{\delta}_{wr} - \Delta\ddot{\delta}_{wr}) < 0 \end{aligned} \quad (41)$$

$\dot{V}_1(e_1, \dot{e}_1)$ expressed in (17), and used in (41), can be rewritten as

$$\begin{aligned}\dot{V}_1(e_1,\dot{e}_1) &= K_0 e_1\dot{e}_1 + (e_1+\dot{e}_1)(\dot{e}_1 + \ddot{y}_{ref} - \\ &\quad f_1(x_1,x_2,x_3,x_4,\Delta\delta_{wr}+\delta_{wrdes})) \\ &= K_0 e_1\dot{e}_1 + (e_1+\dot{e}_1)(-K_0 e_1 - K_1(e_1+\dot{e}_1) - \\ &\quad \lambda_{12}(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr})) \\ &= -K_0 e_1^2 - K_1(e_1+\dot{e}_1)^2 - \\ &\quad \lambda_{12}(e_1+\dot{e}_1)(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr})\end{aligned} \tag{42}$$

with

$$f_1(x_1,x_2,x_3,x_4,\Delta\delta_{wr}+\delta_{wrdes}) = f_1(x_1,x_2,x_3,x_4,\delta_{wrdes}) + \lambda_{12}(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) \tag{43}$$

and λ_{12} is expressed as

$$\lambda_{12} = \frac{f_1(x_1,x_2,x_3,x_4,\delta_{wr}) - f_1(x_1,x_2,x_3,x_4,\delta_{wrdes})}{\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}} \tag{44}$$

Remark 3: If $\xi = (\delta_{wrdes} \quad \dot{\delta}_{wrdes})^T$, then the expression $f_1(x_1,x_2,x_3,x_4,\delta_{wr}) - f_1(x_1,x_2,x_3,x_4,\delta_{wrdes})$ in (44) becomes equal to zero. This implies that, for this condition, we have $\lambda_{12}=0$, and (42) becomes equal to equation (19).

Using (42), equation (40) can be rewritten as

$$\begin{aligned}\dot{V}_2(e_1,\dot{e}_1,\Delta\xi) &= \dot{V}_1(e_1,\dot{e}_1) + \Delta\dot{\xi}^T W\Delta\xi + \Delta\xi^T W\Delta\dot{\xi} \\ &= -K_0 e_1^2 - K_1(e_1+\dot{e}_1)^2 - \\ &\quad \lambda_{12}(e_1+\dot{e}_1)(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) + \\ &\quad K_2\Delta\delta_{wr}\Delta\dot{\delta}_{wr} + (\Delta\delta_{wr}+\Delta\dot{\delta}_{wr})(\Delta\dot{\delta}_{wr}+\Delta\ddot{\delta}_{wr})\end{aligned} \tag{45}$$

A solution to obtain $\dot{V}_2(e_1,\dot{e}_1,\Delta\xi)<0$ is to have

$$\begin{aligned}\Delta\dot{\delta}_{wr}+\Delta\ddot{\delta}_{wr} &= -K_2\Delta\delta_{wr} - K_3(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) \\ &\quad +\lambda_{12}(e_1+\dot{e}_1)\end{aligned} \tag{46}$$

with $K_3>0$.

This solution is verified by introducing the value of $(\Delta\dot{\delta}_{wr}+\Delta\ddot{\delta}_{wr})$ expressed in (46) to equation (45) as

$$\begin{aligned}\dot{V}_2(e_1,\dot{e}_1,\Delta\xi) &= -K_0 e_1^2 - K_1(e_1+\dot{e}_1)^2 - \\ &\quad \lambda_{12}(e_1+\dot{e}_1)(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) + \\ &\quad K_2\Delta\delta_{wr}\Delta\dot{\delta}_{wr} + (\Delta\delta_{wr}+\Delta\dot{\delta}_{wr})(-K_2\Delta\delta_{wr} - \\ &\quad K_3(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) + \lambda_{12}(e_1+\dot{e}_1))\end{aligned} \tag{47}$$

From (47), the following equation is finally obtained:

$$\begin{aligned}\dot{V}_2(e_1,\dot{e}_1,\Delta\xi) &= -K_0 e_1^2 - K_1(e_1+\dot{e}_1)^2 - \\ &\quad K_2\Delta\delta_{wr}{}^2 - K_3(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr})^2 < 0\end{aligned} \tag{48}$$

Equation (48) shows that for $(\Delta\dot{\delta}_{wr}+\Delta\ddot{\delta}_{wr})$ expressed as in (46), $V_2(e_1,\dot{e}_1,\Delta\xi)$ is a Lyapunov function since it satisfies C_7 and C_8. The control input u_r that verifies (46) can now be computed.

From (13), we have

$$\ddot{\delta}_{wr} = \frac{-N_1 X_5 + u_r}{J_r} = \frac{-B_r\dot{\delta}_{wr} + u_r}{J_r} \tag{49}$$

Knowing that $\Delta\ddot{\delta}_{wr} = \ddot{\delta}_{wr} - \ddot{\delta}_{wrdes}$, we can write

$$\begin{aligned}\Delta\ddot{\delta}_{wr} &= \ddot{\delta}_{wr} - \ddot{\delta}_{wrdes} \\ &= \frac{-B_r\dot{\delta}_{wr}+u_r}{J_r} - \ddot{\delta}_{wrdes}\end{aligned} \tag{50}$$

From (49), $\Delta\ddot{\delta}_{wr}$ in (50) can be expressed as

$$\begin{aligned}\Delta\ddot{\delta}_{wr} &= -K_2\Delta\delta_{wr} - K_3(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) - \Delta\dot{\delta}_{wr} \\ &\quad +\lambda_{12}(e_1+\dot{e}_1)\end{aligned} \tag{51}$$

Using equations (50) and (51), we can write

$$\begin{aligned}\frac{-B_r\dot{\delta}_{wr}+u_r}{J_r} - \ddot{\delta}_{wrdes} &= -K_2\Delta\delta_{wr} - K_3(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) - \Delta\dot{\delta}_{wr} \\ &\quad +\lambda_{12}(e_1+\dot{e}_1)\end{aligned} \tag{52}$$

From (52), the rear-wheel steering control input u_r is computed as

$$\begin{aligned}u_r &= J_r(-K_2\Delta\delta_{wr} - K_3(\Delta\delta_{wr}+\Delta\dot{\delta}_{wr}) - \Delta\dot{\delta}_{wr} \\ &\quad +\lambda_{12}(e_1+\dot{e}_1) + \ddot{\delta}_{wrdes}) + B_r\dot{\delta}_{wr}\end{aligned} \tag{53}$$

With u_r computed as in (53), $V_2(e_1,\dot{e}_1,\Delta\xi)$ is a Lyapunov function and $(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr}) = (0,0,0,0)$ is a global asymptotically stable equilibrium point for the system.

4. SIMULATION RESULTS

The proposed strategy is tested using a co-simulation between CarSim, a professional simulator used by automobile manufacturers, and Matlab-Simulink software, as in (Haddad, A., et al., 2014).

In this test, an overactuated autonomous vehicle is circulating with an initial constant speed of 60 km/h, and performing a double lane-change maneuver on a dry asphalt road (friction coefficient $\mu_{\max}$ = 1.2). At t=3.6s, a drop of efficiency is created at the front-wheel steering actuator. Two scenarios are then considered.

In the first scenario, the vehicle is controlled using its front-wheel steering system only. It can be seen in Fig. 4 that the front-wheel steering controller is not able to ensure alone the lateral stability of the vehicle. The vehicle exceeds the limits of the road at t=4.58s.

In the second scenario, fault detection (FD) system is used to monitor the lateral deviation. When this deviation violates the acceptable security margins (determined based on the width of the road) at t=4.16s, the FD system activates the rear-wheel steering controller presented in section 3. The rear-wheel steering system is then able to maintain the lateral stability of the global system in the presence of the component fault (see Fig. 4).

5. CONCLUSIONS

In this paper, a fault tolerant control strategy is developed for a 2WS4WD autonomous vehicle. It is demonstrated that this strategy can ensure the vehicle's path tracking in the presence of an unknown actuator fault.

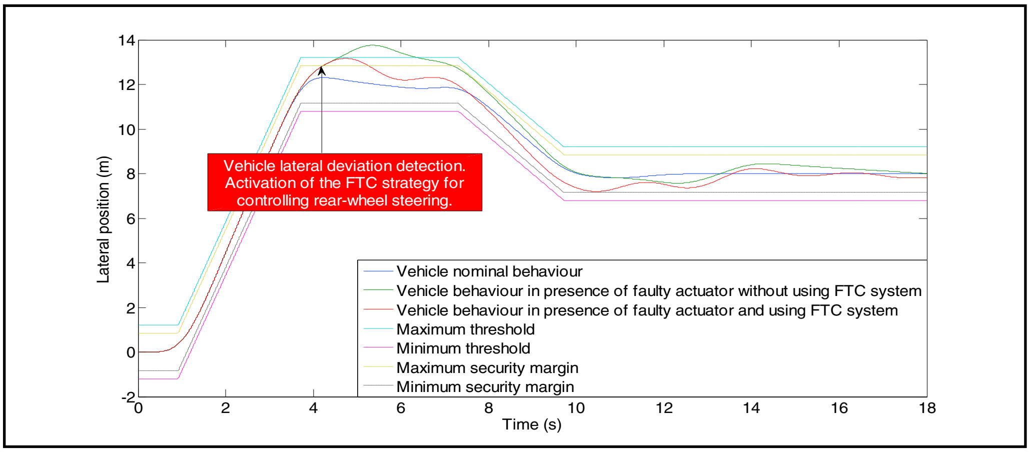

Fig. 4. Vehicle lateral behaviour when performing a double lane-change maneuver

This strategy is developed using the backstepping technique, and consists of dividing the control strategy into two loops: the outer one and the inner one. In the outer loop, the steering position needed in order to obtain the nominal vehicle performances is computed. In the inner loop, the rear-wheel steering system is controlled in order to follow the desired reference calculated in the outer loop. When a vehicle lateral deviation is detected, the elaborated algorithm is activated in order to maintain the global system's lateral stability. The efficiency of this strategy is illustrated using a co-simulation between Carsim and Matlab-Simulink softwares.

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