Conor D'Arcy

Equations of Optimization

February 24th 2014

Overview

Abstract:
The following equations serve to calculate the optimal price of a good/service for maximum revenue or profit.

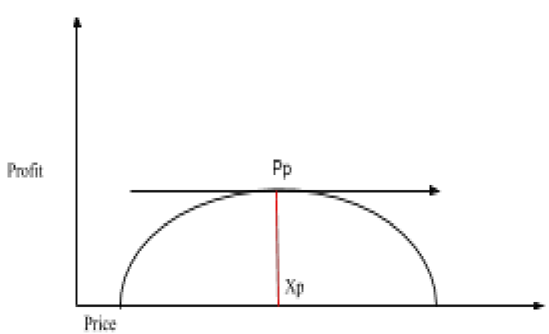
Known Variables:
B is the total production cost per individual good/service
max is the maximum price one person out of 100 will pay for a good/service.

Necessary Assumptions:
The purchasing behavior of customers follows one of three forms: Parabolic Regression, Linear Regression, or Exponential Regression. To establish which form to use, a study must be conducted in which customers are asked if they would purchase an item for [0,max] with (max/20) step size. After graphing the data, the regression that best fits the plotted data points should be used. Prices of the regressions follow: Parabolic > Linear > Exponential.

Parabolic:
Newly released items that are popular and selling well. A change in price results in little change of consumer action.

Linear:
Items that are still relevant, but not as popular as their original release. A change in price results in an equal change in consumer action.

Exponential:
Items that are no longer relevant in the marketplace. A small change in price results in a large change in consumer action.



Exponential Regression:

Where:

$max = Price\ one\ person\ per\ 100\ will\ pay$

$M = \sqrt[max]{.01}$

$B = Production\ cost\ per\ unit$

$L = \%\ of\ population\ who\ will\ purchase\ item$

Constraints:

$max \geq 0$

$M:\ [0,1)\ \cup\ (1,\infty)$

$B \geq 0$

Proof:

Generic equation of exponential regression: $y = a(1-b)^x$

$L = 100(1-b)^x$

Assume: 1 buyer at 5 dollars and 100% population purchase at a price of 0

$1 = 100(1-b)^5$

$.01 = (1-b)^5$

$b = 1 - \sqrt[5]{.01}$

Thus:

$b = 1 - \sqrt[max]{.01}$

$Revenue = (Volume)(Pricesold)$

$Revenue = [100(1-(1-\sqrt[max]{.01}))^x](x)$

$Revenue = 100(\sqrt[max]{.01}^x)(x)$

$Revenue = 100x(\sqrt[max]{.01})^x$

$\frac{dR}{dx} = 100(\sqrt[max]{.01})^x + 100x[ln(\sqrt[max]{.01})(\sqrt[max]{.01})^x]$

$Substitute\ \sqrt[max]{.01}\ with\ M$

$\frac{dR}{dx} = 100(M)^x + 100x[ln(M)(M)^x]$

$0 = 100(M)^x + 100x[ln(M)(M)^x]$

$0 = M^x(1 + ln(M)^x)$

$M^x \neq 0$

Therefore:

$0 = 1 + ln(M)^x$

$0 = 1 + xln(M)$

Thus:

$x = \frac{-1}{ln(M)}$

Where x is the optimal price for revenue

Ultimately:

$$Revenue = 100\left(\frac{-1}{ln(M)}\right)(M)^{\frac{-1}{ln(M)}}$$

$$Profit = (Volume)(Pricesold) - (Volume)(Pricemade)$$

$$Profit = 100x(M)^{x} - 100B(M)^{x}$$

$$\frac{dP}{dx} = 100(M)^{x} + [100xln(M)(M)^{x}] - 100Bln(M)(M)^{x}$$

$$0 = 100(M)^{x} + [100xln(M)(M)^{x}] - 100Bln(M)(M)^{x}$$

$$0 = (M)^{x} + [xln(M)(M)^{x}] - Bln(M)(M)^{x}$$

$$0 = (M)^{x}[1 + xln(M) - Bln(M)]$$

$$M^{x} \neq 0$$

Therefore:

$$0 = 1 + xln(M) - Bln(M)$$

$$-1 = ln(M)(x - B)$$

Thus:

$$x = \frac{-1}{ln(M)} + B$$

Where x is the optimal price for profit

Ultimately:

$$Profit = 100\left(\frac{-1}{ln(M)} + B\right)(M)^{\frac{-1}{ln(M)}+B} - 100B(M)^{\frac{-1}{ln(M)}+B}$$

Final Equations:

$$Optimal\ Profit\ Price = \left(\frac{-1}{ln(M)}\right) + B$$

$$Optimal\ Revenue\ Price = \left(\frac{-1}{ln(M)}\right)$$

$$Profit = 100\left(\frac{-1}{ln(M)} + B\right)(M)^{\frac{-1}{ln(M)}+B} - 100B(M)^{\frac{-1}{ln(M)}+B}$$

$$Revenue = 100\left(\frac{-1}{ln(M)}\right)(M)^{\frac{-1}{ln(M)}}$$

Linear Regression:

Where:

$max = Price\ one\ person\ per\ 100\ will\ pay$

$B = Production\ cost\ per\ unit\ and\ \geq 0$

$Optimal\ Profit\ Price = \frac{max+B}{2}$

$Optimal\ Revenue\ Price = \frac{max}{2}$

$Revenue = 25max$

$Profit = 100(\frac{max+B}{2}) + \frac{100B}{Max}(\frac{max+B}{2}) - 100B - \frac{100}{Max}(\frac{max+B}{2})^2$

Proof:

Generic equation of a line: $y = mx + b$

$y = mx + 100$

$0 = m(max) + 100$

$-100 = m(max)$

$m = \frac{-100}{max}$

$Revenue = (Volume)(Price)$

$Revenue = (\frac{-100x}{max} + 100)(x)$

$Revenue = \frac{-100x^2}{max} + 100x$

$\frac{dR}{dx} = \frac{-200x}{max} + 100$

$0 = \frac{-200x}{max} + 100$

$\frac{-100max}{-200} = x$

$x = \frac{max}{2}$

Where x is the optimal price for revenue

Ultimately,

$Revenue = 25max$

$Profit = \frac{-100x^2}{max} + 100x - [(\frac{-100x}{max} + 100)B]$

$\frac{dP}{dx} = \frac{-200x}{max} + 100 + 100B$

$0 = \frac{-200x}{max} + 100 + 100B$

$\frac{-100max-100B}{-200} = x$

$x = \frac{max+B}{2}$

Where x is the optimal price for profit

Ultimately,

$Profit = 100(\frac{max+B}{2}) + \frac{100B}{Max}(\frac{max+B}{2}) - 100B - \frac{100}{Max}(\frac{max+B}{2})^2$