Strassen's Algorithm for Matrix Multiplication

1 A Simple Divide-and-Conquer Algorithm

$$\begin{bmatrix} r & s \\ t & u \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & g \\ f & h \end{bmatrix}$$

The values of r, s, t, and u:

$$r = ae + bf$$
$$s = ag + bh$$
$$t = ce + df$$
$$u = cg + dh$$

Complexity: Total of 8 multiplications and 4 additions.

2 Strassen's Magic Idea

With 7 multiplications and 10 additions, compute 7 help variables:

$$p_1 = a(g - h)$$
$$p_2 = (a + b)h$$
$$p_3 = (c + d)e$$
$$p_4 = d(f - e)$$
$$p_5 = (a + d)(e + h)$$
$$p_6 = (b - d)(f + h)$$
$$p_7 = (a - c)(e + g)$$

With 8 more additions, compute r, s, t, and u:

$$r = p_5 + p_4 - p_2 + p_6$$
$$s = p_1 + p_2$$
$$t = p_3 + p_4$$
$$u = p_5 + p_1 - p_3 - p_7$$

Complexity: Total of 7 multiplications and 18 additions.