

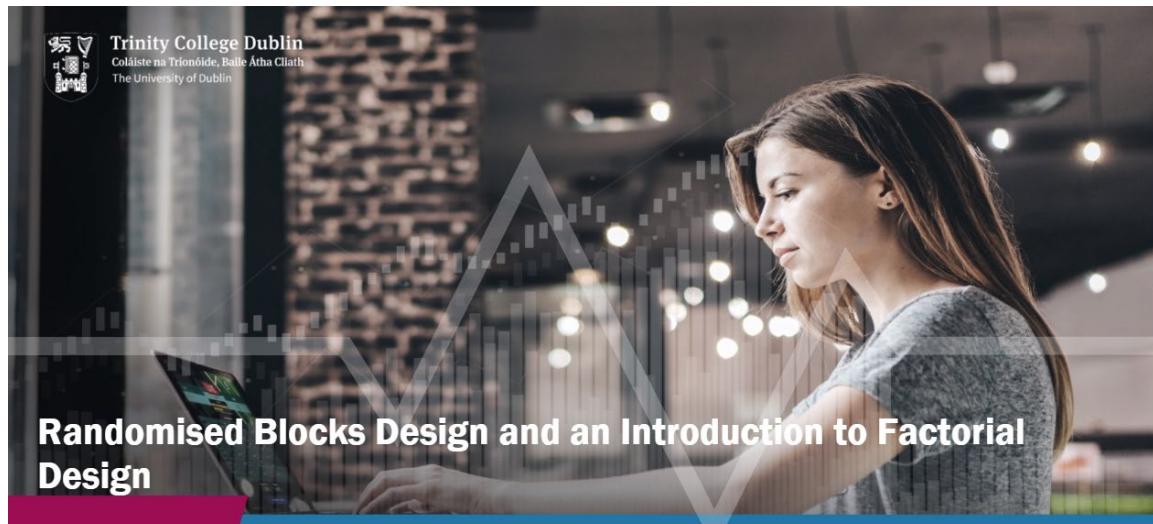
Randomised Blocks Design and an Introduction to Factorial Design

Contents

Slide 1:	Introduction.....	3
Slide 2:	Randomised Blocks Design	3
Tab 1:	Treating Crops with Fertiliser to Improve Yield	4
Tab 2:	Treating Long Spools of Rubber to Improve Abrasion Resistance	5
Slide 3:	Reducing Yield Loss in the Manufacture of Sulfanilimide.....	6
Tab 1:	Reducing Yield Loss at filtration stage.....	7
Slide 4:	Exercise	8
Slide 5:	Answers	9
Slide 6:	Randomisation Procedure	14
Tab 1:	Results of Randomisation Procedure.....	15
Slide 7:	Randomised Blocks Analysis	15
Slide 8:	Initial Data Analysis	16
Slide 9:	Interaction Plot.....	16
Slide 10:	Formal Analysis: Analysis of Variance.....	17
Slide 11:	Multiple Comparisons.....	18
Slide 12:	Testing Variation Between Block Means	19
Slide 13:	To Block or Not?.....	20
Slide 14:	To Block or Not to Block?	21
Slide 15:	Model for Analysis: Yield Loss	22
Tab 1:	Estimating the Model	23
Slide 16:	Decomposing Total Variation	24
Slide 17:	Expected Mean Squares.....	25
Slide 18:	Deletion Residuals.....	26
Tab 1:	Calculating Deletion Residuals.....	27
Slide 19:	An Example of a 3 X 3 Factorial Design	28
Slide 20:	Study Design and Results	29
Slide 21:	Knowledge Check	30
Slide 22:	Solutions.....	30
Tab 1:	Response	31
Tab 2:	Experimental Factors.....	31
Tab 3:	Factor Levels.....	32

Tab 4:	Treatments.....	32
Tab 5:	Treatment Structure	33
Tab 6:	Experimental Unit	33
Tab 7:	Unit Structure.....	34
Tab 8:	Replication	34
Slide 23:	Experiment Design Issues	35
Slide 24:	Initial Data Analysis of Experiment	36
Tab 1:	Assessing Chance Variation.....	37
Slide 25:	Formal Analysis of Experiment.....	37
Slide 26:	Mathematical Version of the Statistical Model.....	38
Tab 1:	Detailed Mathematical Notation	39
Slide 27:	Decomposing Total Variation	40
Slide 28:	Estimating the Model.....	41
Tab 1:	Summary and Interaction Tables	41
Slide 29:	Reporting Interaction.....	42
Slide 30:	Estimating Sigma	43
Slide 31:	Using Minitab to Produce an Analysis of Variance Table	43
Slide 32:	Diagnostic Plots	44
Slide 33:	Summary	45

Slide 1: **Introduction**



Trinity College Dublin
Coláiste na Trionóide, Baile Átha Cliath
The University of Dublin

Randomised Blocks Design and an Introduction to Factorial Design

 Presenter: James Ng
Duration: 41:12
School: Computer Science and Statistics

Hello and welcome. My name is James Ng and I will lead you through this presentation on randomized blocks design and an introduction to factorial design.

We will introduce the basic principles of randomized blocks design and factorial design and their applications through case studies.

Slide 2: **Randomised Blocks Design**



Randomised Blocks Design

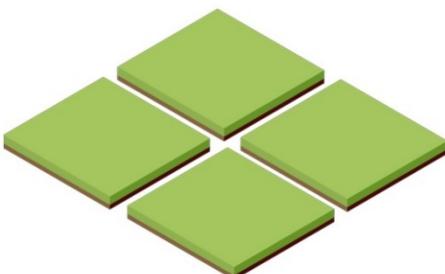
2 of 33

Treating Crops with Fertiliser to Improve Yield

Treating Long Spools of Rubber to Improve Abrasion Resistance

Introduction

- Dividing experimental units into response-homogeneous sets of units came about through agricultural research.
 - Experimental plots were plots in fields and sets of response-homogeneous plots were called blocks.



 Click each tab to learn more. Then, click Next to continue.

The idea of dividing the experimental units into response-homogeneous sets of units arose, like most of the basic ideas of experimental design, in agricultural research, where the experimental units were plots in fields and sets of response-homogeneous plots were referred to as blocks.

Let's consider two examples in agriculture and manufacturing that demonstrate the use of randomised blocks design.

Click each tab to learn more. Then click next to continue.

Tab 1: Treating Crops with Fertiliser to Improve Yield

Randomised Blocks Design		2 of 33
Treating Crops with Fertiliser to Improve Yield	Treating Long Spools of Rubber to Improve Abrasion Resistance	<input checked="" type="checkbox"/>
Treating Crops with Fertiliser to Improve Yield <ul style="list-style-type: none"> Four different fertilisers are tested on several different fields. <ul style="list-style-type: none"> Each field is divided into four plots (experimental units) to form one block. The fields constitute blocks. The experimental factor is fertiliser. Treatments are assigned at random to plots within blocks. This minimises misinterpreting unknown factors affecting yields within blocks. Using blocks in varying locations ensures comparisons between fertilisers are generalisable. <ul style="list-style-type: none"> This is valuable if fields are in different locations. 		

Consider an experiment to compare the use of four different fertilisers to improve crop yield. One approach is to run the experiment on several different fields, with each field being divided into four plots, the experimental units, and the four treatments being assigned at random to the four plots in each field. Frequently, different fields have different fertility, particularly if they are in different parts of the country. However, by allowing fertilisers to be compared within each field and then combining these comparisons across fields, the differences between fields do not affect comparisons between fertilisers.

In this case, the fields constitute blocks. The experimental factor is Fertiliser, with four "levels". Random assignment of fertilisers to plots within blocks minimises the chances that an unknown factor affecting yields within blocks will be misinterpreted as a fertiliser effect. Even if the effects of some unknown factor are aligned with the fertiliser effects in one block, the separate randomisations make it unlikely that this will happen in all blocks.

Another benefit of the use of different fields as blocks is that the comparisons between fertilisers are generalisable to the different conditions in the different fields. This is particularly valuable if the fields are in different locations.

Tab 2: Treating Long Spools of Rubber to Improve Abrasion Resistance

Randomised Blocks Design
2 of 33

Treating Crops with Fertiliser to Improve Yield	Treating Long Spools of Rubber to Improve Abrasion Resistance
--	--

Treating Long Spools of Rubber to Improve Abrasion Resistance

- A randomised blocks experiment assesses four treatments for improving abrasion resistance by:
 - Cutting a single piece of the rubber roll into four experimental units to form one block
 - Assigning treatments at random to the four units
 - Repeating with several other pieces from the roll to form several blocks

Block 1	Block 2	Block 3	Block 4	etc.
B A	C A	A D	D B	
C D	B D	C B	C A	

- Blocking allows for anticipated variation patterns along the length of the spool.
- Randomisation allows for unanticipated sources of variation within blocks.

A specific type of rubber used in the manufacture of car tyres is produced by extrusion in a continuous sheet which is then spooled into a roll. An important feature of the rubber is its abrasion resistance and four new treatments intended to improve the abrasion resistance are being assessed. As it is known that abrasion resistance varies along the length of the roll, a randomised blocks experiment is proposed that involves cutting a single piece into four experimental units constituting one block, assigning treatments at random to the four units, repeating with several other pieces from the roll to form several blocks.

Blocking allows for anticipated variation patterns along the length of the spool of rubber, permitting more precise comparisons of the treatment effects. Randomisation allows for unanticipated sources of variation within blocks, for example, side to side, diagonal or any other consistent pattern of variation, avoiding possible bias that could arise if the treatments were applied to the same positions in all blocks.

Slide 3:

Reducing Yield Loss in the Manufacture of Sulfanilimide.



Reducing Yield Loss in the Manufacture of Sulfanilimide

3 of 33

- A Davies et al. case study¹ examined how to reduce yield loss in the manufacture of the drug Sulfanilimide.
 - Its manufacture involves the chlorosulfonation of acetanilide.

Acetanilide Chlorosulfonation

This three-step process involves:

- Blending acetanilide from different stocks
- Producing a liquid with suspended solids through the reaction with chlorosulfonic acid
- Filtering the liquid to recover the powdered end product



Reducing Yield Loss in a Chemical Process



Click the tab to learn more. Then, click Next to continue.

Let's now look at the Davies et al.'s case study on reducing yield loss in a chemical process. Part of a process for manufacturing sulfanilamide, a drug used in treating yeast infections, involved the chlorosulfonation of acetanilide, a predecessor of paracetamol. The chlorosulfonation process had three steps:

- acetanilide was blended from different stocks
- reaction with chlorosulfonic acid resulted in an intermediate liquid product with suspended solids and
- the liquid was filtered to recover powdered end product.

Click the Reducing Yield Loss in a Chemical Process tab to learn more. Then click next to continue.

Tab 1: Reducing Yield Loss at filtration stage

Reducing Yield Loss in a Chemical Process

Yield Loss at Filtration Stage

-

Case Study on Reducing Yield Loss



The problem:

- A yield loss at the filtration stage



The proposal:

- To improve the yield by using different blends of acetanilide



The plan:

- To prepare five different blends
- To use each blend in successive process runs, in random order
- To replicate the experiment at later times (blocks)

There was a problem with yield loss at the filtration stage. It was thought that yield might be improved by using different blends of the input acetanilide, although an alternative view was also expressed, on chemical grounds. An experiment was conducted to test this possibility. To avoid the possibility that the yield might be affected by a time trend in the process, the experiment was run in blocks, as follows:

- five different blends were prepared,
- samples of the five blends were used in five successive process runs, in random order, and
- the experiment was replicated at later times.

Slide 4: **Exercise**



Exercise

4 of 33

- Can you identify the following?

Response

Experimental Factor(s)

Factor Levels

Treatments

Experimental Unit

Observational Unit

Unit Structure

Treatment Assignment

Replication



Take time to think about your answers.

For the case study on yield loss reduction, can you identify the following?

the response

The experimental factor(s)

The factor levels

The treatments

An experimental unit

An observational unit

Unit structure

Treatment assignment and

Replication

Take time to think about your answers. Then click next to continue.

Slide 5: **Answers**



Answers

Solutions

- Did you answer correctly?

Response

Experimental Factor(s)

Factor Levels

Treatments

Experimental Unit

Observational Unit

Unit Structure

Treatment Allocation

Replication



 Click each tab to learn more. Then, click Next to continue.

Click each tab to find out if you were correct. Then click next to continue.

Tab 1.1: Response



Answers

Solutions

- Did you answer correctly?

Response

Experimental Factor(s)

Factor Levels

Treatments

Experimental Unit

Observational Unit

Unit Structure

Treatment Allocation

Replication

Response

- The response is the yield loss.

 Click each tab to learn more. Then, click Next to continue.

The response is the yield loss.

Tab 1.2: Experimental factor(s)

 **Answers** 5 of 33

Solutions

- Did you answer correctly?

Response	Experimental Factor(s)
Experimental Factor(s)	<ul style="list-style-type: none">The experimental factor is the blend.
Factor Levels	
Treatments	
Experimental Unit	
Observational Unit	
Unit Structure	
Treatment Allocation	
Replication	

 Click each tab to learn more. Then, click Next to continue.

The experimental factor is the blend.

Tab 1.3: Factor Levels

 **Answers** 5 of 33

Solutions

- Did you answer correctly?

Response	Factor Levels
Experimental Factor(s)	<ul style="list-style-type: none">A, B, C, D, and E are the five different blends.
Factor Levels	
Treatments	
Experimental Unit	
Observational Unit	
Unit Structure	
Treatment Allocation	
Replication	

 Click each tab to learn more. Then, click Next to continue.

The factor levels are A, B, C, D, E, the five different blends.

Tab 1.4: Treatments

 **Answers** 5 of 33

Solutions

- Did you answer correctly?

Response	Treatments
Experimental Factor(s)	• The treatments are A, B, C, D and E.
Factor Levels	
Treatments	
Experimental Unit	
Observational Unit	
Unit Structure	
Treatment Allocation	
Replication	

 Click each tab to learn more. Then, click Next to continue.

The treatments are the same as the factor levels for a single factor experiment.

Tab 1.5: Experimental Unit

 **Answers** 5 of 33

Solutions

- Did you answer correctly?

Response	Experimental Unit
Experimental Factor(s)	• An experimental unit is a process run.
Factor Levels	
Treatments	
Experimental Unit	
Observational Unit	
Unit Structure	
Treatment Allocation	
Replication	

 Click each tab to learn more. Then, click Next to continue.

An experimental unit is a process run.

Tab 1.6: Observational Unit


Answers
5 of 33

Solutions

- Did you answer correctly?

Response Experimental Factor(s) Factor Levels Treatments Experimental Unit Observational Unit Unit Structure Treatment Allocation Replication	<p>Observational Unit</p> <ul style="list-style-type: none"> An observational unit is a process run.
--	--

 Click each tab to learn more. Then, click Next to continue.

An observational unit is the same as an experimental unit, a process run.

Tab 1.7: Unit Structure


Answers
5 of 33

Solutions

- Did you answer correctly?

Response Experimental Factor(s) Factor Levels Treatments Experimental Unit Observational Unit Unit Structure Treatment Allocation Replication	<p>Unit Structure</p> <ul style="list-style-type: none"> The unit structure is three blocks of five runs. <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px dashed #D3D3D3; padding: 5px; margin-right: 10px;"> <ul style="list-style-type: none"> The process runs are nested in blocks. Nesting entails performing different runs in different blocks. Each block contains five process runs. </div> <table border="1" style="border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 2px;">Block 1</th> <th style="padding: 2px;">Block 2</th> <th style="padding: 2px;">Block 3</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px;">Run 1</td> <td style="padding: 2px;">Run 6</td> <td style="padding: 2px;">Run 11</td> </tr> <tr> <td style="padding: 2px;">Run 2</td> <td style="padding: 2px;">Run 7</td> <td style="padding: 2px;">Run 12</td> </tr> <tr> <td style="padding: 2px;">Run 3</td> <td style="padding: 2px;">Run 8</td> <td style="padding: 2px;">Run 13</td> </tr> <tr> <td style="padding: 2px;">Run 4</td> <td style="padding: 2px;">Run 9</td> <td style="padding: 2px;">Run 14</td> </tr> <tr> <td style="padding: 2px;">Run 5</td> <td style="padding: 2px;">Run 10</td> <td style="padding: 2px;">Run 15</td> </tr> </tbody> </table> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="border: 1px solid #D3D3D3; padding: 5px; text-align: center;"> Blocks  </div> <div style="border: 1px solid #D3D3D3; padding: 5px; text-align: center;"> Units  </div> </div> </div>	Block 1	Block 2	Block 3	Run 1	Run 6	Run 11	Run 2	Run 7	Run 12	Run 3	Run 8	Run 13	Run 4	Run 9	Run 14	Run 5	Run 10	Run 15
Block 1	Block 2	Block 3																	
Run 1	Run 6	Run 11																	
Run 2	Run 7	Run 12																	
Run 3	Run 8	Run 13																	
Run 4	Run 9	Run 14																	
Run 5	Run 10	Run 15																	

 Click each tab to learn more. Then, click Next to continue.

The unit structure is 3 blocks with 5 runs each, so we have a nesting structure where runs are nested within blocks. Nesting entails that we perform different runs in different blocks.

The unit structure is shown in this slide. We can see that the process runs are nested in blocks where each block contains 5 process runs. We use the symbol shown in this slide to denote that the experimental units are nested in blocks.

Tab 1.8: Treatment allocation

 **Answers** 5 of 33

Solutions

- Did you answer correctly?

Response	Treatment Allocation
Experimental Factor(s)	• The run order of blends within blocks is randomised for treatment allocation.
Factor Levels	
Treatments	
Experimental Unit	
Observational Unit	
Unit Structure	
Treatment Allocation	
Replication	

 Click each tab to learn more. Then, click Next to continue.

For treatment allocation, the run order of blends within blocks is randomised.

Tab 1.9: Replication

 **Answers** 5 of 33

Solutions

- Did you answer correctly?

Response	Replication
Experimental Factor(s)	• The number of replications is three.
Factor Levels	
Treatments	
Experimental Unit	
Observational Unit	
Unit Structure	
Treatment Allocation	
Replication	

 Click each tab to learn more. Then, click Next to continue.

The number of replications is 3.

Slide 6: Randomisation Procedure

Randomisation Procedure

6 of 33

- The simplest randomisation approach is to:
 - Assign five treatments randomly to the first block of five runs and repeat the process for the other two blocks
- Alternatively, randomisation may be done in a single operation.

 Results of Randomisation Procedure

01	Enter numbers 1-15 in Column A of a spreadsheet, headed Run.
02	Enter numbers 1, 2, 3 five times each in Column B, headed Block.
03	Enter letters A-E three times in Column C, headed Blend.
04	Generate 15 random numbers in Column D, headed Random.
05	Sort Blend by Block and by Random. (Exclude Run). Allocate Blends as sorted to Runs.

 Click the tab to learn more. Then, click Next to continue.

The simplest approach to perform randomization is to assign the five treatments randomly to the first block of five runs and then repeating this for two further blocks. Alternatively, it may be done in a single operation, as follows:

- Step 1: Enter numbers 1 to 15 in Column A of a spreadsheet, headed Run,
- Step 2: Enter numbers 1, 2, 3 five times each in Column B, headed Block,
- Step 3: Enter letters A-E three times in Column C, headed Blend,
- Step 4: Generate 15 random numbers into Column D, headed Random, and
- Step 5: Sort Blend by Block, then by Random, (exclude Run), allocate Blends as sorted to Runs.

Click the tab to learn more. Then click next to continue.

Tab 1: Results of Randomisation Procedure

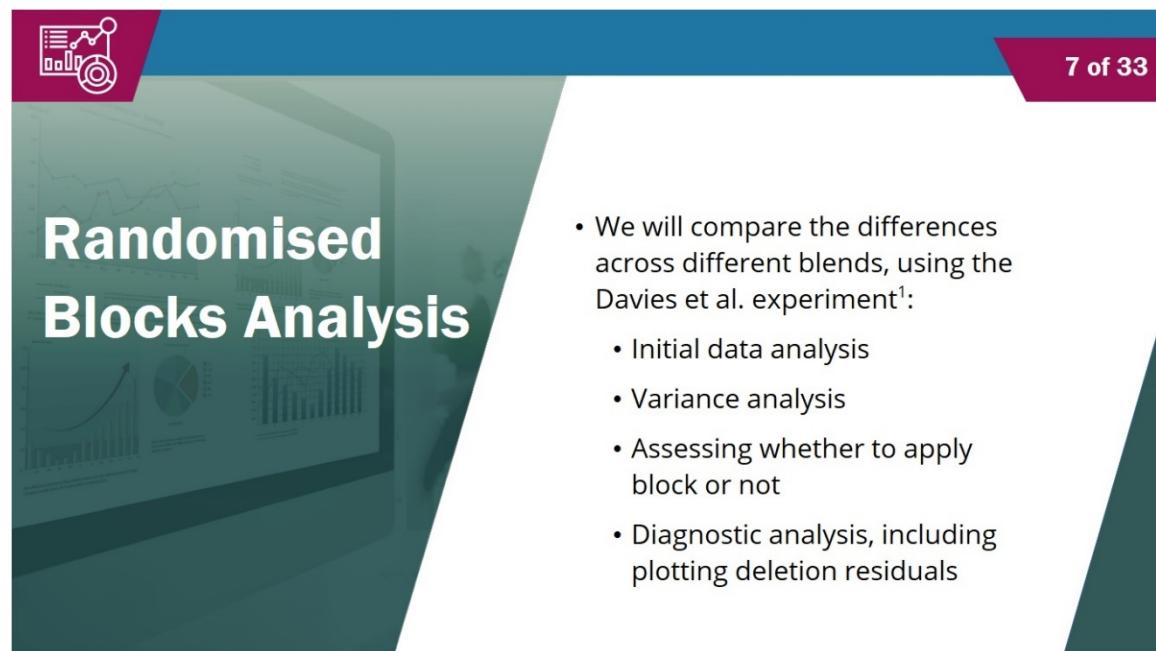
Results of Randomisation Procedure

■

Block	Run	Blend	Loss, per cent
I	1	B	18.2
	2	A	16.9
	3	C	17.0
	4	E	18.3
	5	D	15.1
II	6	A	16.5
	7	E	18.3
	8	B	19.2
	9	C	18.1
	10	D	16.0
III	11	B	17.1
	12	D	17.8
	13	C	17.3
	14	E	19.8
	15	A	17.5

The data resulting from this experiment are shown in the Table. In particular, the percentage loss for each blend in each block is shown.

Slide 7: Randomised Blocks Analysis



7 of 33

Randomised Blocks Analysis

- We will compare the differences across different blends, using the Davies et al. experiment¹:
 - Initial data analysis
 - Variance analysis
 - Assessing whether to apply block or not
 - Diagnostic analysis, including plotting deletion residuals

A randomized blocks analysis allows us to compare the differences across different blends.

We will go through the following steps for a randomized blocks analysis using the same Davies et al. experiment.

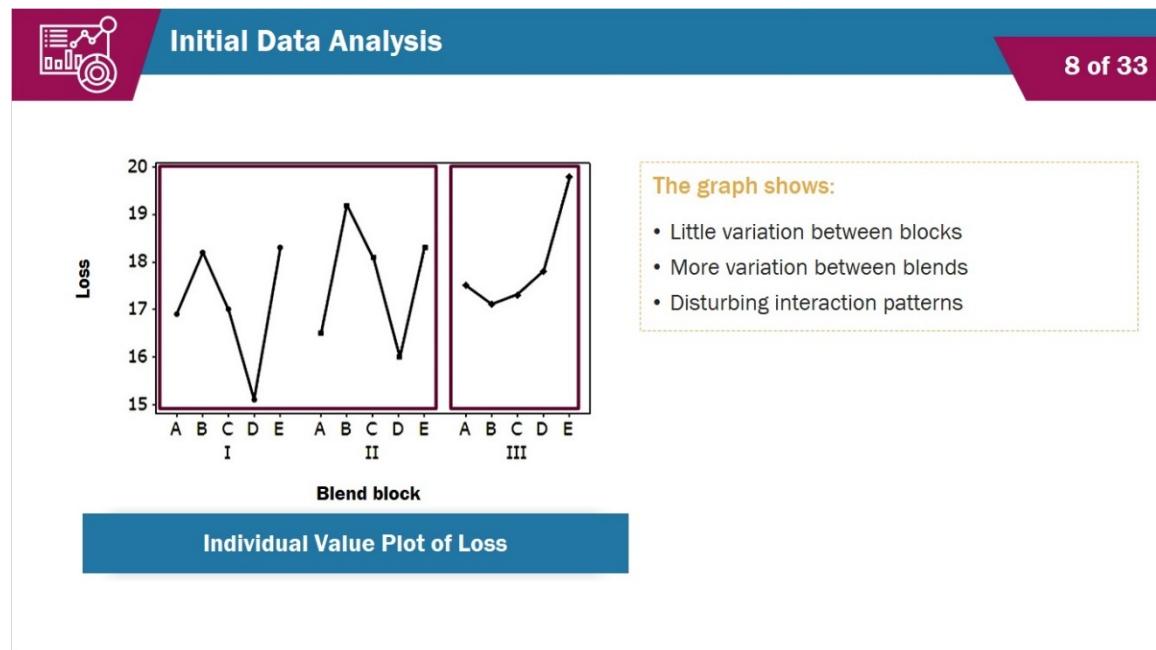
Step 1: Initial data analysis

Step 2: Analysis of variance

Step 3: We want to assess whether we should apply blocking or not and

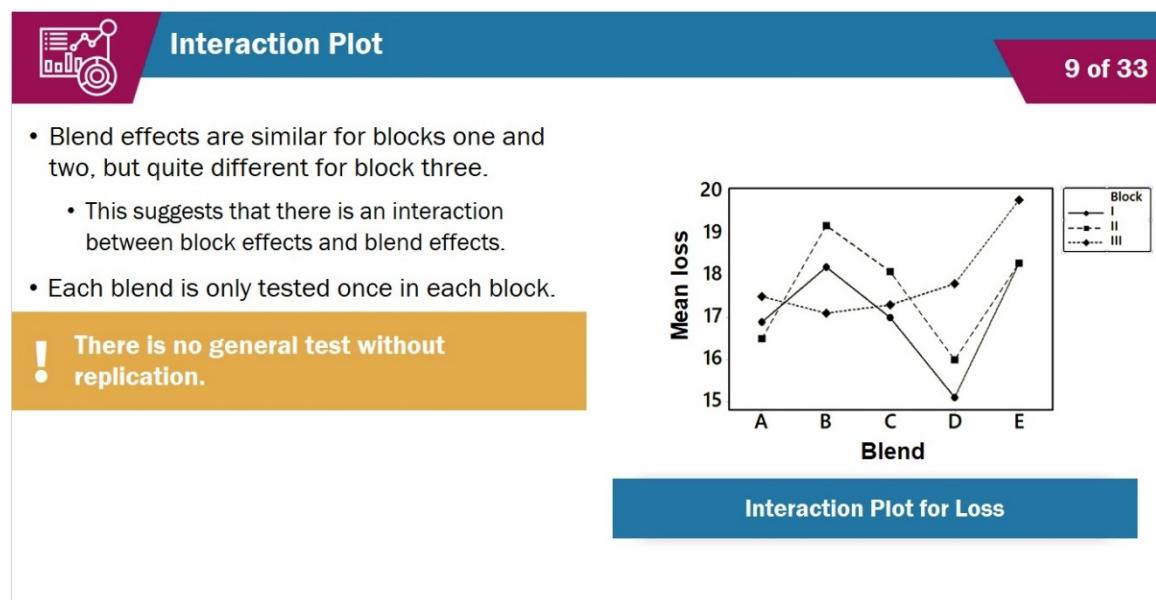
Step 4: diagnostic analysis including plotting deletion residuals

Slide 8: Initial Data Analysis



The figure shows an Individual Values plot of Loss. This shows relatively little variation between blocks and considerably more variation between blends. It also shows a disturbing difference of blend variation pattern between the blocks, with Blocks I and II showing strikingly similar patterns and Block III being strikingly different.

Slide 9: Interaction Plot



The interaction plot shows that the blend effects are similar for Blocks 1 and 2 since the two curves corresponding to Block 1 and 2 are almost parallel. However, the blend effects for Block 3 are quite different compared to Block 1 and 2. Thus, there is evidence suggesting that there is an interaction between block effects and blend effects, that is, the blend effects are not the same across all blocks. However, each blend is only tested once in each block, there is no general test for such interaction without replication.

Slide 10: Formal Analysis: Analysis of Variance



Formal Analysis: Analysis of Variance

- An analysis of variance is required to assess the statistical significance of differences between blends.

Analysis of Variance: Loss Vs Block, Blend					
Source	DF	SS	MS	F	P
Block	2	1.648	0.824	0.94	
Blend	4	11.556	2.889	3.31	0.071
Error	8	6.992	0.874		
Total	14	20.196			

Selected Critical Values for the F Distribution

F(Blends) = 3.3	F-statistic
F _{4,8,0.10} = 2.8	Critical Values
F _{4,8,0.05} = 3.8	
p = 0.07	P-value

F(Blends) is "almost statistically significant".

- The original case study concluded that there was some evidence of blend-to-blend variation.

To assess the statistical significance of differences between blends, an analysis of variance is required. The form of the analysis of variance table is similar to that for the single factor experiment, but with an additional row to represent the variation between blocks. Its interpretation with regard to the experimental factor is also similar.

In the table, the Total sum of squares (SS) measures the total variation in the 15 Loss measurements and has $14 = 15 - 1$ degrees of freedom (DF), the Blocks sum of squares measures the variation between the means of the 3 blocks and has $2 = 3 - 1$ degrees of freedom, the Blends sum of squares measures the variation between the means of the 5 blends and has $4 = 5 - 1$ degrees of freedom and the Error sum of squares measures the variation ascribed to chance. The error degrees of freedom may be calculated as $(3 - 1) \times (5 - 1) = 8$. The justification for this is tedious, however, it may be calculated indirectly from the fact that the Total degrees of freedom is the sum of the three component degrees of freedom.

As an exercise, you should verify the numbers in the table of interactions.

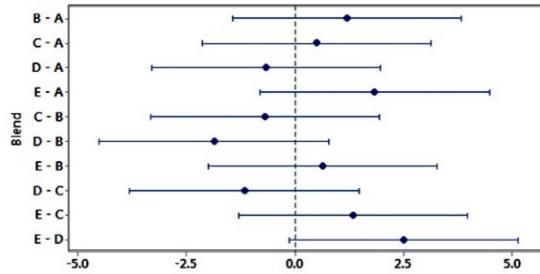
The primary interest is in variation between blend means. From the table of selected critical values for the F distribution, the 10% critical value for F with 4 and 8 degrees of freedom is 2.8 and the 5% value is 3.8. The observed value of 3.3 is between the two. Equivalently, the p-value of 0.071 is between 0.05 and 0.1. According to one convention on the interpretation of significance levels, such a value of F is described as "almost

significant". The conclusion of the authors of the original case study was that "there is some evidence of blend-to-blend variation".

Slide 11: Multiple Comparisons


Multiple Comparisons
11 of 33

- The F-test indicates that variation between blends does not reach the 5% statistical significance level.
 - Thus, confidence intervals for differences between blend means are not calculated.
- The Tukey simultaneous confidence intervals for differences between pairs of means indicate:
- Blends D and E pairwise differences are "almost significant".



Tukey Simultaneous 95% CIs Differences of Means for Loss

Given that the F-test indicates that variation between blends is not statistically significant at the customary 5% level of statistical significance, there is little point in calculating confidence intervals for differences between blend means. For the record, the Tukey simultaneous confidence intervals for differences between pairs of means are shown in the Figure. All of the intervals cover 0, indicating that none of the pairwise differences are statistically significant at the 5% level although that between the Blend D and Blend E means is very close. It may be suggested that it is the latter that makes the F-ratio "almost significant".

Slide 12: Testing Variation Between Block Means



Testing Variations Between Block Means

12 of 33

- In this case study, the mean square for blocks is less than the mean square for chance variation.
 - Blocking appears neither necessary nor effective.

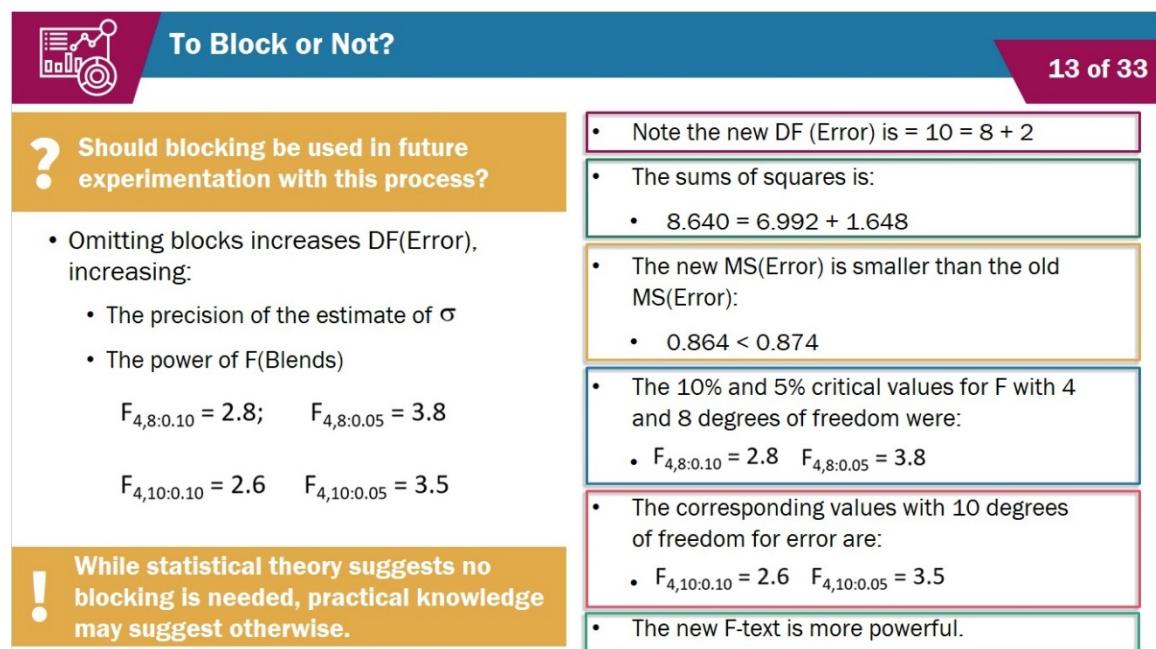
Full Analysis of Testing Variation Between Block Means					
Source	DF	SS	MS	F	P
Block	2	1.648	0.824	0.94	
Blend	4	11.556	2.889	3.31	0.071
Error	8	6.992	0.874		
Total	14	20.196			

Difference Between Blocks Consistent with Chance Variation					
Source	DF	SS	MS	F	P
Blend	4	11.556	2.889	3.34	0.055
Error	10	8.640	0.864		
Total	14	20.196			

Assessing the extent of block-to-block variation is usually regarded as of secondary interest. Here, the F-ratio is less than 1, corresponding to the mean square for blocks being less than the mean square for chance variation. Thus, there appears to be less variation between blocks than would be expected if differences between blocks were consistent with the pure chance variation inherent in the process. Thus, blocking appears to have been neither necessary nor effective in this case.

Further insight may be gained by comparing the full analysis of variance table as shown on the slide with the reduced table that results if blocks are ignored in the analysis, tantamount to a one-way analysis of variance. Note that F(Blends) and the mean squared error are almost unchanged when blocks are deleted from the analysis, supporting the conclusion that blocking was not effective in this case.

Slide 13: To Block or Not?



To Block or Not?

13 of 33

? Should blocking be used in future experimentation with this process?

- Omitting blocks increases DF(Error), increasing:
 - The precision of the estimate of σ
 - The power of F(Blends)

$$F_{4,8:0.10} = 2.8; \quad F_{4,8:0.05} = 3.8$$

$$F_{4,10:0.10} = 2.6 \quad F_{4,10:0.05} = 3.5$$

! While statistical theory suggests no blocking is needed, practical knowledge may suggest otherwise.

- Note the new DF (Error) is $= 10 = 8 + 2$
- The sums of squares is:
 - $8.640 = 6.992 + 1.648$
- The new MS(Error) is smaller than the old MS(Error):
 - $0.864 < 0.874$
- The 10% and 5% critical values for F with 4 and 8 degrees of freedom were:
 - $F_{4,8:0.10} = 2.8 \quad F_{4,8:0.05} = 3.8$
- The corresponding values with 10 degrees of freedom for error are:
 - $F_{4,10:0.10} = 2.6 \quad F_{4,10:0.05} = 3.5$
- The new F-test is more powerful.

This raises the question of whether blocking should be used in future experimentation with this process. Referring to the discussion in the previous session on the comparative advantages of blocking and not blocking, note that the degrees of freedom for chance variation (error) are reduced by the degrees of freedom for variation between blocks. If variation between blocks amounts to no more than chance variation, then it would be better not to block, thereby increasing the degrees of freedom for chance variation, thereby increasing the precision with which the extent of chance variation (\square) is estimated, thereby increasing the power of the F-test for blends.

This can be seen by comparing the Analysis of Variance table with the original table presented previously.

Note that the new DF (Error) is the sum of the old DF (Error) and DF (Blocks)

$$10 = 8 + 2,$$

The sums of squares follow the same relation,

$$8.640 = 6.992 + 1.648$$

while the new MS(Error), a weighted average of the old MS(Error) and the smaller old MS(Blocks), is smaller than the old MS(Error),

$$0.864 < 0.874.$$

It is also instructive to compare the critical values for F corresponding to the two analyses. Recall that the 10% and 5% critical values for F with 4 and 8 degrees of freedom were

$$F_{4,8:0.10} = 2.8$$

and

$$F_{4,8:0.05} = 3.8.$$

The corresponding values with 10 degrees of freedom for error are

$$F_{4,10:0.10} = 2.6$$

and

$$F_{4,10:0.05} = 3.5.$$

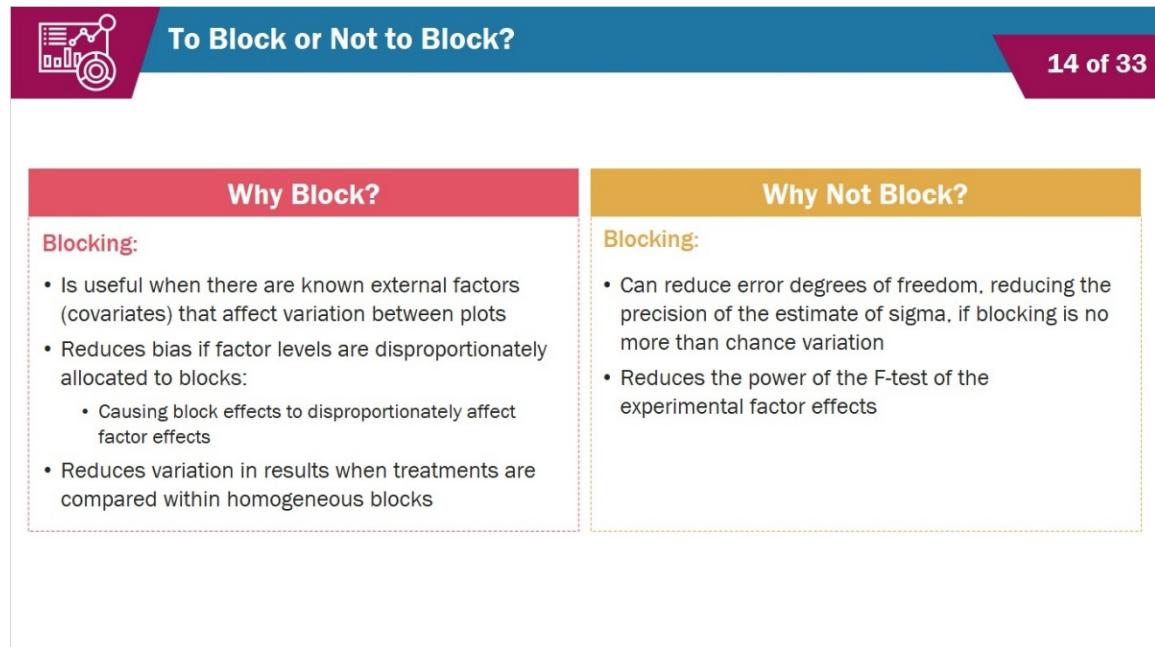
Because the new critical values are smaller than the old, it is easier for the new F-ratio to exceed the critical values. Thus, the new F-test, taking advantage of the additional 2 degrees of freedom for error, is more powerful.

While statistical theory suggests no blocking, practical knowledge may suggest otherwise. We use the quote from Davies et al (1956):

"Although the apparent variation among the blocks is not confirmed (i.e. it might well be ascribed to experimental error), future experiments should still be carried out in the same way.

"There is no clear evidence of a trend in this set of trials, but it might well appear in another set, and no complication in experimental arrangement is involved".

Slide 14: To Block or Not to Block?



To Block or Not to Block?

14 of 33

Why Block?	Why Not Block?
Blocking: <ul style="list-style-type: none"> Is useful when there are known external factors (covariates) that affect variation between plots Reduces bias if factor levels are disproportionately allocated to blocks: <ul style="list-style-type: none"> Causing block effects to disproportionately affect factor effects Reduces variation in results when treatments are compared within homogeneous blocks 	Blocking: <ul style="list-style-type: none"> Can reduce error degrees of freedom, reducing the precision of the estimate of sigma, if blocking is no more than chance variation Reduces the power of the F-test of the experimental factor effects

Why block?

Blocking may be advantageous when there are known external factors (covariates) that affect variation between plots.

Blocking reduces bias in factor effect estimates arising from the unbalanced assignment of treatments to blocks, resulting in "good" blocks and "bad" blocks having unbalanced

influence on factor effect estimates, that is, bias due to block effects disproportionately affecting estimates arising from treatments being disproportionately assigned to blocks.

Blocking reduces residual variation by reducing variation in within-block results when treatments are compared within homogeneous blocks.

Why not block?

If variation between blocks is no more than chance variation, then blocking reduces error degrees of freedom, and so reduces the precision of the estimate of sigma.

Unnecessary blocking reduces the power of the F-test of the experimental factor effects.

Slide 15: Model for Analysis: Yield Loss



Model for Analysis: Yield Loss

15 of 33

- The statistical model assumes that yield loss includes a contribution:
 - From each blend
 - From each block
 - Due to chance variation

$$Y = \mu + \alpha + \beta + \varepsilon$$

The overall mean

The blend effect, above or below the mean, depending on which blend is used

Chance variation

The block effect, above or below the mean, depending on which block is involved



Estimating the Model

☞
Click the tab to learn more. Then, click Next to continue.

Now we are going to look at the statistical model used for the analysis of yield loss. The model assumes that yield loss includes a contribution from each blend, plus a contribution from each block, plus a contribution due to chance variation.

Mathematically, the model is written as $Y = \mu + \alpha + \beta + \varepsilon$,

Where μ is the overall mean

α is the blend effect, above or below the mean, depending on which blend is used,

β is the block effect, above or below the mean, depending on which block is involved,

ε represents chance variation.

We now demonstrate the estimation of the model.

Click the tab to learn more. When you are ready, click next to continue.

Tab 1: Estimating the Model

Estimating the Model

Calculating Blend Means, Block Means and Blend Effects



- Calculate:
 - The **blend means** for the five blends by taking the row means
 - The **block means** for the three blocks by taking column means
 - The **blend effects** by subtracting the overall mean from the blend means

		Blocks			<i>Blend Means</i>	Blend Effects
		I	II	III		
Blends	A	16.9	16.5	17.5	17.0	17.0–17.5 = -0.6
	B	18.2	19.2	17.1	18.2	18.2–17.5 = +0.6
	C	17.0	18.1	17.3	17.5	17.5–17.5 = -0.1
	D	15.1	16.0	17.8	16.3	16.3–17.5 = -1.2
	E	18.3	18.3	19.8	18.8	18.8–17.5 = +1.3
		Block Means	17.1	17.6	17.9	17.5
		Block Effects	17.1–17.5 = -0.4	17.6–17.5 = +0.1	17.9–17.5 = +0.4	

The blend means for the five blends are calculated by taking the row means. For example, the blend mean for Blend A is calculated as $(16.9 + 16.5 + 17.5) / 3 = 17.0$ (ignore rounding). The block means for the three blocks are calculated by taking column means. For example, the mean for Block A is $(16.9 + 18.2 + 17.0 + 15.1 + 18.3)/5 = 17.1$. Finally, the overall mean is calculated by taking the average of all 15 observations, which is 17.5.

The blend effects are computed by subtracting the overall mean from the blend means. For example, the blend effect for Blend A is $17.0 - 17.5 = -0.6$ (ignore the rounding). The block effects are computed by subtracting the overall mean from the block means. For example, the block effect for Block A is $17.1 - 17.5 = -0.4$.

Slide 16: Decomposing Total Variation



Decomposing Total Variation

16 of 33

- The genesis of the decomposition is the elementary decomposition

$$y_{ij} - \bar{y} = (\bar{y}_{i\bullet} - \bar{y}) + (\bar{y}_{\bullet j} - \bar{y}) + [(y_{ij} - \bar{y}) - (\bar{y}_{i\bullet} - \bar{y}) - (\bar{y}_{\bullet j} - \bar{y})]$$

or

Yield loss from run
using Blend i in
Block j

$$y_{ij} - \bar{y} = (\bar{y}_{i\bullet} - \bar{y}) + (\bar{y}_{\bullet j} - \bar{y}) + (y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y})$$

- The equations represent the decomposition of this "total" deviation into the remaining blend, block and residual deviations, after subtracting the deviations corresponding to blend and block from the total.
- As with the one-way analysis of variance, this decomposition is paralleled by the fundamental analysis of variance decomposition.

$$\sum_{\text{all data}} (y_{ij} - \bar{y})^2 = \sum_{\text{all data}} (\bar{y}_{i\bullet} - \bar{y})^2 + \sum_{\text{all data}} (\bar{y}_{\bullet j} - \bar{y})^2 + \sum_{\text{all data}} (y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y})^2$$

$$\text{SSTO} = \text{SS(Blocks)} + \text{SS(Blends)} + \text{SS>Error)$$

The genesis of the decomposition is the elementary decomposition

$$y_{ij} - \bar{y} = (\bar{y}_{i\bullet} - \bar{y}) + (\bar{y}_{\bullet j} - \bar{y}) + [(y_{ij} - \bar{y}) - (\bar{y}_{i\bullet} - \bar{y}) - (\bar{y}_{\bullet j} - \bar{y})]$$

or

$$y_{ij} - \bar{y} = (\bar{y}_{i\bullet} - \bar{y}) + (\bar{y}_{\bullet j} - \bar{y}) + (y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y}),$$

where, y_{ij} represents the yield loss resulting from the run using Blend i in Block j, $y_{ij} - \bar{y}$ is the "total" deviation of that yield loss from the overall average and the equations above represent the decomposition of this "total" deviation into a "blend" deviation, a "block" deviation and the "residual" deviation remaining after subtracting the deviations corresponding to blend and block from the total.

The blend and block deviations are regarded as explaining part of the total deviation and the residual deviation is the unexplained component, regarded as due to chance variation or random "error".

As with the one-way analysis of variance, this decomposition is paralleled by the fundamental Analysis of Variance decomposition

$$\sum_{\text{all data}} (y_{ij} - \bar{y})^2 = \sum_{\text{all data}} (\bar{y}_{i\bullet} - \bar{y})^2 + \sum_{\text{all data}} (\bar{y}_{\bullet j} - \bar{y})^2 + \sum_{\text{all data}} (y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y})^2$$

or, in friendlier notation,

$$\text{SSTO} = \text{SS(Blocks)} + \text{SS(Blends)} + \text{SS>Error)$$

Slide 17: Expected Mean Squares



Expected Mean Squares

17 of 33

- The expected mean squares are calculated in a similar way to one-way analysis of variance.

$$\bullet \text{ EMS(Blends)} = \sigma^2 + J \frac{\sum_i (\mu_{i\bullet} - \mu)^2}{I-1}$$

$$\bullet \text{ EMS(Blocks)} = \sigma^2 + I \frac{\sum_j (\mu_{\bullet j} - \mu)^2}{J-1}$$

$$\bullet \text{ EMS(Error)} = \sigma^2$$

- If there are no differences between blend means, then:

$$\bullet \text{ EMS(Blends)} = \sigma^2 = \text{EMS(Error)}$$

$\bullet F(\text{Blends}) = \frac{\text{MS(Blends)}}{\text{MS(Error)}}$ tests the equality of blend means.

$\bullet F(\text{Blocks}) = \frac{\text{MS(Blocks)}}{\text{MS(Error)}}$ assesses the effectiveness of blocking.

Formulas for the expected (or theoretical) mean squares (EMS) analogous to those arising in the one-way analysis of variance also apply here. Denoting the expected value of y_{ij} by \bar{y}_{ij} and using the usual dot notation to denote blend and block means, it may be shown that

$$\text{EMS(Blends)} = \sigma^2 + J \frac{\sum_i (\mu_{i\bullet} - \mu)^2}{I-1},$$

$$\text{EMS(Blocks)} = \sigma^2 + I \frac{\sum_j (\mu_{\bullet j} - \mu)^2}{J-1},$$

$$\text{EMS(Error)} = \sigma^2.$$

Thus, if there are no differences between blend means,

$$\mu_{1\bullet} = \mu_{2\bullet} = \dots = \mu_{I\bullet} = \mu,$$

then

$$\text{EMS(Blends)} = \sigma^2 = \text{EMS(Error)}.$$

Applying an argument analogous to justify the F test used in the one-way analysis of variance, this suggests using the F-ratio

$$F(\text{Blends}) = \frac{\text{MS}(\text{Blends})}{\text{MS}(\text{Error})}$$

to assess the statistical significance of the observed differences between blend means.

Applying a similar argument in the case of blocks suggests using the F-ratio

$$F(\text{Blocks}) = \frac{\text{MS}(\text{Blocks})}{\text{MS}(\text{Error})}$$

Slide 18: Deletion Residuals

 **Deletion Residuals** 18 of 33

Residuals are:

The difference between the observed and fitted value by the statistical model

- To calculate the standardised residual:
 - Divide by the standard error of the residual, using the usual estimate of σ .



Calculating Deletion Residuals

Deletion residuals are:

The residual based on data with suspect case deleted

- To obtain the standardised deletion residual:
 - Divide by the standard error of the residual, using an estimate of σ with suspect case deleted.

☞ Click tab to learn more. Then, click Next to continue.

We now turn to diagnostic analysis and will introduce the concept of deletion residuals. Recall that residual is defined as the difference between the observed value and the fitted value by the statistical model. Recall that for standardized residual, we divide the residual by the standard error of the residual, using the usual estimate of σ .

On the other hand, the deletion residual is the residual based on data with the suspect case deleted. The standardized deletion residual is obtained by dividing the deletion residual by the standard error of the deletion residual, using an estimate of σ with suspect case deleted.

Let's now look at the details of the calculation of deletion residuals.

Click the tab to learn more. Then click next to continue.

Tab 1: Calculating Deletion Residuals

Calculating Deletion Residuals (1/3)

Calculation Steps



- For each potentially exceptional case:
 1. Delete the case
 2. Fit the ANOVA model using the rest cases to get the deletion fitted model
 3. Use this model to calculate a deletion fitted value
 4. Calculate the deletion residual as:
 - Observed value minus deletion fitted value
 5. Use the deletion estimate of σ to calculate the standard error to standardise



For each potentially exceptional case, we first delete the case and fit the ANOV model using the rest cases to obtain the deletion fitted model. We use this model to calculate a deletion fitted value. Now the deletion residual is calculated as observed value minus deletion fitted value. We then use the deletion estimate of σ to calculate the standard error used to obtain the standardize deletion residual.

Tab 1.1: Calculating Deletion Residuals: Using Deletion Residuals to Accentuate Exceptional Cases

Calculating Deletion Residuals (2/3)

Using Deletion Residuals to Accentuate Exceptional Cases



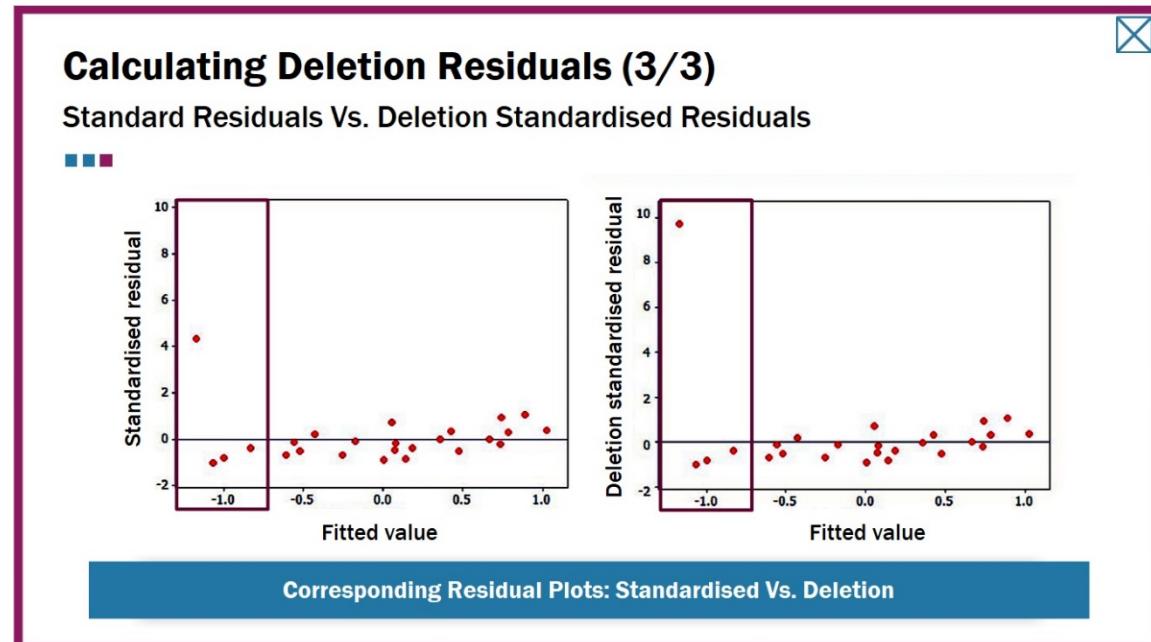
- Given an exceptional case:
 - Deletion residual > residual using all the data
 - Deletion estimate of σ < using all the data
- Deletion standardised residual >> than standardised residual using all the data

! Using deletion residuals accentuates exceptional cases.

Given an exceptional case, the deletion residual will tend to be larger than the residual obtained using all the data. On the other hand, the deletion estimate of σ is smaller than the σ estimated using all the data. Therefore, deletion standard residual tends

to be much larger than standardized residual using all the data for this exceptional case. Thus, using deletion residuals accentuates exceptional cases.

Tab 1.2: Calculating Deletion Residuals: Standardised Residuals Vs. Deletion Standardised Residuals



This slide shows the standardised residuals and deletion standardised residuals. There is one exceptional case, and it can be seen that the deletion standardised residual for this exceptional case is much larger than the standardized residual obtained using all cases.

Slide 19: An Example of a 3 X 3 Factorial Design

 **An Example of a 3 X 3 Factorial Design:** 19 of 33

- Iron-deficiency anaemia is the most common form of malnutrition in developing countries.
 - Cooking food in aluminium pots, instead of traditional iron pots, may be increasing the problem.
- We will now consider a study² to assess the effect on iron content of cooked food of:
 - Cooking pot type

A Aluminium	C Clay	I Iron
----------------	-----------	-----------
 - Food type

M Meat	L Legumes	V Vegetables
-----------	--------------	-----------------
- This is a two-factor 3 x 3 experiment, where each factor has 3 levels.

“Ethiopian children fed food from iron pots had lower rates of anaemia and better growth than children whose food was cooked in aluminium pots.

Provision of iron cooking pots for households in less-developed countries may be a useful method to prevent iron-deficiency anaemia.”¹

Iron-deficiency anaemia is the most common form of malnutrition in developing countries. It has been suggested that the problem has been exacerbated by a change

from cooking food in traditional iron pots to cooking in aluminium pots. One recent study concluded as follows:

“Ethiopian children fed food from iron pots had lower rates of anaemia and better growth than children whose food was cooked in aluminium pots. Provision of iron cooking pots for households in less-developed countries may be a useful method to prevent iron-deficiency anaemia”.

For the remainder of this session we consider this 1999 study to assess the effect on iron content of cooked food of cooking pot type and food type.

The three pot types considered are Aluminium (A), Clay (C) and Iron (I) and three food types are Meat (M), Legumes (L) and Vegetables (V). This was a two-factor 3×3 experiment where each factor has 3 levels.

Slide 20: Study Design and Results


Study Design and Results
20 of 33

- Four samples of each food type, (M, L, and V) were cooked in each pot type (A, C and I).
- The iron content in each sample was measured in milligrams of iron per 100 grams of cooked food.
- All combinations of pairs of factor levels (AM, AL, AV, CM, CL, CV, IM, IL, IV) were used.
- This was a fully crossed design.
 - Every possible class in the cross classification is included.
 - The experiment was replicated four times.

Pot type	Food type		
	Meat	Legumes	Vegetables
Aluminium	1.77 2.36 1.96 2.14	2.40 2.17 2.41 2.34	1.03 1.53 1.07 1.30
Clay	2.27 1.28 2.48 2.68	2.41 2.43 2.57 2.48	1.55 0.79 1.68 1.82
Iron	5.27 5.17 4.06 4.22	3.69 3.43 3.84 3.72	2.45 2.99 2.80 2.92

As part of the study, four samples of each of three food types, Meat (M), Legumes (L) and Vegetables (V), were cooked in each of the pot types, Aluminium (A), Clay (C) and Iron (I). The iron content in each sample, in milligrams of iron per 100 grams of cooked food, was measured. In the experiment, as described, all nine treatments, that is, combinations of pairs of factor levels were used. The full list of coded treatments is AM, AL, AV, CM, CL, CV, IM, IL, IV. Such experiments are said to be *fully crossed*, meaning that every possible class in the cross classification of Pot Type and Food Type is included. Each treatment was used 4 times, that is, the experiment was replicated 4 times.

Slide 21: **Knowledge Check**



Knowledge Check

21 of 33

- Can you identify the following?

Response

Experimental Factor(s)

Factor Levels

Treatments

Treatment Structure

Experimental Unit

Unit Structure

Treatment Assignment

Replication



Take time to think about your answers.

For this experiment, identify the following: response, experimental factors, factor levels, treatments, treatment structure, experimental unit, unit structure, treatment assignment, and replication.

Take some time to consider your answers. Then click next to continue.

Slide 22: **Solutions**



Solutions

22 of 33

Answers

- Did you answer correctly?

Response

Experimental Factor(s)

Factor Levels

Treatments

Treatment Structure

Experimental Unit

Unit Structure

Treatment Assignment

Replication



Click each tab to learn more. Then, click Next to continue.

Click each tab to find out if you answered correctly. Then click next to continue.

Tab 1: Response

 Solutions 22 of 33

Answers

- Did you answer correctly?

Response	Response
Experimental Factor(s)	• Iron content (mg per 100g)
Factor Levels	
Treatments	
Treatment Structure	
Experimental Unit	
Unit Structure	
Treatment Assignment	
Replication	

 Click each tab to learn more. Then, click Next to continue.

The response is iron content (mg per 100g).

Tab 2: Experimental Factors

 Solutions 22 of 33

Answers

- Did you answer correctly?

Response	Experimental Factor(s)
Experimental Factor(s)	• Pot type
Factor Levels	• Food type
Treatments	
Treatment Structure	
Experimental Unit	
Unit Structure	
Treatment Assignment	
Replication	

 Click each tab to learn more. Then, click Next to continue.

There are two experimental factors: pot type and food type.

Tab 3: Factor Levels


Solutions
22 of 33

Answers

- Did you answer correctly?

Response Experimental Factor(s) Factor Levels Treatments Treatment Structure Experimental Unit Unit Structure Treatment Assignment Replication	Factor Levels <ul style="list-style-type: none"> A, C, I L, M, V
---	---

 Click each tab to learn more. Then, click Next to continue.

The factor levels for pot type are A, C, I;

The factor levels for food type for L, M, V.

Tab 4: Treatments


Solutions
22 of 33

Answers

- Did you answer correctly?

Response Experimental Factor(s) Factor Levels Treatments Treatment Structure Experimental Unit Unit Structure Treatment Assignment Replication	Treatments <ul style="list-style-type: none"> AM, AL, AV, CM, CL, CV, IM, IL, IV
---	--

 Click each tab to learn more. Then, click Next to continue.

There are 9 treatments, AM, AL, AV, CM, CL, CV, IM, IL, IV, which correspond to all possible treatment level combinations.

Tab 5: Treatment Structure

 **Solutions** 22 of 33

Answers

- Did you answer correctly?

Response	Treatment Structure
Experimental Factor(s)	• Fully crossed
Factor Levels	
Treatments	
Treatment Structure	
Experimental Unit	
Unit Structure	
Treatment Assignment	
Replication	

 Click each tab to learn more. Then, click Next to continue.

The treatment structure is fully crossed.

Tab 6: Experimental Unit

 **Solutions** 22 of 33

Answers

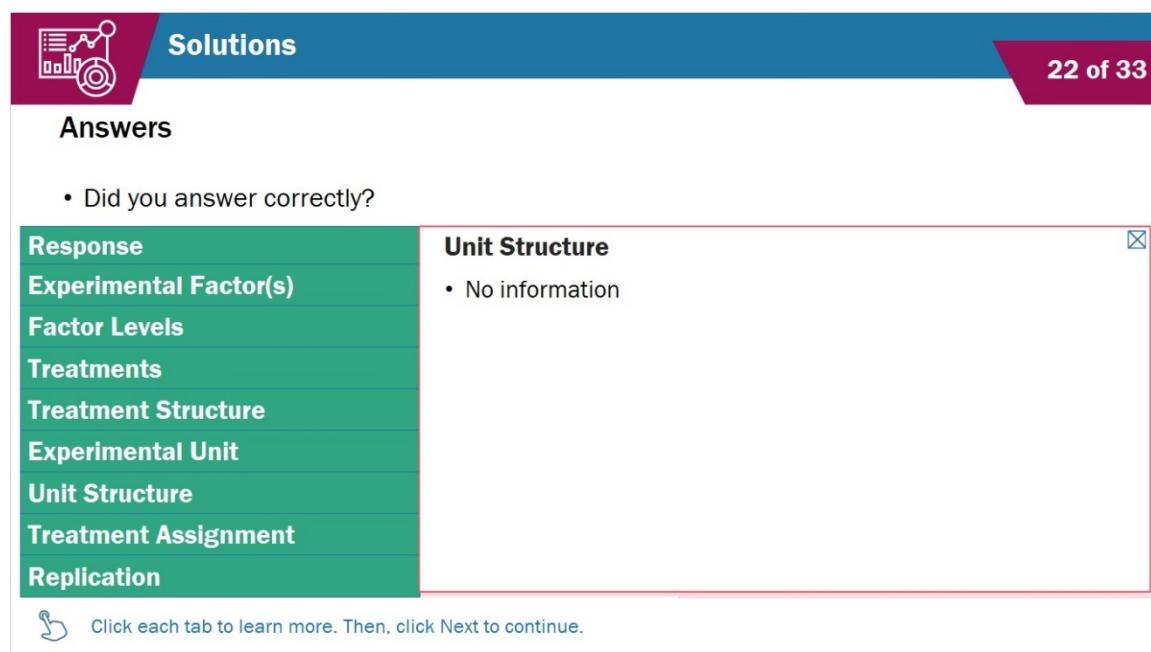
- Did you answer correctly?

Response	Experimental Unit
Experimental Factor(s)	• Cooked meal
Factor Levels	
Treatments	
Treatment Structure	
Experimental Unit	
Unit Structure	
Treatment Assignment	
Replication	

 Click each tab to learn more. Then, click Next to continue.

The experimental unit is a cooked meal.

Tab 7: Unit Structure



Solutions 22 of 33

Answers

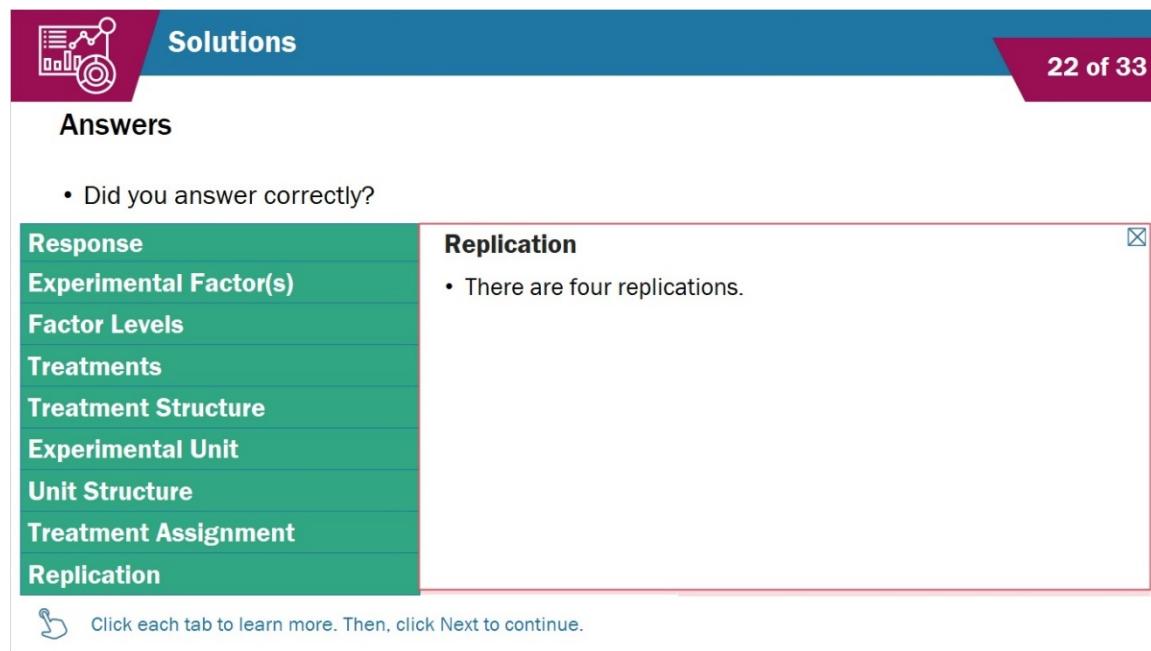
- Did you answer correctly?

Response	Unit Structure
Experimental Factor(s)	• No information
Factor Levels	
Treatments	
Treatment Structure	
Experimental Unit	
Unit Structure	
Treatment Assignment	
Replication	

Click each tab to learn more. Then, click Next to continue.

There is no information given on unit structure and treatment assignment.

Tab 8: Replication



Solutions 22 of 33

Answers

- Did you answer correctly?

Response	Replication
Experimental Factor(s)	• There are four replications.
Factor Levels	
Treatments	
Treatment Structure	
Experimental Unit	
Unit Structure	
Treatment Assignment	
Replication	

Click each tab to learn more. Then, click Next to continue.

The number of replications is 4.

Slide 23: Experiment Design Issues

Experiment Design Issues

23 of 33

- The degree of control in this experiment was not ideal.

Experiment Design Conditions	Implications for Experiment
<ul style="list-style-type: none"> Food was cooked in a laboratory, using a single hotplate at 120 degrees celsius. All nine samples were cooked on each of four successive days. Different spices were used with the different food types. Cooking times were not specified. 	<p>→</p> <ul style="list-style-type: none"> Iron content could be influenced by: <ul style="list-style-type: none"> Different spices Different cooking times Additional factors were not addressed, so we assume: <ul style="list-style-type: none"> A single heat source was used sequentially There were fixed times, fixed temperatures and no spice effect There was random assignment of treatments

The degree of control applied in this experiment was not ideal. Food samples were cooked in a laboratory using a single hotplate set at 120 °C, with all nine treatments being cooked on each of 4 successive days, using standard Ethiopian recipes involving the use of different spices with the different food types. Cooking times were not specified; conceivably, different cooking times were used with the different food types. This leaves open the possibility that iron content was influenced by the different spices associated with the different food types and also by the different cooking times associated with the different food types. Technically speaking, the effects on iron content of using different food types may be confounded with the effects of either or both of using different spices and using different cooking times. These issues were not addressed in the report of this experiment. There is an implicit assumption that these additional factors were not influential.

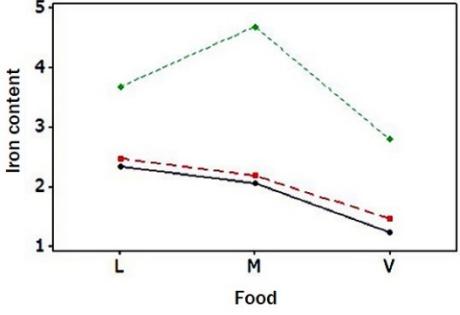
Conceivably, also, there may be unknown time related effects, from time to time within a day and from day to day. The standard approach to controlling such effects is to randomise the order of assignment of treatments to cooking runs, separately on each of the four days, that is, a randomised blocks experiment with days as blocks and cooking runs as experimental units within blocks. As above, there is no report of such randomisation having been implemented and so it is assumed that there were no such time related factors and that the replications were genuine.

Slide 24:

Initial Data Analysis of Experiment

Initial Data Analysis of Experiment
24 of 33

- Initial data analysis produce the following results:



Mean Iron Content Corresponding to the Three Food Types for Each Pot Type

Pot type	Food type		
	Legumes	Meat	Vegetables
Aluminium	2.33	2.06	1.23
Clay	2.47	2.18	1.46
Iron	3.67	4.68	2.79

 Assessing Chance Variation

 Click the tab to learn more. Then, click Next to continue.

The Figure is a plot of pot type profiles showing mean iron content corresponding to the three food types for each pot type. The Table shows the corresponding means.

When aluminium or clay pots are used, the mean iron content is highest for legumes, slightly lower for meat and lowest for vegetables. For each food type, food cooked in clay has marginally higher iron content than food cooked in aluminium. When iron pots are used, all three food types have considerably higher iron content than when either of the other pot types are used but with a considerably bigger difference for meat than for the other food types.

Before drawing definitive conclusions from this informal analysis of the treatment means, the extent of chance variation needs to be assessed.

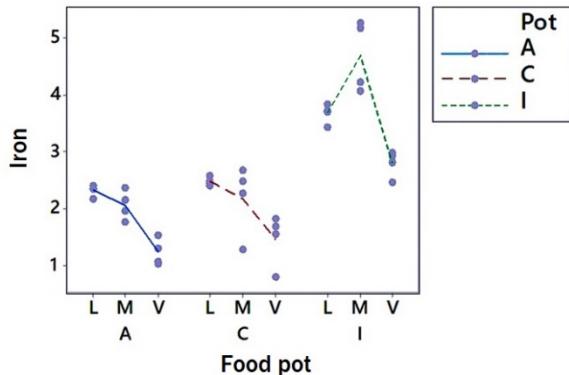
Click the tab to learn more. Then click next to continue.

Tab 1: Assessing Chance Variation

Assessing Chance Variation

Individual Value Plot of Iron

- - The variation between dots within each of the nine treatments suggests:
 - Variation in means outweighs chance variation



Individual Value Plot of Iron

The individual values plot in this slide provides some insight into the extent of chance variation and suggests that the variation in means outweighs chance variation, as reflected in the variation between dots within each of the 9 treatments.

Slide 25: Formal Analysis of Experiment



Formal Analysis of Experiment

25 of 33

- A formal analysis is performed to assess whether the apparent pattern of variation in means outweighs chance variation.

Analysis of Variance Model

Variation in iron content includes:

- A contribution for each food type
- A contribution for each pot type
- A unique contribution for each food type/pot type combination
- A unique contribution due to chance variation

The model minitab syntax = Pot Food Pot*Food

- Pot * Food is the interaction of pot type and food type.

Formal assessment of the suggestion arising from informal analysis that the apparent pattern of variation in means outweighs chance variation may be made using analysis of variance. We now want to validate this claim by performing a formal analysis.

A statistical model such as the following is appropriate.

Variation in iron content includes:

- an overall contribution for each food type
- plus
- an overall contribution for each pot type
- plus
- a unique contribution for each food type / pot type combination
- plus
- a unique contribution for each sample due to chance variation,

The Minitab syntax for the model is given in the bottom of this slide. In particular, the Pot * Food denotes the interaction of pot type and food type.

Slide 26: Mathematical Version of the Statistical Model

 Mathematical Version of the Statistical Model

26 of 33

$Y = \mu + \alpha + \beta + \alpha\beta + \varepsilon$	
μ	The overall mean
α	The food effect, above or below the mean, depending on which food type is used (main effect)
β	The pot effect, above or below the mean, depending on which pot type is involved (main effect)
$\alpha\beta$	The food/pot interaction effect, depending on which food type/pot type combination is used
ε	Chance variation



Detailed Mathematical Notation

👉 Click the tab to learn more. Then, click Next to continue.

In a more mathematical version,

$$Y = \mu + \alpha + \beta + \alpha\beta + \varepsilon$$

where

μ is the overall mean,

α is the average food effect above or below the mean, depending on which food type is used,

β is the average pot effect above or below the mean, depending on which pot type is used,

$\alpha\beta$ (combined to form a single symbol) is the average food/pot interaction effect above or below the mean adjusted for the food affect and pot effect, depending on which food type / pot type combination is used,

and

ε represents chance variation, affecting all experimental units differently and unpredictably.

The food effect, α , and pot effect, β , are referred to as "main effects", to distinguish them from the corresponding "interaction effect", $\alpha\beta$.

Click the tab to view a more detailed mathematical notation for the statistical model then click next to continue.

Tab 1: Detailed Mathematical Notation

Detailed Mathematical Notation

Mathematical Notation of Statistical Model



$$Y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk}$$

Y_{ijk}	The iron content of replicate k of food type i cooked in pot type j
μ	The overall mean
α_i	The main effect of food type i, above or below the mean
β_j	The main effect of pot type j, above or below the mean
$\alpha\beta_{ij}$	The interaction effect of food type i with pot type j
ε_{ijk}	Chance variation



Take time to view the information on this slide.

More detailed mathematical notations of the statistical model are given in this slide. For example, y_{ijk} is the iron content of replicate k of food type i cooked in pot type j, and a_i is the main effect of food type i, above or below the mean.

Take time to view the information on this slide. Then click next to continue.

Slide 27: Decomposing Total Variation

Decomposing Total Variation

27 of 33

- The “total” deviation is decomposed into a food, pot, interaction and residual deviation.

<ul style="list-style-type: none"> Food deviation <ul style="list-style-type: none"> This estimates the main effect of food type i. 	$y_{ijk} - y_{...} = y_{i..} - y_{...}$ $+ y_{.j..} - y_{...}$
<ul style="list-style-type: none"> Pot deviation <ul style="list-style-type: none"> This estimates the main effect of pot type j. 	$+ (y_{ij..} - y_{...}) - (y_{i..} - y_{...}) - (y_{.j..} - y_{...})$ $+ y_{ijk} - y_{ij..}$
<ul style="list-style-type: none"> Interaction deviation <ul style="list-style-type: none"> This estimates the effect of the food type i/pot type j combination. 	$(y_{ij..} - y_{...}) - (y_{i..} - y_{...}) - (y_{.j..} - y_{...})$
<ul style="list-style-type: none"> Residual deviation <ul style="list-style-type: none"> This reflects chance variation, involving the four replicates. 	$= (y_{ij..} - y_{i..} - y_{.j..} + y_{...})$

This model is reflected in the following decomposition of "total" deviation into a "food" deviation, a "pot" deviation, an "interaction" deviation and an "error" or "residual" deviation shown in this slide. The first term on the right, the "food" deviation, is the deviation of the average response using food type i from the overall average and estimates the main effect of food type i. The second term, the "pot" deviation, is the deviation of the average response using pot type j from the overall average and estimates the main effect of pot type j.

The third term, the "interaction" deviation, is the deviation of the average response using food type i with pot type j from the overall average less the "food" deviation and the "pot" deviation. It estimates the interaction effect of food type i and pot type j and represents the unique effect of the food type i / pot type j combination having allowed for the corresponding main effects.

The last term in the decomposition of "total" deviation is the deviation of the response of replicate k using food type i and pot type j from the average of the four replicates using those food and pot types and is assumed to reflect pure chance variation.

To summarise, the first two terms on the right of the decomposition estimate the food and pot *main effects*, respectively. The third term estimates the corresponding *interaction effect*, the combined effect of food type i and pot type j adjusted for the main effects. The last term, involving four replicates each using the same food type, i and the same pot type, j, reflects chance effects.

Slide 28: Estimating the Model

📊
Estimating the Model
28 of 33

Food type					
Pot type		M	L	V	Pot means
	A	2.1	2.3	1.2	1.9
	C	2.2	2.5	1.5	2.0
	I	4.7	3.7	2.8	3.7
Food means	3.0	2.8	1.8	2.5	

Estimates of Pot and Food Main Effects

Pot main effects		Food main effects
1.9 - 2.5 = -0.6		3.0 - 2.5 = +0.5
2.0 - 2.5 = -0.5		2.8 - 2.5 = +0.3
3.7 - 2.5 = +1.2		1.8 - 2.5 = -0.7

💡
Summary and Interaction Tables

👉 Click the tab to learn more. Then, click Next to continue.

The estimates of the Pot main effects and Food main effects are shown in this slide. For each pair of Pot/Food combination, the averages of the observations are shown inside the box. We first compute the Pot means and Food means by taking the row averages and column averages, respectively. The Pot main effects are computed by subtracting the overall mean from the Pot means, and the Food main effects are computed by subtracting the overall mean from the Food means. You should verify the calculations in this slide.

Tab 1: Summary and Interaction Tables

💡
Summary and Interaction Tables
✖

Tables

-

	M	L	V	Pot effects
A	2.1	2.3	1.2	-0.7
C	2.2	2.5	1.5	-0.5
I	4.7	3.7	2.8	1.2
Food effects	0.4	0.3	-0.7	2.5

Interaction Effects		
-0.1	0.2	0.1
-0.2	0.2	0.2
0.6	-0.3	-0.2

💡
Summary of Row and Column Means

⌚ Take time to view the information on this slide and to verify the numbers on the table of interactions.

41

Click the tab to view a summary table with row and column means followed by the table of interactions, calculated using the simplified formula. When you are ready, click next to continue.

Treatment mean – Row mean – Column mean + Overall mean

As an exercise, you should verify the numbers in the table of interactions.

Slide 29: Reporting Interaction



Reporting Interaction

29 of 33

! Focus on patterns of interaction effects when reporting interaction.

Pot type	Food type		
	Legumes	Meat	Vegetables
Aluminium	2.33	2.6	1.23
Change effect	0.14	0.12	0.23
Clay	2.47	2.18	1.46
Change effect	1.20	2.50	1.33
Iron	3.67	4.68	2.79

“

When pot type was changed from aluminium to clay, iron content increased by a relatively small amount, **between 0.12 and 0.23 mg/100g**.

When pot type was changed from clay to iron, the change was much larger, 1.20 and 1.33 mg/100g for legumes and vegetables respectively, and approximately twice that level, 2.5 mg/100g, for meat.

 Take time to view the information on this slide.

While the focus on deviations and effects as discussed above is appropriate for linking with formal analysis of variance, a different approach is needed when reporting interaction. In that case, the focus should be on patterns of interaction effects. This will vary from one study to the next. In this case, a focus on the changes from Aluminium to Clay and from Clay to Iron may be appropriate. The Table shows these changes as calculated from the summary data.

Take time to read how an interaction report might be worded. Then click next to continue.

When pot type was changed from aluminium to clay, iron content increased by a relatively small amount, **between 0.12 and 0.23 mg/100g.**

When pot type was changed from clay to iron, the change was much larger, 1.20 and 1.33 mg/100g for legumes and vegetables respectively, and approximately twice that level, 2.5 mg/100g, for meat.

Slide 30: Estimating Sigma

Estimating Sigma

30 of 33

Treatment	Iron content				S
L/A	2.40	2.17	2.41	2.34	0.01
M/A	1.77	2.36	1.96	2.14	0.06
V/A	1.03	1.53	1.07	1.30	0.05
L/C	2.41	2.43	2.57	2.48	0.01
M/C	2.27	1.28	2.48	2.68	0.39
V/C	1.55	0.79	1.68	1.82	0.21
L/I	3.69	3.43	3.84	3.72	0.03
M/I	5.27	5.17	4.06	4.22	0.39
V/I	2.45	2.99	2.80	2.92	0.06

$\hat{\sigma}^2 = 0.13$
 $\hat{\sigma} = 0.37$

Interaction Change Effects

The table shows calculating the variances of the 9 sets of 4 Iron content measurements, one for each treatment (food type / pot type combination), each of which is subject to pure chance variation. Each such set of 4 provides an estimate of S^2 , based on $4 - 1 = 3$ degrees of freedom. The 9 resulting estimates may be combined into a single improved estimate, based on $9 \times 3 = 27$ degrees of freedom. $\hat{\sigma}^2$ is calculated as the average of the nine S^2 values and $\hat{\sigma}$ is its square root.

Slide 31: Using Minitab to Produce an Analysis of Variance Table

Using Minitab to Produce an Analysis of Variance Table

31 of 33

- The minitab model is:
 - Pot Food Pot*Food

Source	DF	SS	MS	F	P
Pot	2	24.8940	12.4470	92.26	0.000
Food	2	9.2969	4.6484	34.46	0.000
Pot*Food	4	2.6404	0.6601	4.89	0.004
Error	27	3.6425	0.1349		
Total	35	40.4738			

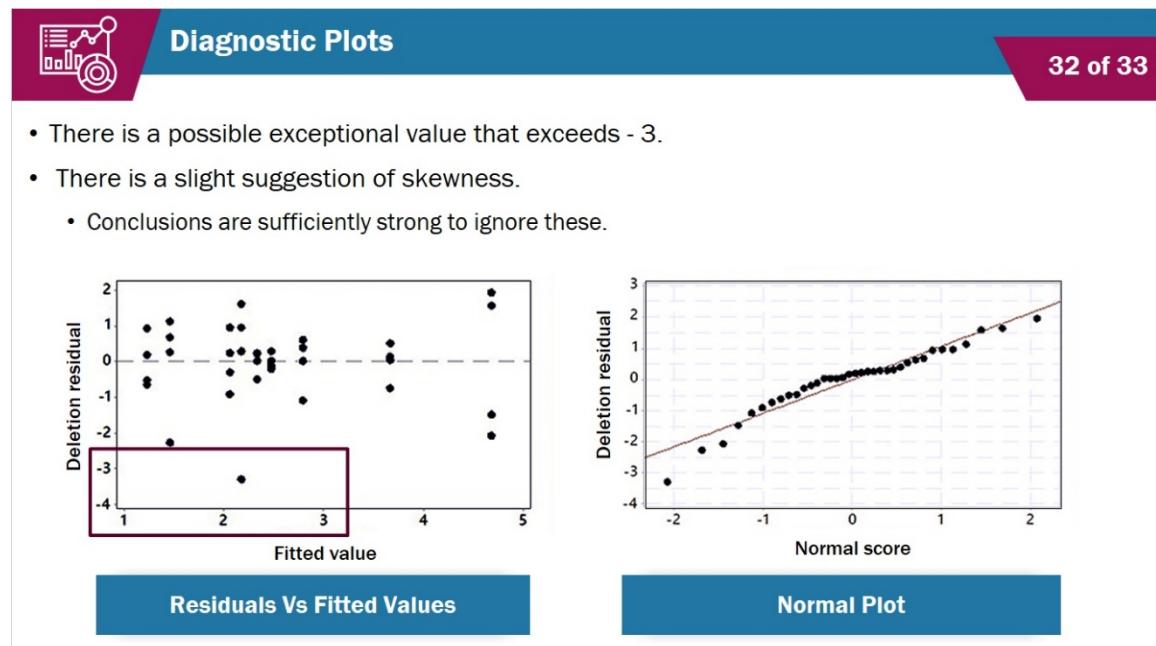
$S = 0.367297$

Analysis of Variance for Iron

Minitab may be used to produce the following Analysis of Variance table. It is seen that all F-ratios are highly statistically significant indicating that the differences in factor main effects and in interaction effects observed earlier are themselves statistically significant.

Note that the value of s is the square root of the error mean square and corresponds to the value of $\hat{\sigma}$ calculated earlier.

Slide 32: Diagnostic Plots



The standard diagnostic plots are shown in Figure 3.3, page 10. There is a possible exceptional point in the Residuals versus Fits plot with a deletion residual value exceeding -3 . The Normal plot suggests some skewness, which would accommodate the "exceptional" point identified in the Residuals versus Fits plot. It may be suggested that a transformation to reduce skewness, such as the logarithmic transformation, should be tried. However, given the high levels of statistical significance noted above, there seems little point in attempting this; the results will not change much.

Slide 33:

Summary



Summary

33 of 33

- Having completed this presentation, you should now be able to:
 - Explain the principles and procedure of randomised blocks design
 - Discuss the advantages and disadvantages of randomised blocks design
 - Differentiate between standard residuals and deletion residuals
 - Explain the principles and procedure of factorial design



Developed by Trinity Online Services CLG with the School of Computer Science and Statistics, Trinity College Dublin, The University of Dublin

Having completed this presentation, you should now be able to:

Explain the principles and procedure of randomised blocks design

Discuss the advantages and disadvantages of randomised blocks design

Differentiate between standard residuals and deletion residuals and

Explain the principles and procedure of factorial design