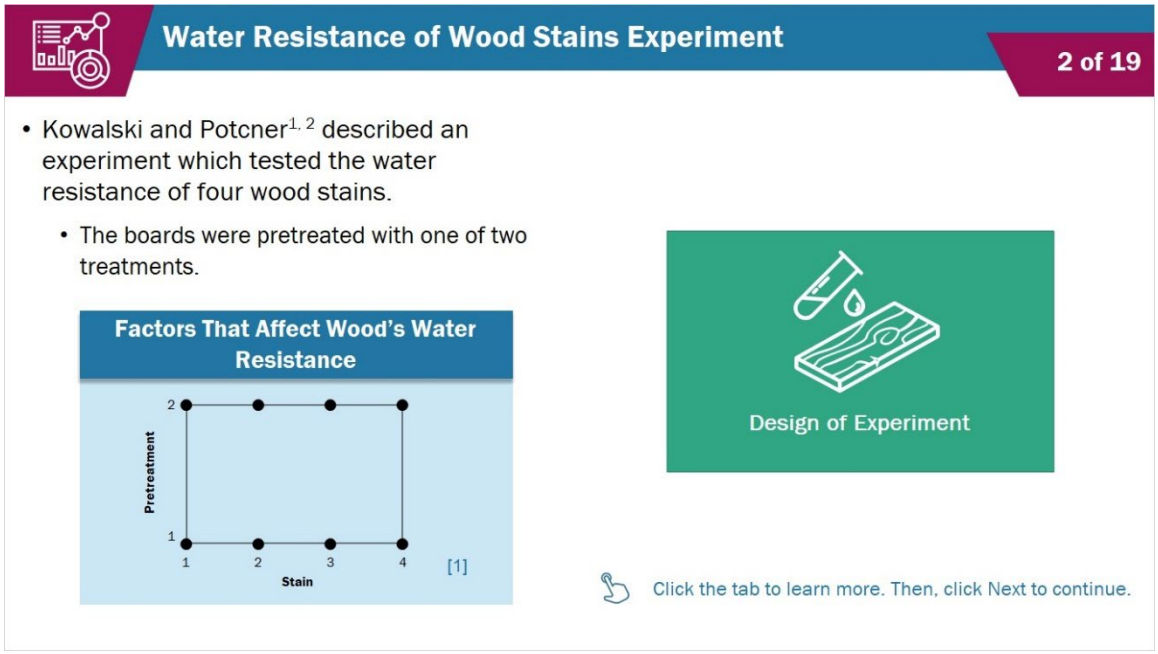
Case Studies on Split Plots Design

Contents

Slide 1: Introduction

Hello, and welcome. My name is James Ng, and I will lead you through this presentation which describes a Split Plots Design case study.

Slide 2: Water Resistance of Wood Stains Experiment



Kowalski and Potcner (2003)[i] described an experiment designed to compare the water resistance properties of four different wood stains used in conjunction with one of two different pre-treatments of the wood.

Reference(s):

1. Kowalski, S.M. and Potcner, K.J. (2003), How to Recognize a Split-Plot Experiment, *Quality Progress*, Vol. 36, No. 11, pp. 60-66.

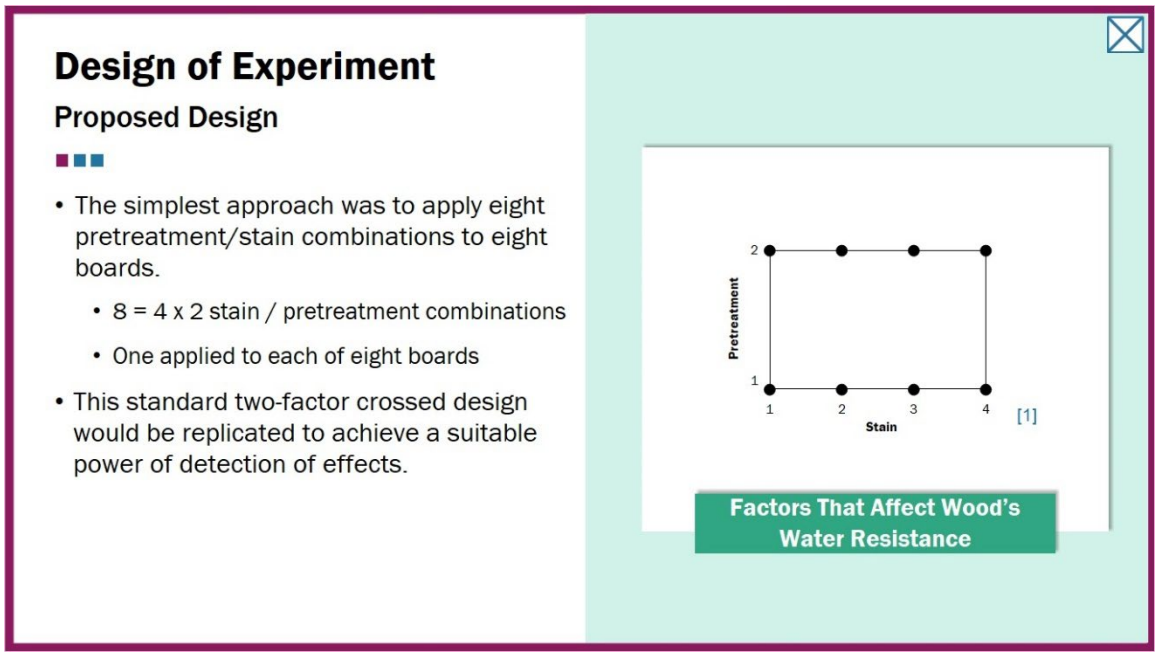
2. Potcner, K.J. and Kowalski, S.M. (2004), How to Analyze a Split-Plot Experiment, *Quality Progress*, Vol. 37, No. 12, pp. 67-74.

Image(s):

1. Kowalski, S.M. and Potcner, K.J. (2003), How to Recognize a Split-Plot Experiment, *Quality Progress*, Vol. 36, No. 11, pp. 60-66.

Click the tab to learn more. When you are ready, click next to continue.

Tab 1: Design of Experiment

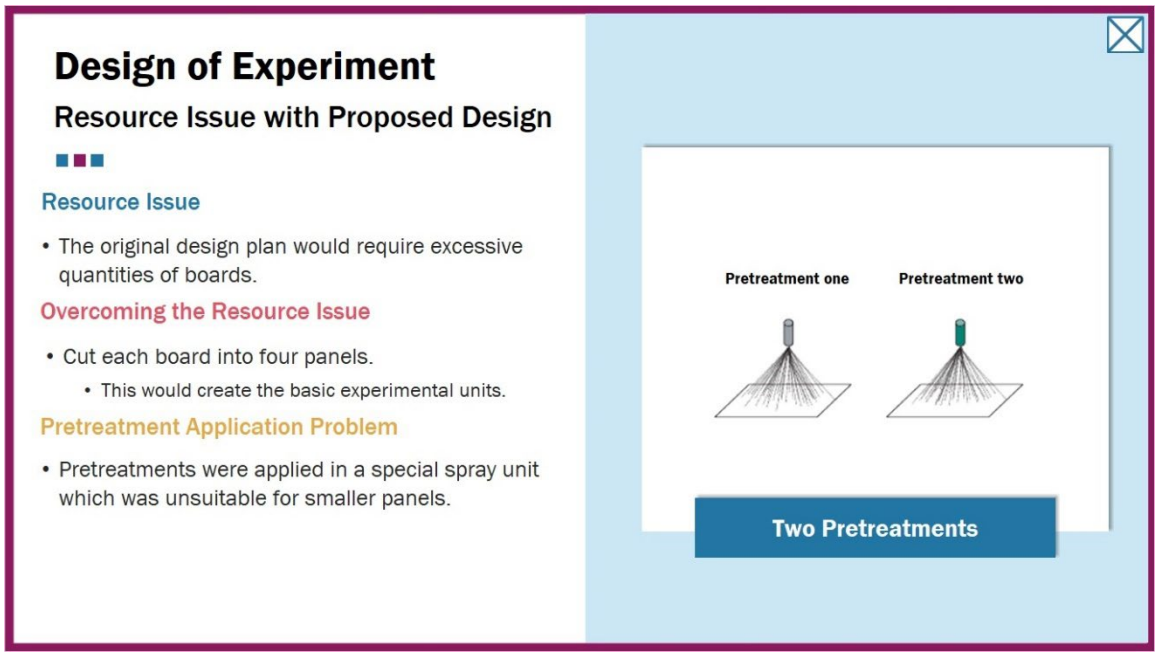


The simplest approach to this problem was to apply the eight pre-treatment / stain combinations to eight boards, in random order, replicating this basic crossed 2-factor design as often as desired, to achieve suitable power of detection of effects, or as often as feasible, given time, logistical and budgetary constraints.

Image(s):

1. Kowalski, S.M. and Potcner, K.J. (2003), How to Recognize a Split-Plot Experiment, *Quality Progress*, Vol. 36, No. 11, pp. 60-66.

Tab 1.1: Resource Issue with Proposed Design

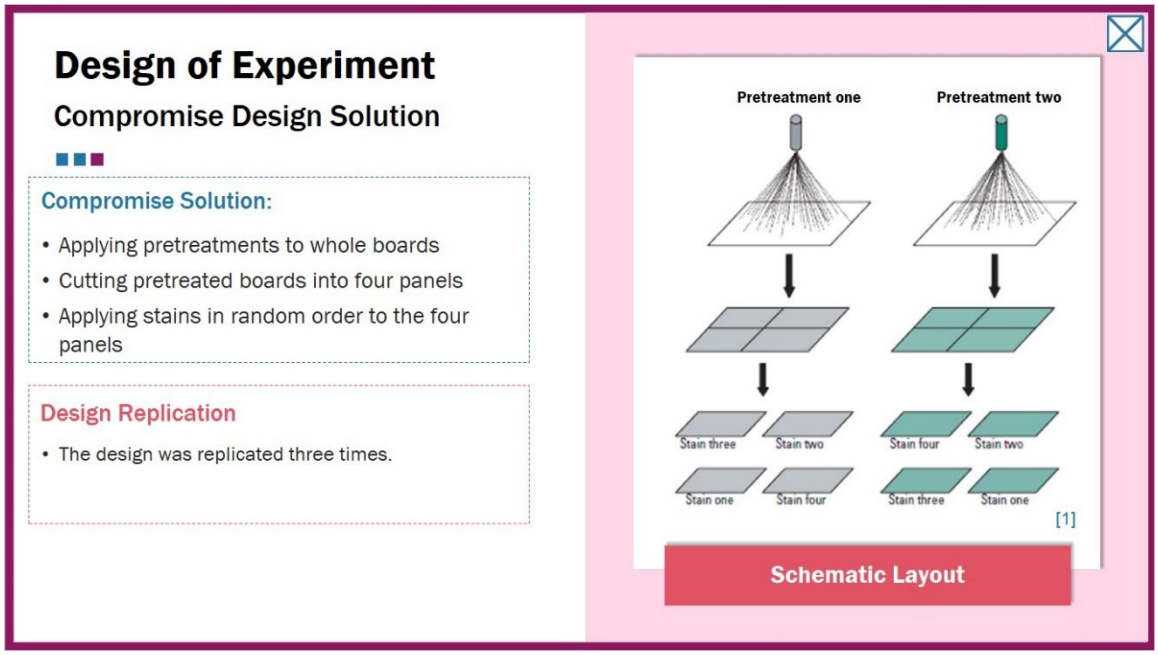


It was quickly realised that this plan would require an excessive quantity of boards. Accordingly, it was decided to cut the boards into four equally sized panels, to serve as the basic experimental units. However, this created a problem with the application of the pre-treatments, which were applied in a special spray unit, designed for whole boards and not feasible for smaller panels.

Image(s):

1. Kowalski, S.M. and Potcner, K.J. (2003), How to Recognize a Split-Plot Experiment, *Quality Progress*, Vol. 36, No. 11, pp. 60-66.

Tab 1.2: Compromise Design Solution

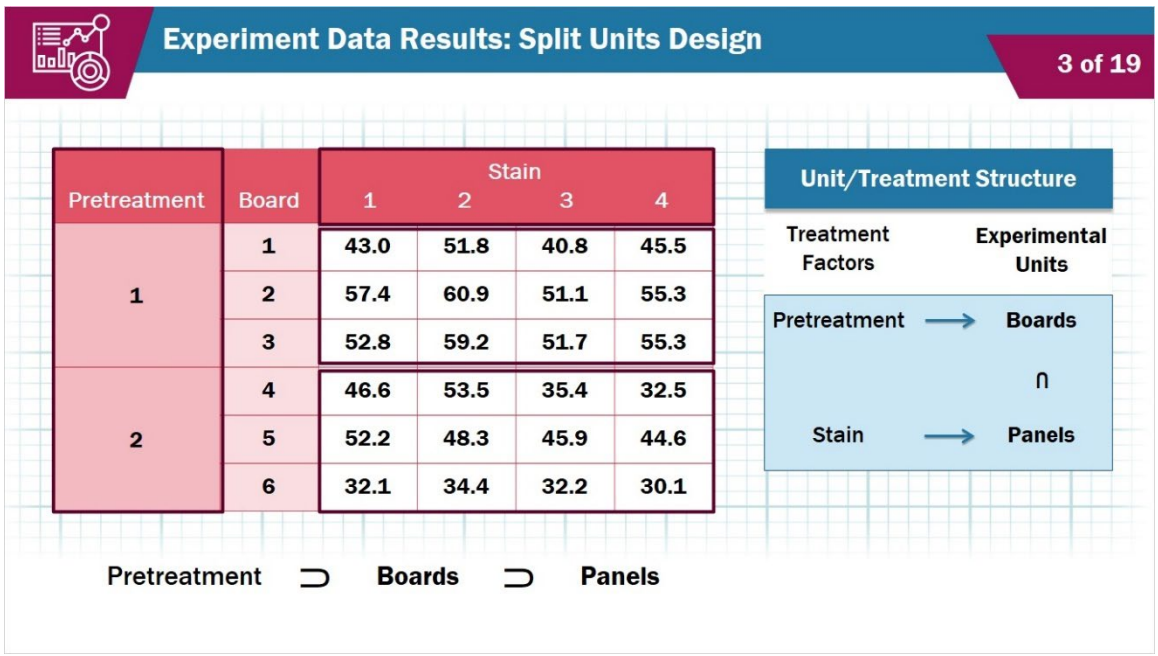


A compromise design was agreed that involved pre-treating each of two randomly selected boards with one of the experimental pre-treatments and then cutting the boards into four panels each and applying the four stains in random order to the four panels from each board. In the actual experiment, the design was replicated three times.

Image(s):

1. Kowalski, S.M. and Potcner, K.J. (2003), How to Recognize a Split-Plot Experiment, *Quality Progress*, Vol. 36, No. 11, pp. 60-66.

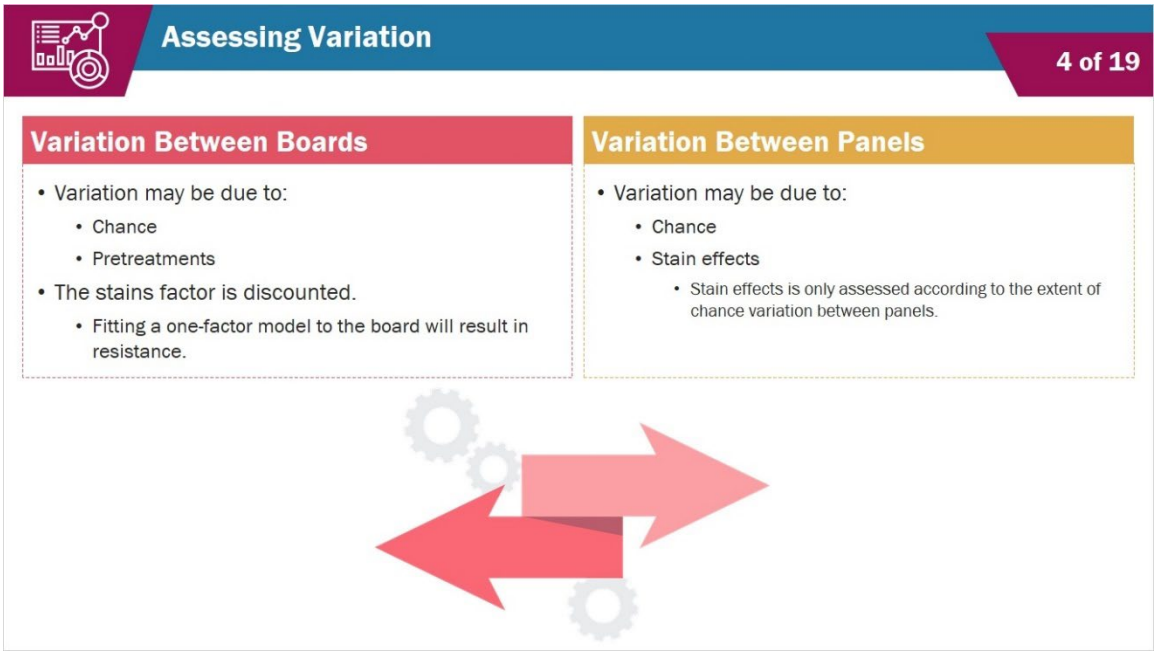
Slide 3: Experiment Data Results: Split Units Design



There were 24 subunits (panels) nested in 6 whole units (boards). There were two treatment factors, Pre-treatment with 2 levels, and Stain with 4 levels. The data are presented in the table.

This is clearly a split units design, with the experimental units for the pre-treatment factor being whole boards while the experimental units for the stain factor were the sets of four panels cut from the boards. The unit / treatment structure is illustrated in this slide.

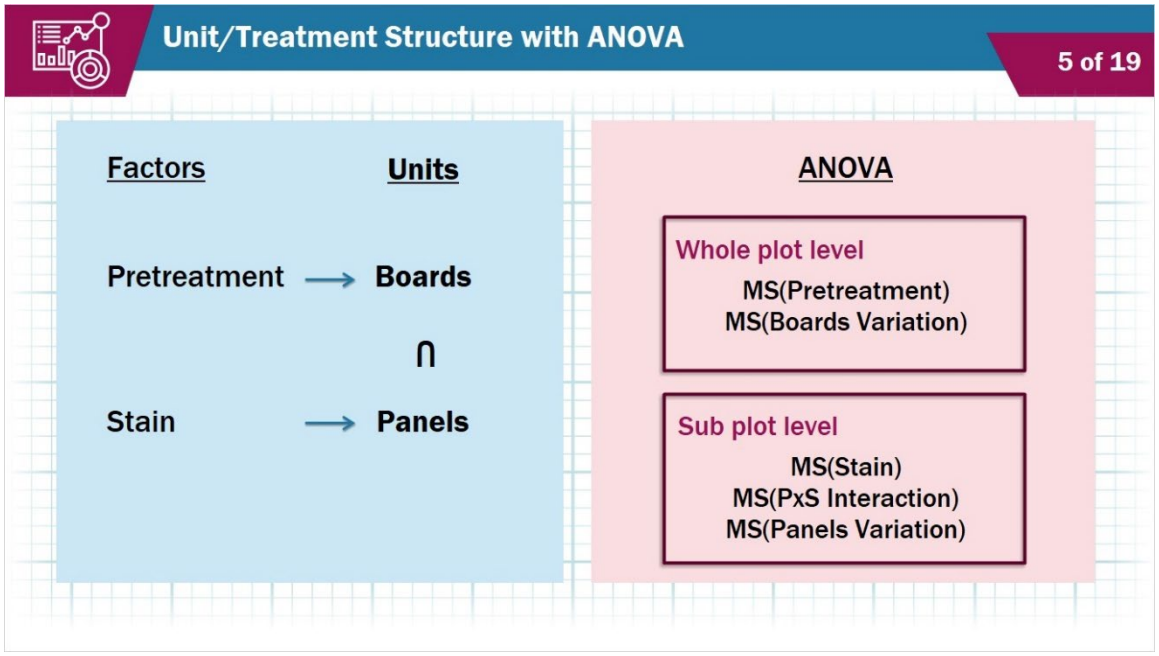
Slide 4: Assessing Variation



Variation between boards is due to either chance or pre-treatment effects. The extent of variation due to pre-treatment effects is assessed by reference to the extent of chance variation between boards. This may be achieved by ignoring the Stains factor for the moment and fitting a one-factor model to the board mean resistances.

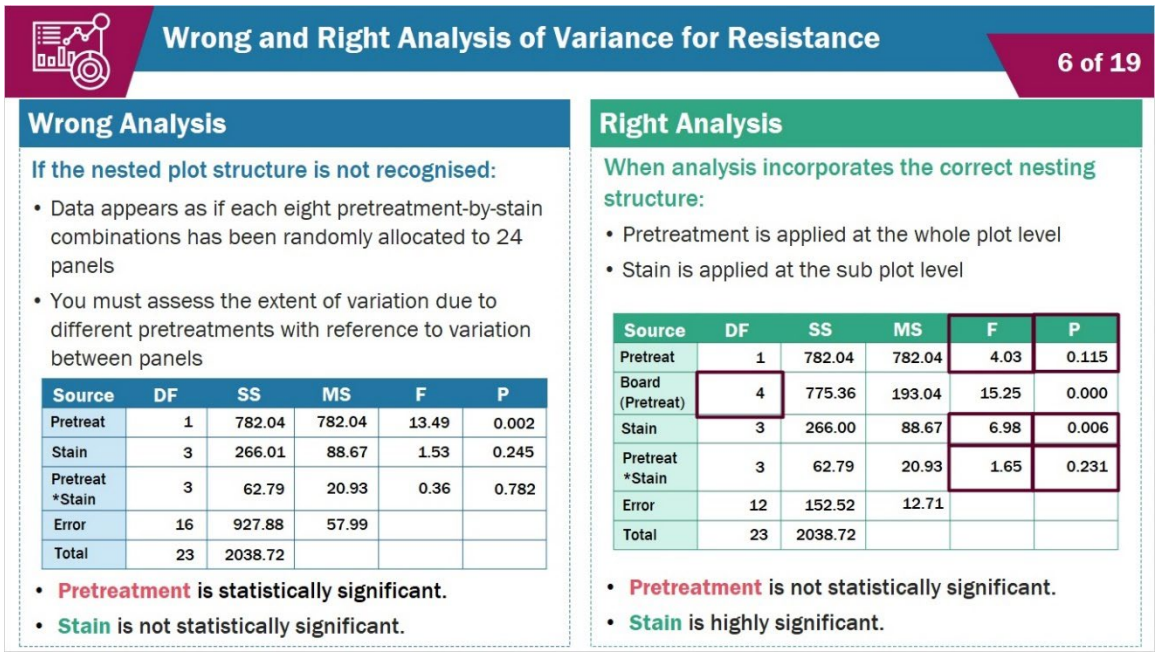
Similarly, variation between panels is due to either chance or stain effects. The extent of variation due to stain effects is assessed by reference to the extent of chance variation between panels.

Slide 5: Unit/Treatment Structure with ANOVA



The unit and treatment structure with ANOVA is shown in this slide. At the whole plot level, there are two mean square terms – mean square due to pre-treatment and mean square due to boards variation. At the subplot level, there are three mean square terms – mean square due to stain, mean square pre-treatment by stain interaction and mean square panels variation.

Slide 6: Wrong and Right Analysis of Variance for Resistance



In the experiment as carried out, 2 pre-treatments were first allocated to 6 boards and 4 stains then allocated to 4 panels within each of the 6 boards, a total of 24 panels. If the

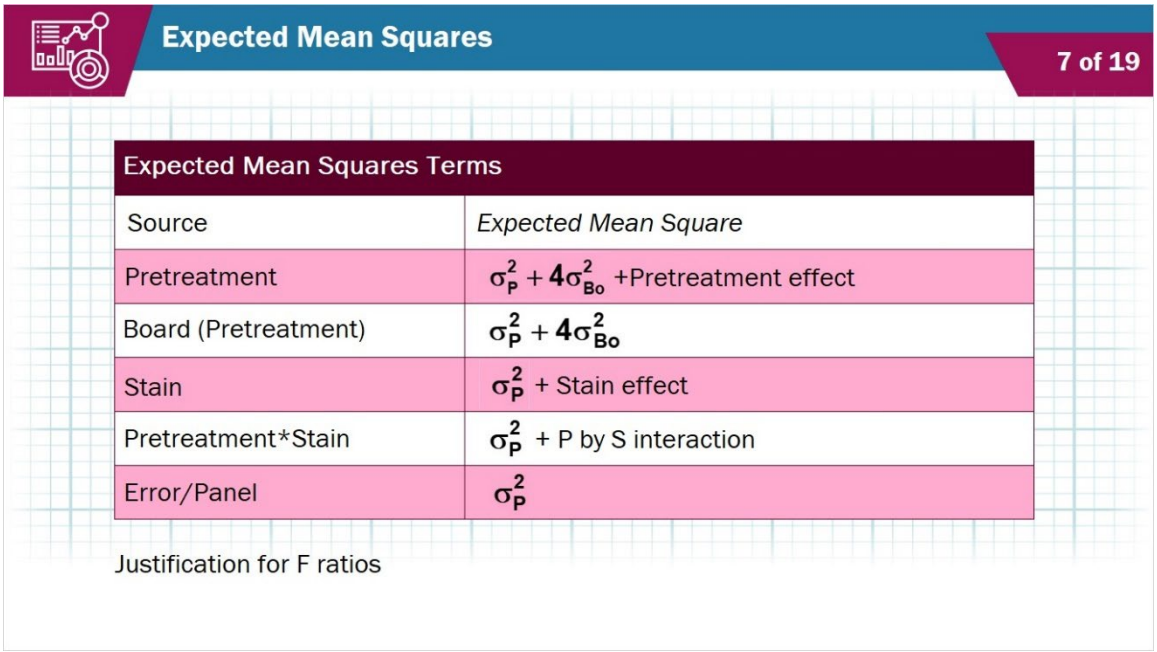
nested plot structure is not recognised, the data will appear as if each of the 8 pre-treatment by stain combinations had been randomly allocated to 24 panels, assuming that it was feasible to pre-treat panels separately. If that were the case, the extent of the variation due to the different pre-treatments should be assessed by reference to variation between panels. The appropriate analysis would be a 2-factor analysis of variance.

The correct analysis incorporates the nesting structure whereby the pre-treatment is applied at the whole plot level and stain applied at the subplot level. The difference in water resistance between the two pre-treatments is not statistically significant, F = 4.03 with a p-value of 0.115, or 11.5%. Note, however, that the degrees of freedom for error in that assessment, 4, as appropriate to the Board(Pre-treatment) error term, is quite small, with the result that the corresponding F test has quite low power. Consequently, the possibility that Pre-treatment does have an effect cannot be ruled out.

The differences between stains are highly statistically significant, with F = 6.98 and a p-value of 0.006 or 0.6%. There appears to be no statistically significant interaction between the two treatments, F = 1.65 with p = 0.231 or 23.1%, so that differences between stains will follow similar patterns for the two pre-treatments.

In summary, according to the right analysis, pre-treatment is not statistically significant whereas it is highly significant in the wrong analysis. On the other hand, stain is highly statistically significant in the right analysis and not significant in the wrong analysis.

Slide 7: Expected Mean Squares

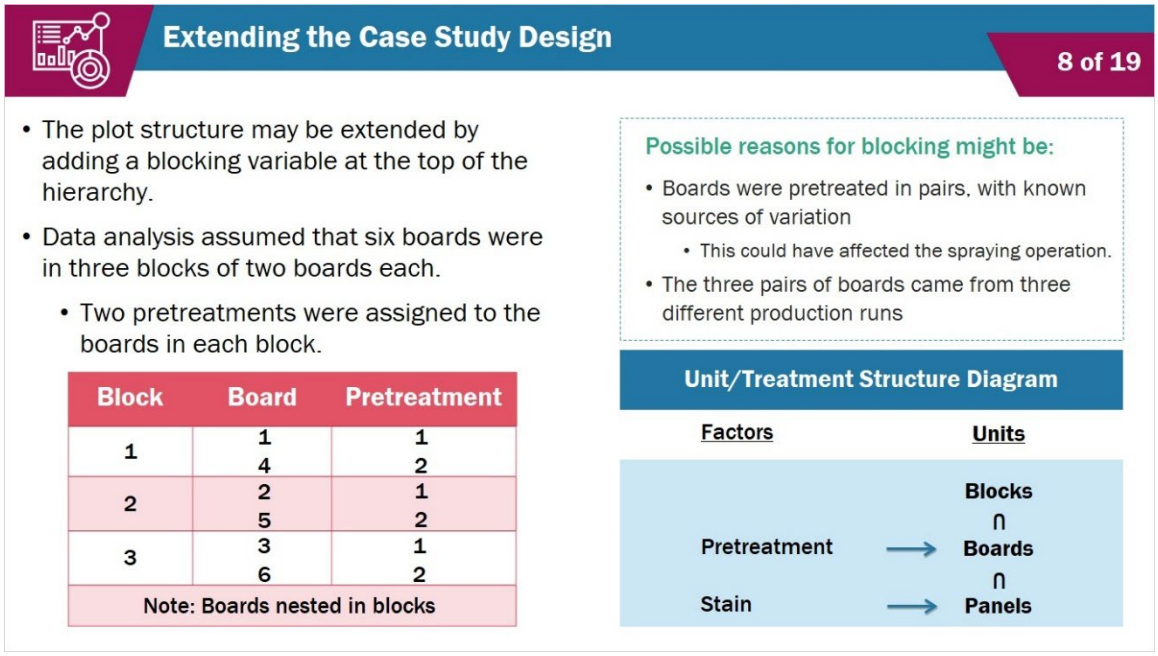


We look at the expected mean squares terms which provide the basis for the computation of the F ratios. For the pre-treatment, the expected mean square is given by the random variation due to panel plus four times the random variation due to boards +

pre-treatment effect. We have to multiply the random variation due to boards by 4 because each board is divided into 4 panels. The expected mean square for board is the sum of panel variation and 4 times board variation. These two expected mean squares provided the basis for testing the pre-treatment effect. In particular, we compute the ratio between mean square for pre-treatment and mean square for board to test the pre-treatment effect.

The expected mean square for stain is given by panel variation plus stain effect. Similarly, the expected mean square for pre-treatment by stain interaction is panel variation plus pre-treatment by stain interaction. Finally, we have the panel variation. These provide the basis for testing stain effect and pre-treatment by stain interaction effect. To test stain effect, we will compute the ratio between stain mean square and panel/error mean square.

Slide 8: Extending the Case Study Design

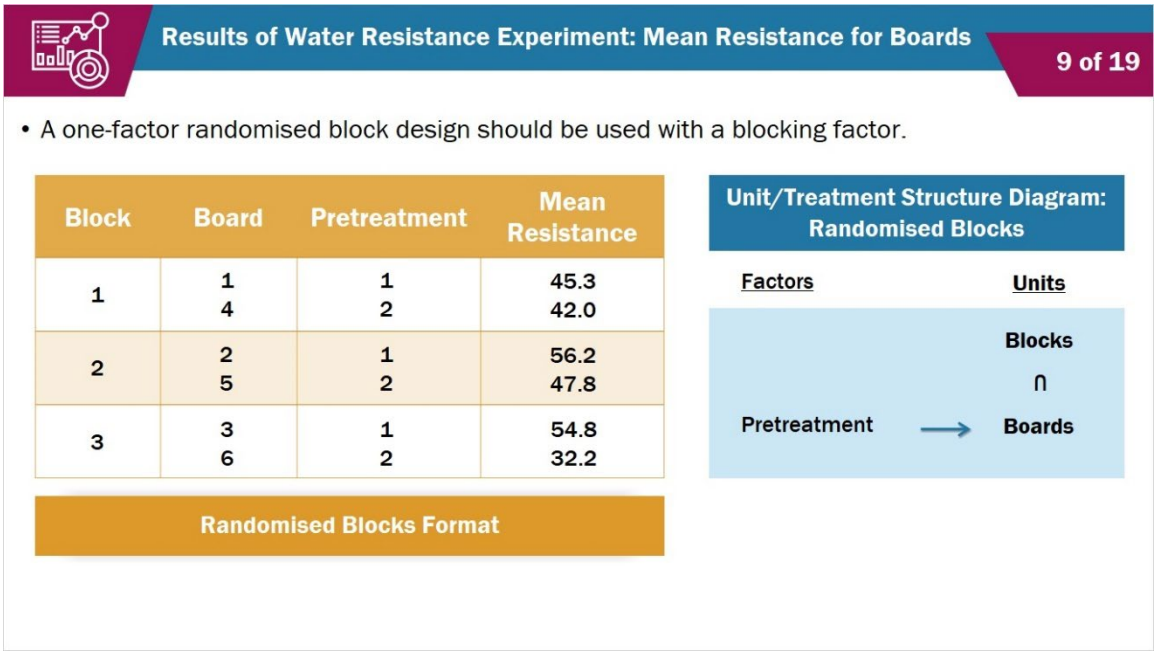


The design for this case study may, in principle, be extended. The plot structure may be extended by, for example, adding a blocking variable at the top of the plot structure hierarchy. An analysis of these data made an assumption that the 6 boards actually formed 3 blocks of 2 boards each, with the two pre-treatments being assigned to the boards in each block as shown in Table. Therefore, the boards are now nested in blocks. A possible reason for blocking could be that boards are pre-treated in pairs and there were known sources of variation such as different operators or different batches of pre-treatment that could affect the spraying operation. Another is the possibility that the three pairs of boards came from three different production runs, although no such reason was quoted in this case.

The unit / treatment structure for this extended design takes the form shown on the right. The intuitive argument assessing the variation due to Pre-treatment effects and Stain effects still applies; the extent of variation due to each of these effects is referred

to the extent of chance variation at the level of the unit structure to which the relevant effects apply, that is, either boards or panels, respectively.

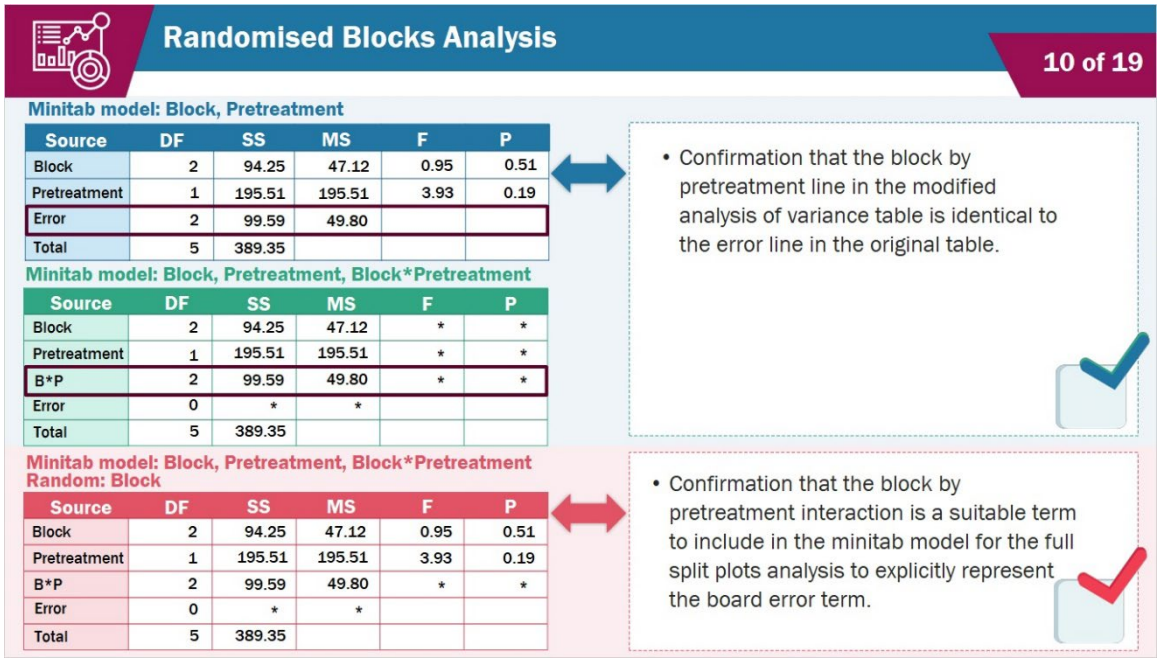
Slide 9: Results of Water Resistance Experiment: Mean Resistance for Boards



Recall that the Minitab model term used to represent boards variation in the previous analysis was Board nested in Pre-treatment, denoted in Minitab notation by Board(Pre-treatment). This was appropriate when the design for Pre-treatment as applied to boards was a simple one-factor design. With the introduction of a blocking factor, this is no longer appropriate and is replaced by a one-factor randomised blocks design. The structure of this is illustrated in the Table, where the response variable is taken to be average water resistance, averaging across stains and thus allowing the focus to be on Pre-treatment.

Since the resistance is averaged in each board, the unit/treatment structure is simplified. In particular, it is reduced to a randomized block structure where the boards are nested within blocks and pre-treatment applied to boards.

Slide 10: Randomised Blocks Analysis



The standard randomised blocks analysis of these data, using the Minitab model

Block Pretreatment

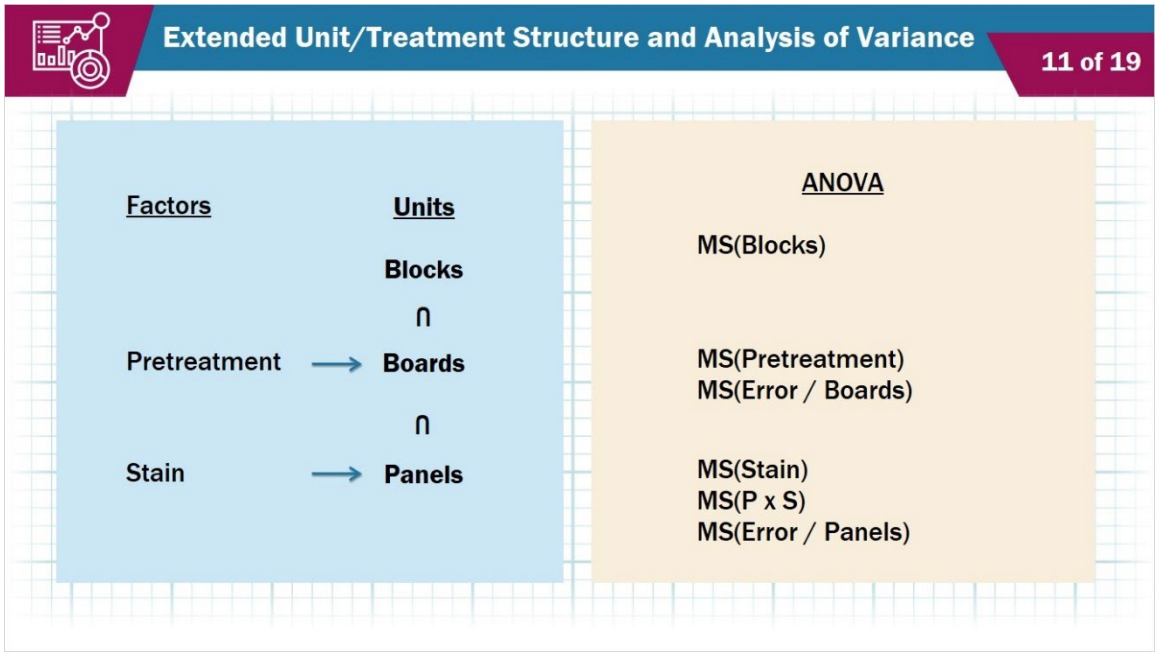
results in the Analysis of Variance shown in the top table. We can augment the model above by adding the Block by Treatment interaction term

Block Pretreatment Block*Pretreatment

resulting in the Analysis of Variance shown in the bottom table. This confirms that the Block by Pre-treatment line in the modified Analysis of Variance table is identical to the Error line in the original table.

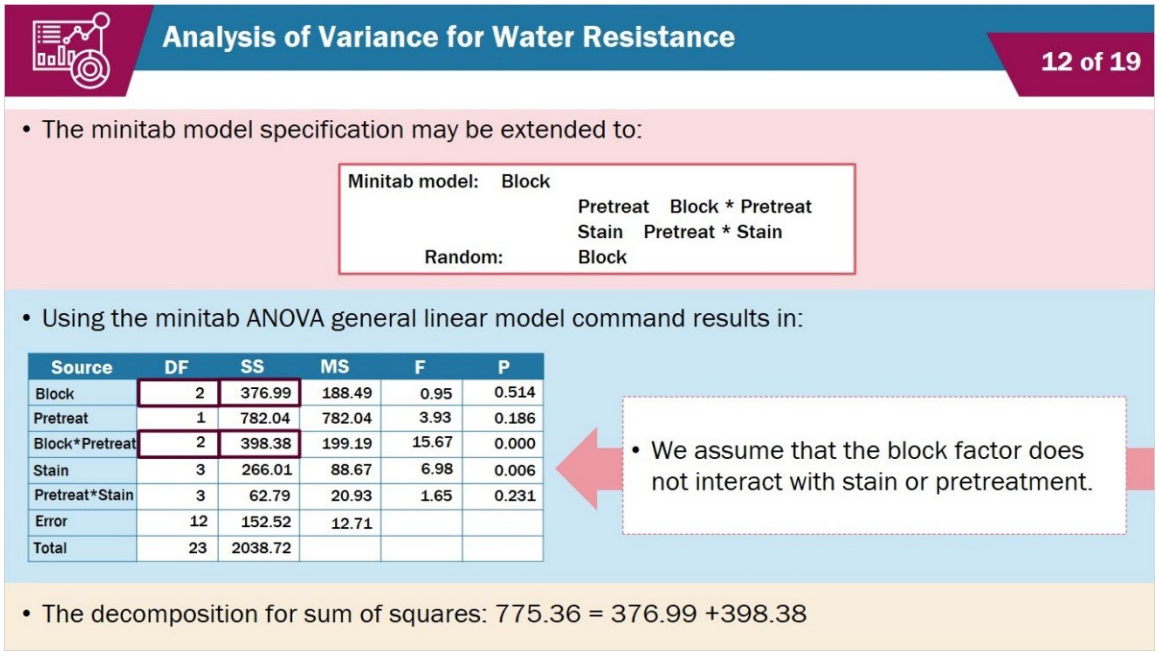
However, Minitab has not recognised it as an error term appropriate for testing the Block and Pre-treatment effects. For this to happen, the Block factor must be declared as a random effects factor. When this is done using the appropriate Minitab dialog, we obtain the Analysis of Variance table shown in the bottom. This confirms that the Block by Pre-treatment interaction is a suitable term to include in the Minitab model for the full split plots analysis to explicitly represent the Board error term.

Slide 11: Extended Unit/Treatment Structure and Analysis of Variance



The extended unit and treatment structure with analysis of variance are shown in this slide. We have boards nested within blocks and panels nested with boards for the unit structure. The pre-treatment is applied to boards and stain applied to panels. The mean square terms provide the basis for testing the effects.

Slide 12: Analysis of Variance for Water Resistance



The Minitab model specification used for fitting the original split units' model may be extended to

Block

Pretreatment Block*Pretreatment

Stain Stain*Pretreatment

Random factor: Block

Fitting this model using the Minitab ANOVA General Linear Model command results in Table shown in this slide. For simplicity, it is assumed that Block does not interact with Stain, in addition to having no interaction with Pre-treatment which is necessary for the Block*Pretreatment term to serve as a pure error term at the Board level.

Note that the 4 degrees of freedom for Boards in the previous analysis have been decomposed into 2 degrees of freedom for Blocks and 2 degrees of freedom for Block*Pretreatment. The corresponding sum of squares has been similarly decomposed.

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775.36 = 376.99 + 398.38
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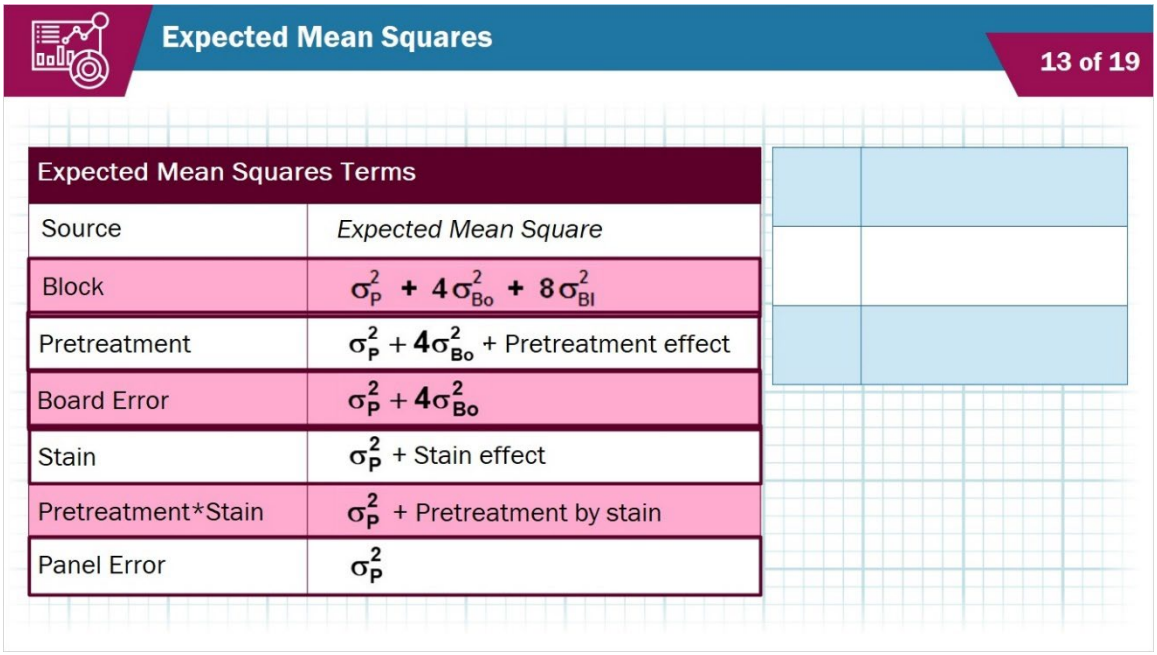
Slide 13: Expected Mean Squares



The expected mean squares terms are shown in this slide.

Here, Board is used to indicate the error term at board level, represented by the Block by Pre-treatment interaction in the analysis of variance, σ_{Bl}^2 is the Block component of variance, σ_{Bo}^2 is the Board component of variance and σ_P^2 is the Panel component of variance.

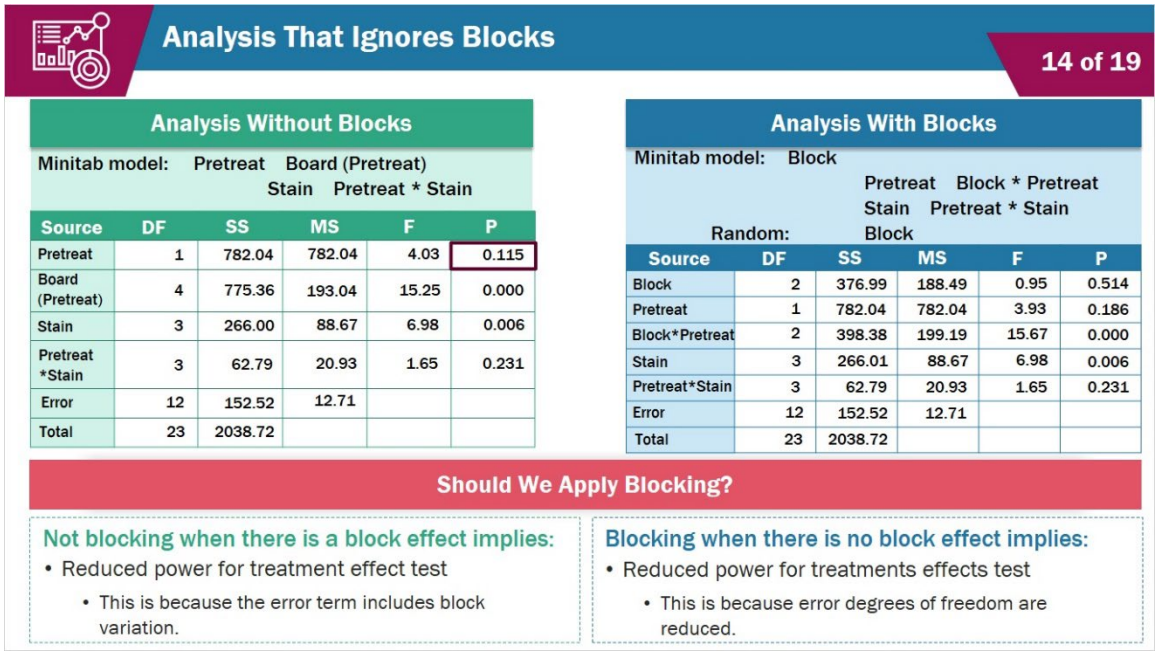
If $\sigma_{Bl}^2 = 0$, that is, there is no component of variance for Blocks or there is no Block effect, then EMS(Block) and EMS(Board) are the same. Thus, the ratio of the corresponding actual mean squares, MS(Block) and MS(Board), provides a test for the hypothesis that $\sigma_{Bl}^2 = 0$.

If Pre-treatment effect is 0, EMS(Pretreatment) and EMS(Board) are the same. Thus, the ratio of MS(Pretreatment) and MS(Board) provides a test for the hypothesis Pretreatment effect = 0.

If Stain effect is 0, EMS(Stain) and EMS(Panel) are the same. Thus, the ratio of MS(Stain) and MS(Panel) provides a test for the hypothesis Stain effect = 0.

A similar justification applies to the test for no interaction.

Slide 14: Analysis Ignoring Blocks

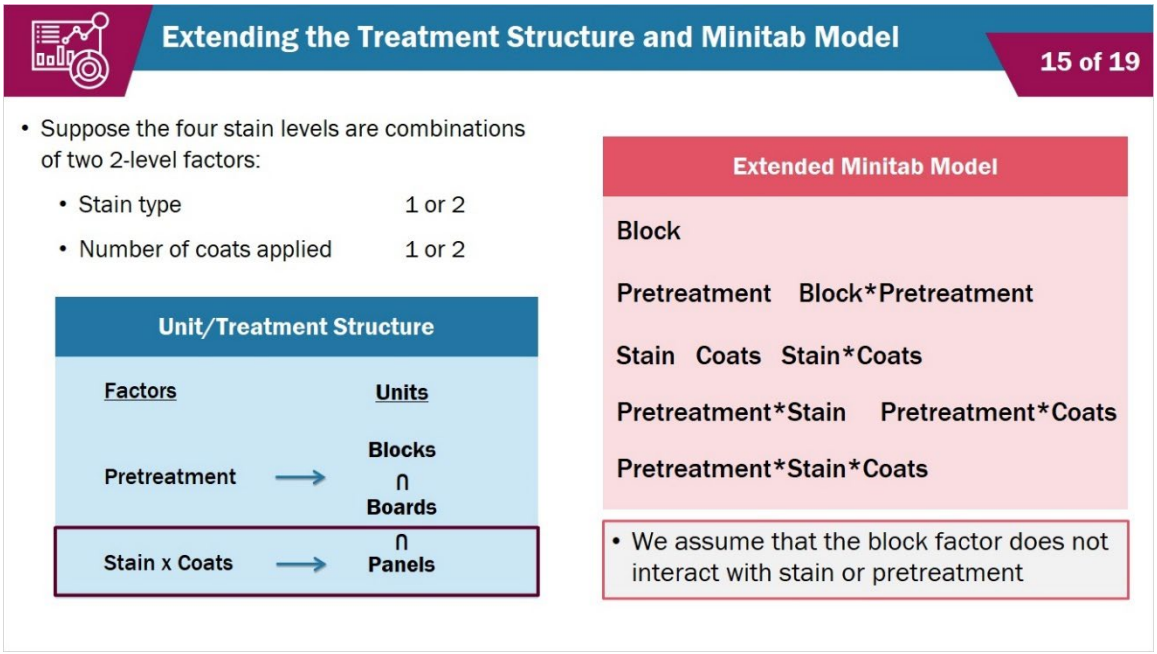


The results for the analysis without blocks are shown in this slide. We note that compared to the analysis with blocks the F-ratio and p-value for pre-treatment are slightly different.

One question that arises is whether or not we should apply blocking. Not blocking when there is a block effect implies reduced power for treatment effects test. This is because the error term will include block variation. A larger error makes rejection of the test more difficult.

Blocking when there is no block effect implies reduced power for treatment effects test. This is because error degrees of freedom will be reduced.

Slide 15: Extending the Treatment Structure and Minitab Model



Suppose now that further information becomes available about the four levels of the Stain factor, that, in fact, these represent the four possible combinations of two 2-level factors,

Stain type, 1 or 2,

number of Coats applied, 1 or 2.

You can see the unit / treatment structure for this design on the left. The only change is in the factor structure at the bottom level of the unit / treatment structure.

Correspondingly, the Minitab model is shown on the right.

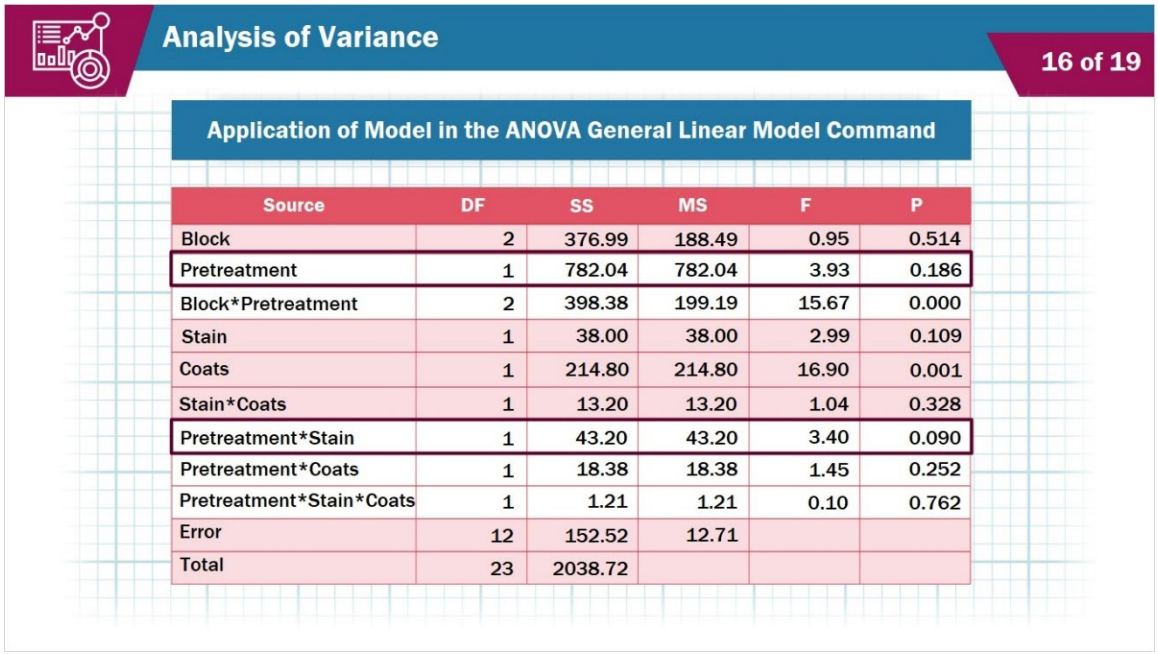
Block

Pretreatment Block*Pretreatment

Stain Coat Stain*Coat Pretreatment*Stain Pretreatment*Coat
Pretreatment*Stain*Coat

Again, for simplicity, it is assumed that the Block factor does not interact with any of the treatment factors.

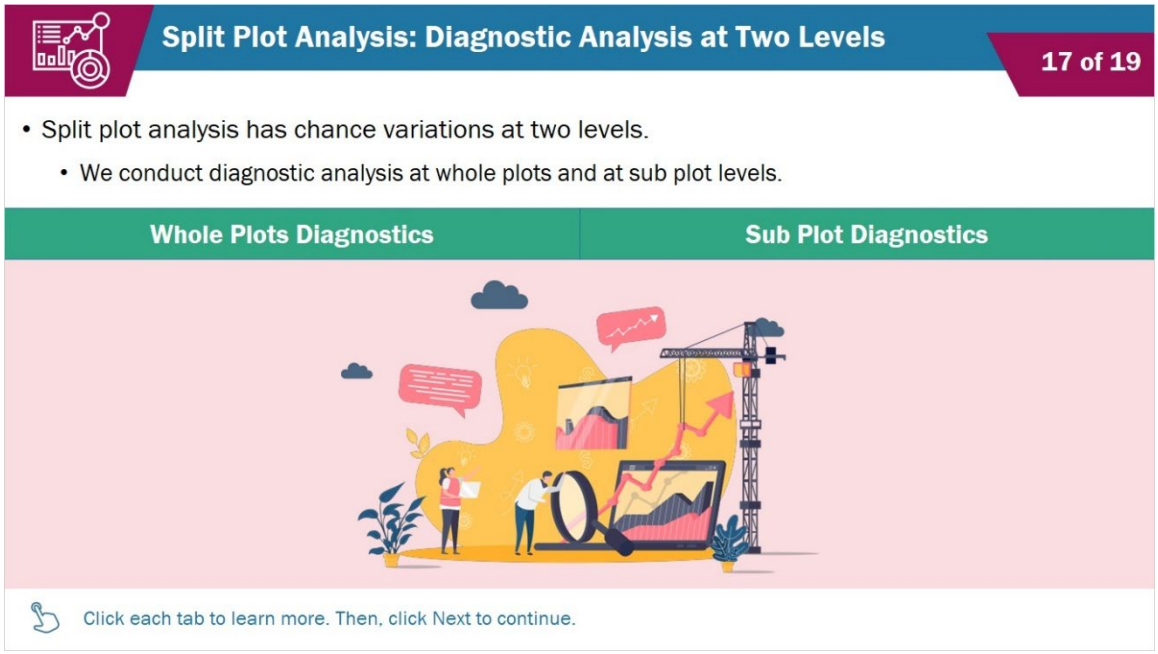
Slide 16: Analysis of Variance



Applying this model in the ANOVA General Linear Model command leads to the analysis of variance table shown in the Table.

The analysis at the levels of Block and Board remains the same as previously. At Panel level, the 3 degrees of freedom for Stain have been decomposed into three single degrees of freedom for (the new) Stain, Coat and Stain*Coat, with a corresponding decomposition of the sum of squares for Stain. The 3 degrees of freedom for the interaction of Pre-treatment with Stain are similarly decomposed.

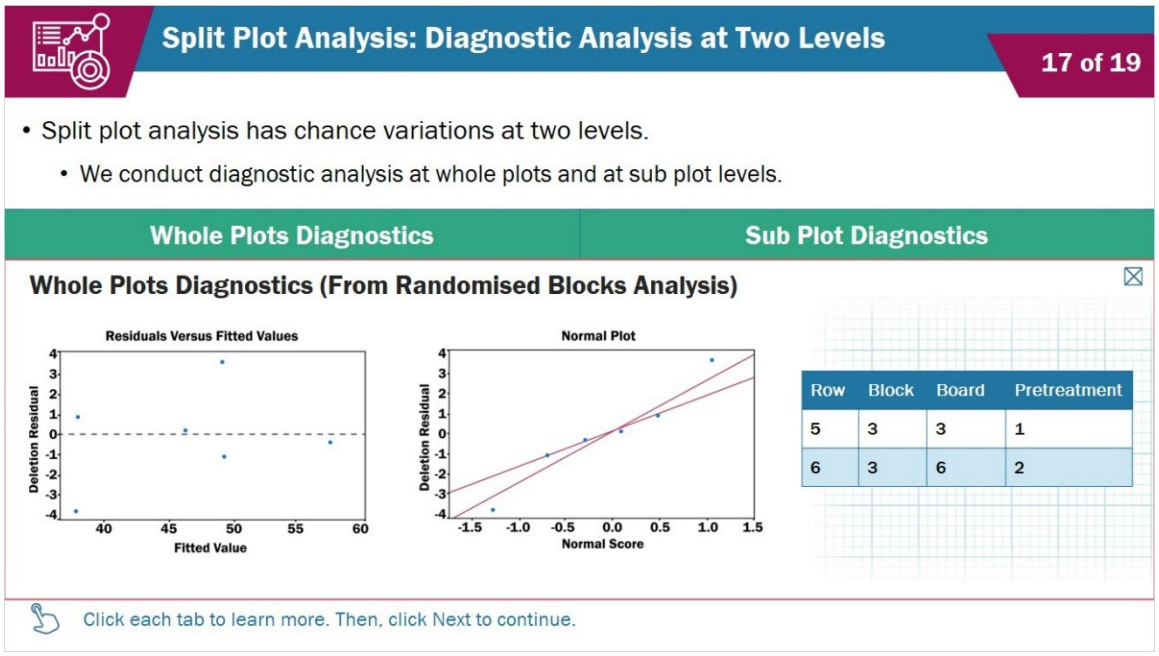
Slide 17: Split Plot Analysis: Diagnostic Analysis at Two Levels



For split-plot analysis, there are chance variations at two levels – the whole plot level and the subplot level. Therefore, we conduct diagnostic analysis at two levels.

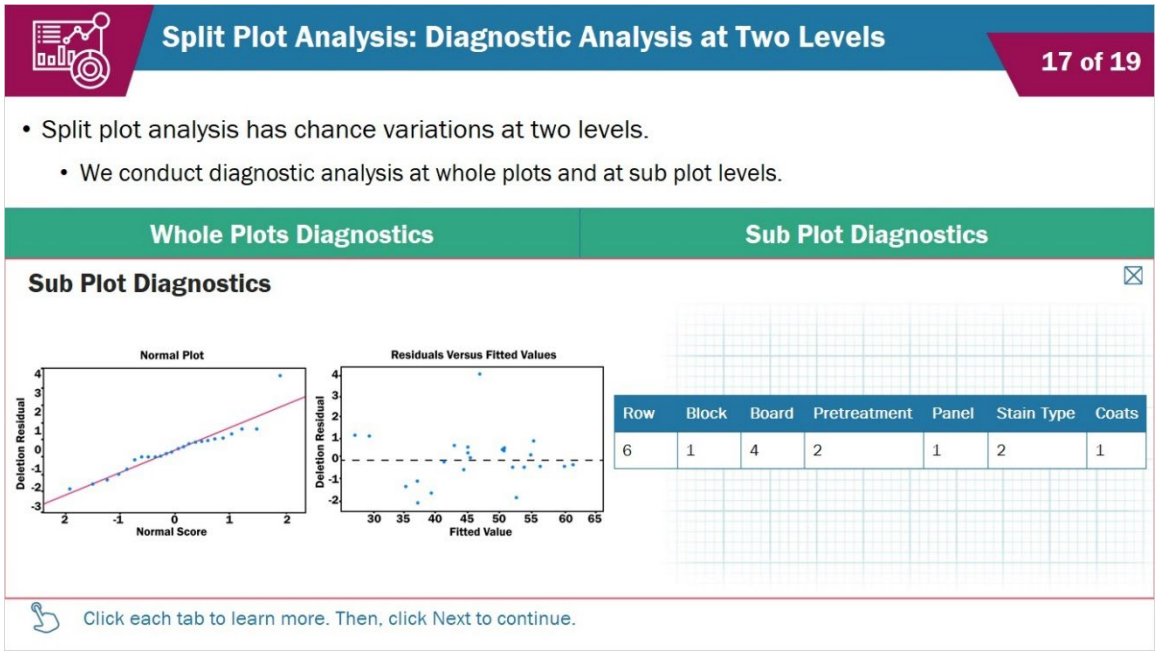
Click each tab to learn more. When you are ready, click next to continue.

Tab 1: Whole Plots Diagnostics



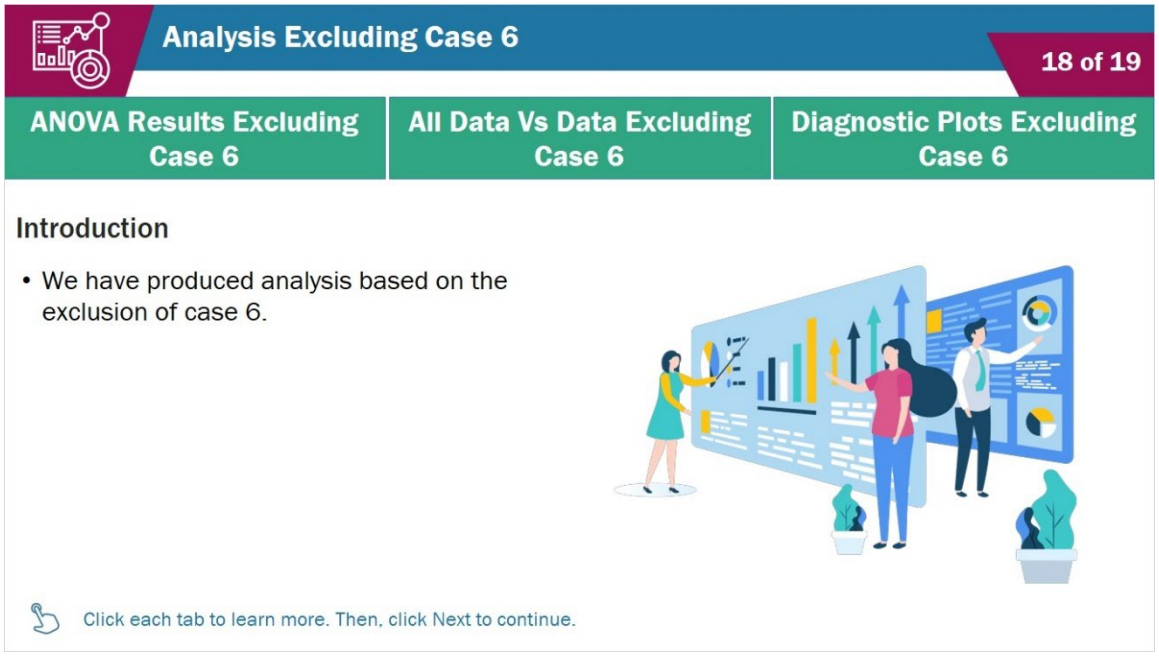
From the residuals versus fitted values plot and the normal plot, we identify two possible outliers which are listed below.

Tab 2: Sub Plot Diagnostics



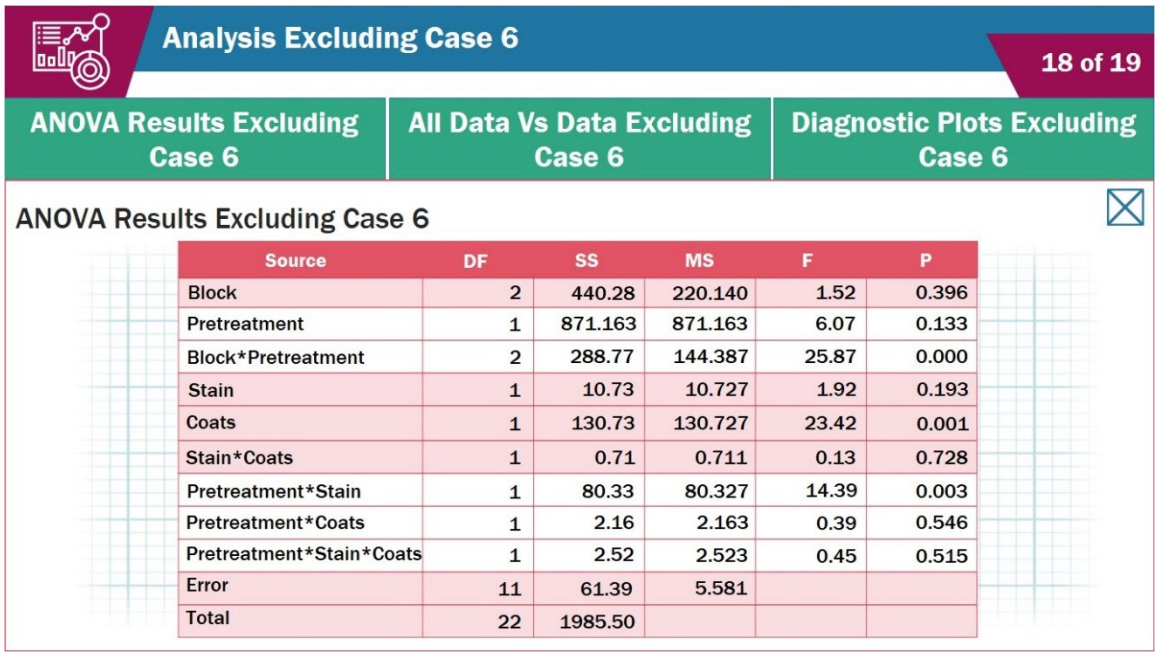
For the subplot diagnosis, we identify one outlier (case number 6) which is listed below.

Slide 18: Analysis Excluding Case 6



We have produced analysis that excludes case 6. Click each tab to learn more. When you are ready, click next to continue.

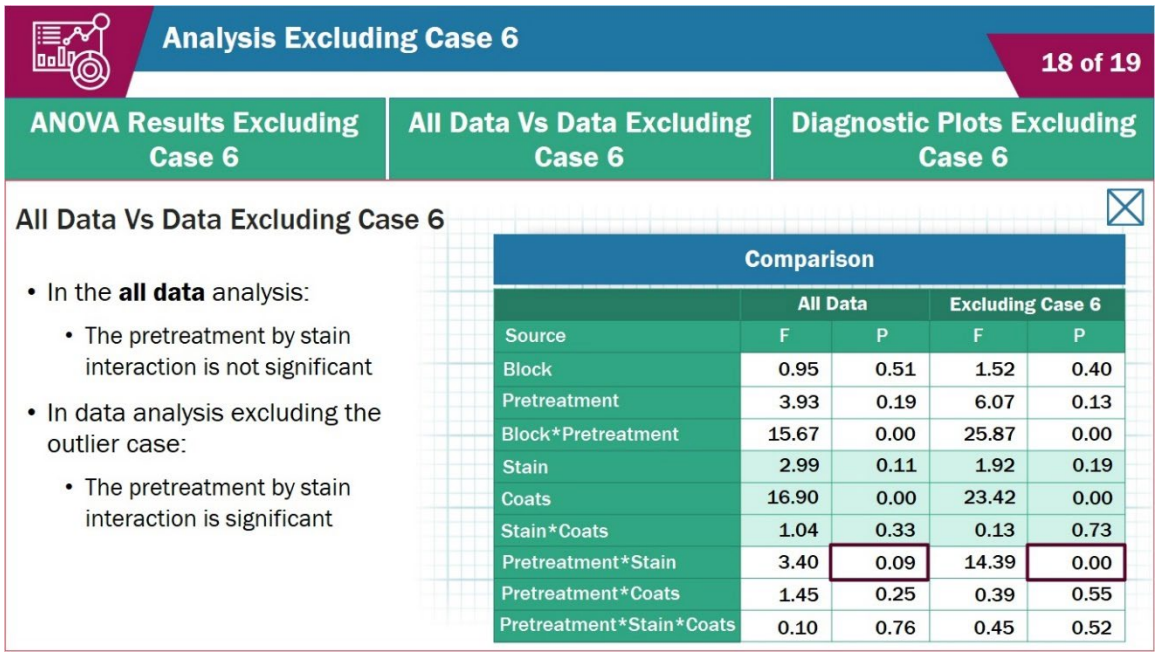
Tab 1: ANOVA Results Excluding Case 6



Source	DF	SS	MS	F	P
Block	2	440.28	220.140	1.52	0.396
Pretreatment	1	871.163	871.163	6.07	0.133
Block*Pretreatment	2	288.77	144.387	25.87	0.000
Stain	1	10.73	10.727	1.92	0.193
Coats	1	130.73	130.727	23.42	0.001
Stain*Coats	1	0.71	0.711	0.13	0.728
Pretreatment*Stain	1	80.33	80.327	14.39	0.003
Pretreatment*Coats	1	2.16	2.163	0.39	0.546
Pretreatment*Stain*Coats	1	2.52	2.523	0.45	0.515
Error	11	61.39	5.581		
Total	22	1985.50			

We refit the model with the outlier case number 6 removed and the ANOVA results are shown in this slide. Take time to review the information on this slide.

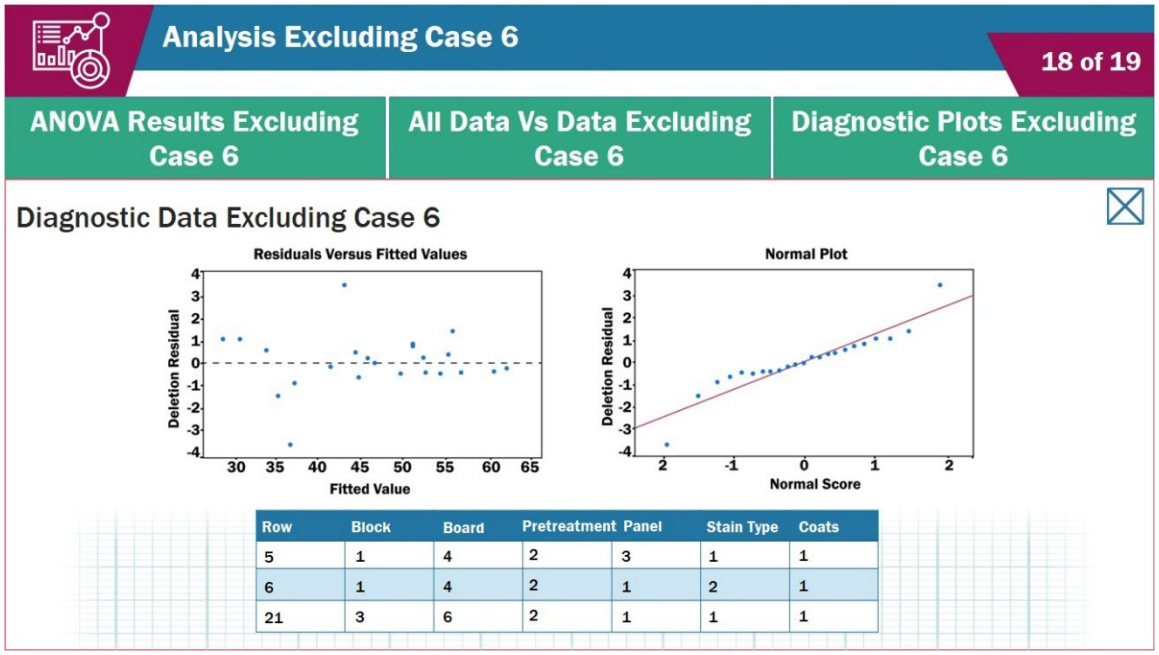
Tab 2: All Data Vs Data Excluding Case 6



Comparison				
	All Data		Excluding Case 6	
Source	F	P	F	P
Block	0.95	0.51	1.52	0.40
Pretreatment	3.93	0.19	6.07	0.13
Block*Pretreatment	15.67	0.00	25.87	0.00
Stain	2.99	0.11	1.92	0.19
Coats	16.90	0.00	23.42	0.00
Stain*Coats	1.04	0.33	0.13	0.73
Pretreatment*Stain	3.40	0.09	14.39	0.00
Pretreatment*Coats	1.45	0.25	0.39	0.55
Pretreatment*Stain*Coats	0.10	0.76	0.45	0.52

We compare the results using all data versus the results from the analysis with case number 6 removed. The F ratios and p-values for all sources are shown. We observe that the results are similar except for the Pre-treatment by Stain interaction. For the Pre-treatment by Stain interaction, the effect is not significant (p-value=0.09) in the analysis using all data whereas it is significant with the outlier case removed (p-value=0).

Tab 3: Diagnostic Plots Excluding Case 6

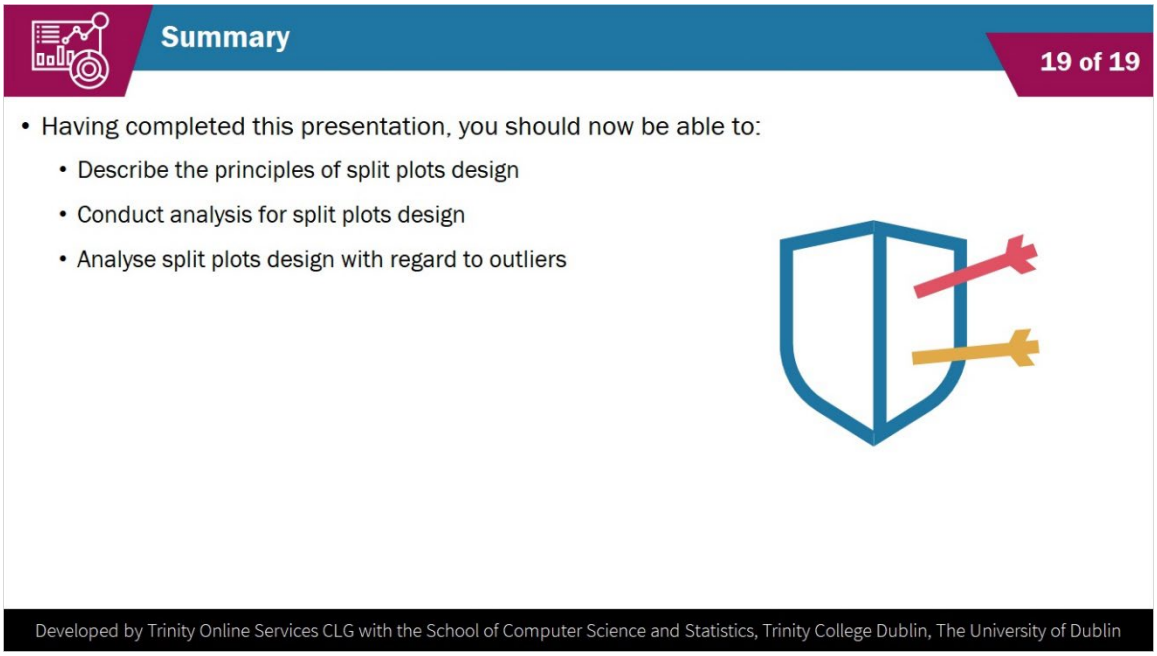


Row	Block	Board	Pretreatment	Panel	Stain Type	Coats
5	1	4	2	3	1	1
6	1	4	2	1	2	1
21	3	6	2	1	1	1

We produce the diagnostic plots after performing the analysis with case number 6 removed. We again identify three potential outliers. However, if we keep removing

outliers from the data and refit the model, we will end up with less and less data which is not desirable. Therefore, we should stop removing further data at this stage.

Slide 19: Summary



Having completed this presentation, you should now be able to:

Describe the principles of split plots design

Conduct analysis for split plots design, and

Analyse split plots design with regard to outliers