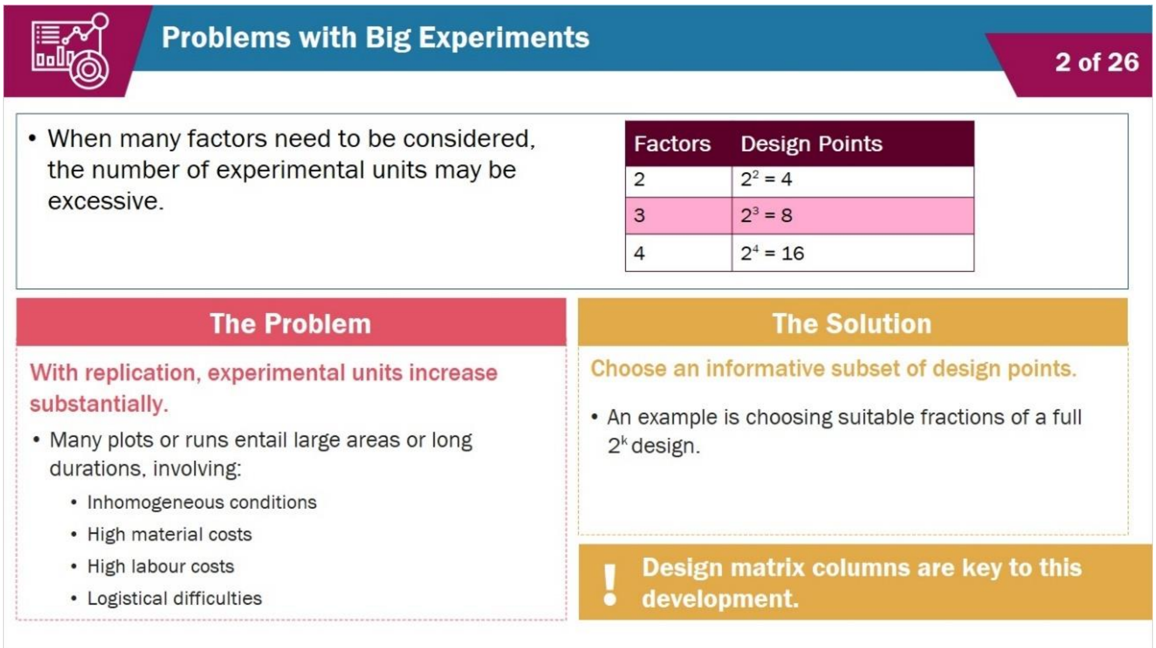
Fractional Factorial Designs and Split Plots Design

Slide 1: Introduction

Hello and welcome. My name is James Ng and I will lead you through this presentation on fractional factorial designs and split plots designs.

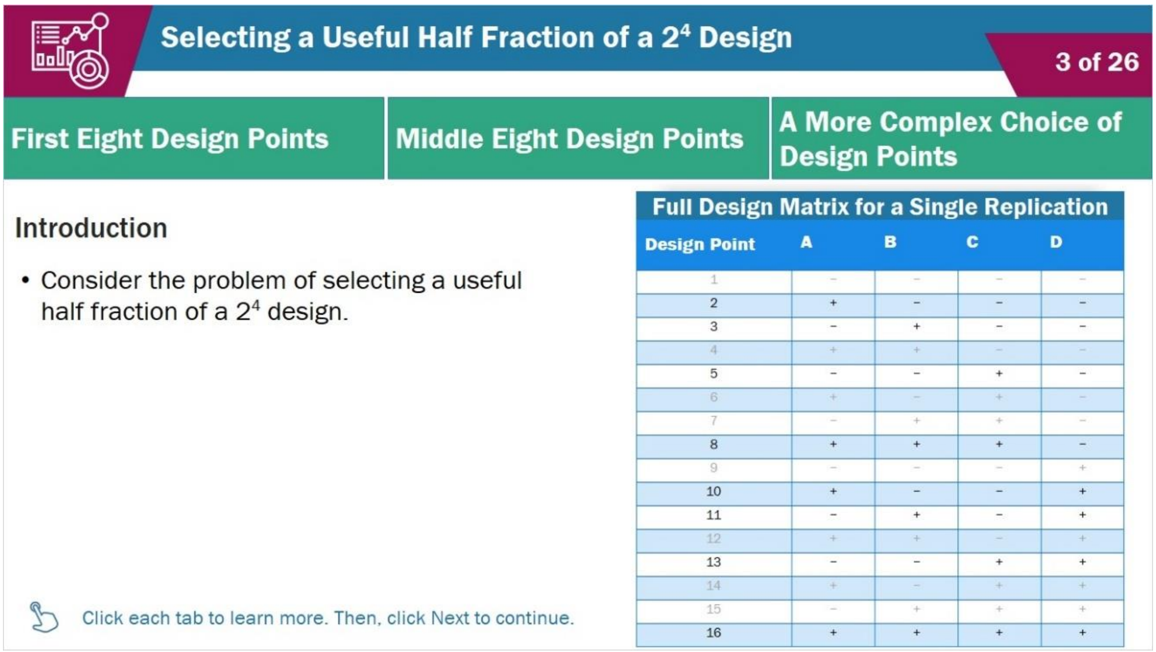
Slide 2: Problems with Big Experiments



When there are many factors to be considered, the number of experimental units may be excessive. With 2 factors, there are 4 design points. With 3 factors, there are 8 design points. With 4 factors, there are 16 design points. With replication, this increases substantially.

Many experimental units (plots, runs) entail large areas or time involving possibly inhomogeneous conditions, high material or labour costs and possibly difficult logistics. A solution is to choose an informative subset of design points. There are well worked methods for doing this in which the design matrix columns play a key role. They involve choosing suitable fractions, for example, half, quarter, eighth, of a full 2^k design. The design matrix columns will play a key role in this development.

Slide 3: Selecting a Useful Half Fraction of a 2^4 Design



Design Point	A	B	C	D
1	−	−	−	−
2	+	−	−	−
3	−	+	−	−
4	+	+	−	−
5	−	−	+	−
6	+	−	+	−
7	−	+	+	−
8	+	+	+	−
9	−	−	−	+
10	+	−	−	+
11	−	+	−	+
12	+	+	−	+
13	−	−	+	+
14	+	−	+	+
15	−	+	+	+
16	+	+	+	+

Consider the problem of selecting a useful half fraction of a 2^4 design. The full design matrix (for a single replication) is shown in this slide.

Click each tab to view the selection of different design points. When you are ready, click next to continue.

Tab 1: **The First 8 Design Points**

First Eight Design Points

- As none of the chosen design points involve factor D at its high level, comparisons of high D with low d are not possible.

Design Point	A	B	C	D
1	–	–	–	–
2	+	–	–	–
3	–	+	–	–
4	+	+	–	–
5	–	–	+	–
6	+	–	+	–
7	–	+	+	–
8	+	+	+	–
9	–	–	–	+
10	+	–	–	+
11	–	+	–	+
12	+	+	–	+
13	–	–	+	+
14	+	–	+	+
15	–	+	+	+
16	+	+	+	+

If the first 8 design points were chosen as a half fraction, it would be impossible to estimate the D main effect or any interaction of factor D with any of the other factors; as none of the chosen design points involve factor D at its high level; comparisons of High D with Low D are not possible.

Tab 2: **The Middle Eight Design Points**

Middle Eight Design Points

- Choosing design points from five to 12 permits an estimate of the D main effect.
- However, Factors C and D are totally confounded in this design.
  - -C is an alias for D.

Design Point	A	B	C	D
1	–	–	–	–
2	+	–	–	–
3	–	+	–	–
4	+	+	–	–
5	–	–	+	–
6	+	–	+	–
7	–	+	+	–
8	+	+	+	–
9	–	–	–	+
10	+	–	–	+
11	–	+	–	+
12	+	+	–	+
13	–	–	+	+
14	+	–	+	+
15	–	+	+	+
16	+	+	+	+

One way of overcoming this particular difficulty is to choose the middle half of the design points, that is, design points 5 to 12. Now, half of the 8 design points chosen involve factor D at it its low level and half at its high level, thus permitting a comparison of low and high D, that is, an estimate of the D main effect. However, note that the sign pattern for factor C in the chosen fraction is precisely the reverse of that for factor D.

Thus, the estimated effect of changing from low D to high D, estimated from data collected using this half fraction design, is the same as the estimate of the effect of changing from high C to low C; the effects of changing factors C and D cannot be distinguished from each other. Factors C and D are said to be *totally confounded* in this design or –C is an alias for D.

Tab 3: A More Complex Choice of Design Points

A More Complex Choice of Design Points

A Design Yielding Unambiguous Estimates

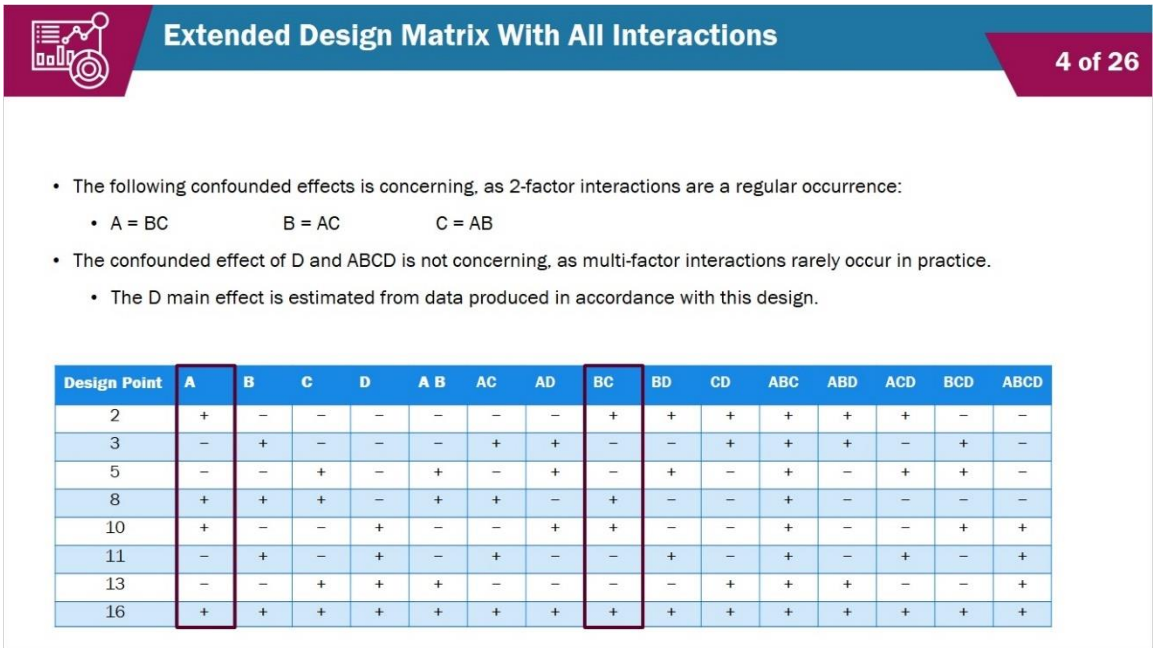
- Simple choices of the first or middle eight design points are inadequate when compared with a more complex choice.
- This design allows apparently unambiguous estimates of all four main effects.
- However, when interactions are considered, complications arise.

Design Point	A	B	C	D
1	–	–	–	–
2	+	–	–	–
3	–	+	–	–
4	+	+	–	–
5	–	–	+	–
6	+	–	+	–
7	–	+	+	–
8	+	+	+	–
9	–	–	–	+
10	+	–	–	+
11	–	+	–	+
12	+	+	–	+
13	–	–	+	+
14	+	–	+	+
15	–	+	+	+
16	+	+	+	+

Simple choices such as those examined above will not do. Consider the more complex choice shown in the design matrix shown in the slide, involving design points 2, 3, 5, 8, 10, 11, 13 and 16 from the full 2^4 design.

This design allows apparently unambiguous estimates of all 4 main effects. However, when interactions are considered, complications arise.

Slide 4: Extended Design Matrix with All Interactions



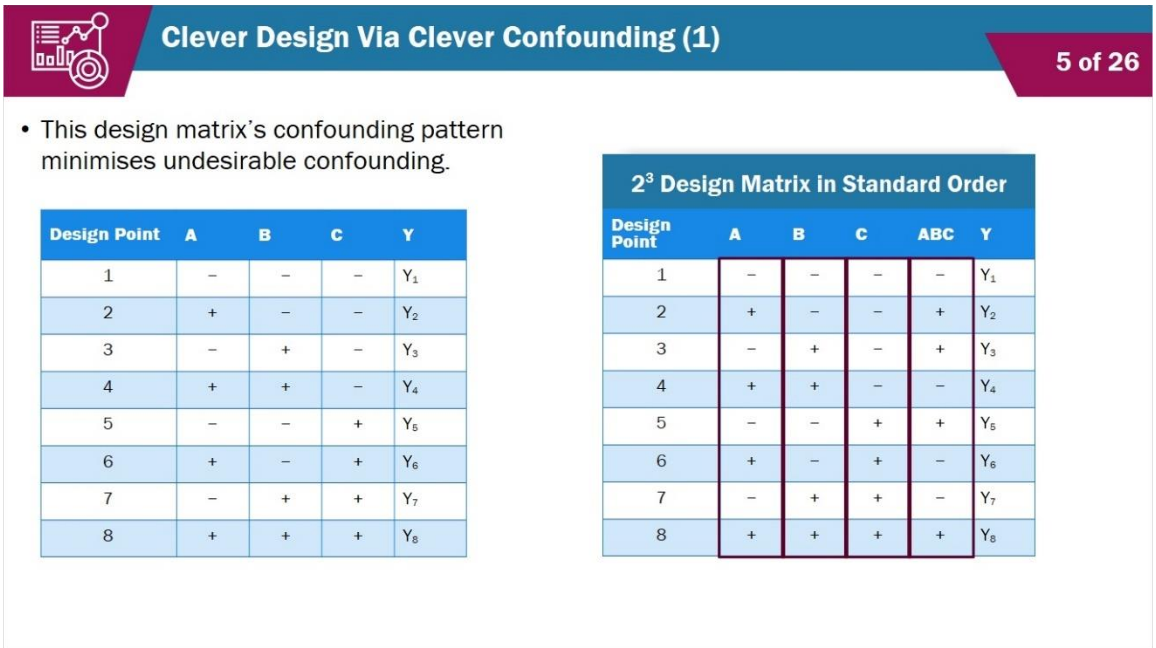
Design Point	A	B	C	D	A B	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD
2	+	−	−	−	−	−	−	+	+	+	+	+	+	−	−
3	−	+	−	−	−	+	+	−	−	+	+	+	−	+	−
5	−	−	+	−	+	−	+	−	+	−	+	−	+	+	−
8	+	+	+	−	+	+	−	+	−	−	+	−	−	−	−
10	+	−	−	+	−	−	+	+	−	−	+	−	−	+	+
11	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
13	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

This slide shows the extended design matrix with all interactions. Focussing on the main effects initially, note that the pattern of signs for the A main effect is identical with that for the BC 2-factor interaction. This means that if the corresponding contrast proved to be statistically significant, it would not be possible to say whether this was due to an A main effect, a BC interaction effect, or some combination of the two; the A main effect and the BC interaction effect are confounded in this design. Further examination shows that B and AC are confounded, C and AB are confounded and D and ABCD are confounded, and so on.

The fact that the other three main effects are confounded with 2-factor interactions is a matter of some concern; as evidenced already in several of the cases considered here, 2-factor interactions are a regular occurrence. Thus, on the basis of data produced in accordance with this design, it is not possible to provide unambiguous estimates of, for example, both the A main effect and the BC interaction effect. In fact, what is estimated by the corresponding contrast, shown here, is the sum of the two effects, A + BC. If either is 0, then the other is unambiguously estimated.

The fact that D and ABCD are confounded in this design is generally regarded as not being of too much concern, on the basis that high order (multi-factor) interactions rarely occur in practice. If this is so, and the four-factor ABCD interaction may be ignored, then the D main effect is estimated unambiguously from data produced in accordance with this design.

Slide 5: Clever Design Via Clever Confounding (1)


Clever Design Via Clever Confounding (1)

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- This design matrix's confounding pattern minimises undesirable confounding.

Design Point	A	B	C	Y
1	–	–	–	Y_1
2	+	–	–	Y_2
3	–	+	–	Y_3
4	+	+	–	Y_4
5	–	–	+	Y_5
6	+	–	+	Y_6
7	–	+	+	Y_7
8	+	+	+	Y_8

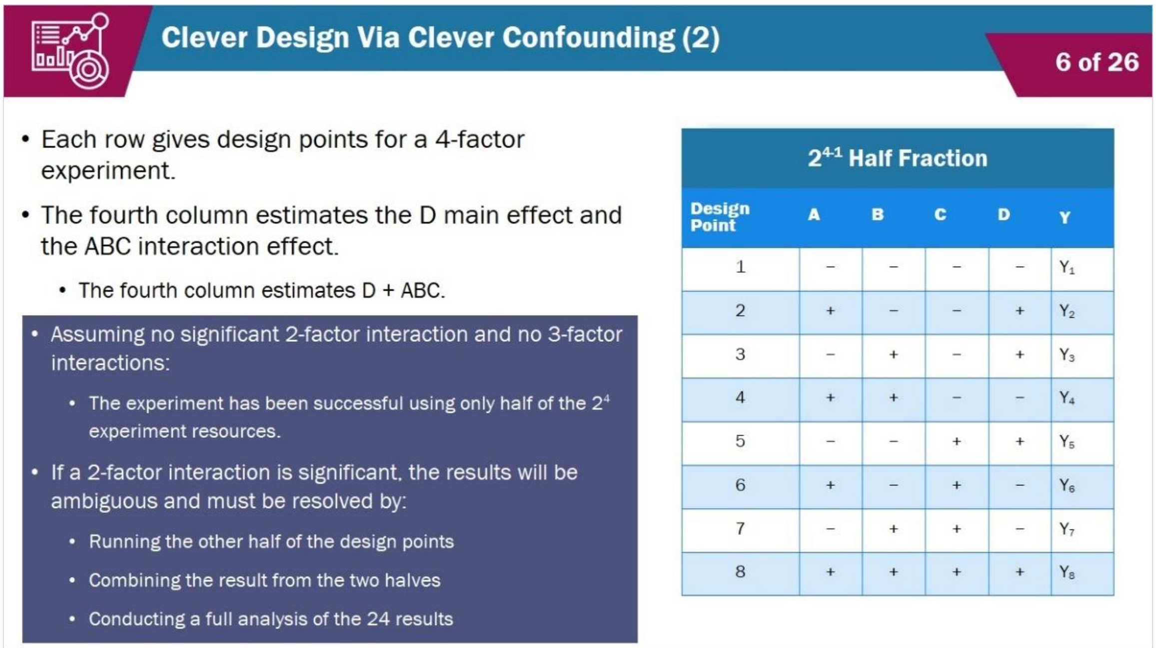
2^3 Design Matrix in Standard Order

Design Point	A	B	C	ABC	Y
1	–	–	–	–	Y_1
2	+	–	–	+	Y_2
3	–	+	–	+	Y_3
4	+	+	–	–	Y_4
5	–	–	+	+	Y_5
6	+	–	+	–	Y_6
7	–	+	+	–	Y_7
8	+	+	+	+	Y_8

The design matrix shows a clever choice of confounding pattern that minimises undesirable confounding.

It was constructed by setting up a full 2^3 design in columns A, B and C and then entering the sign pattern corresponding to the ABC 3-factor interaction in column D.

Slide 6: Clever Design Via Clever Confounding (2)


Clever Design Via Clever Confounding (2)

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- Each row gives design points for a 4-factor experiment.
- The fourth column estimates the D main effect and the ABC interaction effect.
  - The fourth column estimates D + ABC.
- Assuming no significant 2-factor interaction and no 3-factor interactions:
  - The experiment has been successful using only half of the 2^4 experiment resources.
- If a 2-factor interaction is significant, the results will be ambiguous and must be resolved by:
  - Running the other half of the design points
  - Combining the result from the two halves
  - Conducting a full analysis of the 24 results

2^{4-1} Half Fraction

Design Point	A	B	C	D	Y
1	–	–	–	–	Y_1
2	+	–	–	+	Y_2
3	–	+	–	+	Y_3
4	+	+	–	–	Y_4
5	–	–	+	+	Y_5
6	+	–	+	–	Y_6
7	–	+	+	–	Y_7
8	+	+	+	+	Y_8

This gives a perfectly viable 4-factor design matrix, with every row providing valid factor level assignments. It also entails that, when the data become available, the calculation of the D main effect will also provide the value for the ABC interaction, that is, D is confounded with ABC or ABC is an alias for D.

It can be shown that no main effect is confounded with any 2-factor interaction. Thus, if these 8 design points are run, no 2-factor interaction is significant and it is assumed that there are no 3-factor interactions, then the experiment will have been successful while using only half of the resources of a full 2^4 experiment.

On the other hand, if a 2-factor interaction does show as significant, then the results will be ambiguous. The only way to resolve this ambiguity is to run the other half of the design points and combine the result from the two halves and conduct a full analysis of the 2^4 results.

Slide 7: Exercise

Exercise

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- Can you confirm:
- The confounding or aliasing patterns shown in the table?
- That AB = CD?
- What other effects are aliased?

Design Point	A =BCD	B =ACD	C =ABD	D =ABC	Y
1	–	–	–	–	Y_1
2	+	–	–	+	Y_2
3	–	+	–	+	Y_3
4	+	+	–	–	Y_4
5	–	–	+	+	Y_5
6	+	–	+	–	Y_6
7	–	+	+	–	Y_7
8	+	+	+	+	Y_8

Take time to think about your answers.

As an exercise, you should confirm the confounding or aliasing patterns shown in the table. Also, you should confirm that AB = CD. What other effects are aliased?

We will discuss this further during the tutorial session.

Slide 8: Fractional Factorial Designs: Half Fractions and Full Design Matrix

Fractional Factorial Designs: Half Fractions and Full Design Matrix

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We can identify the eight design points from the full design matrix.

First Half Fraction

Design Point	A	B	C	D	Y
1	–	–	–	–	70
2	+	–	–	+	62
3	–	+	–	+	88
4	+	+	–	–	81
5	–	–	+	+	60
6	+	–	+	–	49
7	–	+	+	–	88
8	+	+	+	+	79

Fractional Factorial Designs

Full Factorial Design Matrix

	Design Point	A	B	C	D	Y
1	1	–	–	–	–	70
	2	+	–	–	+	60
	3	–	+	–	+	89
4	4	+	+	–	–	81
	5	–	–	+	+	60
6	6	+	–	+	–	49
7	7	–	+	+	–	88
	8	+	+	+	+	82
	9	–	–	–	+	69
2	10	+	–	–	+	62
3	11	–	+	–	+	88
	12	+	+	–	+	81
5	13	–	–	+	+	60
	14	+	–	+	+	52
	15	–	+	+	+	86
8	16	+	+	+	+	79

Click the tab to learn more. Then, click Next to continue.

Consider a 2^4 experiment with the selected 8 design points on the left and the full design matrix on the right. We can identify the 8 design points from the full design matrix.

Click the tab to learn more. When you are ready, click next to continue.

Tab 1: Fractional Factorial Designs

Fractional Factorial Designs

First and Second Half Fractions

First Half Fraction: Selected Design Points

Design Point	A	B	C	D	Y
1	–	–	–	–	70
10	+	–	–	+	62
11	–	+	–	+	88
4	+	+	–	–	81
13	–	–	+	+	60
6	+	–	+	–	49
7	–	+	+	–	88
16	+	+	+	+	79

- Column A estimates A + BCD

Second Half Fraction: Unselected Design Points

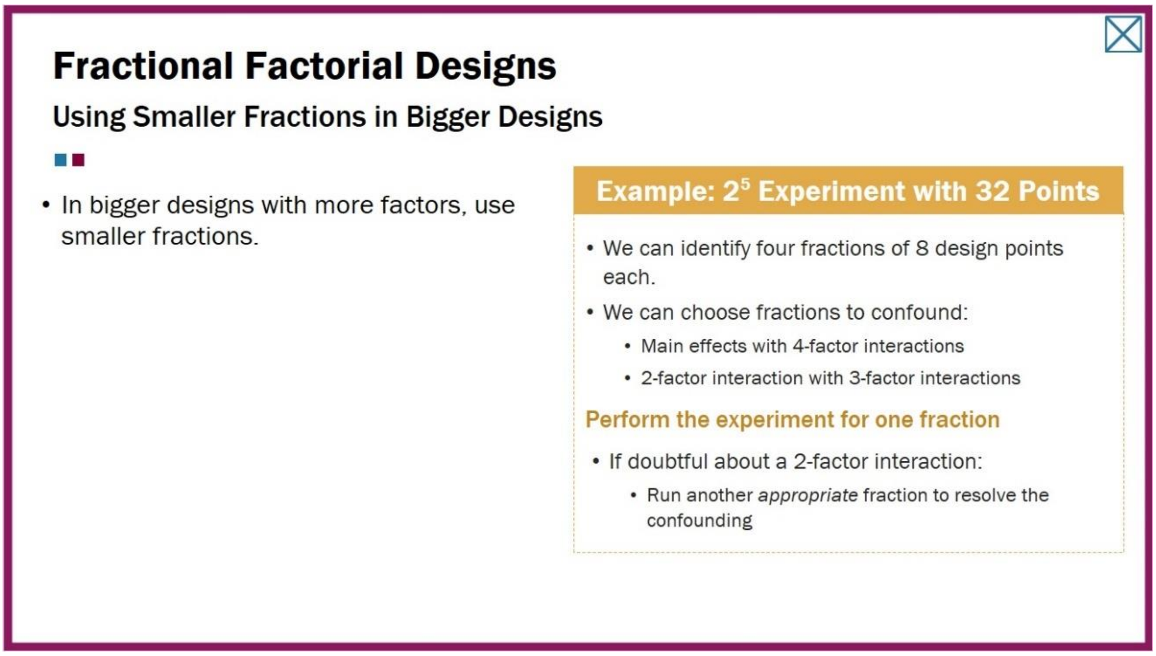
Design Point	A	B	C	D	Y
9	–	–	–	–	69
2	+	–	–	+	60
3	–	+	–	+	89
12	+	+	–	–	81
5	–	–	+	+	60
14	+	–	+	–	52
15	–	+	+	–	86
8	+	+	+	+	82

- Column A estimates A – BCD
- For the full 2^4 design:
  - 1/2[(A + BCD) + (A - BCD)] = A

The selected 8 design points forms the first half fraction. Since A is aliased with BCD, column A in fact estimates A + BCD. For the second half fraction formed using the 8 unselected design points, column A in fact estimates A – BCD.

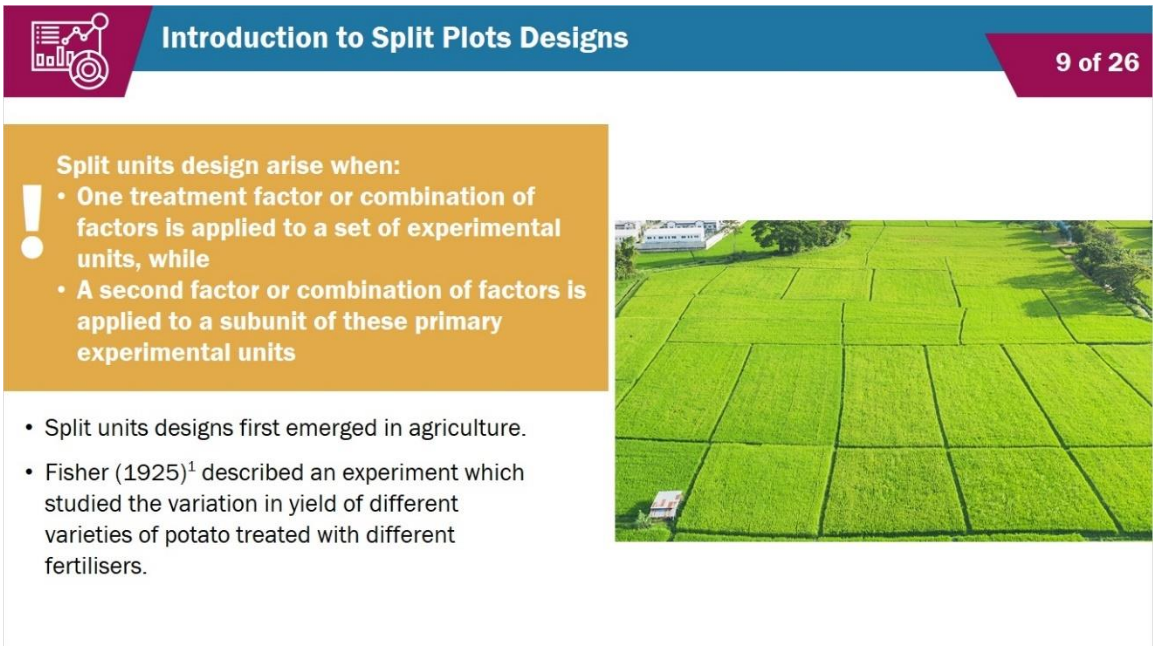
For the full design matrix, column A estimates the average of A + BCD and A – BCD which is equal to A.

Tab 1.1: Using Smaller Fractions in Bigger Designs



With bigger designs with more factors, smaller fractions may be used. For example, with a 2^5 experiment with 32 design points, we could identify 4 fractions of 8 design points each. We could choose fractions to confound main effects with 4-factor interactions, and 2-factor interaction with 3-factor interactions. We perform the experiment for one fraction, and if doubtful about the possibility of a 2-factor interaction, we could run another appropriate fraction to resolve the confounding.

Slide 9: Introduction to Split Plots Design

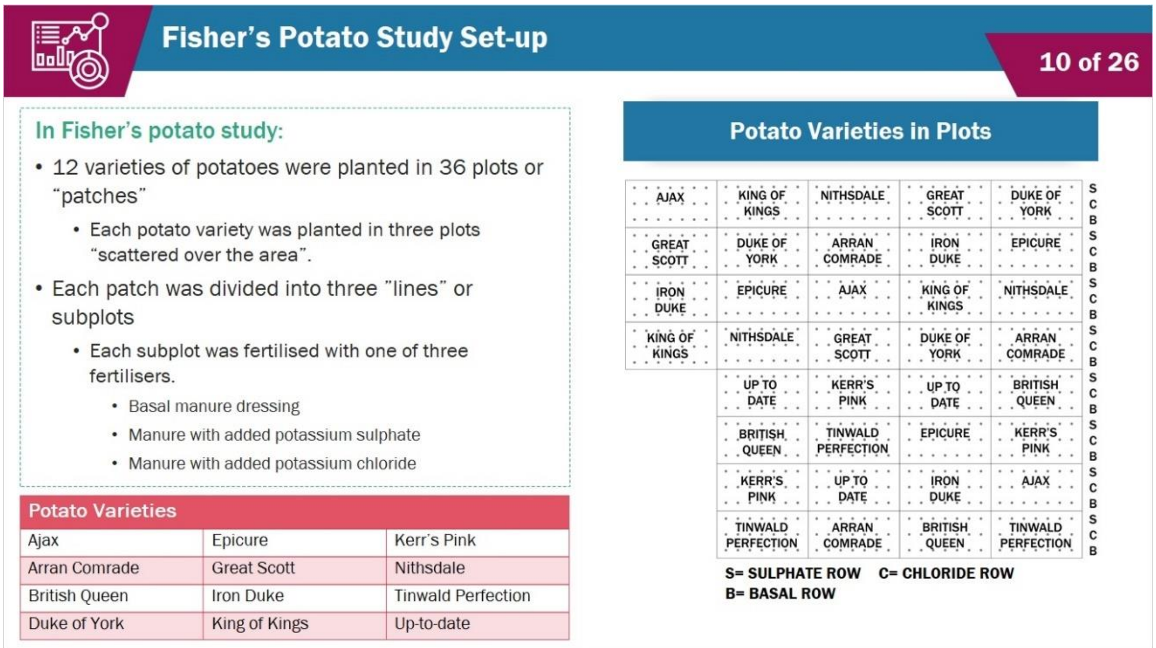


Split units designs arise when one treatment factor or combination of factors is applied to a set of experimental units while a second factor or combination of factors is applied to subunits of these primary experimental units. They arose originally in agriculture where they are referred to as split plots designs.

Fisher (1925)[1] gave an account of an experiment carried out at Rothamsted Experimental Station to study the variation in yield of different varieties of potato treated with different fertilisers.

For the remainder of this session, we will be focussing on this experiment.

Slide 10: Fisher's Potato Study Set-up



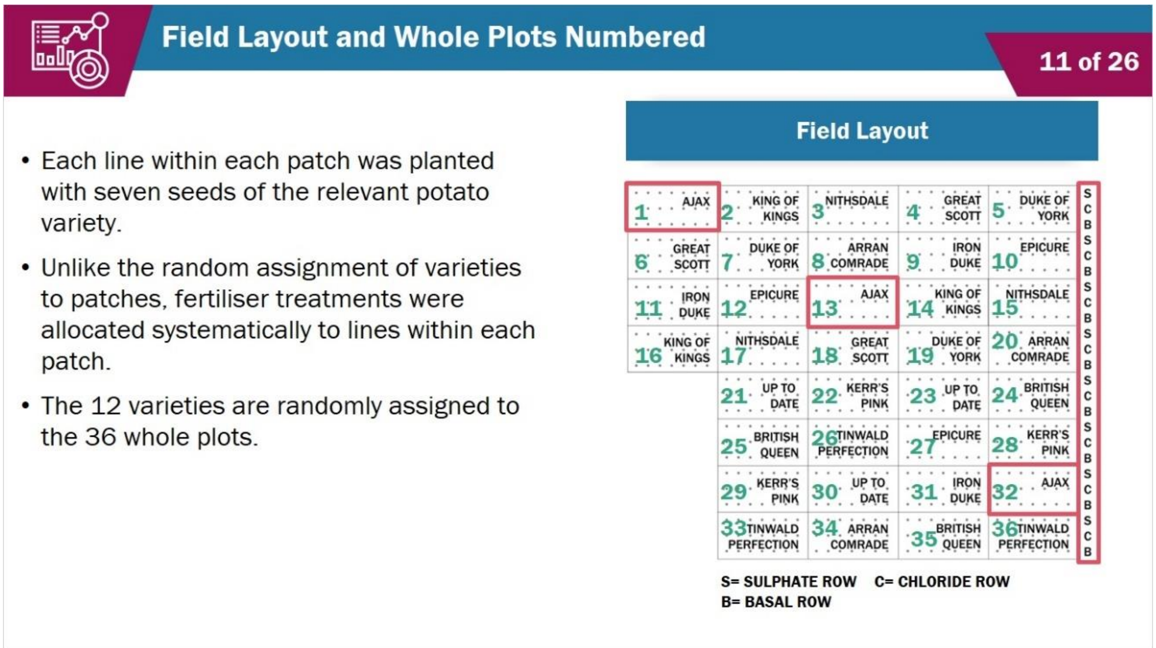
In the experiment, twelve varieties of potatoes were planted in a field divided into 36 "patches", or plots, each variety being planted in three patches "scattered over the area". The potato varieties are listed in this slide.

Each patch was divided into three "lines", or subplots. Within each patch, each line was fertilised with one of three fertilisers;

- a basal dressing of farmyard manure supplemented with superphosphate and ammonium sulphate,
- the basal dressing with added potassium sulphate and
- the basal dressing with added potassium chloride.

[1] Statistical Methods for Research Workers, Oliver & Boyd, 1925; 14th ed. Hafner Publishing Company, 1970; reprinted in Statistical Methods, Experimental Design and Scientific Inference, R.A. Fisher, Oxford University Press, 1990.

Slide 11: Field Layout and Whole Plots Numbered



Each line within each patch was planted with seven seeds of the relevant potato variety. Note that the rows or lines of seven dots represent actual plants within each patch. Also, while having the three patches per variety "scattered over the area" may be taken to approximate random assignment of varieties to patches, the fertiliser treatments were allocated systematically to lines within each patch.

It is convenient to number the patches 1 to 36 as shown in the table.

The 12 varieties are randomly assigned to the 36 whole plots.

Slide 12: Data Results: Yield (lbs Per Plant)

Data Results: Yield (lbs Per Plant)

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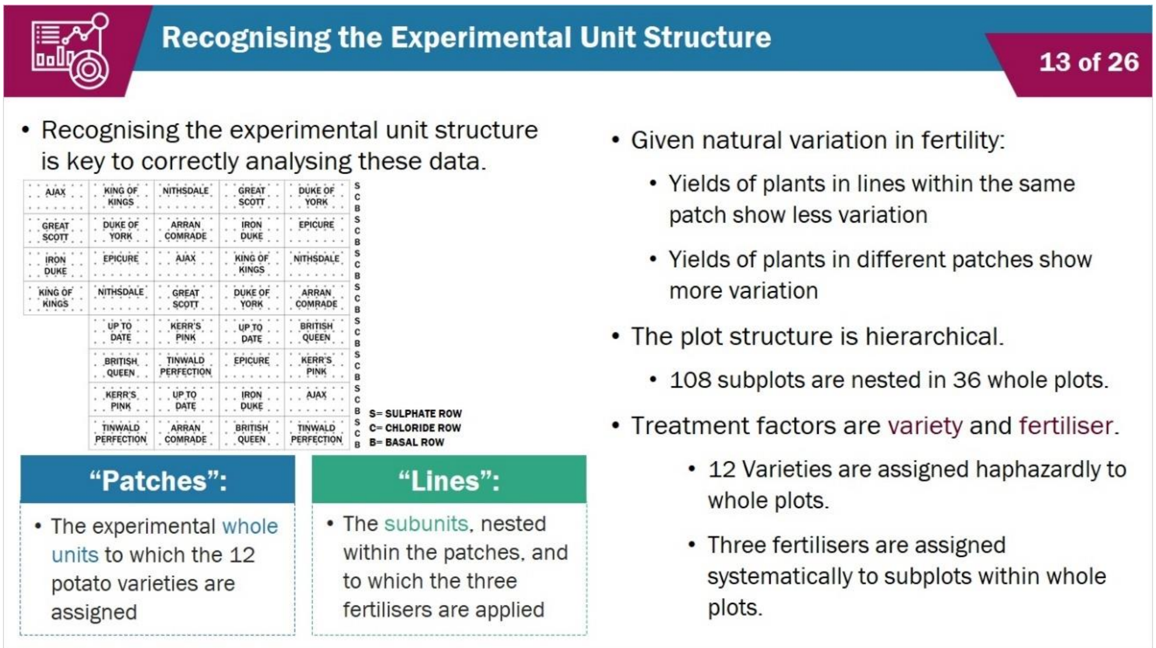
Plot	Mean Yield
1	2.86
13	2.93
32	4.23

Variety	Sulphate			Chloride			Basal		
Ajax	3.20	4.00	3.86	2.55	3.04	4.13	2.82	1.75	4.71
Arran Comrade	2.25	2.56	2.58	1.96	2.15	2.10	2.42	2.17	2.17
British Queen	3.21	2.82	3.82	2.71	2.68	4.17	2.75	2.75	3.32
Duke of York	1.11	1.25	2.25	1.57	2.00	1.75	1.61	2.00	2.46
Epicure	2.36	1.64	2.29	2.11	1.93	2.64	1.43	2.25	2.79
Great Scott	3.38	3.07	3.89	2.79	3.54	4.14	3.07	3.25	3.50
Iron Duke	3.43	3.00	3.96	3.33	3.08	3.32	3.50	2.32	3.29
King of Kings	3.71	4.07	4.21	3.39	4.63	4.21	2.89	4.20	4.32
Kerr's Pink	3.04	3.57	3.82	2.96	3.18	4.32	2.00	3.00	3.88
Nithsdale	2.57	2.21	3.58	2.04	2.93	3.71	1.96	2.86	3.56
Tinwald Perfection	3.46	3.11	2.50	2.83	2.96	3.21	2.55	3.39	3.36
Up to Date	4.29	2.93	4.25	3.39	3.68	4.07	4.21	3.64	4.11

The resulting data is presented in the table where *Yields, in pounds per plant, of twelve varieties of potato each planted in three patches, each patch arranged in three lines, each line treated with one of three fertilisers. The average yield for each plot can be calculated. For example, the average yield for plot 1 is 2.86.*

Slide 13: Recognising the Experimental Unit Structure



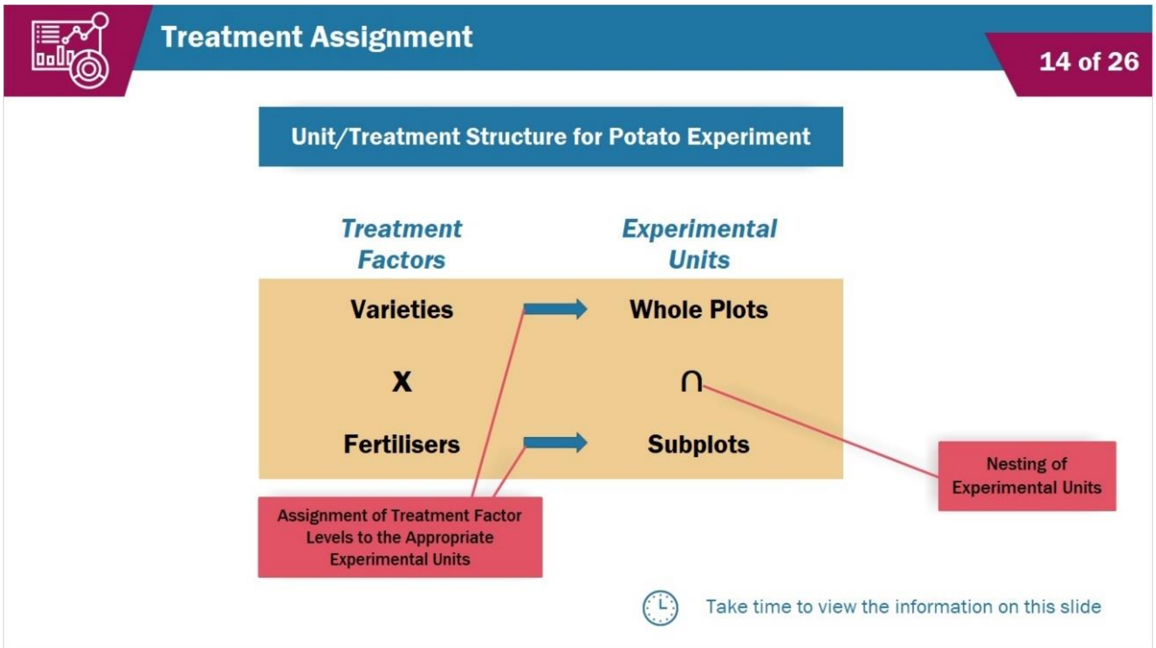
The key to the correct analysis of these data is recognising the experimental unit structure involved. Here, "patches" are the experimental units to which the twelve potato varieties are assigned, and "lines" are the (smaller) experimental units, nested within the patches, to which the three fertilisers are applied. In the standard terminology, the patches are regarded as *whole units* and the lines as *subunits.* Given natural variation in fertility, it may be assumed that yields of plants in lines within the same patch show less variation than yields of plants in different patches.

Note that the subunits are nested in the whole units, meaning that each subunit is part of a unique whole unit or, equivalently, the subunits in a whole unit are different from the subunits in a different whole unit, just as the birds in a nest are different from the birds in a different nest. This unit structure is hierarchical, with two levels, with patches at the top level and lines nested in patches at the second level. There were 108 subunits (lines) nested in 36 whole units (patches).

The treatment factors, Variety and Fertiliser, are applied at different levels of the unit structure. The varieties are assigned to Patches in a haphazard manner, described as "scattered over the area" as noted above. This may be intended to approximate randomised assignment.

The fertilisers are assigned systematically to Lines within each patch, not a randomised assignment.

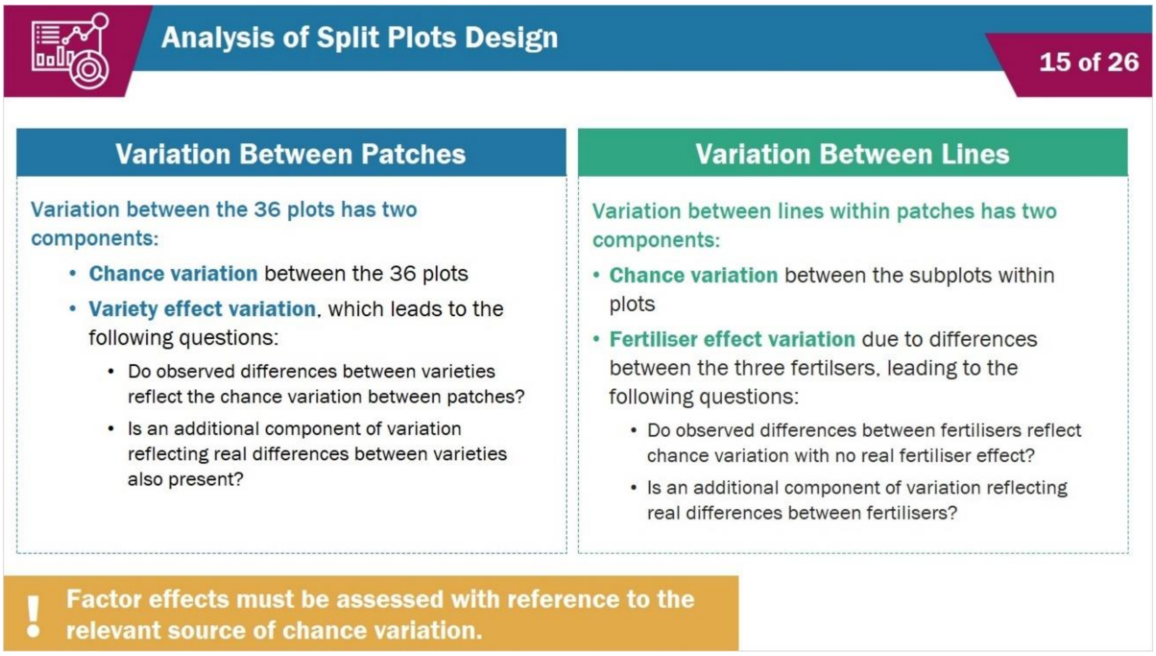
Slide 14: Treatment Assignment



The unit / treatment structure for the Potatoes experiment may be illustrated as in the Unit / Treatment structure diagram shown in this slide. Here, "∩" indicated nesting of experimental units and "→" indicates assignment of treatment factor levels to the appropriate experimental units.

Take time to view this information. When you are ready, click next to continue.

Slide 15: Analysis of Split Plots Design



Variation between the 36 patches has two components,

1. chance variation between the thirty-six patches, that is, between the experimental units to which the varieties are applied, variation that would be present even if only a single variety was used throughout or if there was no variety effect,

and

2. variation due to differences between the twelve varieties, that is, variety effect.

When assessing variety effect, the question to be addressed is whether observed differences between varieties reflects merely the chance component of the variation between the patches, with no real variety effect, or whether an additional component of variation reflecting real differences between the varieties is also present.

Variation between lines within patches also has two components,

1. chance variation between the lines within patches, the experimental units to which the fertilisers are applied,

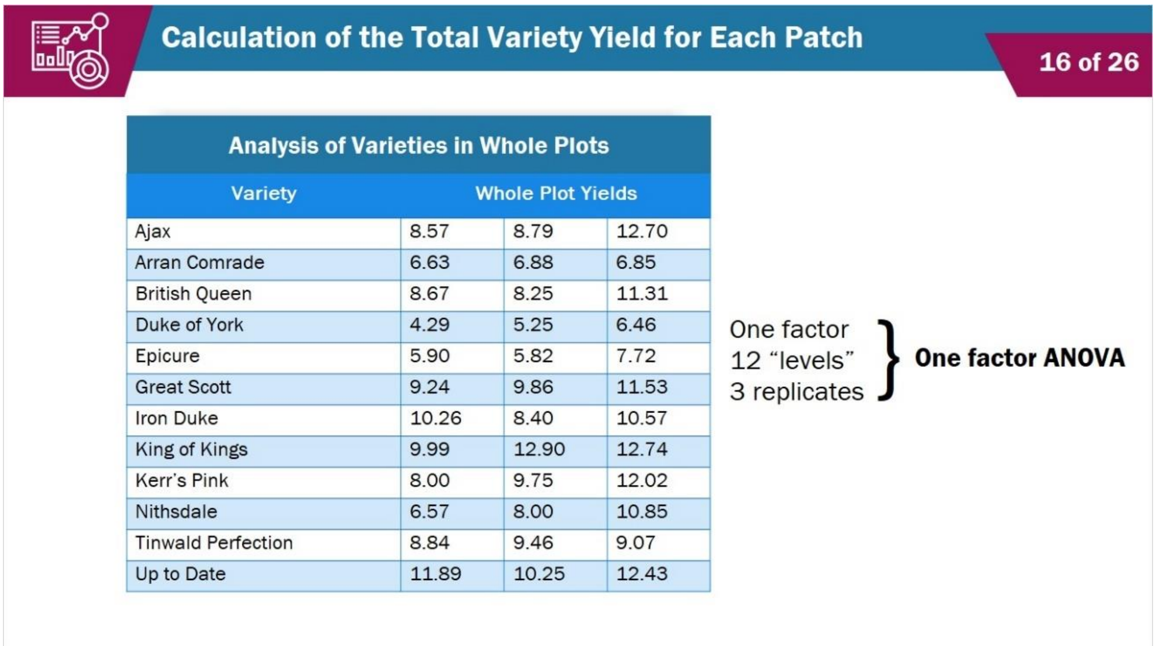
and

2. variation due to differences between the three fertilisers, that is, fertiliser effects.

When assessing fertiliser effects, the question is whether observed differences between fertilisers reflects merely the chance variation between lines within patches, with no real fertiliser effect, or whether there is an additional component of variation reflecting real differences between fertilisers

In both cases, the factor effects must be assessed by reference to the relevant source of chance variation, that is, patches for variety effects and lines within patches for fertiliser effects.

Slide 16: Calculation of the Total Variety Yield for Each Patch

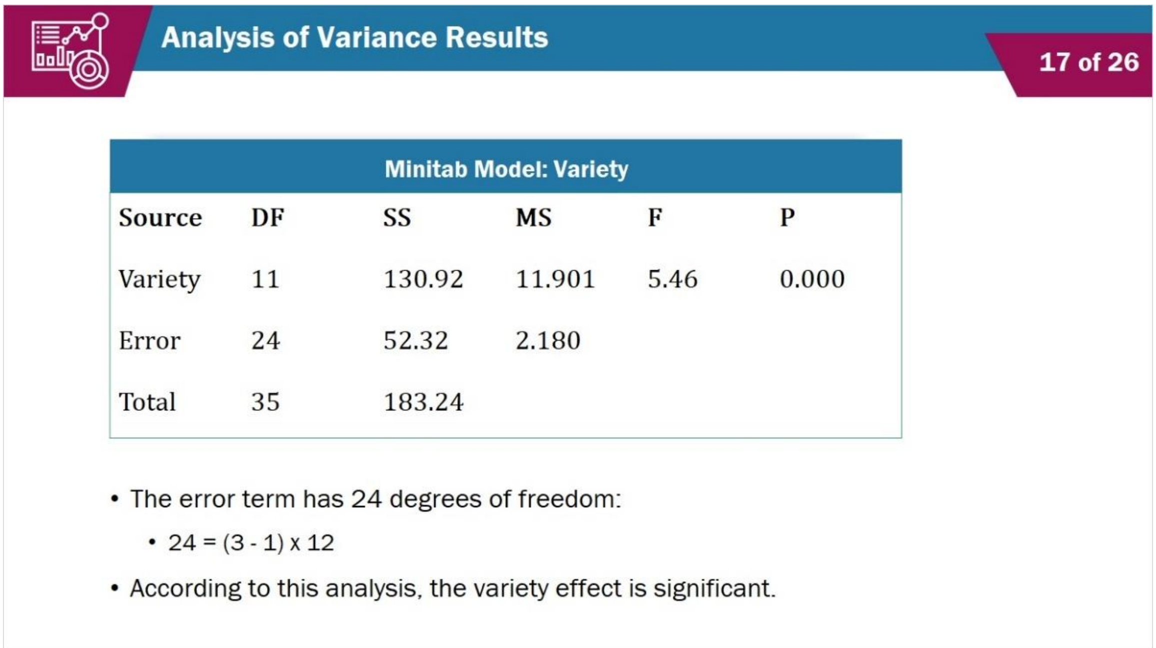


Analysis of Varieties in Whole Plots			
Variety	Whole Plot Yields		
Ajax	8.57	8.79	12.70
Arran Comrade	6.63	6.88	6.85
British Queen	8.67	8.25	11.31
Duke of York	4.29	5.25	6.46
Epicure	5.90	5.82	7.72
Great Scott	9.24	9.86	11.53
Iron Duke	10.26	8.40	10.57
King of Kings	9.99	12.90	12.74
Kerr's Pink	8.00	9.75	12.02
Nithsdale	6.57	8.00	10.85
Tinwald Perfection	8.84	9.46	9.07
Up to Date	11.89	10.25	12.43

To see how the combined assessment can be achieved, it helps to see the separate assessments first. To assess the variety effects, noting that each patch was planted with a single variety so that varieties do not vary between lines, it makes sense to calculate the total variety yield for each patch, that is, sum line yields within patches. The results of doing this are shown in the Table.

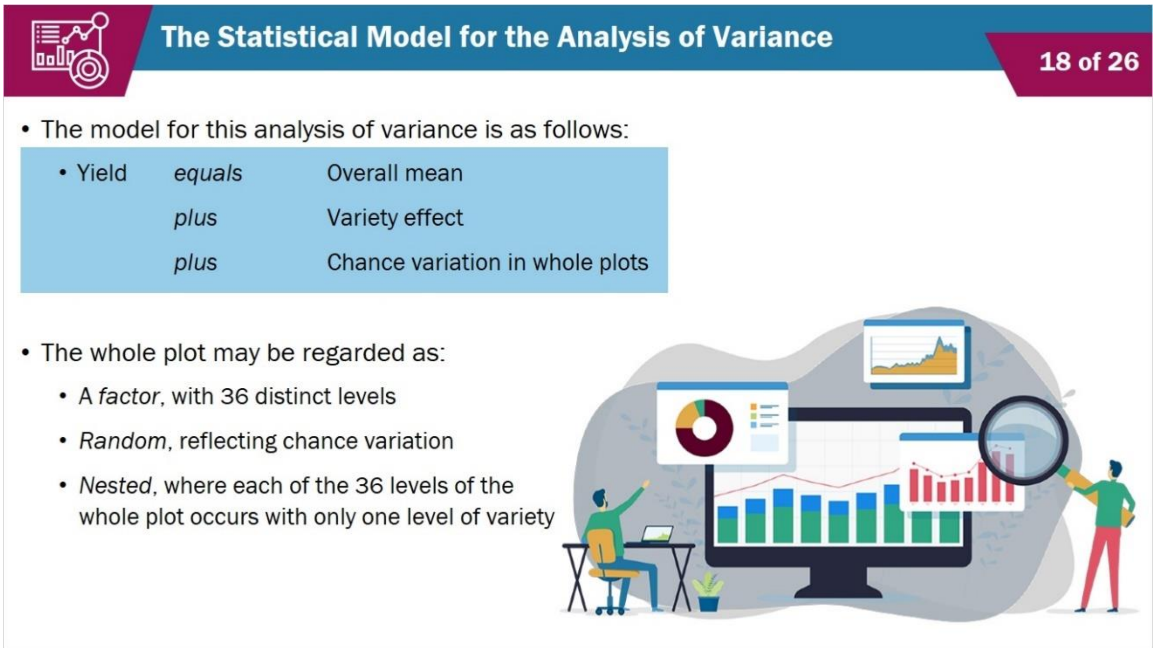
These data may be modelled using one-way analysis of variance.

Slide 17: Analysis of Variance Results



The analysis of variance results is shown in this slide. The Variety mean square estimates pure chance variation between patches plus variation due to differences between varieties, if any. The Error mean square estimates just pure chance variation between patches. Thus, their ratio, the F-value in the Analysis of Variance table, allows an assessment of the extent of difference between varieties relative to pure chance variation, in other words, a test of the statistical significance of differences between varieties. Note that error term has 24 degrees of freedom. Also note that the variety effect is significant according to this analysis.

Slide 18: The Statistical Model for the Analysis of Variance



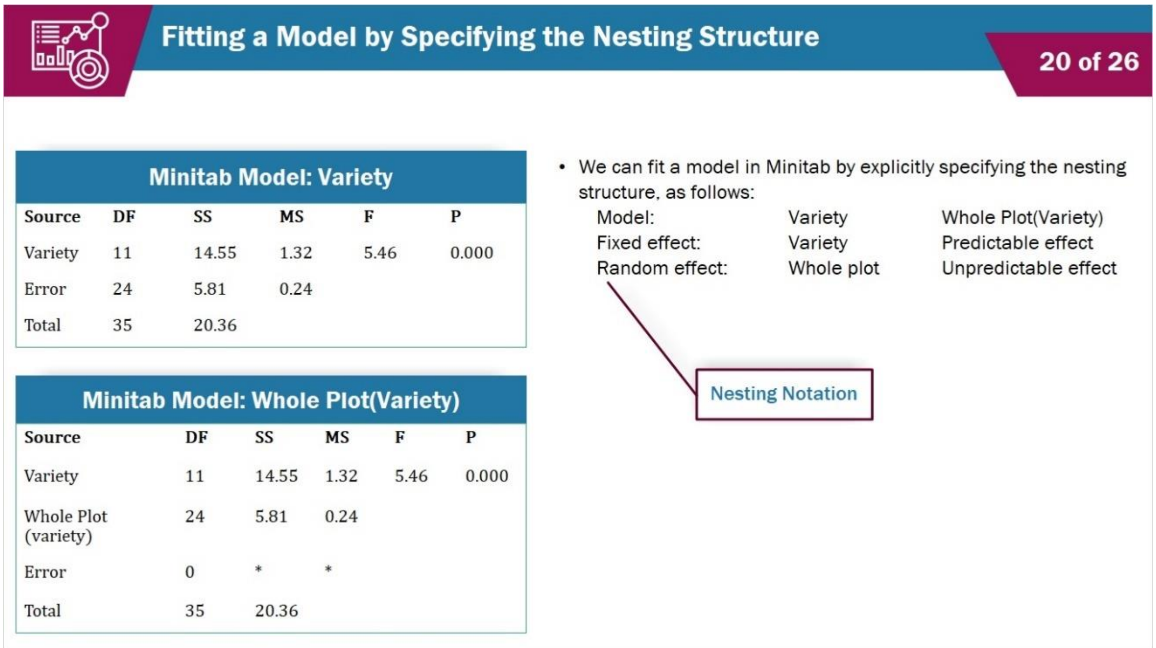
We now look at the statistical model underlying the analysis of variance. We mode the yield as the overall mean plus the variety effect plus chance variation in whole plots. We note that the whole plot may be regarded as a factor with 36 distinct levels. Since the whole plot also reflects chance variation, it is considered a random factor. There is also nesting structure where each of the 36 levels of whole plot occurs with just one level of variety.

Slide 19: Whole Plots Nesting Structure

Variety	Ajax	Arran Comrade	British Queen	Duke of York
Whole Plots	1 13 32	8 20 34	24 25 35	5 7 19
Variety	Epicure	Great Scott	Iron Duke	King of Kings
Whole Plots	10 12 27	4 6 18	9 11 31	2 14 16
Variety	Kerr's Pink	Nithsdale	Tinwald Perf'n	Up to date
Whole Plots	22 28 29	3 15 17	26 33 36	21 23 30

The nesting structure of whole plots is shown in this slide. We can see that whole plot numbers 1, 13, 32 are nested in the variety Ajax. Similarly, whole plot number 8, 20, 34 are nested within Arran Comrade, and so on and so forth.

Slide 20: Fitting a Model by Specifying the Nesting Structure



This top table is the one-way analysis of variance results obtained previously. We can also fit an equivalent model in Minitab by explicitly specifying the nesting structure. In the model definition, we declare the nesting structure whereby whole plot is nested within variety. The variety is declared as a fixed effect whereas the whole plot is declared as a random effect. This ensures that Minitab recognizes the whole plot mean square as an error mean square. The fixed effect can be considered as predictable effect whereas the random effect can be considered as unpredictable effect.

The results obtained from fitting this model are shown in the bottom table. This is effectively the same as the one-way analysis of variance results shown in the above table. With the Whole Plot (Variety) term taking the place of the error term.

Slide 21: Differences Between Fixed and Random Effects

Differences Between Fixed and Random Effects

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Random Effects

A single potato variety is planted in 10 plots.

- The yield variation is not predictable from plot to plot.
  - By regarding plot as a factor with 10 levels, plot is a random effects factor

Fixed Effects

Two potato varieties are planted in five plots each, one high yielding and one low yielding.

- Yield variation between the two sets of five plots is predictable.
  - Therefore, variety is a fixed effects factor.

Let's look at the differences between fixed and random effects. For example, suppose we have a single potato variety planted in 10 plots. The yield variation is not predictable from plot to plot. If we regard plot as a factor with 10 levels, plot is a random effects factor.

In contrast, suppose we have two potato varieties planted in 5 plots each where one variety is high yielding and another low yielding. Now, the yield variation between two sets of 5 plots is predictable. Therefore, variety is a fixed effects factor.

Slide 22: Effects of, and Interactions Between Variety and Fertiliser

Effects of, and Interactions Between, Variety and Fertiliser

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- We can fit a model in Minitab by explicitly specifying the nesting structure with whole plot nested within variety.

Model: Variety	Whole Plot(Variety) Fertiliser Variety*Fertiliser
Fixed effects factors:	Variety, fertiliser
Random effect factor:	Whole plot

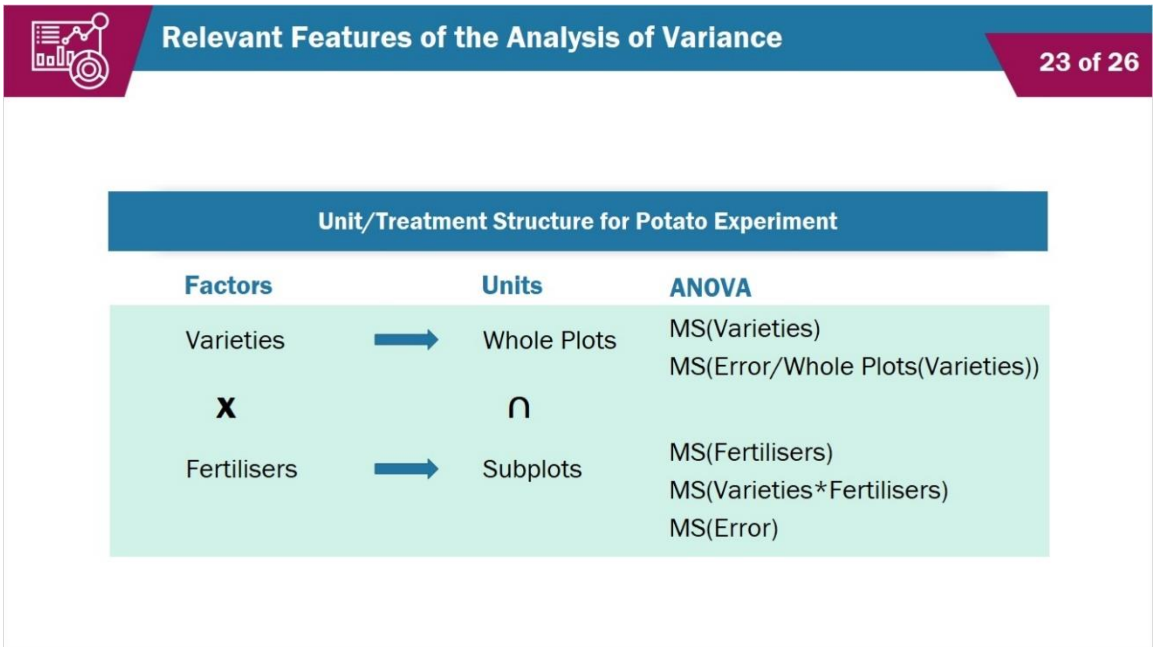
Source	DF	SS	MS	F	P
Variety	11	43.64	3.97	5.46	0.000
Whole Plot (variety)	24	17.44	0.73		
Fertiliser	2	0.35	0.17	1.04	0.362
Variety* Fertiliser	22	2.19	0.10	0.59	0.909
Error	48	8.10	0.17		
Total	107	71.70			

- This analysis shows that the interaction is not statistically significant.
  - There are no statistically significant fertiliser effects.

We now move on to the full analysis which allows us to examine the effects of variety, fertiliser and the interaction between variety and fertiliser. We declare the Minitab model by declaring the nesting structure with whole plot nested within variety. The variety and fertiliser are considered fixed effects factors whereas whole plot is considered a random effects factor.

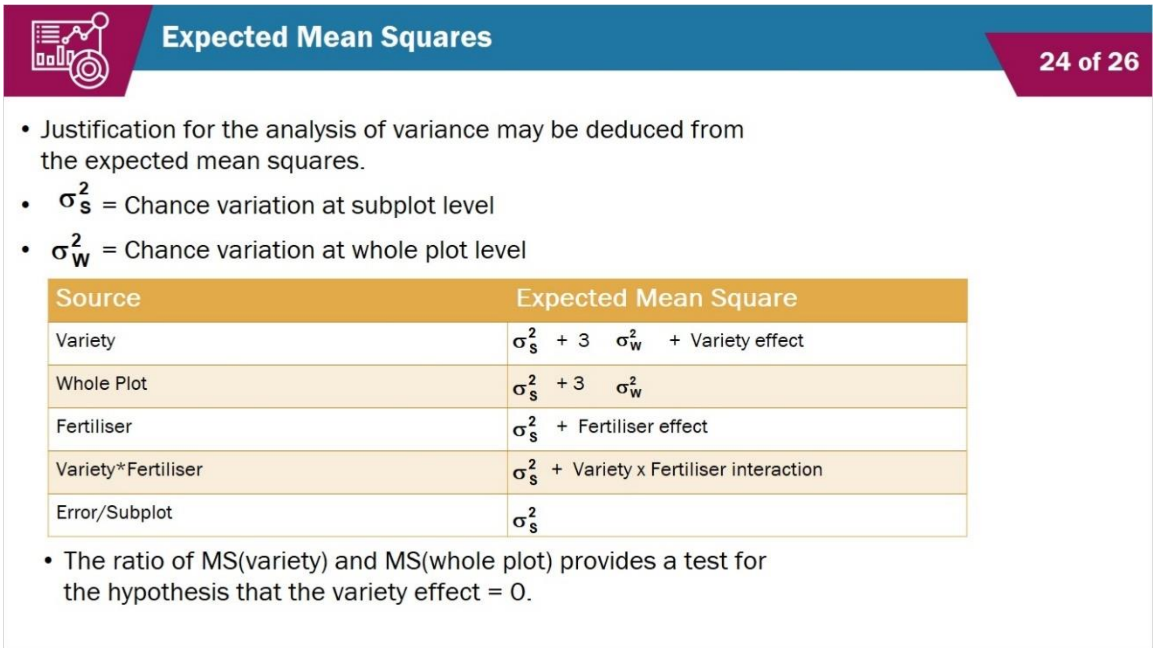
The results are shown in the table here. The analysis shows that the interaction is not statistically significant. Thus, there are no statistically significant fertiliser effects either as main effects or differentially with different varieties.

Slide 23: Relevant Features of the Analysis of Variance



The unit / treatment structure may be extended by showing relevant features of the analysis of variance, as shown in this slide. The factors variety and fertiliser are fully crossed, and we have a nesting structure for the units with subplots nested within whole plots. The effect of varieties is assessed using mean squares for varieties and mean square for whole plots. The fertiliser effect and the varieties by fertiliser interaction effect are assessed using mean square for fertilisers, mean square for varieties and fertilisers interaction, and mean square for subplots.

Slide 24: ## Expected Mean Squares

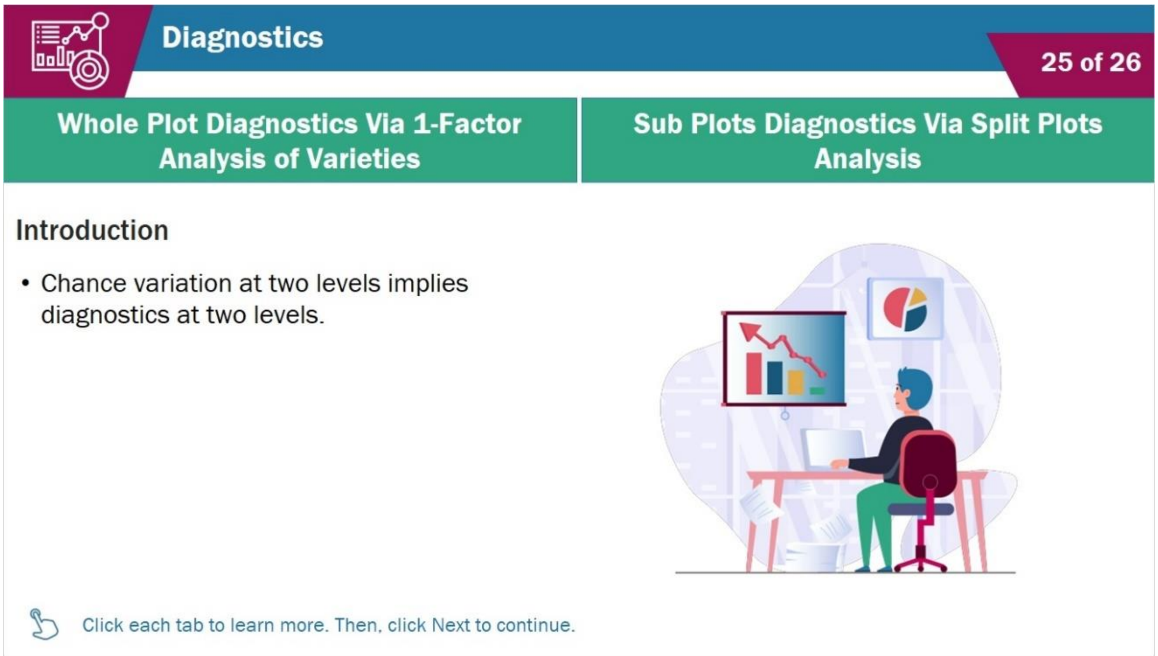


Justification for the analysis of variance may be deduced from the expected mean squares. These expected mean squares are shown in this slide. Here, σ_s^2 refers to chance variation at subplot level whereas σ_W^2 refers to chance variation at whole plot level. If Variety effect is 0, the expected mean squares for Variety and the expected mean square for whole plot are the same. Thus, the ratio of MS(Variety) and MS(whole plot) provides a test for the hypothesis that Variety effect = 0.

If Fertiliser effect is 0, expected mean square for fertiliser and expected mean square for subplot are the same. Thus, the ratio of mean square Fertiliser and mean square subplot provides a test for the hypothesis that Fertiliser effect = 0.

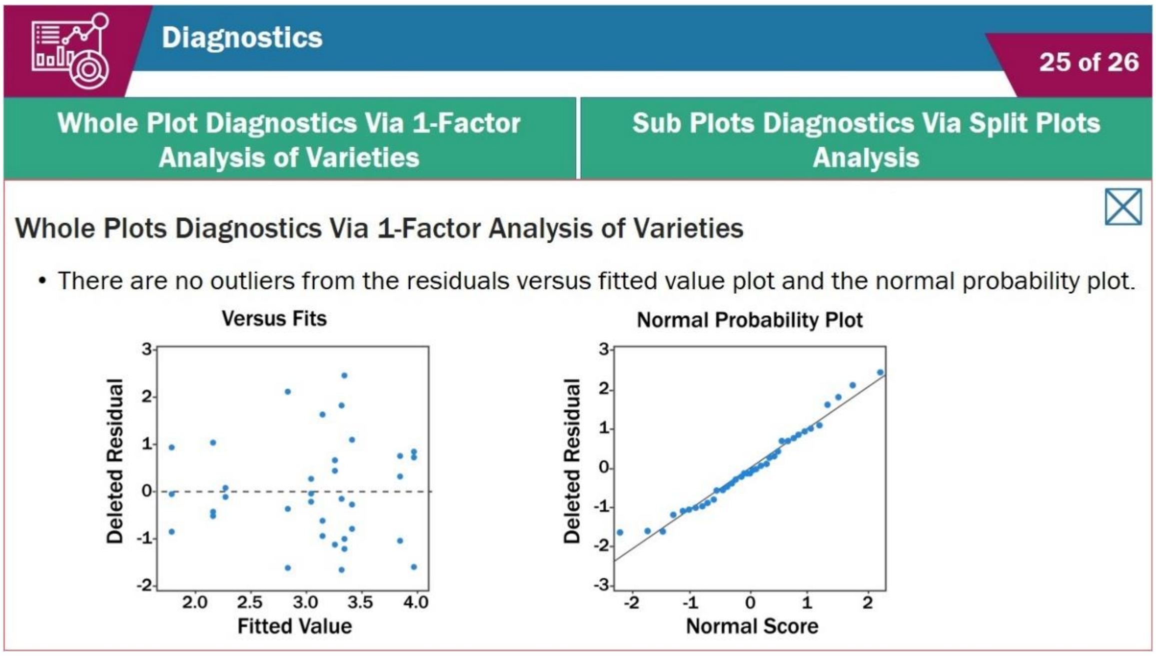
A similar justification applies to the test for no Variety by Fertiliser interaction.

Slide 25: **Diagnostics**



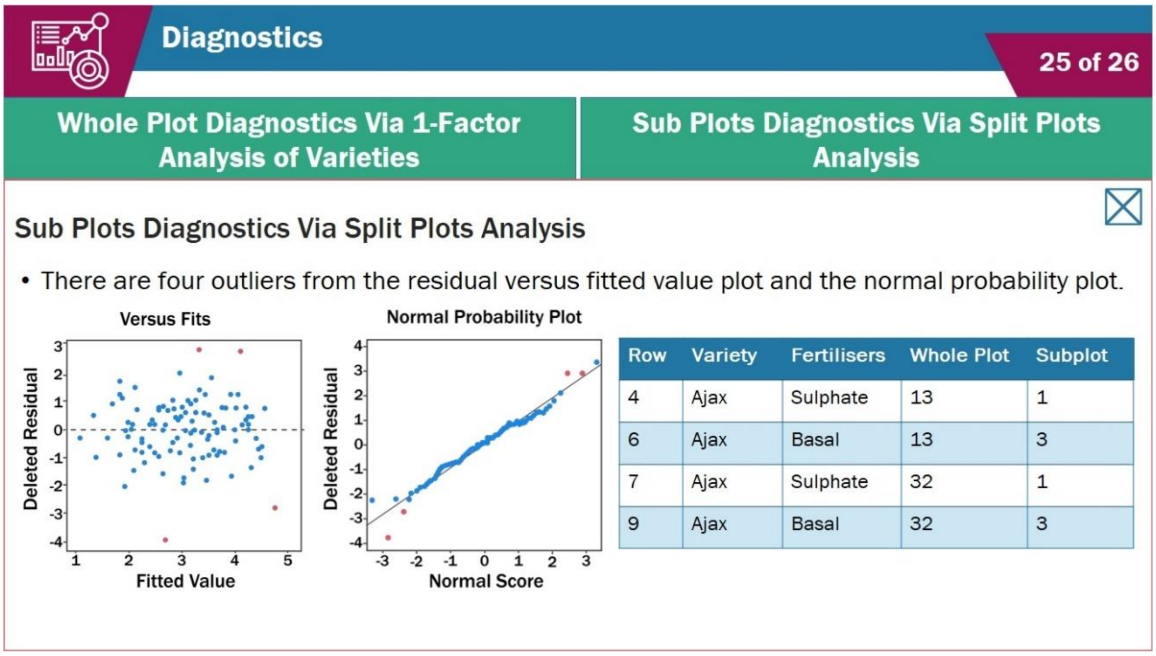
In terms of diagnostics, with chance variation at 2 levels, diagnostics need to be performed at 2 levels.

Tab 1: **Whole Plots Diagnostics Via 1-Factor Analysis of Varieties**



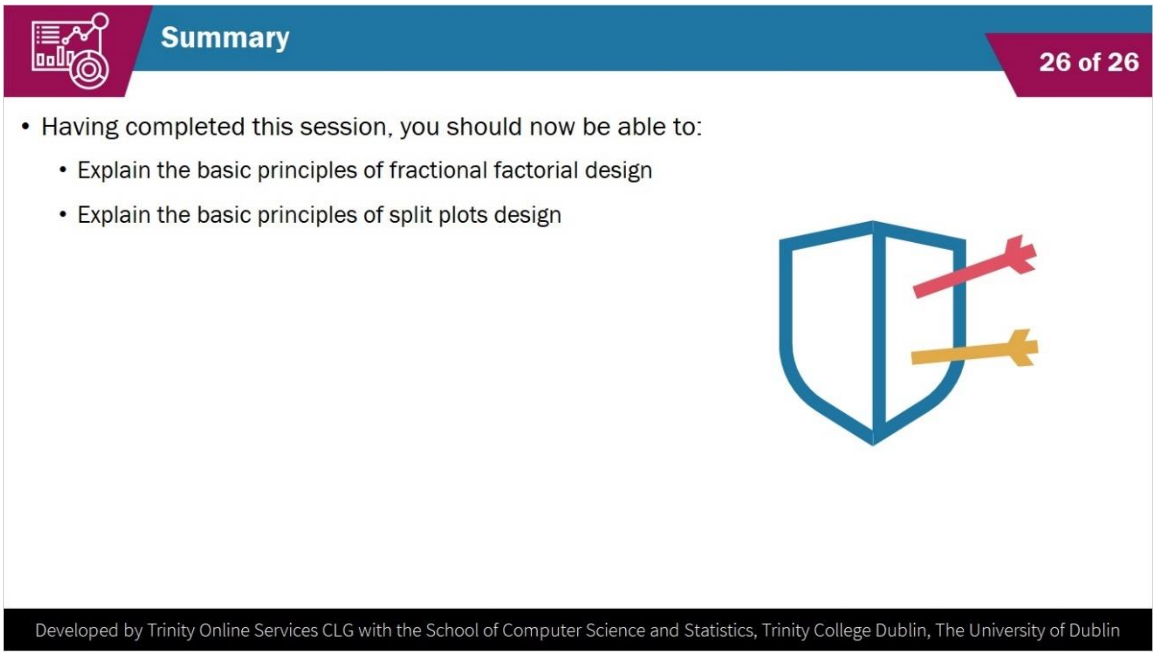
We can perform the whole plots diagnostics via the 1-factor analysis of varieties. We do not observe any outliers from the residuals versus fitted value plot and the normal probability plot.

Tab 2: **Sub Plots Diagnostics Via Split Plots Analysis**



The sub plots diagnostics can be performed based on the split plots analysis. We observe 4 outliers from the residual versus fitted value plot and the normal probability plot. We can identify the corresponding data points which are shown in this slide.

Slide 26: **Summary**



Having completed this session, you should now be able to:

- Explain the basic principles of fractional factorial design
- Explain the basic principles of split plots design