As Sean Carroll stated, mathematics is just determining the underlying structure between sets. * The underlying structure for a set of prospects (options) is the preference relation.

Decision Theory

https://plato.stanford.edu/entries/decision-theory/

Preferences and prospects (outcomes, options). Start from a notion that a preference indicates 'choice worthiness' between two options. Preference is always *comparative*.

We're concerned with ordering the set which seems to be formalised by a binary relation. Taking in two elements and returning some mathematical object that represents ordering. > preference concerns the comparison of options; it is a relation between options. For a domain of options we speak of an agent's preference ordering, this being the ordering of options that is generated by the agent's preference between any two options in that domain.

Ordering operators must defined under:

- Completeness: That for each outcome, there is some relationship (in terms of order) between it all outcomes in the set.
$$\forall A, B \in S$$
- **Transitivity** The argument for transitivity axiom is the *money pump* example.

Preference and preference ordering being two distinct things philosophically speaking. Defining a preference relation may not directly equate to some inner process of choice you possess.

Ordinal Scale

Cardinal Scale

- Interesting way of doing this. If you've an ordering of options $A \wr B \wr C$, create a new option L, that is a lottery between to options. If we want to determine how desired B is relative to C. We say that L is a chance to get either A or C and we change the chance of winning C until we determine an equality of preference with B. If there is a low chance to C but you still deem it just as preferable to B then you much prefer C as an option.
  > - For instance, if you are indifferent between Bangkok and a lottery that provides a very low chance of winning a trip to Cardiff, then you evidently do not regard Bangkok to be much better than Amsterdam, vis-à-vis Cardiff; for you, even a small improvement on Amsterdam,

> > i.e., a lottery with a small chance of Cardiff rather than Amsterdam, is enough to match Bangkok

> The orthodox normative decision theory, expected utility (EU) theory, essentially says that, in situations of uncertainty, one should prefer the option with greatest expected desirability or value.

Numerical representations of prospect ordering, utility functions. The idea of *lotteries* is key to quantifying and comparing preferences.

Decision Making

Formal representation is essential in decision theory. Decision rules only 'work' relative to a formalisation.

Details three levels of abstraction for a decision problem:

1. The decision problem itself.
2. The formalisation of that decision problem.
3. The visualisation of that formalisation.

Formalisations are malleable, in that representation of a decision problem can differ as long as the same information is being captured. Theres an interesting notion of *rival formalisms*.

Ordinal vs Cardinal Scale

In an ordinal scale we just have a set of preference that have no measure of preference, just an ordering. Ordinal scales are mathematically equivalent (in that no set of numbers gives a change in information than the other) if a function transforms the scale but preserves ordering.

In a cardinal scale, I would assume that ordinal transformations would not be cardinal transformations as differences between numbers quantifies preference.

For instance, the scale 4,3,2,1 is that same as 100, 20,10,9 in an ordinal scale but such a transformation converys completely different information in a cardinal scale.

Cardinal scales reflect the differences between measured outcomes.

Preference relation

The rationality involved in the book I'm reading is *instrumental rationality*

> instrumentally rational is to do whatever one has most reason to expect will fulfil ones aim. For instance, if your aim is not to get wet and it is raining heavily, you are rational in an instrumental sense if you bring an umbrella or raincoat when going for a walk.

> Instrumental rationality presupposes that the decision maker has some aim, such as becoming rich and famous, or helping as many starving refugees as possible. The aim is external to decision theory, and it is widely thought that an aim cannot in itself be irrational, although it is of course reasonable to think that sets of aims can sometimes be irrational, e.g. if they are mutually inconsistent.

It seems fundamental to decision theory to solve this preference relation problem. I'm not too sure why.

> we say that a decision is right if and only if its actual outcome is at least as good as that of every other possible outcome.

There is an initial set of outcomes. An agent defines a preference relation on these outcomes. A function that takes a domain of outcomes and converts them to a number for comparison of outcomes.

Lindley refers to this in chapter 3 as a 'numerical measure for consequences'.

Lotteries

Lotteries have outcomes as 'prizes' that are obtained with a given probability.

What's the point in preference ordering of acts? Is this to determine value in the decision tree?

It's a preference ordering of outcomes, not actions.

Savages' Theory

> Savage presents a set of axioms constraining preferences over a set of options that guarantee the existence of a pair of probability and utility functions relative to which the preferences can be represented as maximising expected utility.

The *primitives* are states and outcomes. States are uncertain, the way the world is or will be. Outcomes are good or bad state of affairs that matter to an agent.

Actions are functions from the state space to the outcome space.

> acts are functions from the state space to the outcome space, and the agent's preference ordering is taken to be defined over all such possible functions.

The act $f(s_i)$ denotes the outcome for a state of the world s_i that is actual. The expected utility of act f is

$$U(f) = \sum_i u(f(s_i))P(s_i)$$