Infinite series

- An infinite series is a sum of an infinite sequence of numbers
- Sums are determined with sequences by taking the limit of partial sums as the number of partial sums tends to infinity
- Partial sums could be thought of as generating a new sequence where

$$a_n = s_1 + ...s_n$$

. You can analyse then what that sum approaches

Geometric series

- Are of the form $a + ar + ar^2 + ... + ar^{n-1} + ... = \sum_{n=1}^{\infty} ar^{n-1}$
- a and r are fixed real numbers and $a \neq 0$
- It doesn't seem that r can be greater than 1?
- If $r = 1$ the series diverges $\lim_{n\to\infty} s_n = \pm\infty$ depending on the sign of a
- If $r = -1$ the partial sums alternate between a and 0. This is a divergence.
- If $|r| \neq 1$ we determine convergence or divergence using
- $$s_n = \frac{a(1 - r^n)}{1 - r}, (r \neq 1)$$
- Applies only if the n index starts at 0 or 1 with n-1 as the power
- I've worked through this on paper. When $|r| < 1$ r^n will tend towards 0 so the series converges. If $|r| > 1$ the partial sum will tend towards positive or negative infinity so it diverges —
- If $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$
- If the integral converges or diverges, so does the series. For a sequence of positive terms a_n, if $a_n = f(n)$ where f is continuous and positive and decreasing, that is for each x_1łx_2 in the domain $f(x_1) \geq f(x_2)$ —
- **ratio test** if we've a series $\sum a_n$ and $lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = \rho$, the series converges absolutely if $\rho < 1$, diverges if $\rho > 1$ and is inconclusive if $\rho = 1$
- Going to have to take for granted that we've a bunch of tests for convergence of a series

Power series

- $$\sum_{n=0}^{\infty} c_n(x - a)^n = c_o + c_1(x - a) + ... + c_n(x - a)^n + ...$$

- If we take all the constants to be 1, you get the geometric series that converges to $\frac{1}{1-x}$ (from above)
- If we shift focus to think of a partial sum of highest polynomial n as $P_n(x)$
- If a_n (being the partial sum) is on the y axis and n (or the index of the series) along the x axis. Graphically, each increase along the x is trying to get closer to some incredibly large y value.

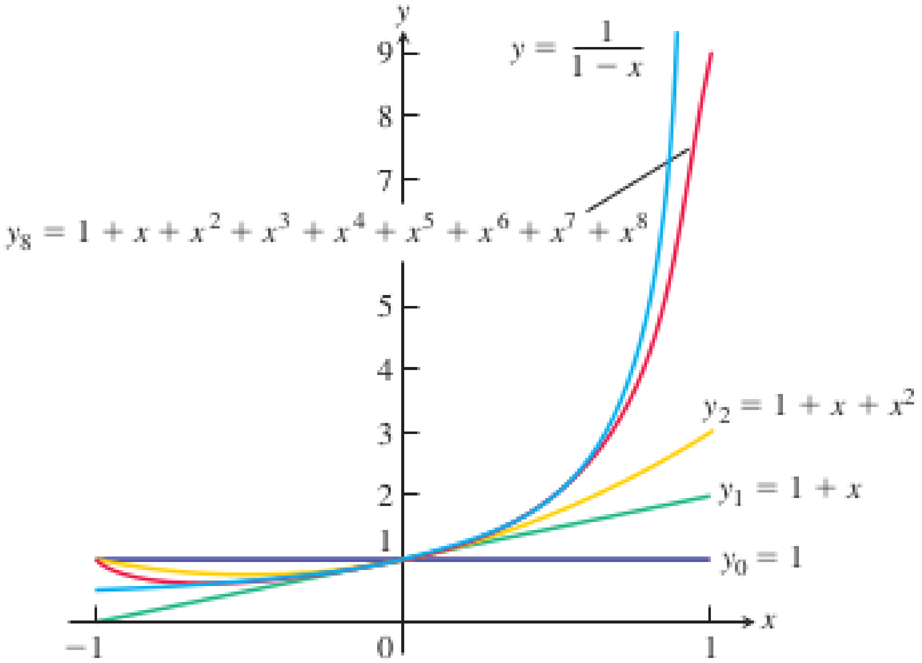

FIGURE 10.14 The graphs of $f(x) = 1/(1-x)$ in Example 1 and four of its polynomial approximations.

- The above has a boundary of plus and minus 1. As x gets closer and closer to 1, $P_n(x)$ approaches the sum $\frac{1}{1-x}$
- power series behaves in 3 possible ways
  - It might converge at a single value
  - converge everywhere
  - converge on some interval, this is like the bounds of -1 and 1 above right?

Taylor Series

- If we take the sum of a power series as a function $f(x) = \sum_{n=0}^{\infty} a_n(x-a)^n$

- There are an infinite number of derivates in the interval of convergence
- If we had a set of these intervals, could we reconstruct the series, or at least parts of it, in turn, approximating a function based on the power series.

Notes

- Is the significance of polynomials in the derivative nature, that it intuitively makes sense that they're approximating a function as each higher order of polynomial means another (more specific) derivative on the initial convergence interval?
- Also, watched a video that details how, similar to converting fractions to decimal places, making addition easier. Converting functions to approximations makes addition easier.