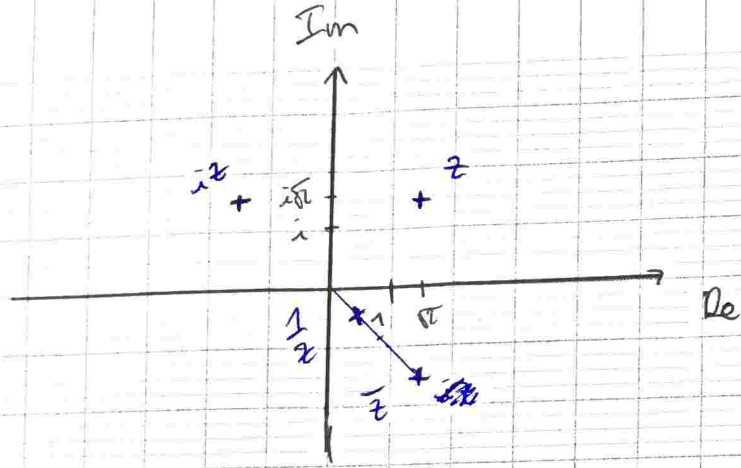
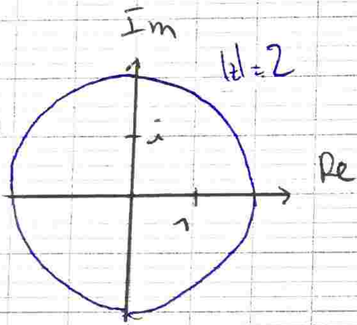
MAT 1 Ex 7

b)

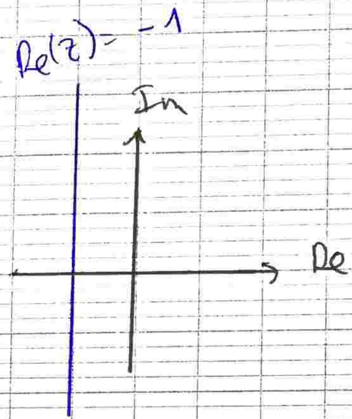


Exercice 8

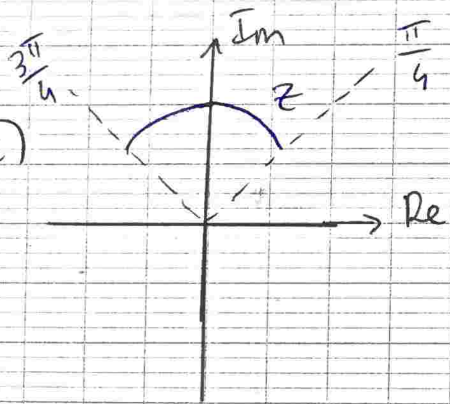
a)



b)



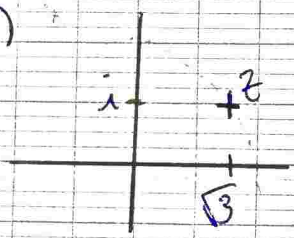
c)



$$\frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$$

$$\frac{11\pi}{4} - 2\pi = \frac{3\pi}{4}$$

d)



$|z| = 2$ et $Im(z) = 1$

donc $|z| = \sqrt{1^2 + a^2}$ $\quad a = Re(z)$

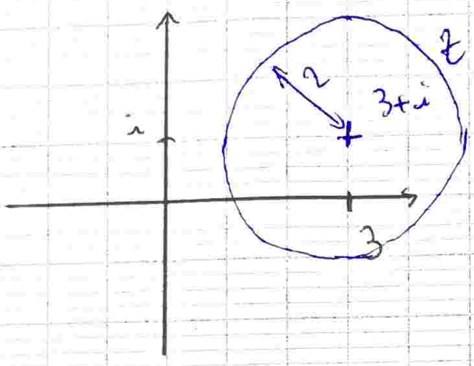
$\Leftrightarrow |z|^2 = 1^2 + a^2$

$\Leftrightarrow a^2 = |z|^2 - 1$

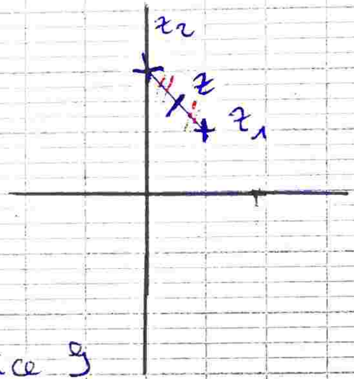
$\Leftrightarrow a = \sqrt{|z|^2 - 1} = \sqrt{4-1}$

$\Leftrightarrow a = \sqrt{3}$

e) $|z-(3+i)|=2 \Leftrightarrow$ La distance entre $3+i$ et z est 2.



f)



$z_1 = 1+i$

$z_2 = 2i$

z est à équidistance de z_1 et z_2.

Exercice 9

Soit $(a,b)\in\mathbb{R}^2$ tq $z=a+bi$

$|z|=\sqrt{a^2+b^2}$ $\quad \left|\frac{1}{z}\right| = \frac{1}{|z|} = \frac{1}{\sqrt{a^2+b^2}}$

$|1-z| = \sqrt{(1-a)^2+b^2}$

D'où : $|z|^2 = \frac{1^2}{|z|^2} = |1-z|^2$

$\Leftrightarrow a^2+b^2 = \frac{1}{a^2+b^2} = (1-a)^2+b^2$

$\Leftrightarrow a^2+b^2 = 1$ et $(1-a)^2+b^2 = 1$

$\Leftrightarrow 1-2a+\underbrace{a^2+b^2}_{1} = 1$

$\Leftrightarrow 2-2a = 1$

$\Leftrightarrow a = \frac{1}{2}$

$\left(\frac{1}{2}\right)^2+b^2 = 1$

$\Leftrightarrow b = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}$

Exercice 10

a) f donne le conjugué de z.
b) f donne z auquel on ajoute $2+i$
c) f donne le double de z.
d) f donne z multiplié par i.
e) f donne z multiplié par $(1+i)$.

Exercice 11

$$\cos(x) = \frac{\overbrace{e^{ix}+e^{-ix}}^{2\cos(x)}}{2} \qquad \sin(x) = \frac{\overbrace{e^{ix}-e^{-ix}}^{\boxed{2i\sin(x)}}}{2i}$$

a) $$\sin(x)^3 = \left(\frac{e^{ix}-e^{-ix}}{2i}\right)^3$$

⚠ NE PAS OUBLIER LE i

$$= -\frac{1}{8i}\left(e^{ix}-e^{-ix}\right)^3$$

$$= -\frac{1}{8i}\left(e^{3ix} - 3e^{2ix}e^{-ix} + 3e^{ix}e^{-2ix} - e^{-3ix}\right)$$

$$= -\frac{1}{8i}\left(e^{3ix} - 3e^{ix} + 3e^{-ix} - e^{-3ix}\right)$$

$$= -\frac{1}{8i}\left(e^{3ix} - e^{-3ix} - 3\left(e^{ix}-e^{-ix}\right)\right)$$

$$= -\frac{1}{8i}\left(2i\sin(3x) - 3 \times 2i\sin(x)\right)$$

$$= -\frac{2i}{8i}\left(\sin 3x - 3\sin x\right)$$

$$= \frac{-\sin 3x + 3\sin x}{4}$$

b) $$\cos^2(3x)\sin(5x) = \left(\frac{e^{3ix}+e^{-3ix}}{2}\right)^2\left(\sin(5x)\right)$$

$$= \frac{1}{4}\left(e^{6ix} + 2\underbrace{e^{3ix}e^{-3ix}}_{1} + e^{-6ix}\right)\sin(5x)$$

$$= \frac{1}{2} + \frac{\cos(6x) \times \sin(5x)}{4}$$

Or, $\cos(b)\sin(a) = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$

D'où : $\cos(6x)\sin(5x) = \frac{1}{2}\left(\sin(5x+6x) + \sin(5x-6x)\right)$

$$= \frac{1}{2}\left(\sin(11x) + \sin(-x)\right)$$

$$= \frac{1}{2}\left(\sin(11x) - \sin(x)\right)$$

Finalement :

$$\cos^2(3x)\sin(5x) = \frac{1}{2} + \frac{1}{8}\left(\sin(11x) - \sin(x)\right)$$

Exercice 12

a) On cherche $z = x + yi$ tq $z^2 = 1 + i\sqrt{3}$

$$z^2 = 1 + i\sqrt{3} \iff \begin{cases} (x+iy)^2 = 1 + \sqrt{3}i \\ |z|^2 = |1 + \sqrt{3}| \end{cases}$$

$$\iff \begin{cases} \underbrace{x^2 - y^2}_{} + \underbrace{2xy}_{}i = \underbrace{1}_{} + \underbrace{\sqrt{3}}_{}i \\ \sqrt{x^2+y^2}^{\,2} = \sqrt{1+3} = \sqrt{4} = 2 \end{cases}$$

partie réelle
partie imaginaire

$$\iff \begin{cases} x^2 - y^2 = 1 \\ 2xy = \sqrt{3} \\ x^2 + y^2 = 2 \end{cases} \qquad \cancel{\iff \begin{cases} \left(\frac{\sqrt{3}}{2y}\right)^2 - y^2 = 1 \\ x = \frac{\sqrt{3}}{2y} \\ \left(\frac{\sqrt{3}}{2y}\right)^2 + y^2 = 2 \end{cases}}$$

$$\iff \begin{cases} x^2 = 1 + y^2 \\ 2xy = \sqrt{3} \\ 1 + y^2 + y^2 = 2 \end{cases} \iff \begin{cases} x^2 = 1 + y^2 = 1 + \frac{1}{2} = \frac{3}{2} \\ 2xy = \sqrt{3} \\ 2y^2 = 1 \iff y = \sqrt{\frac{1}{2}} \end{cases}$$

donc $x = \sqrt{\frac{3}{2}}$ $y = \sqrt{\frac{1}{2}}$