

1 Basic Calculus

Calculus was invented independently by both Leibnetz and Newton

1.1 Differentiation

Notation ‘ Rule - Differentiation of Powers

$$y = x^n$$
$$\frac{dy}{dx} = nx^{n-1} \quad (1)$$

Example 1

$$y = x^2$$
$$\frac{dy}{dx} = 2x^{2-1} = 2x$$

Example 2

$$y = x^2$$
$$\frac{dy}{dx} = 2x^{2-1} = 2x$$

Example 3

$$y = x$$
$$\frac{dy}{dx} = 1x^{1-1} = 1.1$$

Example 4

$$y = 3$$
$$\frac{dy}{dx} = 3.0 = 0$$

Example 5

$$y = 4x^3$$
$$\frac{dy}{dx} = 4 * 3 * x^{3-1} = 12x^2$$

Example 6

$$y = 3x + 3x^4 + 7$$
$$\frac{dy}{dx} = 3 * x^{1-1} + 3 * 4 * x^{4-1} + 7 * 0$$
$$\frac{dy}{dx} = 3 * x^0 + 12 * x^3 + 0$$
$$\frac{dy}{dx} = 3 + 12x^3$$

Example 7

$$y = \frac{1}{x} + 5x - 2x^{\frac{2}{3}} + 3$$

Rewrite the indices.

$$y = x^{-1} + 5x^1 - 2x^{\frac{2}{3}} + 3$$

Differentiate

$$\frac{dy}{dx} = -1 * x^{-1-1} + 5 * x^{1-1} - 2 * \left(\frac{2}{3}\right) * x^{\frac{2}{3}-1} + 0$$

$$\frac{dy}{dx} = -1 * x^{-2} + 5 * x^0 - \left(\frac{4}{3}\right) * x^{\frac{-1}{3}}$$

$$\frac{dy}{dx} = -x^{-2} + 5 - \frac{4x^{\frac{-1}{3}}}{3}$$

1.2 Integration

Integration the reverse process of *differentiation* think of it as *anti-differentiation*

Notation $\int_a^b f(x)$

Rules

Rule	Function	Integral
Multiplication by constant	$\int_a^b cf(x)dx$	$c \int_a^b f(x)dx$
Power Rule ($n \neq -1$)	$\int_a^b x^n dx$	$x^{n+1}/(n+1) + C$
Sum Rule	$\int (f+g)dx$	$\int_a^b f dx + \int_a^b g dx$
Difference Rule	$\int_a^b (f-g)dx$	$\int_a^b f dx - \int_a^b g dx$

1.2.1 Integration Power Rule

$$\int_a^b x dx = \frac{x^{n+1}}{n+1} + C \quad (2)$$

Example Here we will differentiate first before showing the reverse

$f(x) = y = 3x^2 + 5x - 3$
 $f'(x) = \frac{dy}{dx} = 3(2)x^{2-1} + 5(1)x^{1-1} - 3(0) = 6x + 5$ So we know what the result of differentiating is so we can see that the reverse process should produce the original expression.

$$\int_a^b f'(x) = \int_a^b \frac{dy}{dx}$$

$$\int_a^b 6x + 5 = 3x^2 + 5x + C$$

It Could be any value of C .

$$\int_a^b 6x + 5 = 3x^2 + 5x - 3$$

$$\int_a^b 6x + 5 = 3x^2 + 5x + 11$$

Using the reverse of the rule of powers we can see how we could get $3x^2 + 5x$ however we have no way of knowing that -3 was there so we would have to use a constant