

1 Basic Calculus

1.1 Differentiation

Rule - Differentiation of Powers

$$\begin{aligned}y &= x^n \\ \frac{dy}{dx} &= nx^{n-1}\end{aligned}\tag{1}$$

Example 1

$$\begin{aligned}y &= x^2 \\ \frac{dy}{dx} &= 2x^{2-1} = 2x\end{aligned}$$

Example 2

$$\begin{aligned}y &= x^2 \\ \frac{dy}{dx} &= 2x^{2-1} = 2x\end{aligned}$$

Example 3

$$\begin{aligned}y &= x \\ \frac{dy}{dx} &= 1x^{1-1} = 1.1\end{aligned}$$

Example 4

$$\begin{aligned}y &= 3 \\ \frac{dy}{dx} &= 3.0 = 0\end{aligned}$$

Example 5

$$\begin{aligned}y &= 4x^3 \\ \frac{dy}{dx} &= 4 * 3 * x^{3-1} = 12x^2\end{aligned}$$

Example 6

$$\begin{aligned}y &= 3x + 3x^4 + 7 \\ \frac{dy}{dx} &= 3 * x^{1-1} + 3 * 4 * x^{4-1} + 7 * 0 \\ \frac{dy}{dx} &= 3 * x^0 + 12 * x^3 + 0 \\ \frac{dy}{dx} &= 3 + 12x^3\end{aligned}$$

Example 7

$$y = \frac{1}{x} + 5x - 2x^{\frac{2}{3}} + 3$$

Rewrite the indices.

$$y = x^{-1} + 5x^1 - 2x^{\frac{2}{3}} + 3$$

Differentiate

$$\frac{dy}{dx} = -1 * x^{-1-1} + 5 * x^{1-1} - 2 * \left(\frac{2}{3}\right) * x^{\frac{2}{3}-1} + 0$$

$$\frac{dy}{dx} = -1 * x^{-2} + 5 * x^0 - \left(\frac{4}{3}\right) * x^{\frac{-1}{3}}$$

$$\frac{dy}{dx} = -x^{-2} + 5 - \frac{4x^{\frac{-1}{3}}}{3}$$

1.2 Integration

Integration the reverse process of *differentiation* think of it as *anti-differentiation*

Notation $\int_a^b f(x)$

Example Here we will differentiate first before showing the reverse $f(x) = y = 3x^2 + 5x - 3$

$f'(x) = \frac{dy}{dx} = 3(2)x^{2-1} + 5(1)x^{1-1} - 3(0) = 6x + 5$ So we know what the result of differentiating is so we can see that the reverse process should produce the original expression. $\int_a^b f'(x) = \int_a^b \frac{dy}{dx}$

$$\int_a^b 6x + 5 = 3x^2 + 5x - 3$$

Using the reverse of the rule of powers we can see how we could get $3x^2 + 5x$ however we have no way of knowing that -3 was there so we would have to use a constant