1 Basic Calculus

Calculus was invented independently by both Leibnetz and Newton

1.1 Differentiation

Notation ' Rule - Differentiation of Powers

$$y = x^n$$
$$\frac{dy}{dx} = nx^{n-1} \tag{1}$$

Example 1
$y = x^2$
$\frac{dy}{dx} = 2x^{2-1} = 2x$
Example 2
$y = x^2$
$\frac{dy}{dx} = 2x^{2-1} = 2x$
Example 3
$y = x$
$\frac{dy}{dx} = 1x^{1-1} = 1.1$
Example 4
$y = 3$
$\frac{dy}{dx} = 3.0 = 0$
Example 5
$y = 4x^3$
$\frac{dy}{dx} = 4 * 3 * x^{3-1} = 12x^2$
Example 6
$y = 3x + 3x^4 + 7$
$\frac{dy}{dx} = 3 * x^{1-1} + 3 * 4 * x^{4-1} + 7 * 0$
$\frac{dy}{dx} = 3 * x^0 + 12 * x^3 + 0$
$\frac{dy}{dx} = 3 + 12x^3$

Example 7
$y = \frac{1}{x} + 5x - 2x^{\frac{2}{3}} + 3$
Rewrite the indices.
$y = x^{-1} + 5x^1 - 2x^{\frac{2}{3}} + 3$
Differentiate
$\frac{dy}{dx} = -1 * x^{-1-1} + 5 * x^{1-1} - 2 * \left(\frac{2}{3}\right) * x^{\frac{2}{3}-1} + 0$

$\frac{dy}{dx} = -1 * x^{-2} + 5 * x^0 - \left(\frac{4}{3}\right) * x^{\frac{-1}{3}}$

$\frac{dy}{dx} = -x^{-2} + 5 - \frac{4x^{\frac{-1}{3}}}{3}$

1.2 Integration

Integration the reverse process of *differentiation* think of it as *anti-differentiation*
Notation $\int_a^b f(x)$

Rules

Rule	Function	Integral
Multiplication by constant	$\int_a^b cf(x)dx$	c $\int_a^b f(x)dx$
Power Rule ($n \neq -1$)	$\int_a^b xndx$	$x^{n+1}/(n+1) + C$
Sum Rule	$\int(f+g)dx$	$\int_a^b$ f dx + $\int_a^b gdx$
Difference Rule	$\int_a^b(f-g)dx$	$\int_a^b fdx - \int_a^b gdx$
Product Rule		

1.2.1 Integration Power Rule

$$\int_a^b xdx = \frac{x^{n+1}}{n+1} + C \tag{2}$$

Example Here we will differentiate first before showing the reverse
$f(x) = y = 3x^2 + 5x - 3$
$f'(x) = \frac{dy}{dx} = 3(2)x^{2-1} + 5(1)x^{1-1} - 3(0) = 6x + 5$ So we know what the result of differentiating is so we can see that the reverse process should produce the original expression.
$\int_a^b f'(x) = \int_a^b \frac{dy}{dx}$
$\int_a^b 6x + 5 = 3x^2 + 5x + C$

It Could be any value of C.
$\int_a^b 6x + 5 = 3x^2 + 5x - 3$
$\int_a^b 6x + 5 = 3x^2 + 5x + 11$

Using the reverse of the rule of powers we can see how we could get $3x^2 + 5x$ however we have no way of knowing that -3 was there so we would have to use a constant

1.2.2 Intergration by Parts

Involves manipulating the rules into more manageable parts

1. x is easy to *differentiate*
2. dx is easy to *integrate*
3. sometimes you have to use divide into parts more than once

1.2.3 Integration by Substitution

Integration by substitution is reforming the equation in a way which is able to be integrated.