Conors^3 Maths Grinds

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Contents

1 The Prologue

I[1] am just going to outline the rules and formulae which are needed for the GMAT Mathematics test.

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2 Topics

The topics covered

- Geometry - Area and Perimeter of shapes, and Surface Area and Volume.
- Algebra - solve equations
- Arithmetic - working out the results
- Problem Solving - word problems

3 Prerequistes and Definitions

- **Natural Number** - a number which occurs in nature, an integer, a positive whole number e.g. 1,2,3,4,511 etc.,
- **Real Number** - any number which can be plotted on a line e.g $3, 2/3, -0.2, \sqrt{3}$
- **Imaginary Number** - a number which can not be calculated e.g. $\sqrt{-5}$
- **Rational Number** - a number which can be written as a fraction e.g. $4(4/1), 2/3$
- **Irrational Number** - a number which can not be written as a fraction e.g $\sqrt{2}, \pi, 0.271271271271...$

Multiplying
$+a * +b = +ab$
$+a * -b = -ab$
$-a * +b = -ab$
$-a * -b = +ab$

Exponents or Indices Exponents are the 'power' of a number, so the number of times a number is multiplied by itself e.g.
$x^3 = x * x * x$

A Negative exponent equates to 1 divided by that number multiplied by itself e.g.
$x^{-4} = \frac{1}{(x*x*x*x)}$
$10^{-3} = \frac{1}{1000} = 0.001$

Multiplying numbers with exponents you add the exponents
$a^2 * a^3 = a^{2+3} = a^5$

Dividing numbers with exponents you subtract the exponents of the divisor number from the number
$a^3/a^2 = a^{3-2} = a^1 = a$
$a^2/a^3 = a^{2-3} = a^{-1} = 1/a$
$a^x * b^x = ab^x$
$(a/b)^x = a^x/b^x$
$(a^x)^y = (a^y)^x = a^{xy}$

Zero to the power of a number is zero
$0^x = 0 \quad \text{e.g.} \quad 0^1 = 0$

A number to the power of zero is one
$x^0 = 1 \quad \text{e.g.} \quad x^{0=1}$

Ratio - the Ratio of A to B is written as $A/B \ or \ A : B$

Percentage - to get a percentage of a fraction your multiply by 100 so $(3/4) * 100 = 75\%$

4 Geometry

- Lines
- Four-sided figures
- Triangles
- Pythagoras
- Circles
- Volume and Surface Area
- Polygons

4.1 Lines

A Line is said to be 180 degrees, so if you know the angle one makes intersecting a line you know the other side

4.2 Intersecting Lines

The opposite angles in intersecting lines are equal.

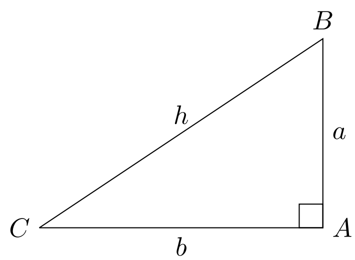
4.3 Line intersecting Parallel Lines

Parallel lines the angles are preserved, i.e. the angles made by the intersecting lines are the same

4.4 Four-sided figures

1. Rectangles
2. Squares
3. Parallelograms
4. Other foursided figures

4.4.1 Area of a Rectangle



Area of a rectangle is the length of the sides multiplied together.

$$Area_{rectangle} = width * height \tag{1}$$

4.4.2 Perimeter of a Rectangle

Is the sum of the 4 sides

$$Perimeter_{Rectangle} = or 2width + 2heigntor Perimeter_{rectangle} = 2(width + height) \tag{2}$$

4.4.3 Square

Well a square is just a rectangle where all the sides are the same size, so all the rules apply but just are simpler.

4.4.4 Parallelogram

A parallelogram is a foursided figure where the the sides are parallel, opposite sides are equal and opposite angles are equal. The only property which changes is the area

$Area_{Parallelogram} = base * perpendicular - height$

$Sum of the Angles in a Parallelogram = 360°$

4.5 Triangles

1. Perimeter of a Triangle equals sum of 3 sides
2. Area of a Triangle equals half the base by perpendicular height
3. The sum of the angles of a triangle equal 180°
4. Equilateral Triangle - all the angles are 60°, and all sides are the same length
5. Isosceles Triangle - 2 angles are the same and 2 sides are the same length

4.5.1 Perimeter of a Triangle

Perimeter of a triangle is the sum of the 3 sides so Perimeter = Side A + Side B + Side C

4.5.2 Area of a Triangle

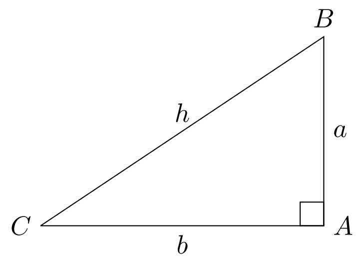
$$Area_{Triangle} = base * PerpindicularHeight/2 \tag{3}$$

4.5.3 Angles in a Triangle

The angles in a triangle equal 180° So if you have two angles you always can deduct the third.

4.6 Pythagoras Theorem

The most important theorem In a Right angle triangle(one angle = 90°), the square on the hypoteneuse (longest side) is equal to the sum of the squares on the other two sides



$$h^2 = a^2 + b^2 \tag{4}$$

so $h = \sqrt{a^2 + b^2}$

4.6.1 Also works for the Circle on the Hypoteneuse

As a result the circle on the hypoteneuse is equal to the sum of the circle of the other two sides.

$$\pi(h/2)^2 = \pi(a/2)^2 + \pi(b/2)^2 \tag{5}$$

You will find that often the numbers used in examples are triangles with sides 5, 4 and 3, 10, 8 and 6 or 50, 40 and 30 which all neatly square etc.

4.7 Circles

4.7.1 Area of a Circle

$$Area_{Circle} = \pi r^2 \tag{6}$$

4.7.2 Circumference of a Circle

$$Circumference_{Circle} = 2\pi r \tag{7}$$

4.8 Sectors of a Circle

A sector of a Circle of is a portion of a circle like a slice of pizza or tart with two straight lines from the centre of the circle out to the edge. The Angle is the angle between the two straignt lines from the centre of the circle.

4.8.1 Area of a Sector of a Circle

$$Area_{SectorofCircle} = (\frac{Angle}{360})\pi r^2 \tag{8}$$

4.8.2 Circumference of a Sector of a Circle

$$Circumference_{SectorofaCircle} = (\frac{Angle}{360})2\pi r \tag{9}$$

4.9 Volume and Surface Area

4.9.1 Rectangular Box

- Volume of a Rectangular Box - Length by Breath by Height
- Surface Area, is six rectangle, two breath by depth plus two breath by height plus two height by depth

$$Volume = width * height * depth \tag{10}$$

$$Surface_Area = 2(width * height) + 2(width * depth) + 2(height * depth) \tag{11}$$

4.9.2 Cylinder

- Volume of a Cylinder is area of the base by the height
- Surface area is 2 circles and a rectangle (from the rolled out tube of the cylinder) height by circumference of the circle

$$Volume_{Cylinder} = h\pi r^2 \tag{12}$$

$$SurfaceArea_{Cylinder} = 2\pi rh + 2(\pi r^2) \tag{13}$$

4.10 Polygons

4.10.1 Area of a Polygon

To get a polygons area, you break it up into triangles or triangles and rectangles, may need pythagoras.

4.10.2 Perimeter of a Polygon

The sum of the lengths of its sides

4.10.3 Sum of angles of a polygon

Sum of the triangles which meet the points of the polygon - i.e. multiples of 180 Or triangle is 180, rectangle 360, adding another side will always be adding another triangle so pentagon is 540, hexagon is 720 and heptagon is 900 and octagon 1060 and so on...

4.11 Coordinate Geometry

4.11.1 Length of a line between two points on the plane

Finding the length of a line on a plane, involves plotting it on an xy axis. The length of a line can be worked out by using Pythagoras. So the length of a line from a (1,-3) and b (-1, 2) is $\sqrt{2^2 + (-5)^2}$.

4.11.2 Slope of a Line

There are two ways which get you the slope of a line one is with the co-ordinates the other with an equation which you resolve to look like y = mx + b where m is the slope of the line

Using the Equation

$$y = mx + B \tag{14}$$

Using Co-ordinates

With co-ordinates (x_1, y_1) and (x_2, y_2)

$$m = \frac{(y_2 - y_1)}{(x_2 - y_1)} \tag{15}$$

5 Algebra

5.1 Solve an Equation one Variable

In this case you just manipulate the equation so as the variable is on one side on its own and what it equals is on the other.

$2x - 9 = 1$
$2x = 1 + 9$
$2x = 10$
$x = 10/2$
$x = 5$

5.1.1 Two Equations with Two Variables

Solved by getting what one variable is as an expression of the other then plug it in

$2x - 3y = 1$
$x + 2y = 11$
$x = 11 - 2y$
$22 - 4y - 3y = 1$
$22 - 7y = 1$
$-7y = 1 - 22$
$-7y = -21$
$7y = 21$
$y = 21/7$
$y = 3$ So since you now know y you can work out x
$x + 3(3) = 11$ $x = 11 - 9 = 2$

5.1.2 Solve Equations by Factoring

The factors of a number or equation are 2 (or more) numbers or equations which when multiplied together will give you that number or equation.

$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$
$a^2 - b^2 = (a + b)(a - b)$
$2a^3 - 4a^2b + 2ab^2 = 2a(a^2 - 2ab + b^2) = 2a(a - b)^2$

To solve an equation by factoring you move all the elements to one side so as it equals to 0, then you try and reduced the equation to what it will. Ultimately you are find what are the possible values for the variable.

5.1.3 Absolute Value

The *Absolute Value* of a number is the square root of a number squared, so it is always a positive value.

The *absolute value* of a number 'a' is written as $|a|$.

$|a| = \sqrt{a^2}$

The absolute value of a number is often used to get its value, ignoring the sign e.g. the length of a line, where the sign is the direction,

6 Colophon

Wouldn't have been possible without Mr Euclid and Mr Pythagoras.

This document was written created using LaTeX.

Initially it was word-processed using text editor www.vim.org and then rendered into pdf using pdfplatex.

The amendments from the original version were made using Version 2.4 of MiKTeX(www.miktex.org) However since using the macbook a lot of late I use TeX Live (www.tug.org/texlive/).