Algorithms Worksheet 2

For each part of a question write the answer and include workings. Each question is worth two marks, there are also two marks for attendance.

1. Solve for $T(n)$ using the ansatz $T(n) = r^n$ for the following two step recursion relations. Solving for r will give two values r_1 and r_2, this means that the general solution will be $T(n) = Ar_1^n + Br_2^n$. Use the two base values to find A and B.

   a) $T(n) = 2T(n-1) + 3T(n-2)$ with $T(0) = 0$ and $T(1) = 4$.

   b) $T(n) = T(n-2)$ with $T(0) = 0$ and $T(1) = 2$.

   Solution:

   For (a) we have
   $$r^2 = 2r + 3 \tag{1}$$
   so $r^2 - 2r - 3 = 0$ or $(r-3)(r+1) = 0$ so
   $$T(n) = 3^n A + (-1)^n B \tag{2}$$
   and the initial conditions give $A + B = 0$ and $3A - B = 4$ so
   $$T(n) = 3^n - (-1)^n \tag{3}$$
   For (b) we get $r^2 = 1$ so
   $$T(n) = A + (-1)^n B \tag{4}$$
   and the initial conditions give $A + B = 0$ and $A - B = 2$ so
   $$T(n) = 1 - (-1)^n \tag{5}$$

2. This question is about the master theorem. Use it to calculate big-Theta for $T(n)$ in each case.

   a) $T(n) = 25T(n/5) + 4n^2$

   b) $T(n) = 20T(n/5) + 4n$

   c) $T(n) = 16T(n/2) + 2n^4$

   Solutions: for the first one $\log_5 25 = 2$ and $c = 2$ so this is the middle case and $T(n) \in \Theta(n^2 \log n)$, for the second $\log_5 20 > 1$ so it is the first case and $T(n) \in \Theta(n^{\log_5 20})$; the last one is in the middle case as well since $\log_2 16 = 4$ and $T(n) \in \Theta(n^4 \log n)$.

3. This question is about quicksort; use the quicksort algorithm to sort the set $\{4, 7, 8, 10, 1, 2, 5, 9, 3, 6\}$ showing all your steps, use the median of triples on the first three entries to find the pivot; you don't need to go through the individual swaps involved in the in-place implementation, just divide the set around the pivot.

   Solution: for example

4	7	8	10	1	2	5	9	3	6
4	1	2	5	3	6	**7**	8	10	9
1	**2**	4	5	3	6	**7**	8	**9**	10
1	**2**	3	**4**	5	6	**7**	8	**9**	10

4. This question is about quicksort in place; perform the first step of quicksort, dividing the set into two, on the set $\{4, 7, 8, 10, 1, 2, 5, 9, 3, 6\}$ using the pivot 7 and individual swaps.

   Solution: the bold numbers are being considered for swapping.

4	7	8	10	1	2	5	9	3	6
4	6	8	10	1	2	5	9	**3**	7
4	**6**	8	10	1	2	5	9	**3**	7
4	6	**8**	10	1	2	5	9	**3**	7
4	6	**3**	10	1	2	5	9	**8**	7
4	6	3	**10**	1	2	5	9	**8**	7
4	6	3	**10**	1	2	5	**9**	8	7
4	6	3	**10**	1	2	**5**	9	8	7
4	6	3	**5**	1	2	**10**	9	8	7
4	6	3	**5**	1	2	**10**	9	8	7
4	6	3	5	**1**	2	**10**	9	8	7
4	6	3	5	1	**2**	**10**	9	8	7
4	6	3	5	1	2	7	9	8	10