Algorithms Worksheet 2

This worksheet contains mostly pen and paper calculations. The solutions should be submitted as plain text, that is .txt, files in other formats, like .rtf, .tex, .doc or .pdf will not be accepted. You can write $x\hat{}n$ and a_n for x^n and a_n. Where applicable, write a short description of how the answer is obtained. The graph in part 6 should be submitted as an eps or pdf file.

1. This question is about solving recursion relations using telescoping. In each case find the value of $T(n)$ by telescoping. Check your answer by substitution, it is permisseable to combine these two steps by using telescoping to come up with an ansatz and then substituting it fix values in the ansatz. Write down the big-Theta for the solution. [25 marks]
   a) $T(n) = T(n-1) + 3$ with $T(0) = 1$
   b) $T(n) = T(n-1) + 3$ with $T(1) = 1$
   c) $T(n) = T(n-1) + 3n$ with $T(1) = 1$
   d) $T(n) = 2T(n-1) + 3$ with $T(0) = 1$
   e) $T(n) = 3T(n-1) + 2$ with $T(0) = 1$

2. This question is about the asymptotic behavior of different functions, in each case give big-Theta for $T(n)$; if $T(n)$ was the worst case run-time this would give big-Oh. There is no need to give any working for this problem. [30 marks]
   a) $T(n) = n^5 + \frac{1}{n} + n(n-1)(n+2)^4$
   b) $T(n) = n^2 \log n + n^3$
   c) $T(n) = 2^n + n!$
   d) $T(n) = \sum_{i=0}^{n} i$
   e) $T(n) = \sqrt{n}n + n$
   f) $T(n) = n^2 / \log n + n$
   g) $T(n) = (n^5 + 345n^4 + 36n)/(n^2 + 2n + 1)$
   h) $T(n) = 1/(n^2 + 2n + 1)$
   i) $T(n) = [(n+1)(n+2)(n+3)]/[(n+4)(n+5)]$
   j) $T(n) = n!/(n-1)!$

3. Solve for $T(n)$ using the ansatz $T(n) = r^n$ for the following two step recursion relations. Solving for r will give two values r_1 and r_2, this means that general solution will be $T(n) = Ar_1^n + Br_2^n$. Use the two base values to find A and B. [10 marks]
   a) $T(n) = 2T(n-1) + 3T(n-2)$ with $T(0) = 0$ and $T(1) = 5$.
   b) $T(n) = T(n-2)$ with $T(0) = 0$ and $T(1) = 2$.

4. This question is about the master theorem. Use it to calculate big-Theta for $T(n)$ in each case. [15 marks]
   a) $T(n) = 25T(n/5) + 4n^2$
   b) $T(n) = 20T(n/5) + 4n$

   c) $T(n) = 16T(n/2) + 2n^4$

5. In the section 1_introduction the actual run time for insert sort was plotted against the size of the set. Using another algorithm whose asymptotic behavior you know, make a similar plot. I use `gnuplot` to do plots and instructions for this are provided in the 1_introduction folder, you can use other plotting packages but please submit the graph as pdf or eps. This can be any algorithm, so binary or linear search would be good. [20 marks]