EMAT10001 Workshop Sheet 1 - outline solutions.

2 October 2013

These solutions are partly based on the solutions in *What is the name of this book* by **Raymond M. Smullyan** which is where the problems came from. The usual bounty for errors applies, email me with corrections.

1. No islander can say 'I am a knave' so B must be lying and is therefore a knave. Since B is lying, C is telling the truth. C is therefore a knight. We don't know what A is and it isn't asked.

2. If D was a knave his statement would be true and that's a contradiction, thus D is a knight, his statement is true and E is a knave. [Notice that if V(D) means D is a knave and V(E) means E is a knave then !(V(D)‖V(E)) is !V(D)&!V(E) where ! means *not*, ‖ *or* and & *and*.]

3. For this to be lie then both parts must be false, that is D must be a knight and E must be a knave. However, if D is knight the statement must be true; this means D can't be a knave and the statement is true. Since D is a knight then E is also a knight. [Here we use two pieces of logic, first !(V(D)‖!V(E))=!V(D)&V(E) and second, if P1‖P2 is true and P1 is false, then P2 must be true.

4. A knight could not say P='All of us are knaves' so F is a knave. This means P must be false and there is at least one knight. If there are two knights G is a knight but Q='Exactly one of us is a knight' is a lie, this is a contradiction, so there must be just one knight, Q is true, hence G is knight and so, by Q, H is a knave.

5. So, as before, F is a knave and there is at least one knight. If G is knight then his statement is true, so H is a knight. If G is knave then H must be a knight because we know there is at least one. This means we know H is a knight even though we don't know what G is.

6. I can't be a knight since a knight can't claim to be a knave. Since I is a knave then the overall statement must be false. Since the first part is true, the second part is false and so J is a knave.

7. If K is a knave then L is a knight and so M must be a knave, the same as K. On the other hand, if K is a knight then L is knave and M must be different from K and is therefore a knave.

8. For this one we have to chase down the different possibilities. If N is a knave then O and P are different. If P is a knave then O is a knight and so P lies and answers yes, if P is a knight then O is a knave and so P again answers yes, this time truthfully. On the other hand if N is a knight then O and P are the same and so P answers yes, truthfully if he is a knight, lyingly if he is a knave.

9. The answer given is yes or no and we are told the traveller is able to deduce the real answer from the reply. If the answer was yes then the traveller wouldnt know what the real answer to the question; the one answering could be a knight, in which case the yes was truthfull, or he or she could be knave, in which case the yes is a lie. Since we are told the traveller learns the answer to the question yes cant be the answer given. If the answer was no then the one who answered is a knave and the other a knight and the traveller learns the real answer is yes.

10. Q can't be a knight, a knight can't claim to be anything but a knight. If Q is not an islander then R's statement is true. Because Q is the person not from the island that means R is the knight and so S is the knave and that's a contradiction since S's statement would then be true. This means Q is the knave, R must be lying and so can't be the knight, he is therefore the person not from the island and S is the knight.

11. Obviously T and U cant both be knights and so if U is a knight T is not, he is however telling the truth. If U is not a knight then then neither is T, which means U is telling the truth.

12. V can truthfully make the statement on Thursdays and make it as a lie on Mondays, W can truthfully make it on Sundays and as a lie on Thursdays, so it must be Thursday, the day they can both say 'Yesterday was one of my lying days'.

13. Again, V can only say 'I lied yesterday' on Mondays and Thursdays. If it is a Monday she is lying and so the second statement must be a lie, which it would be. If she says it on a Thursday she is telling the truth but the second statement would be lie and she can't lie, so it's a Monday.

14. She can never say that.

15. This is different, it is the whole statement that has to be true or false. Remember whats more that !P(&Q)=!P‖!Q. There is no day she can truthfully make this statement, so it must be a lie. On Monday the first part is false, on Wednesday the second part.

16. If the first person is Y, the second is Z and both are telling the truth, which can happen on a Sunday. If the first person is Z the second must be Y and so both are lying, there is no day both lie so this cant happen.

17. Well now it isn't Sunday so one of the two statements must be false. If the first statement is true then the second is false and that leads to a contradiction. This means the first statement is false.