EMAT10001 Exercise Sheet 16.

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Introduction

This worksheet is about multidimensional calculus and the gradient vector. There is the usual bounty for errors and typos, 20p to £2 depending on how serious it is.

Exercise sheet

The difference between the work sheet and the exercise sheet is that the solutions to the exercise sheet won't be given and the problems are designed to be more suited to working on on your own, though you are free to discuss them in the work shop if you finish the work sheet problems. Selected problems from the exercise sheet will be requested as part of the continual assessment portfolio.

1. Find $\partial f/\partial x$ and $\partial f/\partial y$ for
   (a) $f(x,y) = xy \ln xy$
   (b) $f(x,y) = 12x^4y + y^2$
   (c) $f(x,y) = xe^{y^2}$
2. Find $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial z$ for
   (a) $f(x,y,z) = xyz$
   (b) $f(x,y,z) = (x-y)^2 + (y-z)^2 + (z-x)^2$
3. For $f(x,y) = xy$ work out the directional derivative in the $(1,3)$ direction at $(1,1)$; don't forget to normalize the direction vector.
4. The third differential operator is curl; it acts on vector fields to give another vector field. It is only defined in three dimensions and has quite a complicated form

$$\text{curl}\mathbf{v} = \left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}, \frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x}, \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right) \tag{1}$$

   Show $\text{grad}\,(\text{curl}\mathbf{v}) = 0$.

Challenge

First four to email or tell me the answer:

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> + + + + +[< ++ > −] < + + + > + + + + [> + + + < −] > [< + + ++ > −]
< .+ < . > . < . > − >>
+ + + + + > + < [> [>> + > + <<< −] >>> [<<< + >>> −] <<<> [< + > −]
<<<< [> + >>>>> + <<<<<< −] > [< + > −] >>
[<< + >>>>> + <<< −] << [>> + << −] >>>>> .[−]
<<<<<<< . >>>>>> [< + > −] <<< −]                                    (2)
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The start of what?