EMAT10001 Workshop Sheet 1.

2 October 2013

Introduction

For this introductory week we will have some warm up fun with silly logic puzzles. These puzzles are all taken from the book *What is the name of this book* by **Raymond M. Smullyan** and silly though they are, solving them is good practice in logic.

The teaching assistants are here to help, ask for help if you are stuck, discuss the problems together, draw diagrams or truth tables on the board. If you have solved a problem call over a teaching assistant to explain your answer, he or she will pick a random student from the table to give the explanation. The table that solves the most puzzles wins.

Useful facts

On this island **knaves** always tell lies, **knights** always tell the truth. This means, for example, nobody on the island can say 'I am a knave' since that would be a lie if spoken by a knight and the truth if spoken by a knave.

Questions

1. Three islanders named Aoife, Brendan and Caoimhe, or any other three names starting with A, B and C you fancy, are standing together on a railway platform. A stranger passed by and asked Aoife, 'Are you a knight or a knave?' She replied just as a train whistle blew and the stranger didn't hear her. The stranger then asked Brendan 'What did Aoife say?' and Brendan replied 'Aoife said that she is a knave.' Caoimhe then says 'Don't believe Brendan, he is lying.' What are Brendan and Caoimhe?

2. In this problem there are only two people, Donnacha and Emer. Donnacha says 'At least one of us is a knave.' What are Donnacha and Emer?

3. Suppose Donnacha had said 'Either I am a knave or Emer is a knight' then what are Donnacha and Emer?

4. Again we have three islanders, Finn, Gavin and Hazel. Finn says 'All of us are knaves'and Gavin says 'Exactly one of us is a knight'. What are Finn, Gavin and Hazel?

5. Suppose instead Finn had said 'All of us are knaves' as before but Gavin had said 'Exactly one of us is a knave.' Can it be determined what Gavin is? Can it be determined what Hazel is?

6. Two islanders this time: Iseult and Jarlath. Iseult says 'I am a knave, but Jarlath isn't.' What are Iseult and Jarlath?

7. Three islanders again: Kieran, Liam and Maeve. Kieran says 'Liam is a knave' and Liam says 'Kieran and Maeve are the same sort, both knaves or both knights.'. What is Maeve?

8. Three more islanders: Niamh, Orla and Padraig. Niamh says 'Orla and Padraig are the same sort.' Someone then asks Padraig 'Are Niamh and Orla the same sort?' What does Padraig answer?

9. For this puzzle there are three people, Quinn, Richard and Sorca, but this time we know one is a knave, one is a knight and the third is not from the island and can lie or tell the truth. Quinn says 'I am not an islander' and Richard replies 'That is true'. Sorca says 'I am an islander'. What are Quinn, Richard and Sorca?

10. Now there are two people, Tadhg and Una but we don't know if they are islanders or not. Tadhg says 'Una is a knight' and Una says 'Tadhg is not a knight.' Prove that at least one of them is telling the truth, but is not a knight.

11. This next problem takes place on a different island where peoples behavior depends on the day of the week. Violet lies on Mondays, Tuesdays and Wednesdays, but tells the truth on the other days. Wendy lies on Thursdays, Fridays and Saturdays, but tells the truth on the other days. One day Xavier, who knew both well, met Violet and Wendy. Violet said 'Yesterday was one of my lying days' and Wendy added 'Yesterday was one of my lying days too'. Which day is it?

12. Another day Xavier met Violet and she told him 'I lied yesterday' and 'I will lie again two days after tomorrow'. What day of the week is it?

13. On what days of the week can Violet say 'I lied yesterday' and say 'I will lie again tomorrow'?

14. On what days of the week can Violet say 'I lied yesterday and I will lie again tomorrow'?

15. Now Yolanda and Zoe are two more inhabitants of this island, one is like Violet, lying on Mondays, Tuesdays and Wednesdays, the other is like Wendy, lying on Thursdays, Fridays and Saturdays. However, Xavier doesn't know which is which and, what's more, Yolanda and Zoe are twins who enjoy confusing people, so Xavier can't tell them apart. One day he meets them and one says 'I am Yolanda' and the other says 'I am Zoe'. Which one is really Yolanda and which is really Zoe?

16. Another day during the same week Xavier met the twins. One said 'I'm Yolanda' and the said 'If that's true, then I'm Zoe'. Which was which?

End note

What is the name of this book? carries on like this for two hundred pages, solving the final puzzle is equivalent to proving Gödel's theorem, the famous mathematical proof that mathematics is incomplete. One of its later problems was adapted by the logician George Boolos and became known as **the hardest logical puzzle ever**. It says

- Three gods A, B, and C are called, in no particular order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for yes and no are *da* and *ja*, in some order. You do not know which word means which.

See: `http://en.wikipedia.org/wiki/The_hardest_logic_puzzle_ever`