

# Computational Neuroscience 1.2

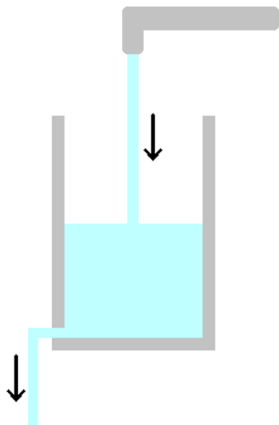
PHPH20007

`github.com/conorhoughton/PHPH20007`

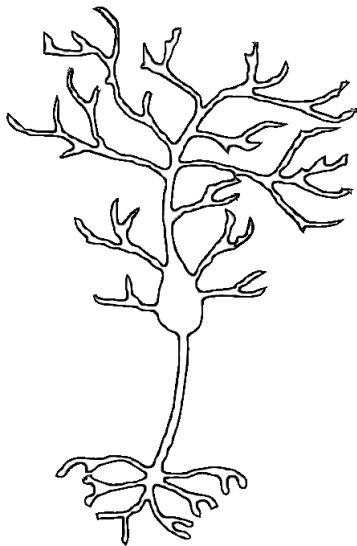
May 2019

## The equations for a leaky bucket

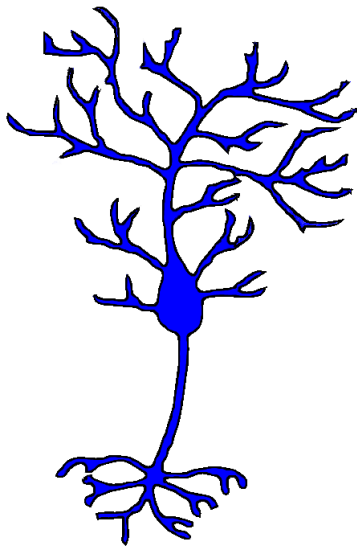
$$\tau \frac{dh}{dt} = \frac{1}{G} i - h$$



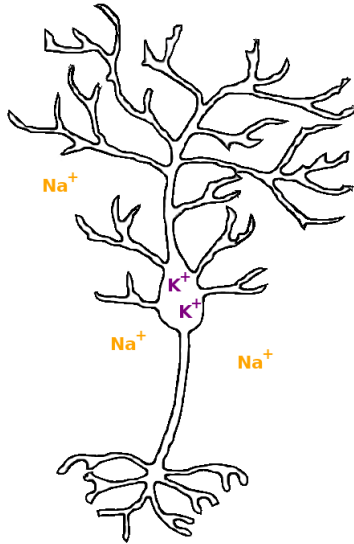
## A neuron



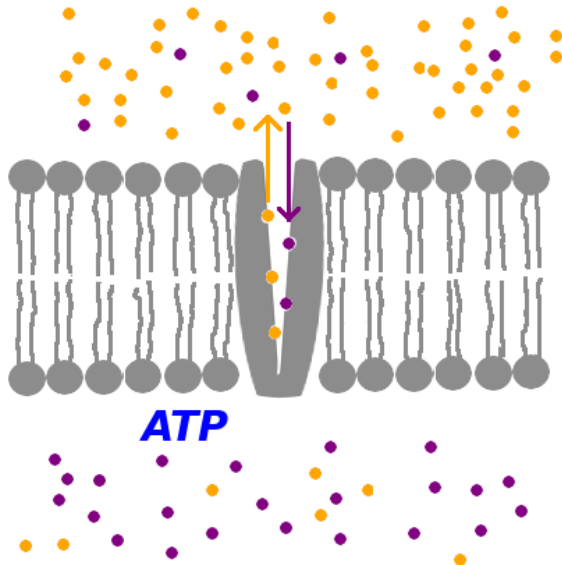
A neuron has an inside and an outside



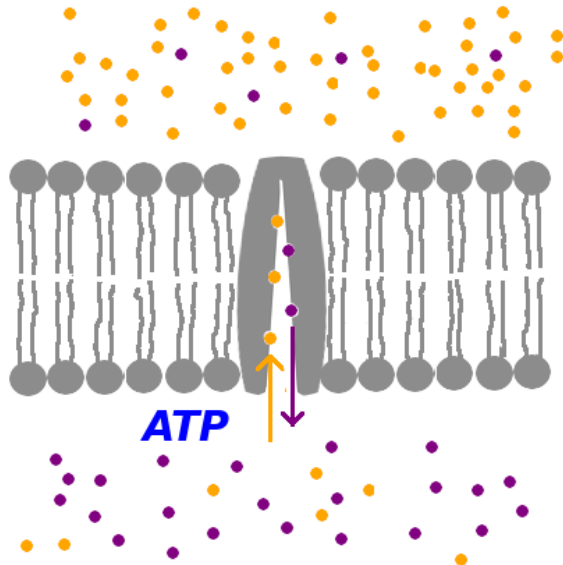
A neuron has an inside and an outside



## Ion pumps



## Ion pumps

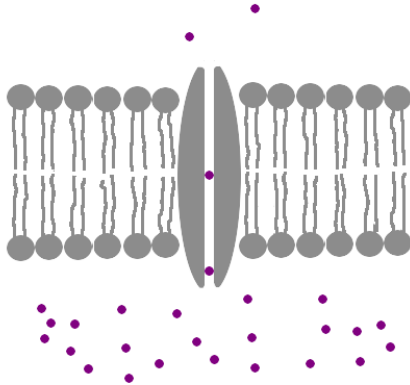


# Potassium inside and sodium outside

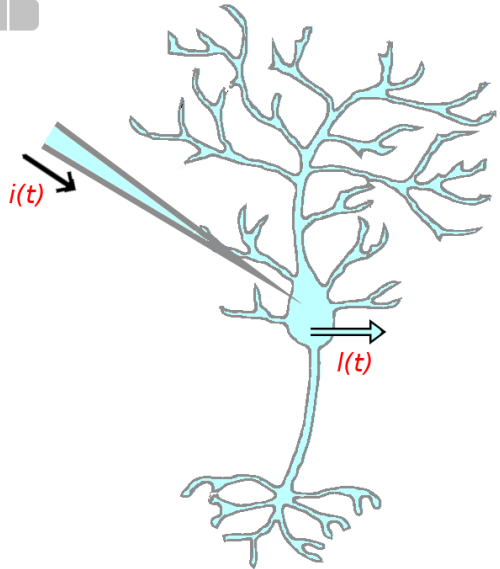
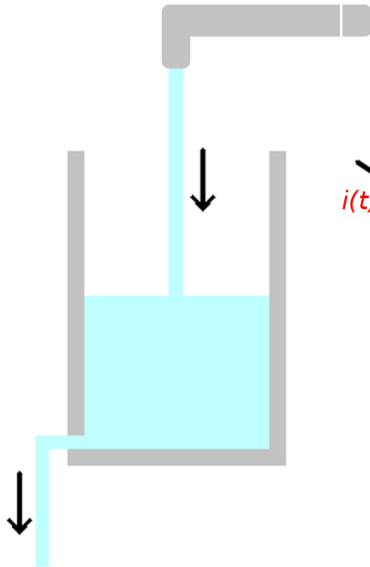




Ions can get through the membrane

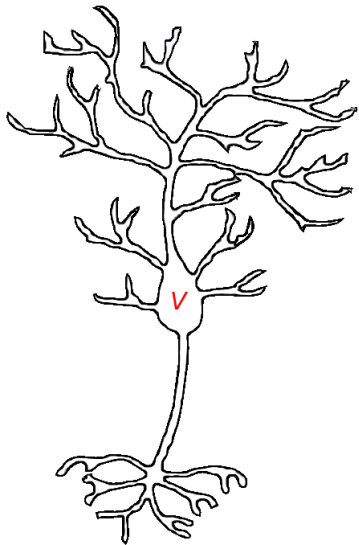


Charge is like water



## Voltage difference

$$V \sim -70\text{mV}$$



## Voltage is like pressure

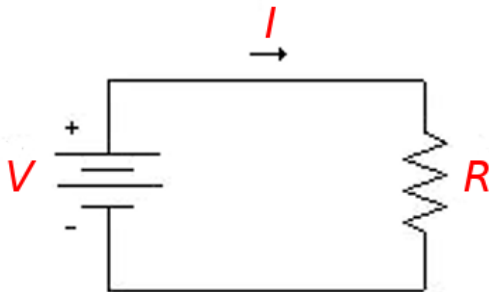
**Voltage** *is defined as the work needed per unit of charge to move a test charge between the two points.*

# Capacitance

$$Q = CV$$

## Ohm's law

$$V = IR$$



# Resistance versus Conductance

$$G = 1/R$$

so

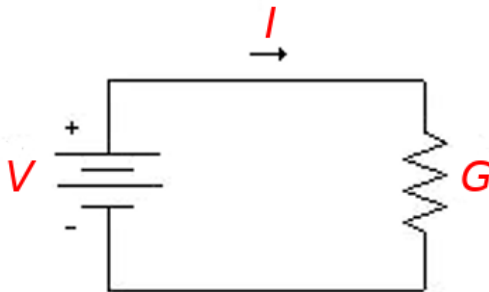
$$I = GV$$



## Voltage and current

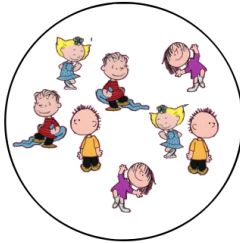
$$I = GV$$

so  $V = 0$  means  $I = 0$ .

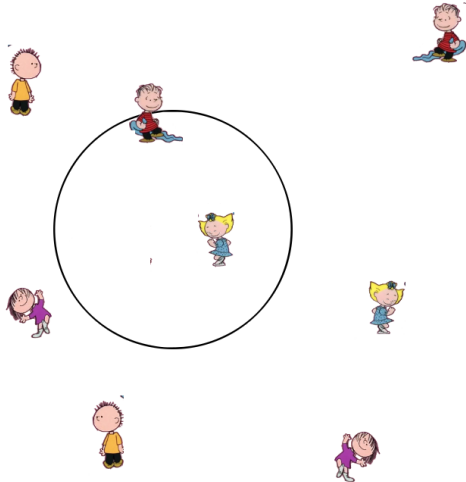




# Chemical gradients



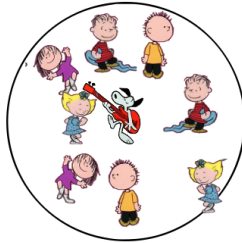
# Chemical gradients



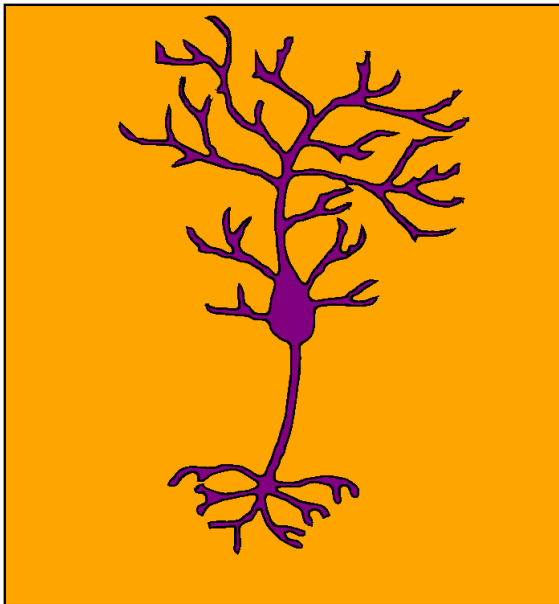
# Chemical gradients



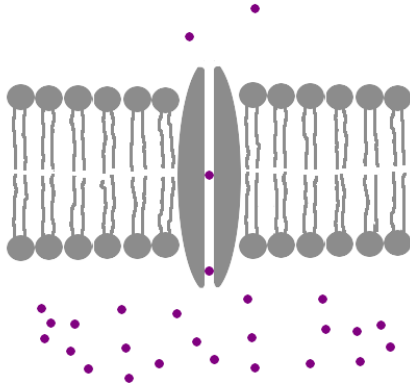
# Chemical gradients



## Chemical gradients



## Chemical gradients



## Leak potential

$$I_l = G(E_l - V)$$

where  $E_l \sim -70\text{mV}$  is the leak reversal potential.

## Leak potential

$$I_l = G(E_l - V)$$

DON'T WORRY TO MUCH ABOUT THE SIGN, it is easy to get confused so I WILL MAKE SURE IT IS RIGHT, this is the expression for the current inward.



## Leak conductance

$$I_l = G(E_l - V)$$

The conductance  $G$  is the conductance of the membrane and is often called the **leak conductance**, in fact, there are other conductance that depend on  $V$ , whereas the leak conductance is constant and sometimes called **passive**. The symbol  $g_l$  is often used:

$$I_l = g_l(E_l - V)$$

# The leaky integrator for neurons

$$\text{rate of change of charge} = \text{current in} - \text{leak out}$$

## The leaky integrator for neurons

$$\frac{dQ}{dt} = i(t) - I_l(t)$$

# Input

For now imagine the input is from an electrode, the electrode input is written:

$$i(t) = I_e(t)$$

## The leaky integrator for neurons

$$C \frac{dV}{dt} = g_l(E_l - V) + I_e(t)$$

# The leaky integrator for neurons

Dividing across by the  $g_l$  we get

$$\tau_m \frac{dV}{dt} = E_l - V + R_m I_e(t)$$

where  $\tau_m$  is called the **membrane time constant** and  $R_m = 1/g_l$  is called the **the membrane resistance**.

## The leaky bucket redux

$$\tau_m \frac{dV}{dt} = E_I - V + R_m I_e(t)$$

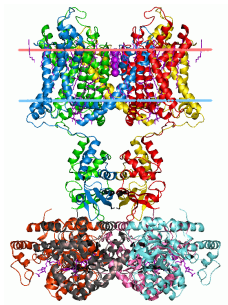
We have seen more-or-less this equation before, all that is really different is the extra constant  $E_I$  and so now how to deal with the equation, for example, if  $I_e$  is constant the solution is

$$V(t) = E_I + R_m I_e + [V(0) - E_I - R_m I_e] e^{-t/\tau_m}$$

# Voltage gated channels

This isn't the full story

$$\tau_m \frac{dV}{dt} = E_I - V + R_m I_e(t) + R_m (\text{currents from the gated channels})$$





## Action potential

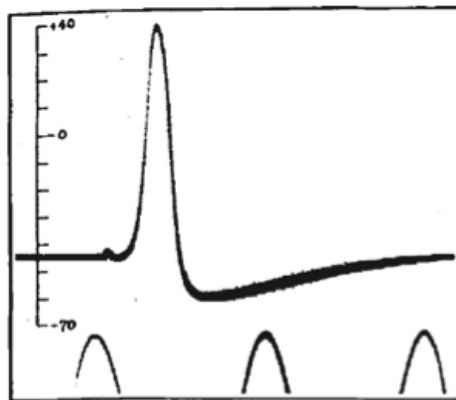


Fig. 2.

ACTION POTENTIAL RECORDED BETWEEN INSIDE AND OUTSIDE OF AXON. TIME MARKER, 500 CYCLES/SEC. THE VERTICAL SCALE INDICATES THE POTENTIAL OF THE INTERNAL ELECTRODE IN MILLIVOLTS, THE SEA WATER OUTSIDE BEING TAKEN AT ZERO POTENTIAL.

## Leaky integrate and fire model

$$\tau_m \frac{dV}{dt} = E_l - V + R_m I_e(t)$$

and put the spike in 'by hand'!

## Leaky integrate and fire model

$$\tau_m \frac{dV}{dt} = E_I - V + R_m I_e(t)$$

if  $V > V_T$  there is a spike and  $V = V_R$ .

## Leaky integrate and fire model

$$\tau_m \frac{dV}{dt} = E_l - V + R_m I_e(t)$$

we call  $V_T$  the threshold and  $V_R$  the reset.

## Leaky integrate and fire model

$$\tau_m \frac{dV}{dt} = E_I - V + R_m I_e(t)$$

with, for example,  $V_T \sim -55$  mV and  $V_R \sim -70$  mV.

## Constant input

$$\tau_m \frac{dV}{dt} = \bar{V} - V$$

where  $\bar{V} = E_I + R_m I_e$  then

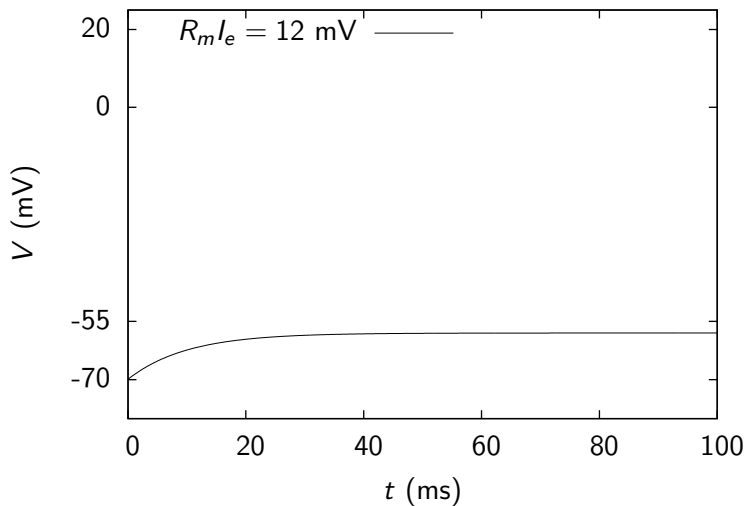
$$V(t) = \bar{V} + [V(0) - \bar{V}]e^{-t/\tau_m}$$

## Constant input

$$V(t) = \bar{V} + [V(0) - \bar{V}]e^{-t/\tau_m}$$

If  $\bar{V} < V_T$  the neuron will never spike!

## Regular spiking



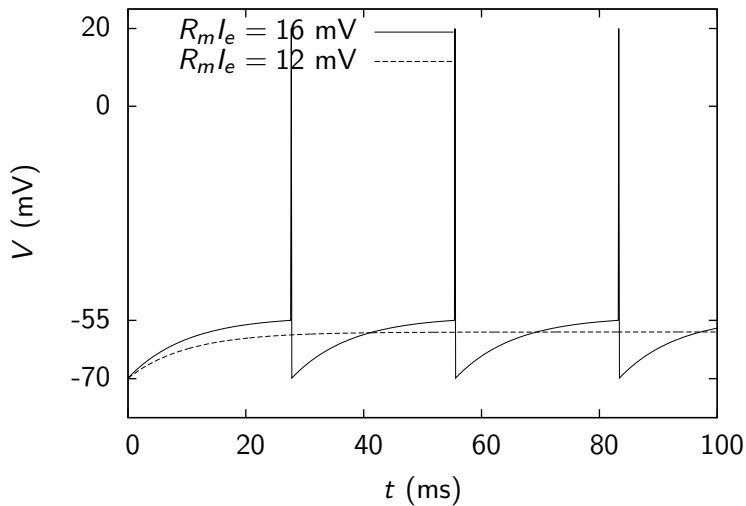


## Constant input

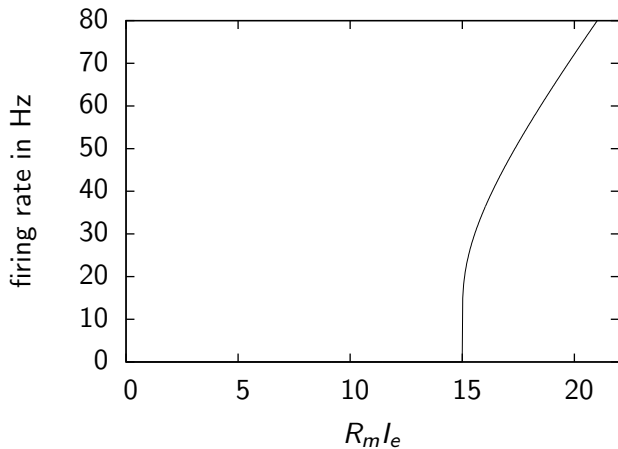
$$V(t) = \bar{V} + [V(0) - \bar{V}]e^{-t/\tau_m}$$

If  $\bar{V} > V_T$  it will approach  $\bar{V}$  but reach  $V_T$  on the way and spike.

## Regular spiking



## f-I curve



## Some observations

- ▶ This tells us that neurons filter their input.
- ▶ The model doesn't have spike rate adaptation but can be extended.
- ▶ Other extensions give more realistic behaviour near threshold.
- ▶ Including the voltage gated channels gives the [Hodgkin-Huxley](#) model.

## Some observations

- ▶ To make networks we also need a synapse model.
- ▶ Lots of synapse models use a similar equation.
- ▶ The equation for the voltage gated channels also use the same equation.

# Numerical calculations

We saw how to integrate the equation numerically. This can be done using MATLAB, or better, Python or Julia. There are also specialized packages like Brian in Python or GENESIS / NEURON / NEST.

# Summary

- ▶ Studied the dynamics of neurons away from threshold.
- ▶ Discussed chemical gradients.
- ▶ Looked at the integrate and fire model.
- ▶ Considered the f-I curve in the integrate and fire model.