

2E2 Tutorial Sheet 12 Second Term¹

27 January 2006

Useful facts:

- Formula for exponentials with pure imaginary arguments

$$\begin{aligned}e^{i\theta} &= \cos \theta + i \sin \theta \\e^{-i\theta} &= \cos \theta - i \sin \theta\end{aligned}\tag{1}$$

- Formula for sine and cos

$$\begin{aligned}\cos \theta &= \frac{e^{i\theta} + e^{-i\theta}}{2} \\ \sin \theta &= \frac{e^{i\theta} - e^{-i\theta}}{2i}\end{aligned}\tag{2}$$

Questions

1. (2) Find the general solutions for the system

$$\frac{dy_1}{dt} = -2y_1 + y_2\tag{3}$$

$$\frac{dy_2}{dt} = y_1 - 2y_2\tag{4}$$

Sketch the phase diagram and describe the node.

2. (3) Find the solution of

$$\frac{dy_1}{dt} = -9y_2\tag{5}$$

$$\frac{dy_2}{dt} = y_1\tag{6}$$

by considering $y_1(0) = r$ and $y_2(0) = 0$, draw the phase diagram.

3. (3) Find the solution of

$$\frac{dy_1}{dt} = y_1 - 3y_2\tag{7}$$

$$\frac{dy_2}{dt} = 3y_1 + y_2\tag{8}$$

for initial conditions $y_1(0) = r$ and $y_2(0) = 0$ write this in real form. Draw the phase diagram.

¹Conor Houghton, houghton@maths.tcd.ie and <http://www.maths.tcd.ie/~houghton/2E2.html>