2E2 Tutorial Sheet 3 First Term[1]

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Useful facts:

- Laplace transform of differenciated functions: if $\mathcal{L}[f(t)] = F(s)$ then

  $$\mathcal{L}(f') = sF - f(0) \tag{1}$$

  and

  $$\mathcal{L}(f'') = s^2F - sf(0) - f'(0) \tag{2}$$

- If there a repeated factor in the fraction the partial fraction expansion looks like:

  $$\frac{1}{(s-a)^2(s-b)} = \frac{A}{s-a} + \frac{B}{(s-a)^2} + \frac{C}{s-b} \tag{3}$$

- $\mathcal{L}(f) = F(s)$ then $\mathcal{L}(e^{at}f) = F(s-a)$, in particular

  $$\mathcal{L}(e^{at}t) = \frac{1}{(s-a)^2}, \tag{4}$$

- The Heaviside function $H_a(t)$ is zero for $t < a$ and one for $t \geq a$, it Laplace transform is given by

  $$\mathcal{L}[H_a(t)] = \frac{e^{-as}}{s} \tag{5}$$

- The shift theorem: if $\mathcal{L}(f) = F(s)$ then

  $$\mathcal{L}[H_a(t)f(t-a)] = e^{-as}F(s) \tag{6}$$

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Questions

1. (2) Using the Laplace transform solve the differential equation

$$f'' - 2f' + f = 0 \tag{7}$$

with boundary conditions $f'(0) = 1$ and $f(0) = 0$.

2. (2) Using the Laplace transform solve the differential equation

$$f'' + f' - 6f = e^{-3t} \tag{8}$$

with boundary conditions $f(0) = f'(0) = 0$.

3. (2) Use Laplace transform methods to solve the differential equation

$$f'' + 2f' - 3f = \begin{cases} 1, & 0 \leq t < c \\ 0, & t \geq c \end{cases} \tag{9}$$

subject to the initial conditions $f(0) = f'(0) = 0$. Notice that the right hand side is $1 - H_1(t)$.

4. (2) Use Laplace transform methods to solve the differential equation

$$f'' + 2f' - 3f = \begin{cases} 0, & 0 \leq t < 1 \\ 1, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases} \tag{10}$$

subject to the initial conditions $f(0) = f'(0) = 0$. You should begin by rewriting the right-hand side in terms of the Heaviside function:

$$H_1(t) - H_2(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 1, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases} \tag{11}$$