2E2 Tutorial Sheet 15 Second Term[1]

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Useful facts:

The idea is to draw the phase diagram by first making an approximate linearization near all the stationary points. The questions below guide you through the steps, but to summarize

1. Convert into y_1, y_2 form.
2. Find the stationary points, these are where $y_1' = y_2' = 0$.
3. Near each stationary point make an approximation. If the stationary point is at $(0, 0)$ then let y_1 and y_2 be small so $\sin y_1 \approx y_1$ and you can discard squares or cubes or higher powers in y_1. If the stationary point is at some other point, say $(a, 0)$, a a constant, then make y_1 be near a, hence $y_1 = a + \eta$ and η small. Now substitute that back in and use η small to make approximations, like dropping powers in η.
4. Now draw the phase diagram near each of the stationary points and then try to join it up to make the whole diagram. The arrows will go left to right above the y_1-axis.

Questions Consider the non-linear differential equation

$$y'' = y - y^2 \tag{1}$$

1. (1) By defining $y_1 = y$ and $y_2 = y_1'$ convert this into two first order equations.
2. (1) The stationary points are the points where $y_1' = y_2' = 0$, find the two stationary points for this equation.
3. (2) Consider the $y_1 = 0$ stationary point, linearize the equations near this point by assuming $y_1 \ll 1$. Solve the corresponding linear equations. What sort of stationary point is this?
4. (2) Consider the $y_1 = 1$ stationary point, linearize the equations near this point by assuming $y_1 = 1 + \eta$ where $\eta \ll 1$. Solve the corresponding linear equations. What sort of stationary point is this?
5. (2) Try and draw the whole phase diagram, first draw in the two stationary points and then try and join the lines, remember the lines don't cross.

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