2E2 Tutorial Sheet 14 Second Term[1]

10 February 2006

Useful facts:

- The first order differential equation $y' = ry + f(t)$ has solution

$$y = Ce^{rt} + e^{rt}\int_0^t f(\tau)e^{-r\tau}d\tau \tag{1}$$

- To solve the inhomogeneous system $\mathbf{y}' = A\mathbf{y} + \mathbf{g}$ substitute

$$\mathbf{y} = f_1\mathbf{x}_1 + f_2\mathbf{x}_2 \tag{2}$$

  where bfx_1 and $\mathbf{x}_2$ are the eigenvectors of A. You should also split $\mathbf{g}$ up as $\mathbf{g} = h_1\mathbf{x}_1 + h_2\mathbf{x}_2$. This will give you two independant first order inhomogeneous equations, one for $\mathbf{x}_1$ and one for $\mathbf{x}_2$.

- Here is an example of converting the second order differential equation $y'' + 2y' + 7y = 0$ into a system first order differential equations. First rename $y_1 = y$ and define $y_2 = y_1'$, now, $y_2' = y_1''$ which we know from the original equation:

$$y_1'' = y'' = -2y' - 7y \tag{3}$$

  but y' is y_2 and y is y_1, hence

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= -7y_1 - 2y_2 \end{aligned} \tag{4}$$

[1]Conor Houghton, houghton@maths.tcd.ie and http://www.maths.tcd.ie/~houghton/ 2E2.html

Questions

1. (2) Find the solution to
$$y' - 2y = -t \tag{5}$$

2. (3) Find the general solution to
$$\begin{aligned} y_1' &= 5y_2 - 23 \\ y_2' &= 5y_1 + 15. \end{aligned} \tag{6}$$
with $y_1(0) = -3$ and $y_2(0) = 5$.

3. (2) Find the general solution to
$$\begin{aligned} y_1' &= y_1 + 3y_2 + e^t \\ y_2' &= 3y_1 + y_2 \end{aligned} \tag{7}$$

4. (1) Rewrite $y'' + 4y' - 3y = 0$ as a system of two first order differential equations.