2E2 Tutorial Sheet 7[1]

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Useful facts:

- Z-transform:
$$\mathcal{Z}\left[(x_n)_{n=0}^{\infty}\right] = \sum_{n=0}^{\infty} \frac{x_n}{z^n} \tag{1}$$
- Linearity:
$$\mathcal{Z}\left[a(x_n) + b(y_n)\right] = aX(z) + bY(z) \tag{2}$$
where $\mathcal{Z}[(x_n)] = X(z)$ and $\mathcal{Z}[(y_n)] = Y(z)$
- The Z-transform of a geometric sequence:
$$\mathcal{Z}\left[(r^n)_{n=0}^{\infty}\right] = \frac{z}{z-r} \tag{3}$$
- Another useful Z-transform:
$$\mathcal{Z}\left[(nr^{n-1})_{n=0}^{\infty}\right] = \frac{z}{(z-r)^2} \tag{4}$$
- Advancing:
$$\mathcal{Z}\left[(x_{n+1})_{n=0}^{\infty}\right] = zX(z) - zx_0 \tag{5}$$
where $\mathcal{Z}[(x_n)] = X(z)$.

Questions

1. (2) Find the Z-transform of the sequence $(2, 0, 1, 0, -3, 0, 0, \ldots)$ where all the other entries are zero.
2. (2) Find the Z-transform of the geometric sequence $(3, 15, 75, 375, \ldots)$.
3. (2) Find the Z-transform of the sequence $(6, 12, 24, \ldots)$ both by considering it the advance of the sequence $(3, 6, 12, 24, \ldots)$ and by applying the formula for geometrical sequences directly. Do you get the same answer?
4. (2) For the difference equation
$$x_{k+1} = -3x_k \tag{6}$$
with $x_0 = 1$ work out $X(z) = \mathcal{Z}[(x_n)]$ by taking the Z-transform of both sides of the equation. Use this to solve the equation.

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