**2E2 Tutorial Sheet 8**[1]

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**Useful facts:**

- Standard Z-transforms

$$\begin{aligned} \mathcal{Z}\left[(r^n)_{n=0}^{\infty}\right] &= \frac{z}{z-r} \\ \mathcal{Z}\left[(nr^{n-1})_{n=0}^{\infty}\right] &= \frac{z}{(z-r)^2} \end{aligned} \tag{1}$$

- Advancing and delaying:

$$\begin{aligned} \mathcal{Z}\left[(x_{n+1})_{n=0}^{\infty}\right] &= zX(z) - zx_0 \\ \mathcal{Z}\left[(x_{n+2})_{n=0}^{\infty}\right] &= z^2X(z) - z^2x_0 - zx_1 \\ \mathcal{Z}\left[(x_{n-n_0})_{n=0}^{\infty}\right] &= \frac{1}{z^{n_0}}X(z) \end{aligned} \tag{2}$$

  where $\mathcal{Z}[(x_n)] = X(z)$ .

- Delta pulse: $(\delta_n)_{n=0}^{\infty} = (1, 0, 0, 0, \ldots)$ and $\mathcal{Z}\left[(\delta_n)_{n=0}^{\infty}\right] = 1$.

**Questions**

1. (2) Use the Z-tranform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = 1 \tag{3}$$

   with $x_1 = 0$ and $x_0 = 0$.

2. (2) Use the Z-tranform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = 3^k \tag{4}$$

   with $x_1 = 0$ and $x_0 = 0$.

3. (2) Use the Z-tranform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = \delta_k \tag{5}$$

   with $x_1 = 0$ and $x_0 = 0$. Remember $\delta_k$ is the unit pulse with $\delta_k = (1, 0, 0, 0, \ldots)$.

4. (2) Use the Z-tranform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = 0 \tag{6}$$

   with $x_1 = 2$ and $x_0 = 3$.

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