2E2 Tutorial Sheet 5 First Term[1]

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Useful facts:

- The formula for complex exponentials:

$$\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned} \tag{1}$$

- Remember $e^{a+b} = e^a e^b$ so,

$$\begin{aligned} e^{a+ib} &= (\cos b + i\sin b)e^a \\ e^{a-ib} &= (\cos b - i\sin b)e^a \end{aligned} \tag{2}$$

- Laplace transform of a periodic function with period c:

$$\mathcal{L}(f) = \frac{1}{1-e^{-cs}}\int_0^c f(t)e^{-st}dt \tag{3}$$

- Integration by parts:

$$\int_a^b u dv = uv]_a^b - \int_a^b v du \tag{4}$$

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/2E2.html

Questions

1. (2) Solve, using Laplace transforms,

$$f'' + 4f = 1 \tag{5}$$

with $f(0) = f'(0) = 0$.

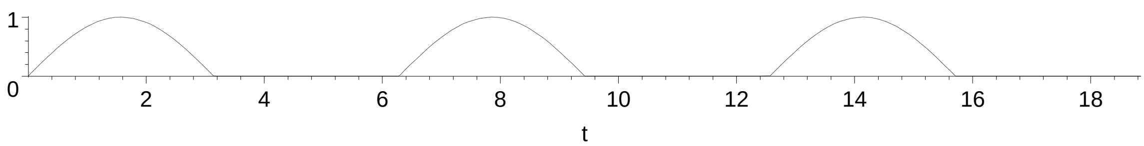
2. (3) Using the Laplace transform solve the differential equation

$$f'' + 6f' + 13f = e^t \tag{6}$$

with boundary conditions $f(0) = 0$ and $f'(0) = 0$.

3. (3) Use the formula for the Laplace transform of a periodic function to find the Laplace transform of a half-rectified wave

$$f(t) = \begin{cases} \sin t & \sin t > 0 \\ 0 & \sin t \leq 0 \end{cases} \tag{7}$$



This is the form a AC current has after going through a diode and is a periodic function with period 2π. To do the integral, the easiest way is probably to split the sine up using

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \tag{8}$$

and then use the usual formula from the log tables for the integral of an exponetential:

$$\int e^{at} dt = \frac{1}{a} e^{at} \tag{9}$$

This works for complex a. Try to get a real answer.