

2E2 Tutorial Sheet 13 Second Term¹

5 February 2006

Useful facts:

- Formula for exponentials with pure imaginary arguments

$$\begin{aligned}e^{i\theta} &= \cos \theta + i \sin \theta \\e^{-i\theta} &= \cos \theta - i \sin \theta\end{aligned}\tag{1}$$

- For $\mathbf{y}' = A\mathbf{y}$ if A has only one eigenvalue λ and only one eigenvector \mathbf{x} the solution is

$$\mathbf{y} = c_1 \mathbf{x} e^{\lambda t} + c_2 (t\mathbf{x} + \mathbf{u}) e^{\lambda t}\tag{2}$$

where \mathbf{u} solves

$$(A - \lambda \mathbf{1})\mathbf{u} = \mathbf{x}\tag{3}$$

and $\mathbf{1}$ is the identity matrix; some people use I or I_2 for this.

Questions

1. (2) An equation system has solution

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2i \end{pmatrix} e^{2it} + c_2 \begin{pmatrix} 1 \\ -2i \end{pmatrix} e^{-2it}\tag{4}$$

Sketch the phase diagram.

2. (3) Find the general solution for the system

$$\frac{dy_1}{dt} = 3y_1 + y_2\tag{5}$$

$$\frac{dy_2}{dt} = -y_1 + y_2\tag{6}$$

3. (3) Find the solution for the system

$$\begin{aligned}y_1' &= 4y_1 + y_2 \\y_2' &= -y_1 + 2y_2.\end{aligned}$$

with initial conditions $y_1(0) = 3$ and $y_2(0) = 2$.

¹Conor Houghton, houghton@maths.tcd.ie and <http://www.maths.tcd.ie/~houghton/2E2.html>