# The Information Carrying Capacity of Spike Trains

J.B.Gillespie, C. Houghton, Trinity College Dublin

## Introduction

It is important to understand the information carrying capacity of spike trains. Existing methods can be difficult to calculate and do not necessarily respect all the properties of the spike train. We believe we have a novel way to improve on this by making use of the metric geometry of spike trains, combined with information theory.

## Method Overview

Each spike train was treated as a continuous time interval, contained in a metric space. A metric was used to obtain the distribution of the neural noise and stimulus signal. The capacity of a spike train was subsequently obtained. Data comprised of spike trains from the primary auditory part of an anethizied zebra finch's forebrain [1].

## The Information Capacity of Spike Trains

Neurons communicate with each other, so it is natural to think of the existence of inter-neuron information channels. Down these channels run electrical impulses; spikes. While the stimulus triggering the release of these spikes can be a mathematically illusive quantity, careful use of channel capacity theory allows us to put a bound on the information flow through such a channel.

## Information Theory and Channel Capacity

Information theory is a mathematical framework for communication. It addresses the problem of reproducing at one point a message transmitted from another. A subset of this, channel capacity theory, allows us to find the maximum amount of information that can be passed between two points, with arbitrarily low error. Associated with each channel is an input, $X_i$, an output, $Y_i$, and a certain amount of inherent noise, $Z_i$ , [2]. If the noise is additive,these are related through:

$$Y_i = X_i + Z_i$$

If the noise has a Gaussian distribution, then the maximum quantity of information that can flow, the capacity of this channel, is given by:

$$C = \frac{1}{2}\log_2\left(1+\frac{\nu^2}{\sigma^2}\right)$$

where $\nu^2$ and $\sigma^2$ represent the variance of input signal and the noise respectively. The indexing '$i$' of the input is used to distinguish sequential information transmissions made through the channel. So if we are able to count the number of '$i$' per second, we will know the number of transmissions per second. Similarly, if we can measure $\nu^2$ and $\sigma^2$ , we can find the capacity of the inter-neuron channel.

## The Useful Metric

How different is one spike train from another? A metric provides a way of answering this [2]. It allows us to find the 'distance' between two spike trains. A 'good' metric should measure a shorter distance between train responses to the same stimuli and a longer one for responses to different stimuli. There exists no unique metric for doing this, and we chose the van Rossum metric [3] for this anlaysis. In order to calculate the van Rossum metric, a spike train:

$$\bar{t} = \{t_1, t_2, .., t_n\}$$

is firstly mapped to a function of time, that is, filtered:

$$\bar{t} \mapsto f(t;\bar{t}) = \left[\sum_{i=1}^{n} h(t-t_i)\right]$$

where $h(t)$ is a kernel. The causal exponential kernel is used here:

$$h(t) = \begin{cases} 0 & t < 0 \\ e^{-\frac{t}{\tau}} & t \geq 0 \end{cases}$$

where $\tau$ is a timescale parametrizing the metric. The $L^2$metric on this space of real functions then induces a metric on the space of spike trains, specifically, if $\bar{t_1}$and $\bar{t_2}$ are two spike trains, then the distance between them is:

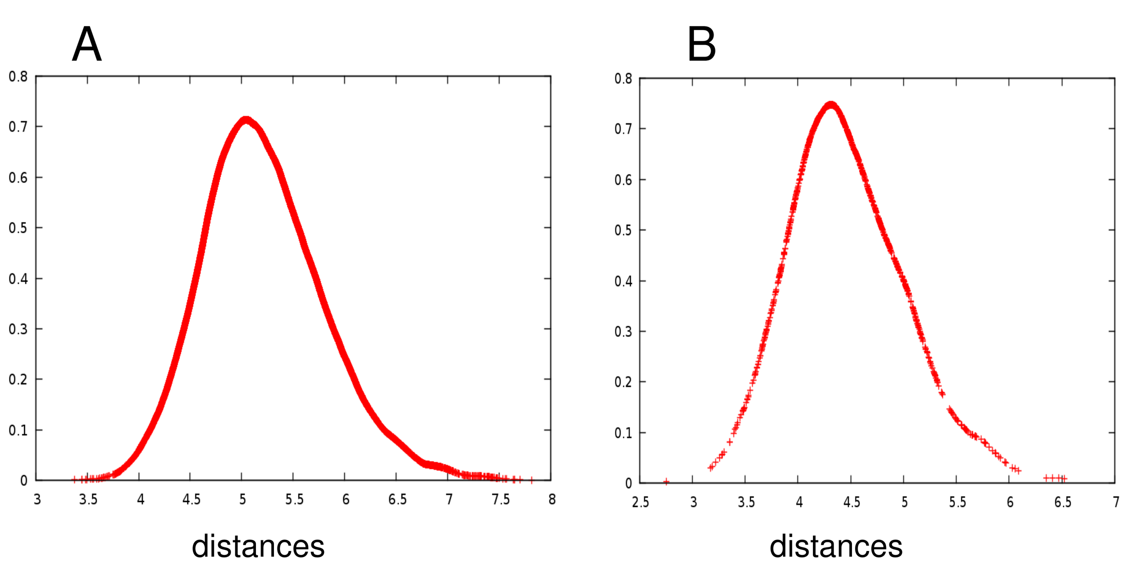
$$d(\bar{t_1},\bar{t_2}) = \sqrt{\int [f(t;\bar{t_1}) - f(t;\bar{t_1})]^2 dt}$$

For a large data set of spike trains, the resulting probability density function of the distances can be plotted using kernel density estimation.

## The Chi Distribution

In the presence of noise, applying the van Rossum metric to sets of trains will lead to a set of distances with an associated distribution. If the noise is indeed additive and Gaussian then this distribution should be a chi:

$$f_\chi(x;k,\sigma) = \frac{1}{\sigma^k 2^{k/2-1}\Gamma(\frac{k}{2})} x^{k-1} e^{-x^2/2\sigma^2}$$




Above are the resulting chi density functions of distance distributions after the van Rossum metric was applied to a set of 200 spike trains. The left distribution, A, occured when trains corresponding to different stimuli were compared, while B when trains corresponding to the same stimuli were compared.

## Metric Geometry Meets Channel Theory

It is possible to re-write the equation for $C$ in terms of metric distances:

$$C = \frac{1}{2}log_2\left(\frac{\mu_d^2}{\sigma_d^2}\right)$$

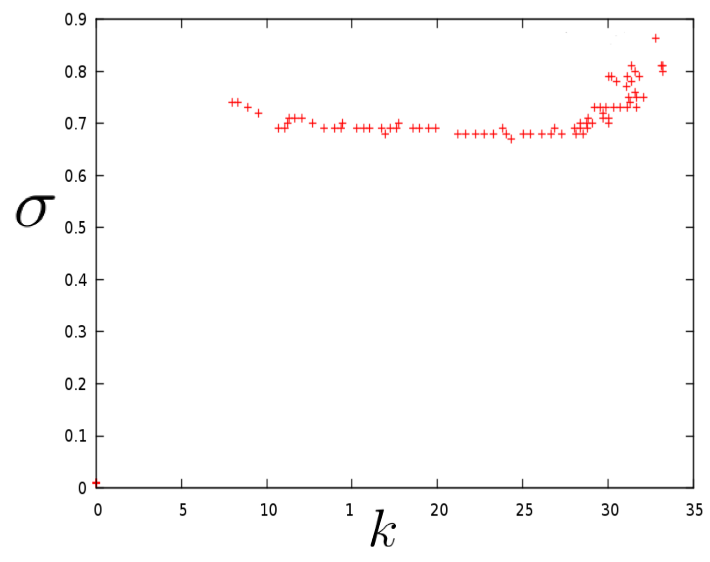
where $\mu_d^2$ represents the variance of the received signal distance distribution and $\sigma_d^2$ is just the variance of the noise distance distribution. By calculating the variance of the distribution of distances between spike trains, where the trains corresponded to presentations of different stimuli, we can find $\mu_d^2$ , the signal distance variance. Hence we have all the quantities needed to calculate the capacity of the channel per transmission. With $k$ , the number of transmissions per second, the capacity of the channel in bits per second can be obtained.

## Results

Of the data considered, the results show a neuron from field L of an anesthetized adult male zebra finch has a capacity of around 13 to 15 bits per second. Our values are of orders similar to those measured biologically [4].

The channel output variable had a metric chi distribution, indicating the neuron was using the channel efficiently.

No correlations in the noise variance, $\sigma$ , were detected as the dimension of the spike train, $k$ , was decreased:




Above shows the absence of correlation,in the form of jitter, between the dimension of the noise, $k$ , and the standard deviation of the underlying Gaussian variable, $\sigma$ .

## Conclusion

By treating each spike train as being contained in a metric space, and reformulation of the results of channel theory in terms of this metric space, the channel capacity of a neuron from field L can be calculated.

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## References

[1] R.Naragan, G.Grana, K.Sen. *Journal of Neurophysiology*,96; 25--258, 2006.
[2] CE.Shannon. *Bell Syst.Tech.J*, 27; 379--423, 1948.
[3] JD.Victor, KP Purpura. *Journal of Neurophysiology*, 76(2); 1310--1326, 1996.
[4] M.van Rossum. *Neural Computation*, 13; 751--763, 2001.
[5] A.Borst, FE.Theunissen. *Nature Neuroscience*, 2; 947--957, 1999.

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