

2E2 Tutorial Sheet 5 First Term¹

12 November 2005

Useful facts:

- The formula for complex exponentials:

$$\begin{aligned}e^{i\theta} &= \cos \theta + i \sin \theta \\e^{-i\theta} &= \cos \theta - i \sin \theta\end{aligned}\quad (1)$$

- Remember $e^{a+b} = e^a e^b$ so,

$$\begin{aligned}e^{a+ib} &= (\cos b + i \sin b)e^a \\e^{a-ib} &= (\cos b - i \sin b)e^a\end{aligned}\quad (2)$$

- Laplace transform of a periodic function with period c :

$$\mathcal{L}(f) = \frac{1}{1 - e^{-cs}} \int_0^c f(t)e^{-st} dt \quad (3)$$

- Integration by parts:

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du \quad (4)$$

Questions

1. (2) Solve, using Laplace transforms,

$$f'' + 4f = 1 \quad (5)$$

with $f(0) = f'(0) = 0$.

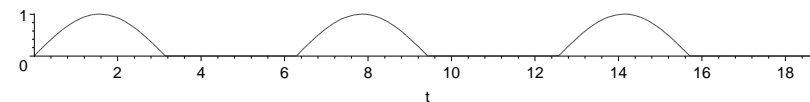
2. (3) Using the Laplace transform solve the differential equation

$$f'' + 6f' + 13f = e^t \quad (6)$$

with boundary conditions $f(0) = 0$ and $f'(0) = 0$.

3. (3) Use the formula for the Laplace transform of a periodic function to find the Laplace transform of a half-rectified wave

$$f(t) = \begin{cases} \sin t & \sin t > 0 \\ 0 & \sin t \leq 0 \end{cases} \quad (7)$$



This is the form a AC current has after going through a diode and is a periodic function with period 2π . To do the integral, the easiest way is probably to split the sine up using

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \quad (8)$$

and then use the usual formula from the log tables for the integral of an exponential:

$$\int e^{at} dt = \frac{1}{a} e^{at} \quad (9)$$

This works for complex a . Try to get a real answer.

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