2E2 Tutorial Sheet 11[1]

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Useful facts:

- The stationary point, or critical point, is the point where $y_1' = y_2' = 0$, in linear examples where $\mathbf{y}' = A\mathbf{y}$ this happens only at $y_1 = y_2 = 0$. Because $\mathbf{y}'$ is determined by $\mathbf{y}$ the stationary point is the only place lines can cross.
- If you are asked to name the stationary point, you are asked to say what the behaviour around it it, we have seen so far improper and proper inward and outward nodes and saddlepoints, next week we will see spiral and circle nodes.
- When drawing the phase diagram, put the eigenvectors in first, if the eigenvector has a negative eigenvalues it goes inwards, if it has positive, it goes out. When the eigenvectors are done, it is simple to add the other trajectories.
- One eigenvalue positive one negative gives a saddlepoint. Two unequal positive eigenvalues gives an outward improper node, if they are both negative the node is inward, if they are equal the node is proper.

[1] Conor Houghton, `houghton@maths.tcd.ie` and `http://www.maths.tcd.ie/~houghton/2E2.html`

Questions

1. (2) For the system

$$\begin{aligned}\frac{dy_1}{dt} &= -3y_1 + 2y_2 \\ \frac{dy_2}{dt} &= -2y_1 + 2y_2\end{aligned}$$

The solution is

$$\mathbf{y} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^t + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-2t} \tag{1}$$

Sketch the phase diagram and and describe the stationary point.

2. (2) For the system

$$\frac{dy_1}{dt} = 3y_1 + y_2 \tag{2}$$

$$\frac{dy_2}{dt} = y_1 + 3y_2 \tag{3}$$

The solution is

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t}. \tag{4}$$

Sketch the phase diagram and and describe the stationary point.

3. (4) Find the general solutions for the system

$$\frac{dy_1}{dt} = 2y_1 - y_2 \tag{5}$$

$$\frac{dy_2}{dt} = -4y_2 \tag{6}$$

Sketch the phase diagram and and describe the stationary point.