# A van Rossum metric with a synapse-like filter

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## Abstract

A new spike train metric is constructed based on a simple model of synaptic conductances which includes binding site depletion. Including binding site depletion in the metric means that a given individual spike has a smaller effect on the distance if it occurs soon after other spikes. The metric proves effective at classifying neuronal responses by stimuli in the sample data set of electro-physiological recordings from the primary auditory area of the zebra finch fore-brain. This shows that this is an effective metric for these spike trains suggesting that in these spike trains the significance of a spike is modulated by its proximity to previous spikes. This modulation is a putative information-coding property of spike trains.

## How is information coded?

Spike trains are unreliable: in the primary sensory areas for example, spike patterns and precise spike times will vary between responses to the same stimuli. Nonetheless, information about sensory inputs is communicated in the form of spike trains. A challenge in understanding spike trains is to assess the significance of individual spikes in encoding information.

## Metrics

One approach to this is to define a spike train metric, allowing a distance to be calculated between pairs of spike trains [5, 2, 4]. A good metric will measure a short distance between responses to the same stimuli and a longer distance between responses to different stimuli: this indicates that the distance measured between two spike trains is related to the information they encode.

## Evaluating metrics

A metric can be evaluated by performing distance-based clustering and calculating how accurately it clusters responses by stimulus. Using this comparison to optimize a parameter in a family of metrics then gives a measurement of how information is coded in the spike trains.

- Ten spike train responses to each of 20 songs.
- Cluster responses using a metric.
- Assess accuracy of clustering.
- Interpret metric features.

Clustering accuracy is measured using transmitted information [5].

$$\tilde{h} = \frac{1}{n}\sum_{ij} N_{ij}\left(\ln N_{ij} - \ln\sum_k N_{kj} - \ln\sum_k N_{ik} + \ln n\right) / \ln s.$$

where $N$ is the confusion matrix, square matrix whose $ij$th entry, $N_{ij}$ is the number of responses from stimulus $i$ which are closest, on average, to the responses from stimulus $j$. $n$ is the number of responses and $s$ the number of stimuli. $\tilde{h} = 1$ for perfect clustering.

## A new metric

A new van Rossum-like metric is defined by filtering of the spike train with a new map:

$$\text{spike train} \to f(t)$$

where $f(t)$ is the solution of

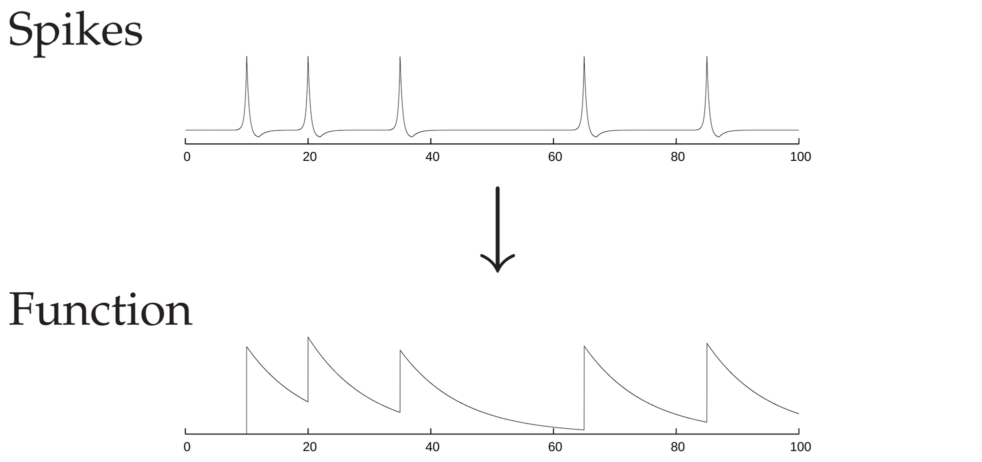
$$\tau\frac{d}{dt}f = -f$$

with discontinuities

$$f \to (1-\mu)f + 1$$

at the spike times. The usual $L^2$ metric on the space of functions then induces a metric on the space of spike trains:

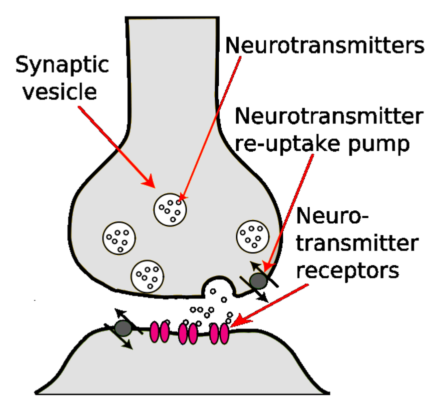
$$d = \sqrt{\int dt[\delta f(t)]^2}$$




## Motivation

The filter mimicks the short term dynamics of synaptic conductance, modelling rapid binding and stochastic unbinding of neurotranmitter to gates in the synaptic cleft [1]

- $\tau$ is the time-scale for unbinding.
- $\mu$ quantifies the effect of the depletion of available binding sites.




When a spike arrives at the terminal button the synaptic cleft is rapidly flooded with neurotransmitter. The neurotransmitter binds to receptors in ligand gated channels, opening them and causing a change of the potential in the dendritic spine. The concentration of neurotransmitter falls quickly. Compared to the unbinding timescale, there is a only a significant concentration in the cleft for a short time. Modeling the rise profile of the conductance in a metric suggests it is not significant. However, the extent to which it rises does, this is what is modelled in the new metric.

## References

[1] Dayan P, Abbott LF. *Theoretical Neuroscience*. MIT Press, 2001.

[2] Hunter JD, Milton JG, Thomas PJ, Cowan JD. *Journal of Neurophysiology*, 80:1427–1438, 1998.

[3] Narayan R, Graña G, Sen K. *Journal of Neurophysiology*, 96:252–258, 2006.

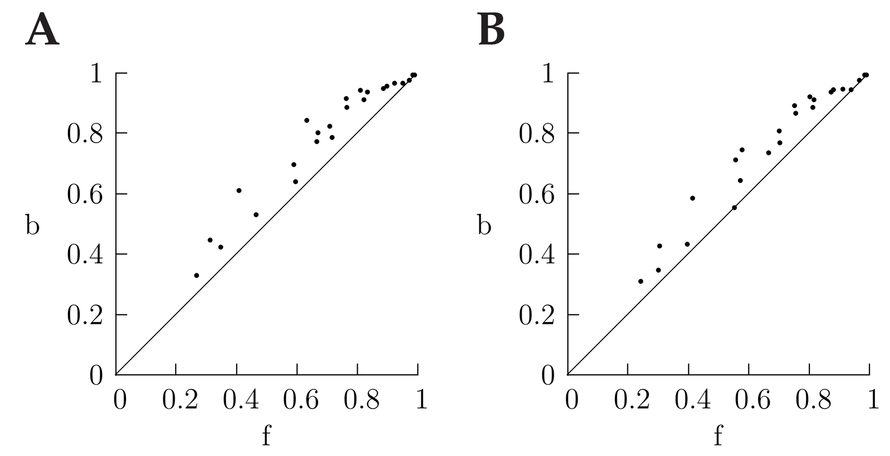
[4] van Rossum M. *Neural Computation*, 13:751–763, 2001.

[5] Victor JD, Purpura KP. *Journal of Neurophysiology*, 76(2):1310–1326, 1996.

## To appear

Studying spike trains using a van Rossum metric with a synapse-like filter. Conor Houghton, Journal of Computational Neuroscience (2008).

## Results



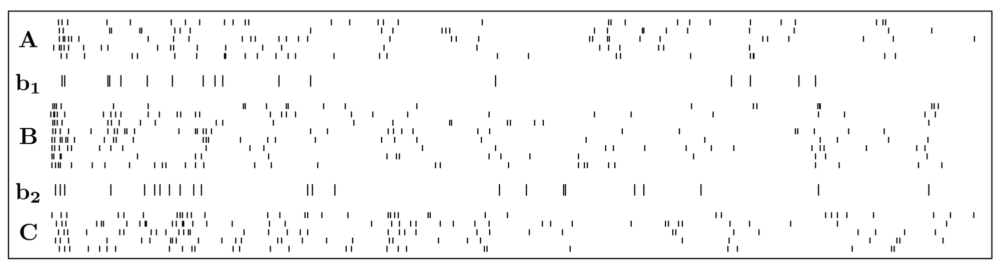

Comparing the old metric to the new. In **A** and **B** each dot corresponds to a single site with the $\tilde{h}$ value calculated using the new metric plotted on the horizonal axis and the value calculated using the old metric on the vertical. The old metric does not include the effect of binding site depletion and corresponds to $\mu = 0$ here. The '$x = y$' line is added for clarity, any dot above the line has a better $\tilde{h}$ value if the f-metric is used. In **A** optimal parameter values were used for each site for both the metrics. In **B** the same, average, values were used for each site.

## Conclusions

The van Rossum metric corresponds to $\mu = 0$; for the data examined here there is a modest but definate improvement if a non-zero value of $\mu$ is used.

- Isolated spikes are weighted more heavily.
- Synaptic conductances track information content.

## Misclassified spike trains



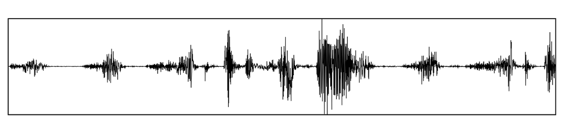

Here, rasters are plotted for one second of response by one site to three different songs. The rasters marked **A**, **B** and **C** are responses to different songs for which, in each case, the spike train is correctly classified by both the metrics. The rasters marked $b_1$ and $b_2$ each show one response to song B. The raster marks are double height for these responses, this is done for clarity and does not reflect any property of these spike trains. $b_1$ and $b_2$ are both incorrectly classified by the old metric but are both correctly assigned to B by the new metric.

## Discussion

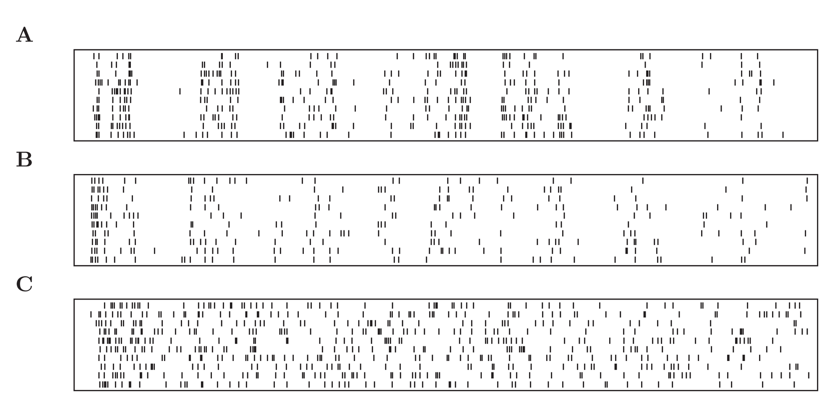
The information processing step that is novel to the metric, the modulation of the significance of spike times, could occur naturally in real neuronal systems in the precisely the mechanism that originally inspired the metric, through the depletion of binding sites in synapses. It would not be surprising to find, as indicated here, that aspects of the neural code are apparent in the post-synaptic voltage response, it is striking, however, that the depletion of binding sites appears to be a significant detail.

## Data

The metrics have been applied electro-physiological data recorded from the primary auditary area of zebra finch during playback of conspecific songs [3].



×20




Ten responses are recorded to each song; responses for three different cells are shown here.

The recordings were taken from field L of anesthetized adult male zebra finch and data was collected from sites which showed enhanced activity during song playback. In the ascending auditory pathway, area field L is afferent to the song system and is considered the oscine analogue of the primary auditory cortex. 24 sites are considered here; of these, six are classified as single-unit sites and the rest as consisting of two to five units. The average spike rate during song playback is 15.1 Hz with a range of across sites of 10.5-33 Hz.

**Acknowledgements:** Thanks to the Human Frontiers Science Programme and to Science Foundation Ireland for grants 06/RFP/BIM020 and 08/RFP/MTH1280 and to Kamal Sen for useful discussion and for the use of the data analysed here.

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