2E2 Tutorial Sheet 6 First Term[1]

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Useful facts:

- Laplace transform of a periodic function with period c:

$$\mathcal{L}(f) = \frac{1}{1-e^{-cs}} \int_0^c f(t)e^{-st}dt \tag{1}$$

- Integration by parts:

$$\int_a^b udv = uv\Big]_a^b - \int_a^b vdu \tag{2}$$

- The convolution:

$$f * g(t) = \int_0^t f(\tau)g(t-\tau)d\tau \tag{3}$$

- The convolution theorem, for two functions $f(t)$ and $g(t)$

$$\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g) \tag{4}$$

Questions:

1. (2) Verify the formula $\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g)$ in the case where $f = \exp(2t)$ and $g = \exp(2t)$.

2. (3) Find the convolution $(f * g)(t)$ when $f(t) = t$, $g(t) = e^{2t}$ $(t \geq 0)$.

3. (3) Use the convolution theorem to find the function $f(t)$ with

$$\mathcal{L}(f) = \frac{1}{s^2(s-4)}. \tag{5}$$

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