# 2E2 Tutorial Sheet 4 First Term[1]

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**Useful facts:**

- The Heaviside function $H_a(t)$ is zero for $t < a$ and one for $t \geq a$, it Laplace transform is given by
$$\mathcal{L}[H_a(t)] = \frac{e^{-as}}{s} \tag{1}$$
- The Dirac delta function $\delta(t-a)$ is zero everywhere except $t = a$ where it is infinite. It can be thought of as the $b$ goes to zero limit of
$$\delta_b(t-a) = \frac{1}{b}\left(H_a(t) - H_{a+b}(t)\right) \tag{2}$$
- A delta function evaluates an integral:
$$\int_0^\infty \delta(t-a)f(t)dt = f(a) \tag{3}$$
- The Laplace transform: $\mathcal{L}(\delta(t-a)) = e^{-as}$.
- The shift theorem: if $\mathcal{L}(f) = F(s)$ then
$$\mathcal{L}[H_a(t)f(t-a)] = e^{-as}F(s) \tag{4}$$
- The formula for complex exponentials:
$$\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned} \tag{5}$$
- Remember $e^{a+b} = e^a e^b$ so,
$$\begin{aligned} e^{a+ib} &= (\cos b + i\sin b)e^a \\ e^{a-ib} &= (\cos b - i\sin b)e^a \end{aligned} \tag{6}$$

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**Questions**

1. (4) Use Laplace transform methods to solve the differential equation
$$f'' + 2f' - 3f = \delta(t-1) \tag{7}$$
subject to the initial conditions $f(0) = 0$, $f'(0) = 1$.

2. (4) Using the Laplace transform solve the differential equation
$$f'' + 6f' + 13f = 0 \tag{8}$$
with boundary conditions $f(0) = 0$ and $f'(0) = 1$ and get your answer into a real form.