

2E2 Tutorial Sheet 7 Solutions¹

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Questions

1. (2) Find the Z-transform of the sequence $(2, 0, 1, 0, -3, 0, 0, \dots)$ where all the other entries are zero.

Solution: So, use the formula

$$\mathcal{Z}[(x_n)] = \sum_{n=0}^{\infty} \frac{x_n}{z^n} \quad (1)$$

to get

$$\mathcal{Z}[(2, 0, 1, 0, -3, 0, 0, \dots)] = 2 + \frac{1}{z^2} - \frac{3}{z^4} \quad (2)$$

2. (2) Find the Z-transform of the geometric sequence $(3, 15, 75, 375, \dots)$.

Solution: This sequence has the form 3×5^n so we can use the formula for the geometric sequence to get

$$\mathcal{Z}[(3 \times 5^n)] = 3\mathcal{Z}[(5^n)] = \frac{3z}{z-5} \quad (3)$$

3. (2) Find the Z-transform of the sequence $(6, 12, 24, \dots)$ both by considering it the advance of the sequence $(3, 6, 12, 24, \dots)$ and by applying the formula for geometrical sequences directly. Do you get the same answer?

Solution: Now this example is advanced by one step, so we use the formula

$$\mathcal{Z}[(x_{k+1})] = zX(z) - zx_0 \quad (4)$$

where $X(z) = \mathcal{Z}[(x_n)]$. In this case we have

$$\mathcal{Z}[(3, 6, 12, 24, \dots)] = \frac{3z}{z-2} \quad (5)$$

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so

$$\mathcal{Z}[(6, 12, 24, \dots)] = z \frac{3z}{z-2} - 3z = \frac{3z^2}{z-2} - 3z = \frac{6z}{z-2} \quad (6)$$

Working directly, this is the sequence 6×2^n and so the Z-transform is

$$\mathcal{Z}[(6 \times 2^n)] = 6\mathcal{Z}[(2^n)] = \frac{6}{z-2} \quad (7)$$

which is, of course, the same answer.

4. (2) For the difference equation

$$x_{k+1} = -3x_k \quad (8)$$

with $x_0 = 1$ work out $X(z) = \mathcal{Z}[(x_n)]$ by taking the Z-transform of both sides of the equation. Use this to solve the equation.

Solution: So taking the Z-transform of both sides we get

$$zX - z = -3X \quad (9)$$

So

$$X = \frac{z}{z+3} \quad (10)$$

Now

$$x_k = (-3)^k \quad (11)$$