2E2 Tutorial Sheet 13 Second Term[1]

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Useful facts:

- Formula for exponentials with pure imaginary arguements

$$\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned} \tag{1}$$

- For $\mathbf{y}' = A\mathbf{y}$ if A has only one eigenvalue λ and only one eigenvector $\mathbf{x}$ the solution is

$$\mathbf{y} = c_1\mathbf{x}e^{\lambda t} + c_2(t\mathbf{x} + \mathbf{u})e^{\lambda t} \tag{2}$$

where $\mathbf{u}$ solves

$$(A - \lambda\mathbf{1})\mathbf{u} = \mathbf{x} \tag{3}$$

and $\mathbf{1}$ is the identity matrix; some people use I or I_2 for this.

Questions

1. (2) An equation system has solution

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2i \end{pmatrix} e^{2it} + c_2 \begin{pmatrix} 1 \\ -2i \end{pmatrix} e^{-2it} \tag{4}$$

Sketch the phase diagram.

2. (3) Find the general solution for the system

$$\frac{dy_1}{dt} = 3y_1 + y_2 \tag{5}$$

$$\frac{dy_2}{dt} = -y_1 + y_2 \tag{6}$$

3. (3) Find the solution for the system

$$\begin{aligned} y_1' &= 4y_1 + y_2 \\ y_2' &= -y_1 + 2y_2. \end{aligned}$$

with initial conditions $y_1(0) = 3$ and $y_2(0) = 2$.