2E2 Tutorial Sheet 17 Second Term[1]

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Useful facts:

- Substitute
$$y = \sum_{n=0}^{\infty} a_n t^n \tag{1}$$
and work out each term in the equation, for example, by differenciating
$$y' = \sum_{n=0}^{\infty} a_n n t^{n-1} \tag{2}$$
or by multiplying
$$ty = \sum_{n=0}^{\infty} a_n t^{n+1} \tag{3}$$
- Do a change of index so all the terms have the same power as the term with the highest power, remove terms from the sums to get the same summation range.
- All coefficients are zero, this gives the recursion relation and, often, some other, conditions.

Note: If
$$y = \sum_{n=0}^{\infty} a_n t^n \tag{4}$$
then, by setting $t = 0$
$$y(0) = a_0. \tag{5}$$
Similarily,
$$y' = \sum_{n=0}^{\infty} n a_n t^{n-1} \tag{6}$$
and, by setting $t = 0$
$$y'(0) = a_1. \tag{7}$$
On the other hand if no initial condition is given then for a first order equation a_0 is arbitrary and for a second order equation a_0 and a_1 are both arbitrary, so when you write out the non-zero terms, there are a_0's and a_1's appearing. In Q1 the arbitrary constant a_0 has been renamed A, this is just to make the solution look nicer.

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Questions:

1. (2) Assuming the solution of
$$(1-t)y' + y = 0 \tag{8}$$
has a series expansion about $t = 0$ work out the recursion relation. Write out the first few terms and show that the series $a_2 = 0$ so the series actually terminates to give $y = A(1-t)$ for arbitrary A.

2. (2) Assuming the solution of
$$(1-t^2)y' - 2ty = 0 \tag{9}$$
has a series expansion about $t = 0$, work out the recursion relation.

3. (2) Assuming the solution of
$$y'' - 3y' + 2y = 0 \tag{10}$$
has a series expansion about $t = 0$, by substitution, work out the recursion relation. If $y(0) = 1$ and $y'(0) = 0$ what are the first three non-zero terms.

4. (2) Assuming the solution of
$$y'' - 3t^2 y = 0 \tag{11}$$
has a series expansion about $t = 0$ work out the recursion relation and write out the first four non-zero terms if $y(0) = 1$ and $y'(0) = 1$.