2E2 Tutorial Sheet 7 Solutions[1]

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Questions

1. (2) Find the Z-transform of the sequence $(2, 0, 1, 0, -3, 0, 0, \ldots)$ where all the other entries are zero.

   Solution: So, use the formula

$$\mathcal{Z}[(x_n)] = \sum_{n=0}^{\infty} \frac{x_n}{z^n} \tag{1}$$

   to get

$$\mathcal{Z}[(2, 0, 1, 0, -3, 0, 0, \ldots)] = 2 + \frac{1}{z^2} - \frac{3}{z^4} \tag{2}$$

2. (2) Find the Z-transform of the geometric sequence $(3, 15, 75, 375, \ldots)$.

   Solution: This sequence has the form 3×5^n so we can use the formula for the geometric sequence to get

$$\mathcal{Z}[(3 \times 5^n)] = 3\mathcal{Z}[(5^n)] = \frac{3z}{z-5} \tag{3}$$

3. (2) Find the Z-transform of the sequence $(6, 12, 24, \ldots)$ both by considering it the advance of the sequence $(3, 6, 12, 24, \ldots)$ and by applying the formula for geometrical sequences directly. Do you get the same answer?

   Solution: Now this example is advanced by one step, so we use the formula

$$\mathcal{Z}[(x_{k+1})] = zX(z) - zx_0 \tag{4}$$

   where $X(z) = \mathcal{Z}[(x_n)]$. In this case we have

$$\mathcal{Z}[(3, 6, 12, 24, \ldots)] = \frac{3z}{z-2} \tag{5}$$

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   so

$$\mathcal{Z}[(6, 12, 24, \ldots)] = z\frac{3z}{z-2} - 3z = \frac{3z^2}{z-2} - 3z = \frac{6z}{z-2} \tag{6}$$

   Working directly, this is the sequence 6×2^n and so the Z-transform is

$$\mathcal{Z}[(6 \times 2^n)] = 6\mathcal{Z}[(2^n)] = \frac{6}{z-2} \tag{7}$$

   which is, of course, the same answer.

4. (2) For the difference equation

$$x_{k+1} = -3x_k \tag{8}$$

   with $x_0 = 1$ work out $X(z) = \mathcal{Z}[(x_n)]$ by taking the Z-transform of both sides of the equation. Use this to solve the equation.

   Solution: So taking the Z-transform of both sides we get

$$zX - z = -3X \tag{9}$$

   So

$$X = \frac{z}{z+3} \tag{10}$$

   Now

$$x_k = (-3)^k \tag{11}$$