$\frac{1}{2}\frac{1}{2}+\frac{1}{4}$

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$X \backslash Y$	a	b
1	$\frac{1}{4}$	$\frac{1}{12}$
2	$\frac{1}{12}$	$\frac{1}{4}$
3	$\frac{1}{12}$	$\frac{1}{4}$

$\frac{1}{4}+\frac{1}{6}=\frac{3+2}{12}=\frac{5}{12}$

$\frac{1}{2}+\frac{1}{12}=\frac{7}{12}$

$\frac{1}{7}$

$X|Y=a$

$3/5$
$1/5$
$1/5$

$X|Y=b$

1	$\frac{1}{7}$
2	$\frac{3}{7}$
3	$\frac{3}{7}$

$\tilde{X}(Y=a)=1$ $\qquad$ $\tilde{X}(Y=b)=\{2,3\}$ with equal pr

Fano $\quad H(P_e)+P_e\log|\mathcal{X}| \geq H(\hat{X}|X)$

P_e $\qquad p$

X	
1	$1/3$
2	$1/3$
3	$1/3$

$\frac{1}{4}+\frac{1}{12}=$

$=$

$Y|X=1$

a	b
$\frac{3}{4}$	$\frac{1}{4}$

$p(X=1)=\frac{1}{3}$ & if that happens $\qquad$ $p(\text{error}|X=1)$

$p(X=2)=\frac{1}{3}$ $\qquad$ $Y|X=2$

$\frac{1}{4}$	$\frac{3}{4}$

$p\,\text{error}|X=2)=$

still need to choose $\hat{X}$ from. $Y=b$

$p(X=3)=\frac{1}{3}$ $\qquad$ $p\,\text{error}|X=3)=\frac{1}{2}$.

$$P_e=\frac{1}{3}\left(\frac{1}{4}+\frac{5}{8}+\frac{5}{8}\right)= \cancel{\tfrac{1}{2}}\ \frac{1}{3}\left(\frac{6}{4}\right)=\frac{1}{2}.$$

$H(P_e) = -\log\frac{1}{2} = 1$

$P_e \log|\mathcal{X}| = \frac{1}{2}\log 3$

$H(X|\hat{X})$ $\quad X|\hat{X}=1 \quad \hat{X}=1 \Rightarrow Y=a \Rightarrow$

X	
1	3/5
2	1/5
3	1/5

$\therefore\ H(X|\hat{X}=1) = -\frac{3}{5}\log\frac{3}{5} - \frac{2}{5}\log\frac{1}{5}$

~~$H(X|\hat{X})$~~ $\quad X|\hat{X}=2 \quad \hat{X}=2 \Rightarrow Y=b$

X	
1	1/7
2	3/7
3	3/7

$H(X|\hat{X}=2)$

$H(X|\hat{X}=2) = -\frac{1}{7}\log\frac{1}{7} - \frac{6}{7}\log\frac{3}{7}.$ $\quad$ simil. $\hat{X}=3$

$$H(X|\hat{X}) = \frac{1}{3}\overbrace{\left[-3/5\log 3 + \tfrac{3}{5}\log 5 + \tfrac{2}{5}\log 5\right]}^{H(X|\hat{X}=1)} + \frac{2}{3}\underbrace{\left[\tfrac{1}{7}\log 7 - \tfrac{6}{7}\log 3 + \tfrac{6}{7}\log 7\right]}_{H(X|\hat{X}=2)=H(X|\hat{X}=3)}$$

$= -\frac{1}{5}\log 3 + \frac{1}{3}\log 5 + \frac{2}{3}\log 7 - \frac{4}{7}\log 3$

≈ 1.42

$H(P_e) + P_e \log D(\,) = 1.79$