2E2 Tutorial Sheet 1[1]

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Useful formulae:

- The Laplace transform of $f(t)$:
$$\mathcal{L}(f) = \int_0^\infty f(t)e^{-st}dt \tag{1}$$

- Linearity:
$$\mathcal{L}(af + bg) = a\mathcal{L}(f) + b\mathcal{L}(g) \tag{2}$$
where a and b are constants.

- Integration by parts:
$$\int_a^b udv = uv]_a^b - \int_a^b udv \tag{3}$$

- Table of Laplace transforms: $\mathcal{L}(1) = 1/s$,
$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}} \tag{4}$$
and
$$\mathcal{L}\left(e^{at}\right) = \frac{1}{s-a} \tag{5}$$

- The first shift theorem: if $\mathcal{L}[f(t)] = F(s)$ then
$$\mathcal{L}\left[f(t)e^{at}\right] = F(s-a) \tag{6}$$

- Laplace transform and differenciation: if $\mathcal{L}[f(t)] = F(s)$ then
$$\mathcal{L}(f') = sF - f(0) \tag{7}$$

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Questions

1. (1) Using the linearity of the Laplace transform, calculate the Laplace transform of
$$f(t) = 2 - \frac{t}{2} \tag{8}$$
2. (1) Using the linearity of the Laplace transform, calculate the Laplace transform of
$$f(t) = 2e^{2t} + 3t + 4e^{-4t} \tag{9}$$
3. (2) The hyperbolic sine is defined as
$$\sinh x = \frac{e^x - e^{-x}}{2} \tag{10}$$
using the linearity of the Laplace transform, show that
$$\mathcal{L}(\sinh at) = \frac{a}{s^2 - a^2} \tag{11}$$
4. (2) Using the shift theorem find the Laplace transform of
$$f(t) = e^{2t}t^2 \tag{12}$$
5. (2) Using the formula for the Laplace transform of the differential find $\mathcal{L}(f')$ where $f = t^2$, check your answer by differentiating f directly and then working out its Laplace transform.