2E2 Tutorial Sheet 10[1]

15 January 2006

Useful facts:

- Solving a linear differential equation: for A a 2×2 matrix with eigenvalues λ_1 and λ_1 and corresponding eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$ then if

$$\mathbf{y}' = A\mathbf{y} \tag{1}$$

  the solution is

$$\mathbf{y} = C_1\mathbf{x}_1 e^{\lambda_1 t} + C_2\mathbf{x}_2 e^{\lambda_2 t} \tag{2}$$

  where C_1 and C_2 are arbitrary constants.

- If initial condition are given, just set $t = 0$ to find $y_1(0)$ and $y_2(0)$ and this should give simultaneous equations for C_1 and C_2.

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Questions

1. (2) Find the general solution for the system

$$\frac{dy_1}{dt} = 3y_1 + y_2 \tag{3}$$
$$\frac{dy_2}{dt} = y_1 + 3y_2 \tag{4}$$

2. (3) Find the solution of the system

$$\frac{dy_1}{dt} = 3y_1 + 4y_2 \tag{5}$$
$$\frac{dy_2}{dt} = 4y_1 - 3y_2 \tag{6}$$

   with $y_1(0) = 2$ and $y_2(0) = -1$.

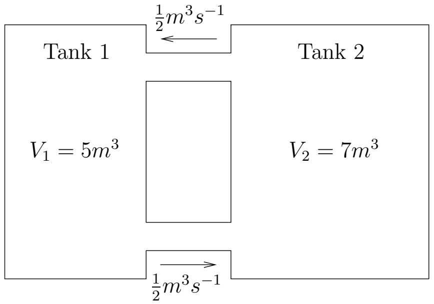

Figure 1: Two containers with flow between them.

3. (3) As illustrated in Fig. 1, two large containers are connected and American style sandwich spead is pumped between them at a rate of $1/2m^3s^{-1}$. One container has volume $5m^3$, the other $7m^3$. Both are full of spread. Initially the smaller container contains pure jam, the second container has $5m^3$ of jam and $2m^3$ of peanut butter. Assume perfect mixing and so on.
   (i) Write down the differential equation for $y_1(t)$ and $y_2(t)$, the amount of peanut butter in the first and second container.
   (ii) Solve it to find $y_1(t)$ and $y_2(t)$ explicitly.
   (iii) Use the initial data to find the values of the constants in the solution.