2E2 Tutorial Sheet 1, Solutions[1]

16 October 2005

1. (1) Using the linearity of the Laplace transform, calculate the Laplace transform of

$$f(t) = 2 - \frac{t}{2} \tag{1}$$

Solution: So, split it up using linearity

$$\begin{aligned} \mathcal{L}\left(2 - \frac{t}{2}\right) &= 2\mathcal{L}(1) - \frac{1}{2}\mathcal{L}(t) \\ &= \frac{2}{s} - \frac{1}{2s^2} \end{aligned} \tag{2}$$

2. (1) Using the linearity of the Laplace transform, calculate the Laplace transform of

$$f(t) = 2e^{2t} + 3t + 4e^{-4t} \tag{3}$$

Solution: So, split it up using linearity

$$\begin{aligned} \mathcal{L}\left(2e^{2t} + 3t + 4e^{-4t}\right) &= 2\mathcal{L}\left(e^{2t}\right) + 3\mathcal{L}(t) + 4\mathcal{L}\left(e^{-4t}\right) \\ &= \frac{2}{s-2} + \frac{3}{s^2} + \frac{4}{s+4} \end{aligned} \tag{4}$$

3. (2) The hyperbolic sine is defined as

$$\sinh x = \frac{e^x - e^{-x}}{2} \tag{5}$$

using the linearity of the Laplace transform, show that

$$\mathcal{L}(\sinh at) = \frac{a}{s^2 - a^2} \tag{6}$$

Solution: Well, just write it out

$$\begin{aligned} \mathcal{L}(\sinh(at)) &= \mathcal{L}\left(\frac{e^{at} - e^{-at}}{2}\right) = \frac{1}{2}\mathcal{L}(e^{at}) - \frac{1}{2}\mathcal{L}(e^{-at}) \\ &= \frac{1}{2}\frac{1}{s-a} - \frac{1}{2}\frac{1}{s+a} \\ &= \frac{1}{2}\frac{s+a-(s-a)}{s^2-a^2} = \frac{a}{s^2-a^2} \end{aligned} \tag{7}$$

[1] Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/2E2.html`

4. (2) Using the shift theorem find the Laplace transform of

$$f(t) = e^{2t}t^2$$

Solution: Recall the first shift theorem says

$$\mathcal{L}\left(e^{-at}f(t)\right) = F(s-a) \tag{8}$$

where $\mathcal{L}(f) = F(s)$. Now, we know that

$$\mathcal{L}\left(t^2\right) = \frac{2!}{s^3} = \frac{2}{s^3} \tag{9}$$

so, by the shift theorem

$$\mathcal{L}\left(e^{2t}t^2\right) = \frac{2}{(s-2)^3} \tag{10}$$

5. (2) Using the formula for the Laplace transform of the differential find $\mathcal{L}(f')$ where $f = t^2$, check your answer by differentiating f directly and then working out its Laplace transform.

Solution: Well

$$\mathcal{L}(f) = \mathcal{L}(t^2) = \frac{2}{s^3} \tag{11}$$

and $f(0) = 0$ so

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0) = \frac{2}{s^2} \tag{12}$$

Doing it by differenciating first, we have $f' = 2t$ so

$$\mathcal{L}(f') = \mathcal{L}(2t) = \frac{2}{s^2} \tag{13}$$

As expected, this is same answer.