

2E2 Tutorial Sheet 4 First Term¹

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Useful facts:

- The Heaviside function $H_a(t)$ is zero for $t < a$ and one for $t \geq a$, its Laplace transform is given by

$$\mathcal{L}[H_a(t)] = \frac{e^{-as}}{s} \quad (1)$$

- The Dirac delta function $\delta(t - a)$ is zero everywhere except $t = a$ where it is infinite. It can be thought of as the b goes to zero limit of

$$\delta_b(t - a) = \frac{1}{b} (H_a(t) - H_{a+b}(t)) \quad (2)$$

- A delta function evaluates an integral:

$$\int_0^\infty \delta(t - a) f(t) dt = f(a) \quad (3)$$

- The Laplace transform: $\mathcal{L}(\delta(t - a)) = e^{-as}$.
- The shift theorem: if $\mathcal{L}(f) = F(s)$ then

$$\mathcal{L}[H_a(t)f(t - a)] = e^{-as}F(s) \quad (4)$$

- The formula for complex exponentials:

$$\begin{aligned} e^{i\theta} &= \cos \theta + i \sin \theta \\ e^{-i\theta} &= \cos \theta - i \sin \theta \end{aligned} \quad (5)$$

- Remember $e^{a+b} = e^a e^b$ so,

$$\begin{aligned} e^{a+ib} &= (\cos b + i \sin b)e^a \\ e^{a-ib} &= (\cos b - i \sin b)e^a \end{aligned} \quad (6)$$

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Questions

1. (4) Use Laplace transform methods to solve the differential equation

$$f'' + 2f' - 3f = \delta(t - 1) \quad (7)$$

subject to the initial conditions $f(0) = 0$, $f'(0) = 1$.

2. (4) Using the Laplace transform solve the differential equation

$$f'' + 6f' + 13f = 0 \quad (8)$$

with boundary conditions $f(0) = 0$ and $f'(0) = 1$ and get your answer into a real form.