

2E2 Tutorial Sheet 8¹

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Useful facts:

- Standard Z-transforms

$$\begin{aligned}\mathcal{Z}[(r^n)_{n=0}^\infty] &= \frac{z}{z-r} \\ \mathcal{Z}[(nr^{n-1})_{n=0}^\infty] &= \frac{z}{(z-r)^2}\end{aligned}\tag{1}$$

- Advancing and delaying:

$$\begin{aligned}\mathcal{Z}[(x_{n+1})_{n=0}^\infty] &= zX(z) - zx_0 \\ \mathcal{Z}[(x_{n+2})_{n=0}^\infty] &= z^2X(z) - z^2x_0 - zx_1 \\ \mathcal{Z}[(x_{n-n_0})_{n=0}^\infty] &= \frac{1}{z^{n_0}}X(z)\end{aligned}\tag{2}$$

where $\mathcal{Z}[(x_n)] = X(z)$.

- Delta pulse: $(\delta_n)_{n=0}^\infty = (1, 0, 0, 0, \dots)$ and $\mathcal{Z}[(\delta_n)_{n=0}^\infty] = 1$.

Questions

1. (2) Use the Z-transform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = 1\tag{3}$$

with $x_1 = 0$ and $x_0 = 0$.

2. (2) Use the Z-transform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = 3^k\tag{4}$$

with $x_1 = 0$ and $x_0 = 0$.

3. (2) Use the Z-transform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = \delta_k\tag{5}$$

with $x_1 = 0$ and $x_0 = 0$. Remember δ_k is the unit pulse with $\delta_k = (1, 0, 0, 0, \dots)$.

4. (2) Use the Z-transform to solve the difference equation

$$x_{k+2} - 8x_{k+1} + 15x_k = 0\tag{6}$$

with $x_1 = 2$ and $x_0 = 3$.

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