2E2 Tutorial Sheet 2 First Term[1]

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Useful facts:

- Laplace transform of differenciated functions: if $\mathcal{L}[f(t)] = F(s)$ then

$$\mathcal{L}(f') = sF - f(0) \tag{1}$$

and

$$\mathcal{L}(f'') = s^2 F - sf(0) - f'(0) \tag{2}$$

- Partial fractions: assume

$$\frac{a}{(s-b)(s-c)} = \frac{A}{s-b} + \frac{B}{s-c} \tag{3}$$

multiply across by $(s-b)(s-c)$

$$a = A(s-c) + B(s-b) \tag{4}$$

and choose $s = c$ and $s = b$ to get A and B.

- Similarily,

$$\frac{a}{(s-b)(s-c)(s-d)} = \frac{A}{s-b} + \frac{B}{s-c} + \frac{C}{s-d} \tag{5}$$

then multiply across by $(s-b)(s-c)(s-d)$ and choose s equal to b, c and d to get A, B and C.

- Finally, it doesn't matter if there is a polynomial in s above the line:

$$\frac{as+e}{(s-b)(s-c)(s-d)} = \frac{A}{s-b} + \frac{B}{s-c} + \frac{C}{s-d} \tag{6}$$

then multiply across by $(s-b)(s-c)(s-d)$ and choose s equal to b, c and d to get A, B and C.

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Questions

1. (2) Find the Laplace transform of both sides of the differential equation

$$2\frac{df}{dt} = 1$$

with initial conditions $f(0) = 4$. By solving the resulting equations find $F(s)$. Based on the Laplace trasforms you know, decide what $f(t)$ is.

2. (2) Using the Laplace transform solve the differential equation

$$f'' - 4f' + 3f = 1 \tag{7}$$

with boundary conditions $f(0) = f'(0) = 0$. You will need to do partial fractions.

3. (2) Using the Laplace transform solve the differential equation

$$f'' - 4f' + 3f = 0 \tag{8}$$

with boundary conditions $f(0) = 1$ and $f'(0) = 1$.

4. (2) Using the Laplace transform solve the differential equation

$$f'' - 4f' + 3f = 0 \tag{9}$$

with boundary conditions $f(0) = 1$ and $f'(0) = 0$.