

Laboratory 1: Polarised Light

Conor Kirby

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1 ABSTRACT

The purpose of these experiments were to investigate various properties of polarised light. The relationship between a Polaroid film with the angle ϕ between the Polaroid axis and the plane of polarisation of the laser beam was investigated. Malus' Law is verified using plots of light intensity vs ϕ and $\cos^2 \phi$ showing the linear relationship. The reflectance of the light ray is measured as a function of the incidence angle with the light being reflected off a glass prism. Brewster's angle is experimentally measured to be $55^\circ \pm 1^\circ$ which has a percentage accuracy of 1.79%. However experimental error obvious from the data effects the accuracy of the results; unwanted extra light rays coming from the lab and instrumental error being the main sources of error. Using Snell's Law the refractive index of glass was determined to be 1.428 which fits within the desired range.

2 INTRODUCTION

The purpose of this report was to experimentally investigate and verify various properties of polarised light. Namely Malus' Law, the reflectance of light by measuring the intensity of light that is reflected of a glass prism for both p and s polarised waves, Brewster's Angle and with this the refractive index of the glass.

What is the point of the experiment apart from verifying a few laws? Polarisation of light has many real world applications so it is well worth gaining a deeper understanding. One example are optical devices such as polarising filters for cameras and liquid crystal displays which are used in electronic displays. Another major example is communications and data transmission; fibre optics plays a major part in telecommunication and makes use of polarisation effects of light specifically total internal reflection of light. Other applications include biomedical imaging and even sunglasses. Evidently there are many useful "real world" applications of polarised light.

3 THEORY

3.1 POLARISED LIGHT

Light is a form of electromagnetic wave. Unpolarised light can be thought of like an lot of waves oscillating in a number of different planes. However when we polarise light (such as pass it through a polarising filter) we confine it to one plane.

An article on BBC Science focus simply explains the polarisation of light very effectively. *"Imagine holding a skipping rope so that it passes between the posts of a picket fence, and waving it up and down. The vertically aligned waves will be able to pass between the gap in the fence posts. But if you wave it from side to side, the waves will be blocked."*[1]

In this experiment we polarise a helium-neon laser to the vertical plane, to be used as a source of linearly polarised light.

3.2 MALUS' LAW

When a plane polarised beam is then polarised to an axis of our choosing (the vertical axis) the intensity of the light will decrease unless the angle ϕ between its E-vector and the axis of polarisation in the polaroid is zero. How do we know what this new intensity is? We can use **Malus' Law** to determine the transmitted intensity, only the component of the E-vector along the polarisation axis will be transmitted:

$$E_t = E_0 \cos \phi \quad (3.1)$$

$$I_t = |E_t|^2 = I_0 \cos^2 \phi \quad (3.2)$$

Where I_0 is the initial intensity and I_t is the transmitted intensity.

3.3 EXTERNAL REFLECTION OF POLARISED LIGHT [5]

When light is incident upon an interface between a medium with a refractive index n_1 to a medium with a more dense medium n_2 , ($n_2 > n_1$) external reflection occurs. We will be looking at the reflectance which is the relative intensity reflected from the interface with respect to the incident wave. The Fresnel equations provide the relationship between the electric fields of the reflected and transmitted waves compared to the incident one, considering both polarization components.

It is important for us to consider angle of the plane of polarisation with respect to the incidence plane. The reflectance depends significantly on this. The two linear polarised components are the s-polarised component and the p-polarised component. The s-polarised component refers to the polarisation of the waves electric field normal to the plane of incidence which implies that the magnetic field is in the plane of incidence. The p-polarisation is when the electric field is oscillating in the plane of incidence.

3.3.1 FRESNEL FORMULAS FOR THE REFLECTANCE OF S AND P-COMPONENT:

$$R_s = \left(\frac{n_1 \cos \alpha - n_2 \cos \beta}{n_1 \cos \alpha + n_2 \cos \beta} \right)^2 \quad (3.3)$$

$$R_p = \left(\frac{n_1 \cos \beta - n_2 \cos \alpha}{n_1 \cos \beta + n_2 \cos \alpha} \right)^2 \quad (3.4)$$

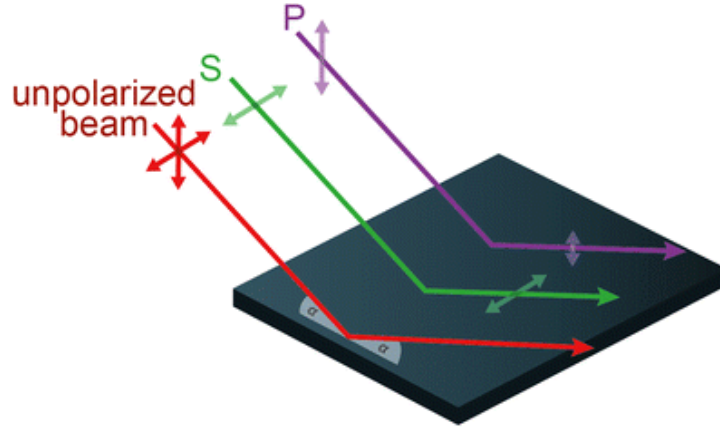


Figure 3.1: Visual representation of the s and p components of a linearly polarised wave.[4]

3.4 BREWSTER'S ANGLE

Brewster's angle refers to a particular angle of incidence for which the reflectance of the p-polarised component is exactly zero. This results in the reflected light being completely s-polarised.

It can be seen from the plot of reflectance vs angle of incidence that Brewster's Angle is approximately between 50 and 60 degrees. The purpose of this experiment is to do our own plot like this and determine the exact value for Brewster's Angle experimentally.

A key application of Brewster's angle is in the design and manufacture of anti-reflective coatings. Engineers can use this to minimise reflections and it is used in lenses, eyeglasses, camera lenses and other optical devices to reduce glare.

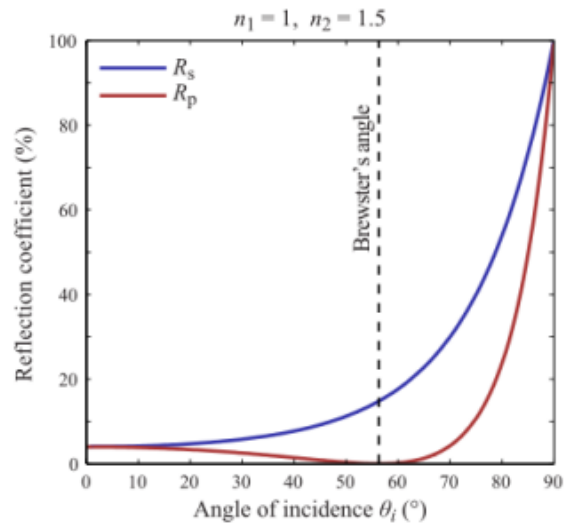


Figure 3.4: Reflectance vs angle of incidence [2]

3.5 SNELL'S LAW

Just to quickly explain the relation between Brewster's Angle and Snell's Law and how it can be used to get the refractive index of the media. Snell's Law gives us:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (3.5)$$

Where θ_1 is Brewster's Angle, θ_2 is the angle of refraction, and n_1 and n_2 are the refractive indices. To get the angle of refraction in terms of Brewster's angle; $\theta_2 = 90 - \theta_1$. This turns

Snell's Law into:

$$n_1 \sin \theta_1 = n_2 \cos \theta_1 \quad (3.6)$$

Therefore, after we determine Brewster's Angle we can determine the refractive index of the glass since we know the refractive index of air is very close to 1.

$$n_2 = n_1 \tan \theta_1 \quad (3.7)$$

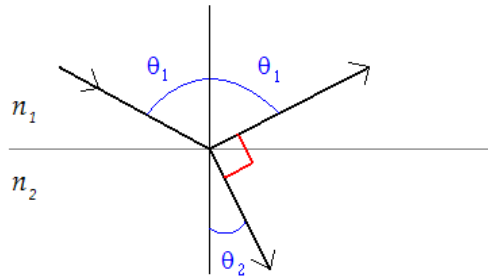


Figure 3.2: This figure shows how Snell's Law and Brewster's angle are related. [3]

4 APPARATUS

The apparatus consisted of a Helium/Neon Gas Laser with Brewster windows as a source of monochromatic light. There is also a polaroid and a general purpose digital voltmeter (DVM) is used to measure the voltage developed by the photodiode. The silicon photodiode is mounted on a swivelling arm to allow ability to align for the reflected light. The intensity of light is proportional to the output voltage measured by the multimeter. Figure 4.1 shows the apparatus.

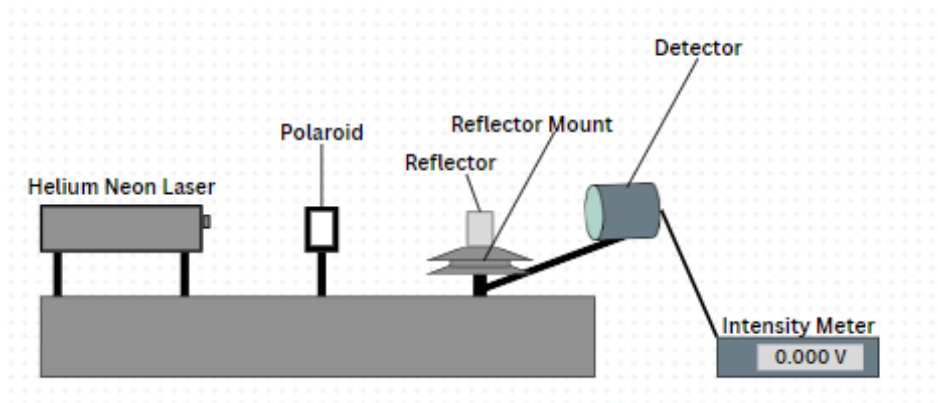


Figure 4.1: Apparatus for polarised light experiment

5 METHODOLOGY

5.1 EXPERIMENT 1 - MALUS' LAW

In this section of the experiment the reflector is removed and the detector is rotated such that the laser beam is incident on the detector. The angular setting on the polaroid is then varied such that the maximum transmission is obtained. This is then taken as the relative angle of 0° . The polaroid is then rotated at 10° intervals until rotated 180° . The intensity was noted at each interval. Malus' Law is then verified by plotting graphs of the intensity vs ϕ and the intensity vs $\cos^2 \phi$.

5.2 EXPERIMENT 2(A) - S REFLECTANCE

The glass prism is then put back into its holder and it is ensured that the laser beam was incident on the bare glass face. This ensures that the plane of polarisation is perpendicular to the plane of incidence since the light is s-polarised.

The scale on the prism table is then set to 0 deg and rotated until the reflected beam coincides with the spot where the beam comes through the Polaroid. This ensures that the prism is not reflecting the beam up or down. Note this angle and use it as a reference point for further readings. Rotate the table through 90 deg to check that the point of incidence on the prism does not change significantly, ie. that the axis of rotation is co-aligned with the face of the prism.

The table is then rotated in 10° increments, each time moving the detector to receive the beam. The angle of incidence and intensity is noted at each increment.

5.3 EXPERIMENT 2(B) - P REFLECTANCE AND BREWSTER'S ANGLE

Experiment 2 is then repeated, this time the half wave plate is placed between the laser and the polaroid. The purpose of the half wave plate is to p-polarise the electromagnetic wave instead. This way we can analyse the reflectance of light when it is polarised parallel to the plane of incidence of the light.

Similarly to the last experiment, it is ensured that the beam is reflected in the horizontal plane only by aligning the beam so it is reflected onto the centre of the polaroid. This angle is then taken as the reference angle and the prism is rotated in 10deg increments until as close to 90deg as you can get, taking note of the intensity of light each time. Because of our preliminary knowledge for the value of Brewster's angle we can take more values between 50 and 60 degrees for increased accuracy.

The reflectance is then plot against the angle of incidence and the graph is used to determine Brewster's angle. The refractive index pf the glass can then be determined using Snell's Law ($n_1 \sin \alpha = n_2 \sin \beta$) and Brewster's angle.

6 RESULTS AND DISCUSSIONS

Possible experimental error in this section could be due to light pollution from the lab. In an ideal world the experiment would be done in complete darkness, so the only thing the detector is detecting is the laser beam. There also may be experimental error due to the prism being dirty or lightly chipped.

6.1 EXPERIMENT 1: MALUS' LAW

Malus' Law was experimentally verified graphically. Initially the angle ϕ is plot against the intensity of the laser beam.

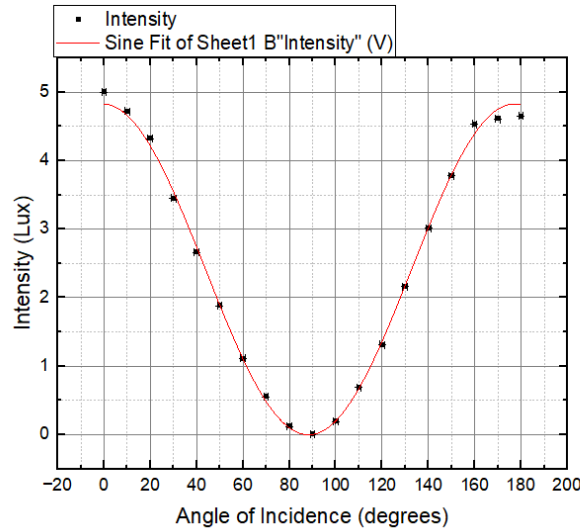


Figure 6.1: Intensity of the electromagnetic wave vs angle of incidence.

The line of best fit for figure 6.1 is a cosine fit. This behaviour arises from the \cos^2 term of Malus' Law. From the plot we can deduce that the intensity is at a maximum at 0 and 180 degrees, and that the intensity of light drops to zero at 90 degrees. Since Malus' Law states that the intensity varies linearly with the \cos^2 of the angle of incidence, we must confirm that there is a linear independence between the intensity of light and $\cos^2 \phi$.

Figure 6.2 confirms Malus' Law since there is a linear fit with pearson's R of 0.9975. Evidently the relation between the data is not perfect however this may be due to experimental error and extra light rays from the lab hitting the detector. Nevertheless the plot still proves that the intensity of light increases linearly with the cosine squared of the angle of incidence.

6.2 EXPERIMENT 2(A): S REFLECTANCE

In experiment 2 we investigate the effect of the angle of incidence on the reflectance of s-polarised light. What we expect to see is a slow initial increase and a sharp increase around

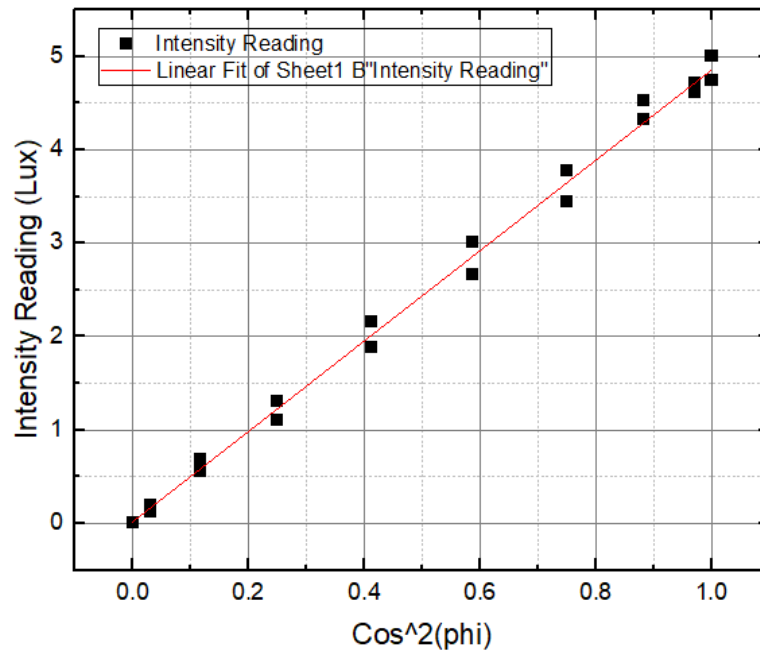


Figure 6.2: Intensity of the electromagnetic wave vs \cos^2 of the angle of incidence.

$\phi = 40 - 60$ degrees.

In figure 6.3 we can confirm what we expected to see. Initially there is a gradual increase and between 40 and 60 the slope increases. An exponential line of best fit is used. A couple of data points deviate more from the line of best fit. This may possibly be due to experimental or environmental error. The error bars are not shown in the plot since they are so small however all error propagation of the reflectance are calculated and shown in the appendix.

6.3 EXPERIMENT 2(B): P REFLECTANCE AND BREWSTER'S ANGLE

The aim of this section of the experiment is to investigate the relationship between the reflectance of p-polarised light and the angle of incidence. From this we should be able to determine the value for Brewster's Angle.

However we had a bit of trouble once we added in the half wave plate. Our results didn't seem extremely accurate. This may be due to experimental error or instrumental error. However we still investigated the data anyway.

It can be seen in figure 6.4 that the data is not 100% correct. Firstly Brewster's angle occurs when p-polarised light is incident such that there is no reflectance of the light wave. However this is not the case in our plot. If the experiment was to be repeated I would take more data points closer to Brewster's angle (56°) to get a more accurate result. For arguments sake I will take the minimum point in the graph as Brewster's angle: (56°), which is accurate to 1.79%.

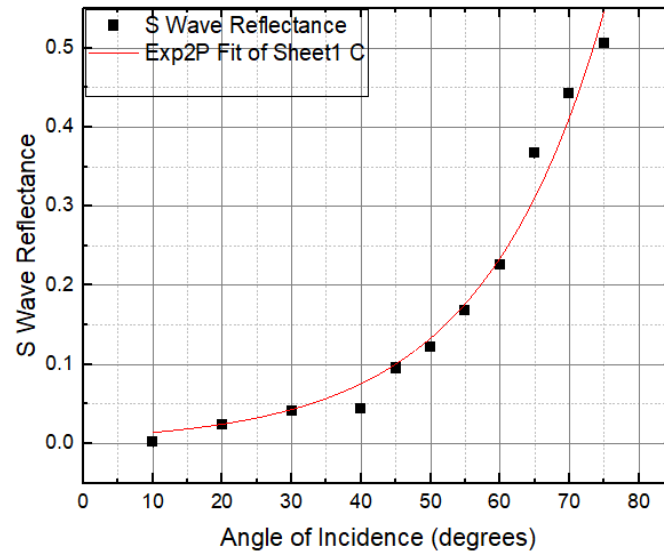


Figure 6.3: Reflectance of s-polarised light waves vs angle of incidence

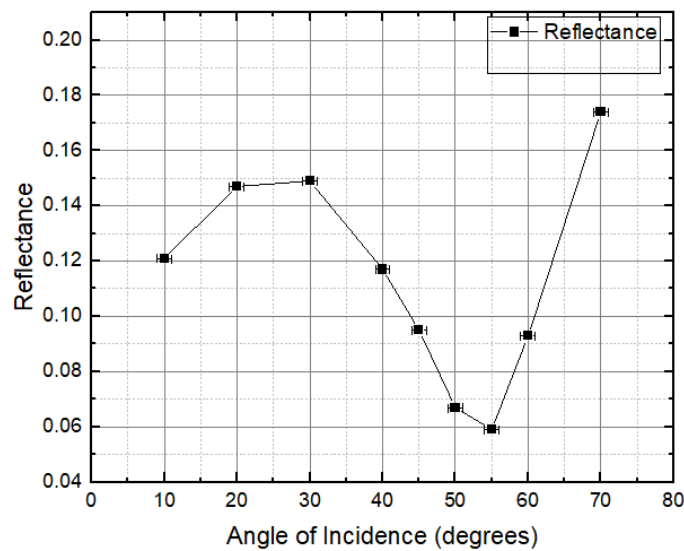


Figure 6.4: Reflectance of p-polarised light waves vs angle of incidence.

Using equation 3.7 we can determine the refractive index of the glass.

$$n_2 = \tan 55$$

$$n_2 = 1.428$$

7 CONCLUSIONS

In this lab we set out to verify Malus' Law and investigate the reflectance of s-polarised and p-polarised light. In experiment 1 Malus' Law was successfully confirmed, the intensity of light increased linearly with \cos^2 of the angle of incidence ϕ .

The s-reflectance was observed to have an exponential relation with the angle of incidence. Slight deviation in the data is due to light pollution from the lab. If repeating, the data would be more data if done in a completely dark room.

The half wave plate was then added in and the reflectance of p-polarised light was investigated. However due to instrumental error and possibly calibration error with the equipment our data was not extremely accurate. What was expected was a plot of R_p vs angle of incidence and around 56 degrees a drop of reflectance to zero. On our plot the reflectance did not drop to zero. If the experiment was to be repeated more data would have been collected around Brewster's Angle for accuracy and the experiment should take place in complete darkness. Final results deduced Brewster's angle to be $55^\circ \pm 1^\circ$ and the refractive index of glass to be 1.428 (unsure how error propagates through a tan function so rather than putting in an error that is most likely incorrect, just going to neglect the error) which is within the range for refractive indices of glass.

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8 APPENDIX

8.1 FIGURES

- Figure 3.1: Visual representation of the s and p components of a linearly polarised wave.[4]
- Figure 3.4: Reflectance vs Angle of Incidence [2]
- Figure 3.2: This figure shows how Snell's Law and Brewster's angle are related. [3]
- Figure 4.1: Apparatus for polarised light experiment
- Figure 6.1: Intensity of electromagnetic waves vs angle of incidence
- Figure 6.2: Intensity of the electromagnetic wave vs \cos^2 of the angle of incidence.
- Figure 6.3: Reflectance of s-polarised light waves vs angle of incidence ϕ
- Figure 6.4: Reflectance of p-polarised light waves vs angle of incidence.

8.2 ERROR PROPAGATION

Intensity of light/ voltage has constant error: $\pm 0.00001V$

Angle of incidence - constant error: ± 2 deg

$\cos^2 \phi$ - unsure how to propagate error through a cosine function so only going to propagate it through the square.

$$\frac{\partial z}{z} = \sqrt{\left(\frac{\partial x}{x}\right)^2 + \left(\frac{\partial y}{y}\right)^2}$$

When x and y are the same (ie. x^2) we get:

$$\delta x^2 = x^2 \sqrt{2} \delta x$$

For reflectance $\left(\frac{I_r}{I_0}\right)$:

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

8.3 DATA

Malus' Law			
$\phi (^{\circ}) \pm 1 (^{\circ})$	$\cos^2 \phi$	$\cos^2 \phi$ Error	Intensity (Lux) $\pm 1e-5$
0	1	2.8284	5
10	0.9698	2.7430	4.72
20	0.883	2.4975	4.32
30	0.75	2.1213	3.45
40	0.5868	1.6597	2.66
50	0.4132	1.1687	1.88
60	0.25	0.7071	1.11
70	0.11698	0.3309	0.55
80	0.0302	0.0854	0.12
90	0	0.0000	0.002
100	0.0302	0.0854	0.19
110	0.11698	0.3309	0.69
120	0.25	0.7071	1.31
130	0.4132	1.1687	2.16
140	0.5868	1.6597	3.01
150	0.75	2.1213	3.78
160	0.883	2.4975	4.53
170	0.9698	2.7430	4.61
180	1	2.8284	4.65

S-Reflectance				
$\phi (^{\circ}) \pm 1(^{\circ})$	$I_r(\text{Lux}) \pm 1e-5$	$I_o(\text{Lux}) \pm 1e-5$	R_s	Er R_s
10	0.0077	4.92	0.00156	2.033E-06
20	0.1186		0.02411	2.033E-06
30	0.2029		0.04124	2.034E-06
40	0.2193		0.04457	2.035E-06
45	0.4693		0.09539	2.042E-06
50	0.6024		0.12243	2.048E-06
55	0.8277		0.16823	2.061E-06
60	1.1116		0.22592	2.084E-06
65	1.8064		0.36715	2.165E-06
70	2.1783		0.44274	2.223E-06
80	2.4872		0.50553	2.277E-06

S-Reflectance				
$\phi(^{\circ}) \pm 1(^{\circ})$	$I_r(\text{Lux}) \pm 1\text{e-}5$	$I_o(\text{Lux}) \pm 1\text{e-}5$	R_p	Er R_p
10	0.5175	4.277	0.121	2.351E-06
20	0.6116		0.143	2.356E-06
30	0.6373		0.149	2.358E-06
40	0.5004		0.117	2.350E-06
45	0.4063		0.095	2.346E-06
50	0.2866		0.067	2.342E-06
55	0.2523		0.059	2.341E-06
60	0.3978		0.093	2.346E-06
70	0.7442		0.174	2.365E-06