Gamma-ray Spectroscopy

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1 ABSTRACT

The spectra of Co^{60}, Cs^{137}, Na^{22}, and KCl were all successfully investigated. All peaks were analysed and the effects of backscattering, compton scattering, gamma ray production, pair annihilation, and photoelectric absorption were observed and investigated further. Varying the pm voltage led to the conclusion that $E_0 \propto V^n$ where n = 7.94434. In the final section of the experiment using relative count rates the efficiency of detection, ϵ, was determined to be 21.07%. For the KCl sample, the predicted activity (0.1048 cps) was compared with the obtained count rate (0.18 cps). There is an obvious discrepancy here which may be due to error in the calculations or experimental error with the MAESTRO II software.

2 INTRODUCTION & THEORY

In this experiment, we aim to understand gamma ray emission and how it interacts with matter. As we will see that the way it interacts with matter will be the way that we detects the gamma rays. Conclusions can also be drawn by analysing the gamma ray spectra of multiple radioactive sources, Co^{60}, Cs^{137}, Na^{22}, and KCl, all with well known energies which are used to calibrate the MAESTRO II software. Doing this we aim to determine the energies of gamma ray sources and the resolution of which they can be measured.

While analysing the spectra we could draw conclusions on different parts peaks and troughs which were due to pair production and annihilation, photoelectric absorption, and Compton scattering.

2.1 Compton Scattering

In this experiment, Compton scattering can be observed when a high energy gamma-ray interacts with an electron. During this interaction the photon is scattered at an angle and loses some of it's energy which is given to the now excited electron that is ejected from the atom. Using the conservation of energy and conservation of momentum laws the energy of the scattered photon can be found to be:

$$h\nu' = h\nu/[1 + h\nu(1 - \cos)\theta/(m_0c^2)] \tag{2.1}$$

Where $h\nu$ is the energy of the incident photon and m_0 is the rest mass of the electron. When the photon is backscattered (scattered at an angle $\theta = 180\,\text{deg}$) the minimum energy of the photon occurs as:

$$h\nu' = \frac{h\nu}{1 + \frac{2h\nu}{m_0c^2}} \tag{2.2}$$

Meanwhile the energy of the scattered electron ($h\nu - h\nu'$) varies from zero to a maximum of:

$$E_e = h\nu(1 - \frac{1}{1 + \frac{2h\nu}{m_0c^2}}) \tag{2.3}$$

2.2 Pair Production

If the gamma-ray photon exceeds the energy of twice the rest mass energy of an electron (2 x 511 keV), pair production may occur where the gamma ray creates an electron positron pair, with any excess energy appearing as kinetic energy. This pair production process happens in close vicinity to the nucleus and so the positron annihilates with an electron leading to the emission of two gamma-ray photons. This annihilation is where we will detect two photons of energy 511 keV in the gamma ray spectrum. Two photons are produced to conserve both energy and momentum. They travel in opposite direction to ensure that the total momentum after the annihilation remains zero.

2.3 Photoelectric Absorption

Photoelectric absorption is another key feature in these gamma ray spectra. It occurs when a gamma ray is completely absorbed in an atomic collision, an electron in ejected, usually from one of the innermost shells, and the gamma ray is replaced by photoelectric energy. This photoelectron has an energy as described in 2.4, where E_b is the binding energy of the electron in it's original shell.

$$E_e = E_\gamma - E_b \tag{2.4}$$

When the electron vacancy is filled an x-ray is emitted, but it is very unlikely to undergo photoelectric absorption itself so the total energy of both photoelectrons is E_γ, when analysing the spectra the peak that we see is what we call a **total energy peak**. The gamma ray can be determined directly from this peak.

In our Co^{60} sample pair production occurs which emits two gamma ray photons with a possibility that both undergo photoelectric absorption, however it is more likely that this will only occur with one of them. When analysing the spectra for Co^{60} we can expect to find a **sum peak** of energy $E_1 + E_2$, the sum of the energies of the photons.

3 EXPERIMENTAL SETUP

In this experiment four radioactive samples are needed. Co^{60}, Cs^{137}, Na^{22}, and KCl. All of which is handled by the lab technician when swapping the samples in different parts of the experiment.

He places the samples into the **scintillator** which is a sodium iodide crystal containing a fraction of a percent of thallium. The visible photons escape the crystal since sodium iodide is transparent to these visible photons, typically 3eV in energy.
The thing to understand about this experiment is that it is very inefficient. In the crystal about 1 MeV of particle energy creates about 40,000 photons with an average energy of 12%, already inefficient. However only approximately 75% of these photons reach the photocathode of the **photomultiplier**, which can be seen in figure 3.1.

The **photocathode** is the light sensing element. This section of the photon's journey is also extremely inefficient. Of the 30,000 photons that made it through the scintillator, only about 6,000 will eject an electron to produce a photoelectron. These are then accelerated towards the dynode through a potential difference of 100V. Secondary electron emission takes place on striking the dynode, where the primary electron emits about 4 electrons and then these go on to strike another dynode, and essentially these photoelectrons that go through the 11 stage tube will give rise to millions of electrons ready for collection at the anode.

At the anode a voltage pulse of magnitude Q/C is produced (where Q is the total charge and C is the capacitance of the anode). This pulse height, and Q, is proportional to the energy of the charged particle which was stopped in the scintillator. For photoelectric absorption of both the gamma-ray and the x-ray this energy equally that of the gamma-ray

4 METHODOLOGY

4.1 EXPERIMENT 1

The lab technician puts in the Cs^{137} source into the scintillator. It emits 662 keV gamma-rays (important information for calibration).
Initially the MAESTRO II has to be set up. The target voltage is set to 600V. In the Amplifier tab the fine gain is set to 0.55. Finally the shaping time should be set to it's default, 0.75uSec.

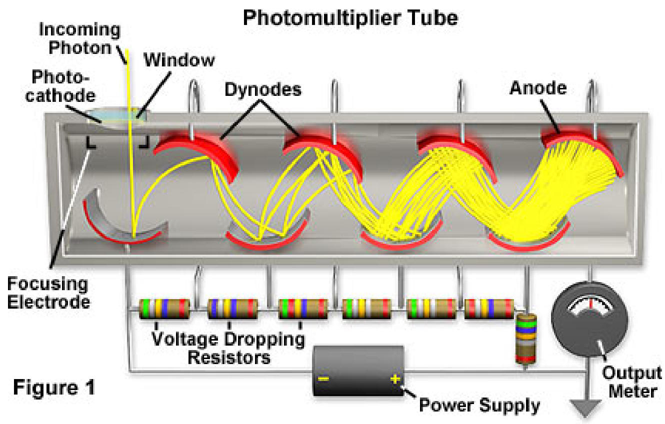

Figure 3.1: PM Tube [2]

Then the energy axis needs to be calibrated. Initially the "Destroy Calibration" is selected. Then to zero the energy axis, the cursor is moved to channel zero and enter 0.00 when "Calibrate" is selected.
The spectra is then analysed looking for key features such as the Compton Edge, Backscattering, and the total energy peak. Values for the full width half maximum, the energy of the peak and hence the resolution (4.1) are taken for a variety of PM voltage, from the limit of detection of low voltage to a maximum of 700V. At the end of the experiment the voltage must be set back to 600 V.

$$R = \frac{FWHM}{H_0} \tag{4.1}$$

4.2 Experiment 2

In this experiment the spectra for Cs^{137}, Co^{60}, *and* Na^{22} are analysed. We looked to identify features such as photoelectric absorption, Compton effect, and pair production. However each time the source is replaced, the MAESTRO II software needs to be recalibrated since the other sources do now exactly match the energy calibration made with the Cs^{137}. Each time we recalibrate the energy axis the error range is determined.
A secondary aim of this experiment is to investigate the dependence of resolution, R, on gamma ray energy. So we measure the resolution (4.1) for all single peaks, and try to determine how R varies with the peak energy at a fixed photomultiplier voltage.

4.3 Experiment 3

From the previous experiment Na^{22} pair annihilation can be seen at the strongest peak 511 keV in the gamma ray spectrum. There is also a weaker sum peak at 1022 keV corresponding to the detection of the two rays. Using this information and the count rates for the total energy peak ($2A\epsilon$) and the sum peak ($A\epsilon^2$), the efficiency of the spectrometer can be deduced. (A is the activity of the sample and ϵ is the efficiency of detection). The same is done for the Cobalt sample for the two gamma ray energies.
Finally the KCl sample is put into the scintillator and left overnight. The aim is to measure the activity of the sample. The software is left running for at least a day to obtain a reasonable signal-to-noise ratio. The count rate is then determines and then compared with the given values for abundance ratio, half life, and branching ratio.

5 Results & Discussions

5.1 Experiment 1

The data in table 8.1 in the appendix is the data obtained by varying the pm voltage from a lowest detectable voltage and to see the knock-on effects on other variables such as H_0, FWHM, and the resolution. From the data we can immediately see a relation between V and H0 but to investigate further we graphed the data.

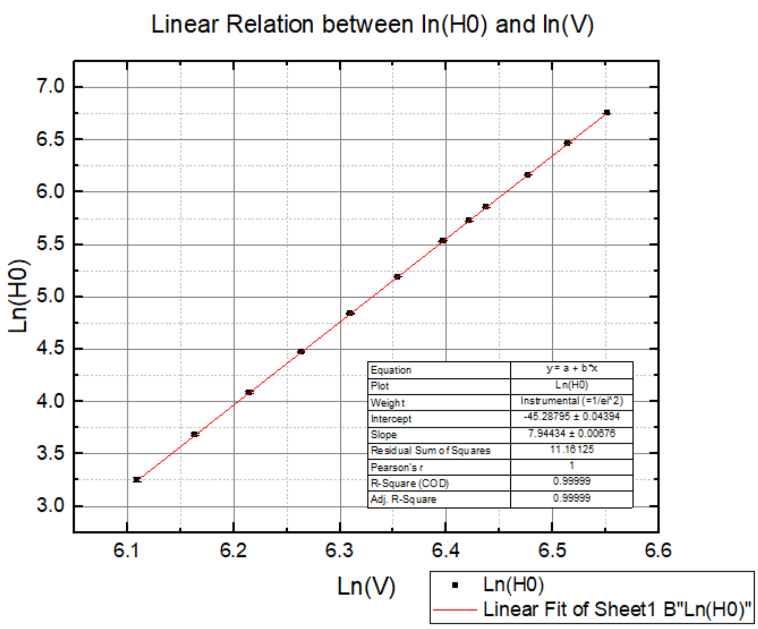

Figure 5.1: Investigating the relationship between pm voltage and H0

The plot shows an obvious linear relation between ln(H0) and ln(V). The equation of the line of best fit (with a Pearson's r of 1):

$$y = (7.94434 \pm 0.00676)x + (-45.28795 \pm 0.04394) \quad (5.1)$$

Since the linear relation is between ln(H_0) and ln(V), we can say that H_0 is proportional to V^n and from the slope we can deduce n to be 7.944.
An attempt was made to deduce a relationship between FWHM and V and R and V, which can be seen in the appendix in figures 8.1 and figure 8.2. However due to two extreme outliers in the data it is hard to make a direct relationship. Although until the two outliers it looks like the logarithm of FWHM and V are linearly proportional. If the experiment were to be repeated it would be worth trying to identify the source of the outliers whether it is error or something experimental.

5.2 Experiment 2

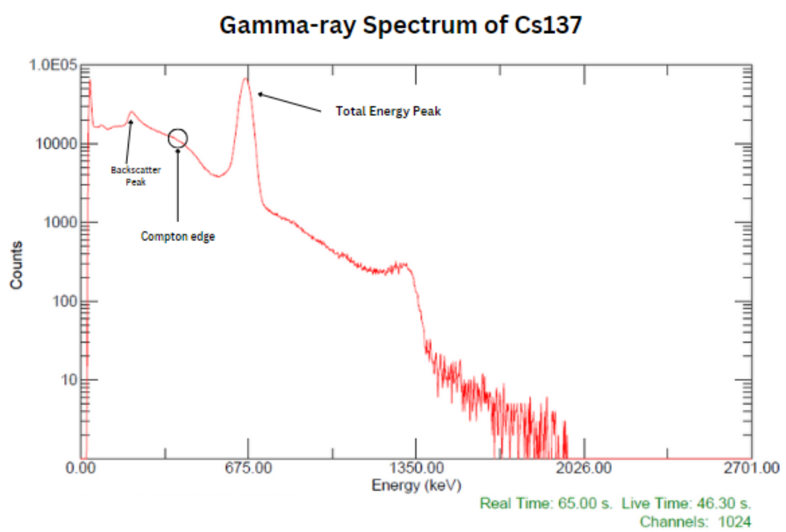

Figure 5.2: γ-ray spectrum of Caesium 137

Figure 5.2 shows the gamma ray spectrum of our caesium sample. The backscatter peak, Compton edge and total energy peak have been clearly labelled. The total energy peak appears as expected at approximately 662 keV.

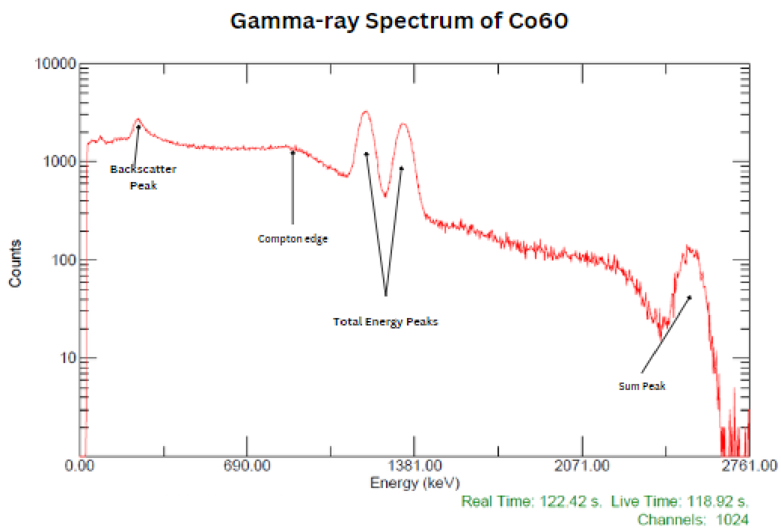

Figure 5.3: γ-ray spectrum of Cobalt 60

From cobalt's gamma-ray spectrum in figure 5.3 we can easily identify the backscatter peak, compton edge, and the two total energy peaks at the predicted values of approximately 1173 keV and 1333 keV. These are due to deexcitation of Ni^{60} (which comes from the beta decay of Co^{60}). While the nickel is becoming de-excited it emits two gamma rays, one when the nucleus decays from the first excited state to a state of lower energy at 1173 keV, and the second when the nucleus decays from the lower energy state to the ground state at 1333 keV. [1]

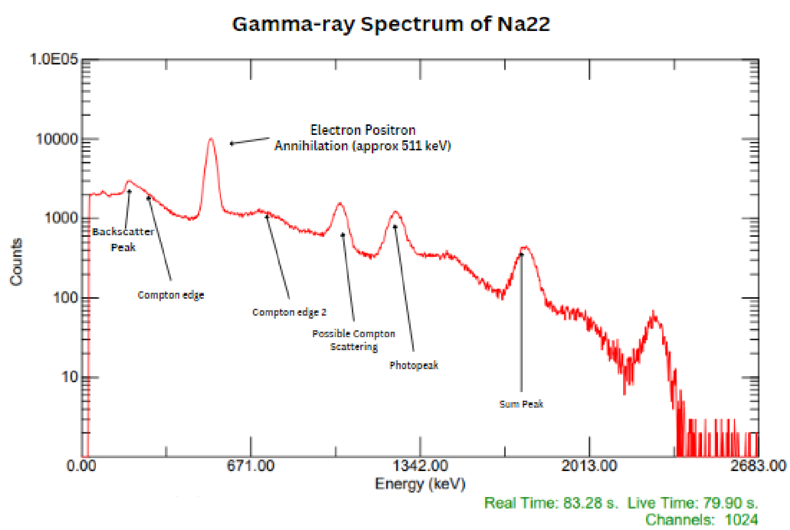

Figure 5.4: γ-ray spectrum of Sodium 22

The γ ray spectrum of our sodium sample, figure 5.4, shows many different peaks which took a bit of research to conclude what each is. At 511 keV the energy peak is due to the **electron positron pair annihilation**. This produces gamma ray photons in opposite directions to conserve energy and momentum. The peak at 1274 keV is due to gamma ray produced when the Ne^{22} excited state de-excites to it's ground state. There is a **sum peak** at 511 keV + 1274 keV = 1785 keV (approximately) which is also known as a coincidence peak and occurs when both the 511 keV annihilation gamma ray and the 1274 keV gamma ray are simultaneously detected. [3]

5.3 Experiment 3

Using relative count rates of the total energy peak and the sum peak in the sodium spectrum the efficiency of detection can be determined.

$$\text{Count Rate of total energy peak} = 2A\epsilon$$
$$2512 = 2A\epsilon$$

$$\text{Count Rate of Sum Peak} = A\epsilon^2$$
$$264.63 = A\epsilon^2$$

$$\epsilon = 0.2106927$$

The efficiency of detection is 21.07% at 511 keV.

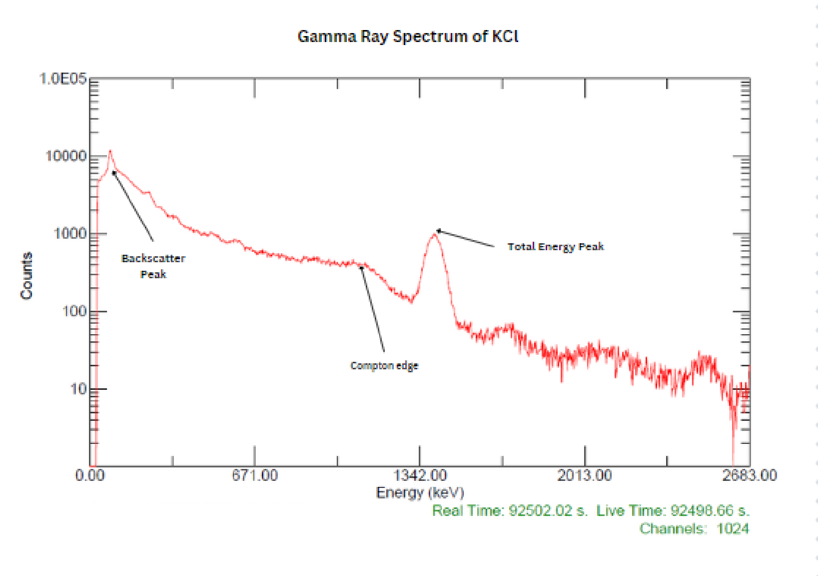

Figure 5.5: γ-ray spectrum of KCl

Figure 5.5 shows the gamma ray spectrum with the expected energy peak at approximately 1446 keV. Next to compare the measured count rate, 0.18 cps, with the count rate found with the predicted count rate:

$$m = 3.625g$$
$$abundance = 0.011\% \Rightarrow abundant \quad mass = 3.988x10^{-7}kg$$
$$A = N\lambda = N\frac{ln(2)}{t_{\frac{1}{2}}}$$
$$N = \frac{3.988x10^{-7}x6.022x10^{23}}{39.96} = 6.01x10^{15}$$
$$A = 6.01x10^{15}\frac{ln(2)}{1.26x10^{9}(365.25)(24)(60)(60)} = 0.1048$$

Hence the predicted count rate is 0.1048 cps. There is an error of 41.7% which is not extremely accurate. However the count rates are comparable however there is error due to experimental error or calculations.

6 Conclusions

In the first part the PM voltage was varied to see the effect on the spectrum. We could conclude from graphing ln(H_0) vs ln(V) that $H_0 \propto V^n$ and from the slope of the graph n is determined to be 7.944.

This experiment was successful in analysing the gamma-ray spectra of $Cs^{137}, Co^{60}, Na^{22}, and KCl.$ In all spectra, the for the gamma ray energies were all close to their expected values within the ranges of experimental error. All peaks were analysed and we gained further insight on features such as the pair production process in the decay of Sodium, and the two photoenergy peaks in the cobalt gamma-ray spectrum which was due to the two step decay process with two gamma photons of different energies. In the sodium spectrum the sum peak was determined to be the sum of the 511 keV electron positron gamma ray and the gamma-ray emitted from the decay process at 1274 keV, hence the peak is when the two gamma rays are simultaneously detected.

Using relative count rates the efficiency of detection was determined to be 21.07% at 511 keV. After this we compared the predicted and obtained count rates for KCl. The predicted activity is 0.1048 cps however the count rate obtained is 0.18 cps. The error may be hidden in the calculations or could be due to experimental error.

Overall the experiment allowed us to successfully analyse the gamma ray spectra of multiple sources and gain a deeper understanding of how each decay process occurs and the cause of all the peaks in the spectra.

7 References

References

[1] In: *https://www.chemeurope.com/en/encyclopedia/Cobalt-60.html* ().

[2] Michael W. Davidson Mortimer Abramowitz. “Hamamatsu”. In: *https://hamamatsu.magnet.fsu.edu/articles/photomultipliers.html* ().

[3] Carlos Pallacio. “Some effects on polymers of low-energy implanted positrons”. In: *https://www.researchgate.net/figure/Simplified-decay-scheme-of-the-radioactive-isotope-22-Na-22-Na-decays-to-the-excited_fig2_292334939* (January 2008).

8 Appendix

8.1 Error

Error of the energy is 0.005 KeV
Error of the FWHM is 0.005

$$\begin{aligned} Q &= a + b + \ldots + c - (\delta x + \delta y + \ldots + \delta z) \\ \delta Q &= \sqrt{(\delta a)^2 + (\delta b)^2 + \ldots + (\delta c)^2 + (\delta x)^2 + (\delta y)^2 + \ldots + (\delta z)^2} \\ \delta ln(y) &= \frac{ln(y + dy) - ln(y - dy)}{2} \end{aligned} \tag{8.1}$$

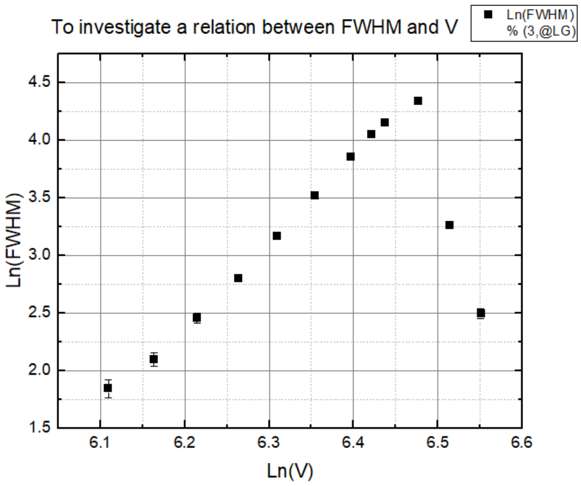

Figure 8.1: ln(FWHM) vs ln(V)

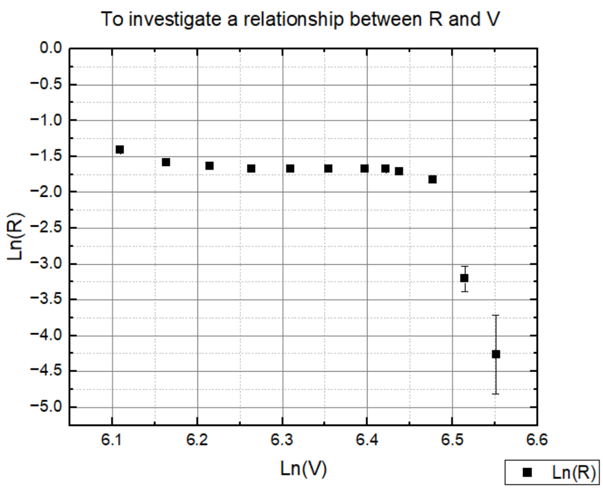

Figure 8.2: ln(R) vs ln(V)

8.2 Data

Experiment 1				
PM Voltage (V)	H_0	Peak Energy (KeV) $\pm$ 0.005	FWHM $\pm$ 0.005	Resolution $\pm$ 0.00707
450	25.87	68.24	6.31	0.2439
475	39.79	104.96	8.13	0.2043
500	59.67	157.38	11.63	0.1949
525	87.8	231.56	16.47	0.1876
550	126.31	333.14	23.82	0.1886
575	179.98	474.68	33.84	0.1880
600	252.6	666.2	47.4	0.1876
615	306.93	809.5	57.36	0.1869
625	349.37	921.46	63.44	0.1816
650	475.85	1255.04	76.94	0.1617
675	644.67	1700.3	26.13	0.0405
700	859.03	2265.65	12.12	0.0141

Table 8.1: Varying PM Voltage

ln V	ln h0	ln fwhm	dln fwhm	ln R	dLn R
6.109247583	3.253084	1.842135677	0.000792	-1.41095	0.028994
6.163314804	3.683616	2.095560924	0.000615	-1.58805	0.034616
6.214608098	4.088829	2.453587967	0.00043	-1.63524	0.03629
6.263398263	4.475062	2.801540544	0.000304	-1.67352	0.037707
6.309918278	4.838739	3.170525564	0.00021	-1.66821	0.037508
6.354370041	5.192846	3.521643535	0.000148	-1.6712	0.03762
6.396929655	5.531807	3.858622229	0.000105	-1.67318	0.037695
6.421622268	5.72662	4.049347196	8.72E-05	-1.67727	0.037849
6.43775165	5.856132	4.150094577	7.88E-05	-1.70604	0.038955
6.476972363	6.165103	4.343025897	6.5E-05	-1.82208	0.043754
6.514712691	6.468739	3.26308408	0.000191	-3.20565	0.176231
6.551080335	6.755804	2.494856981	0.000413	-4.26095	0.550775

Table 8.2: Values for experiment 2

Cs^{137}	Counts	Energy KeV		
E_0	251.34	662.89		
Compton Edge	135.41	357.14		
Backscattering	78.28	206.46		
Co^{60}	**Peak 1**			
	Count	Energy KeV	FWHP	Res
Peak Energy before calibration	437.64	1154.26		
Peak Energy	436.44	1176.79	47.78	0.109477
Error	1.2	-22.53		
	Peak 2			
	Count	Energy KeV	FWHP	Res
Peak Energy before calibration	437.64	1303.06		
Peak Energy	494.06	1332.16	29.01	0.058718
Error	-56.42	-29.1		
Compton Edge	315.04	849.47		
Backscattering	87.87	236.93		
Na^{22}	Count	Energy KeV	Count rate (cps)	
Peak Energy before calibration	195.7	527.68		
Peak Energy	195.71	512.86	2512	
Error	-0.01	14.82		
Compton Edge	268.69	704.11		
Sum Peak	390.63	1023.66	264.63	
K^{40}	Count	Energy KeV		
Peak Energy before calibration	534.46	1400.57		
Peak Energy	534.46	1452.69		
Error	0	-52.12		
Compton Edge	427	1160.61	FWHM	18.43
Initial Peak	31.85	86.56	Count Rate	0.18
Count Rate	1.97			
Total Counts	796795			
Total Time	92502.02			
Overall Count Rate	8.613811893			