SENIOR FRESH EXPERIMENTAL PHYSICS LABORATORY REPORT

Laboratory 3: Cornu Method for Young's Modulus and Poisson's Ratio

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1 ABSTRACT

In this experiment, it is attempted to use Cornu's Method by passing monochromatic light from a sodium lamp to cause interference between a Perspex beam and a glass plate to determine values for Poisson's Ratio and Young's Modulus.

The following are the values for Poisson's Ratio for each mass 100g, 200g, 300g: $\sigma_{100g} = \frac{R_1}{R_2} = 0.279 \pm 0.0175, \sigma_{200g} = 0.341 \pm 0.0111, \sigma_{300g} = 0.312 \pm 0.019$.
The values of Young's Modulus are as follows $Y_{100g} = (2.7 \pm 0.166127532)\, GPa$,
$Y_{200g}\,(2.48 \pm 0.075713184)\, GPa, Y_{300g} = (1.78 \pm 0.044583363)\, GPa$.

It can be concluded that the experiment was partially successful and the values for Young's Modulus and Poisson's Ratio are relatively accurate. However due to the experimental setup there is a lot of error. Because of the fringes oscillating back and forth it is quite hard to get an accurate measurement and it becomes almost entirely subjective one you get past the third or forth fringe. If the experiment was to be repeated a more accurate and efficient was needs to be used to measure the fringes.

2 INTRODUCTION AND THEORY

The objective for this experiment is to determine Young's modulus and Poisson's ration for perspex by an interference method.
Figure 2.1 shows the apparatus used in this experiment. A perspex beam is placed on the support, adjustable weights are hung on the end to bend the beam and it can be seen that R_1 is the longitudinal radius of curvature and that b is the thickness of the beam. A glass plate is placed on top of the beam and a sodium lamp beside it so that Newton's rings can be seen through the travelling microscope.

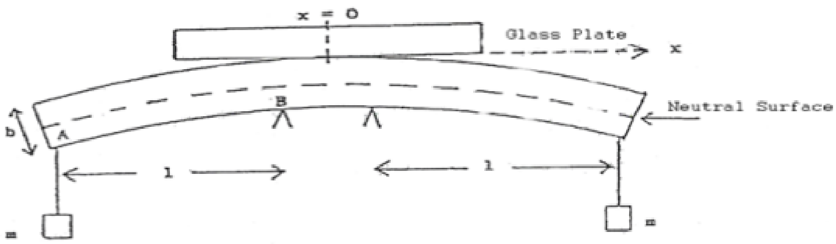

Figure 2.1: Apparatus[3]

2.1 YOUNG'S MODULUS

Essentially Young's modulus is the measure of ability of a material to withstand changes in length, in other words how easily it can stress and strain. Stress in the force applied per unit area and strain is extension per unit length. In fact Young's modulus is defined as the ratio of stress to strain.

In this experiment we have masses hanging off each side of a Perspex beam. You can imagine that the beam is compressed below and extended above. The internal bending moment is given as:

$$\frac{YAk^2}{R_1} \tag{2.1}$$

Where Y is Young's modulus, A is the cross sectional area of the perspex beam, k is the radius of gyration and R_1 is the radius of curvature of the bending perspex. However, when the beam is in equilibrium the two bending moments are equal such that:

$$mgl = \frac{YAk^2}{R_1} \tag{2.2}$$

Rearranging, and with $k = \frac{b}{\sqrt{12}}$, we can find Young's modulus to be:

$$Y = \frac{mglR_1}{Ak^2} = \frac{12mglR_1}{Ab^2} \tag{2.3}$$

2.2 Poisson's Ratio

Poisson's Ratio is defined as:

$$\sigma = \frac{lateral strain}{longitudinal strain} \tag{2.4}$$

The difference between longitudinal and lateral strain can be seen in figure 2.2. Lateral strain is the ratio of change in the diameter of the wire to its change in diameter in longitudinal direction. Longitudinal strain is the ratio of change in the length of the wire to the original length of the wire.

Poisson's ration is essentially how much a material tends to contract laterally when stretched longitudinally.

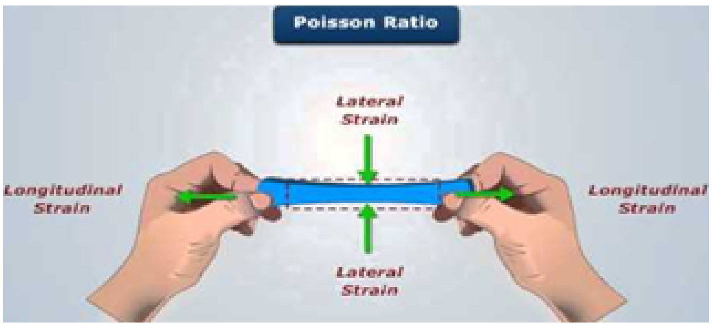

Figure 2.2: Lateral strain vs longitudinal strain.[1]

2.3 R_1, R_2 and d

R_1 and R_2 can be found using Newton's rings due to the interference of almost monochromatic light reflected vertically from both the lower surface of the glass plate and the top surface of the perspex beam. The observed fringes are contours of constant distance d.
In section 2.2 we talk about how a longitudinal strain will cause a lateral strain. From this it is trivial to conclude that the beam will take the shape of a hyperbolic parabaloid. By geometrical construction it can be shown that $\sigma = \frac{R_1}{R_2}$ and that the loci of points of constant distance d are given by

$$\frac{x^2}{R_1} - \frac{y^2}{R_2} = 2\left(d - d_0\right) \tag{2.5}$$

Taking $d = d_0$ at the "origin" $(x, y)= (0, 0)$. The fringes are also loci of points of constant d, this equation implies that the fringes form the two pairs of hyperbolae given in figure 2.3. The common asymptotes making an angle θ are given by

$$x^2 R_2 = y^2 R_1$$
$$\cot^2\theta = \frac{R_1}{R_2} = \sigma$$

This means that when y = 0, equation 2.5 implies that

$$x^2 = 2R_1\left(d - d_0\right)$$

and on a fringe

$$2d = N\lambda$$

where N is an integer, we get

$$x^2 = R_1\left(N\lambda - 2d_0\right) \tag{2.6}$$

3 Methodology

Firstly the mercury lamp is turned on so that it can heat up. Then the apparatus is set up as such in figure 2.1. The perspex sheet is placed on the knife edges and initially 100g weights are places on the hooks at each end. A glass slide, being held by a retort stand is placed at 45 deg to the perspex sheet to reflect light up to the travelling microscope. It is placed directly over the centre of the perspex sheet. Then the travelling microscope was moved around in the x-y plane until the fringes like in figure 3.1 is observed. The microscope is also brought down until the fringes as clear as possible. Any lights in the lab close to the setup may need to be turned off so that there is a better contrast to see the fringes. It is also ensured that the cross-hairs of the microscope line up so that it is in the centre of the fringes.

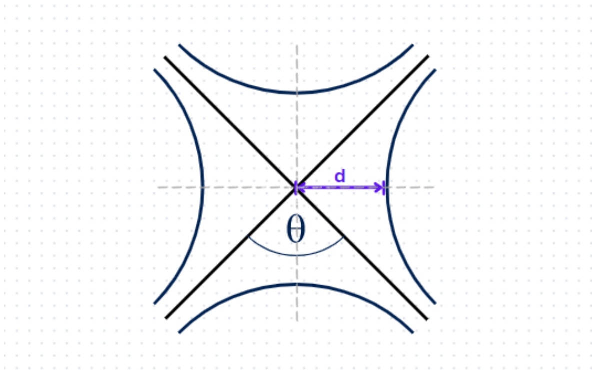

Figure 2.3: Fringes forming two sets of hyperbolae

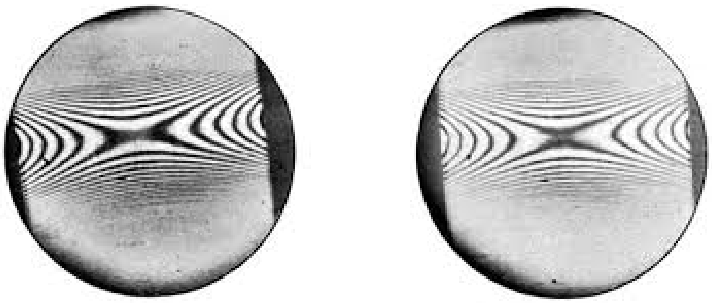

Figure 3.1: Fringes due to interference of light from the mercury lamp reflected vertically from both the lower surface of the glass plate and the top surface of the perspex beam.[2]

This distance when the microscope is at the centre of the fringes is d_0. To take the measurement of the fringes we found that the most accurate was is to go to the first fringe on the right and take a measurement. Then go to the first fringe on the left and take a measurement again and then subtract the two distances and divide by two to get the distance from d_0. This is done for until it becomes to hard to precisely measure where the fringes are. And then the whole process is repeated with 200g weights and again with 300g weights. This is the set of x values obtained.

The perspex beam is then rotated 90 degrees along with the rest of the apparatus excluding the microscope. Then the same method to obtain the x values is used to obtain the y values.

4 Results and Discussion

To begin we took measurements of the beam to be used to calculate Young's Modulus. Using a ruler and a micrometer we measured b = $\left(7.24x10^{-3} \pm 5x10^{-6}\right)$m, l = (0.142 ± 0.0005)m and w = (0.04 ± 0.0005)m. Then it was easy to calculate A = $(28.96 \pm 0.024525) * 10^{-5}$m and $b^2 = (52.4 \pm 0.0512) * 10^{-6}m$ Using the fact that the wavelength of the sodium yellow light is 589.3 nm, x^2 and y^2 were plot against $N\lambda$ to determine R_1 and R_2.

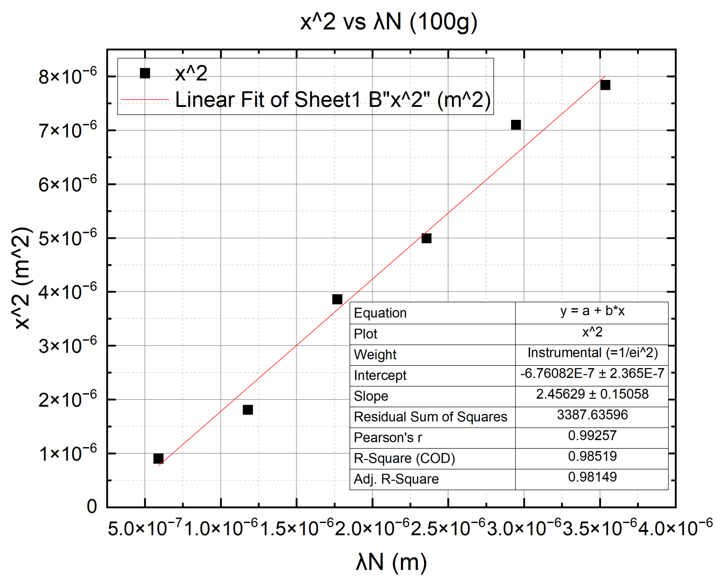

Figure 4.1: Using the slope of this plot we can determine R_1 to be 2.45629 ± 0.15058 m

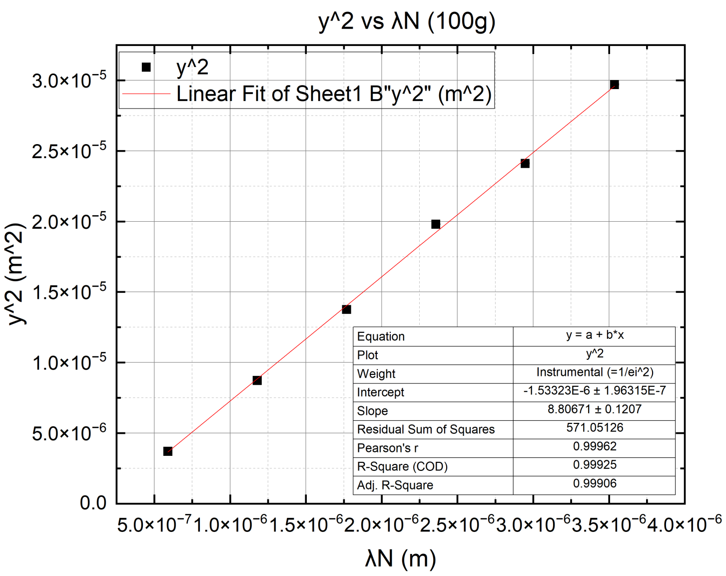

Figure 4.2: Using the slope of this plot we can determine R_2 to be 8.80671 ± 0.1207 m

Using Excel to compute σ and $\delta\sigma$ we can calculate Poisson's Ratio, to three significant figures, to be

$$\sigma_{100g} = \frac{R_1}{R_2} = 0.279 \pm 0.0175$$

Again with calculations done in Excel, using equation 2.3 we calculated Young's Modulus to be

$$Y_{100g} = (2.7 \pm 0.166127532)\, GPa$$

The following formula for error propagation was used to determine the error:

$$\Delta Y = Y\sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta R_1}{R_1}\right)^2 + \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}$$

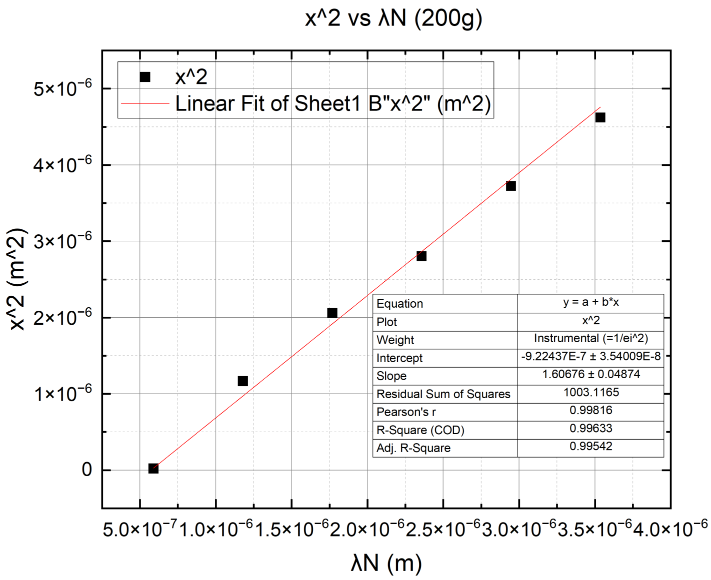

Figure 4.3: Using the slope of this plot we can determine R_1 to be 1.60676 ± 0.04874 m

Using Excel to compute σ and $\delta\sigma$ we can calculate Poisson's Ratio, to three significant figures, to be

$$\sigma_{200g} = 0.341 \pm 0.0111$$

Using R_1 and R_2 obtained from figures 4.3 and 4.4, and with calculations done in Excel, using equation 2.3 we calculated Young's Modulus to be

$$Y_{200g}\,(2.48 \pm 0.075713184)\, GPa$$

Using Excel to compute σ and $\delta\sigma$ we can calculate Poisson's Ratio, to three significant figures, to be

$$\sigma_{300g} = 0.312 \pm 0.019$$

Using R_1 and R_2 obtained from figures 4.5 and 4.6, and with calculations done in Excel, using equation 2.3 we calculated Young's Modulus to be

$$Y_{300g} = (1.78 \pm 0.044583363)\, GPa$$

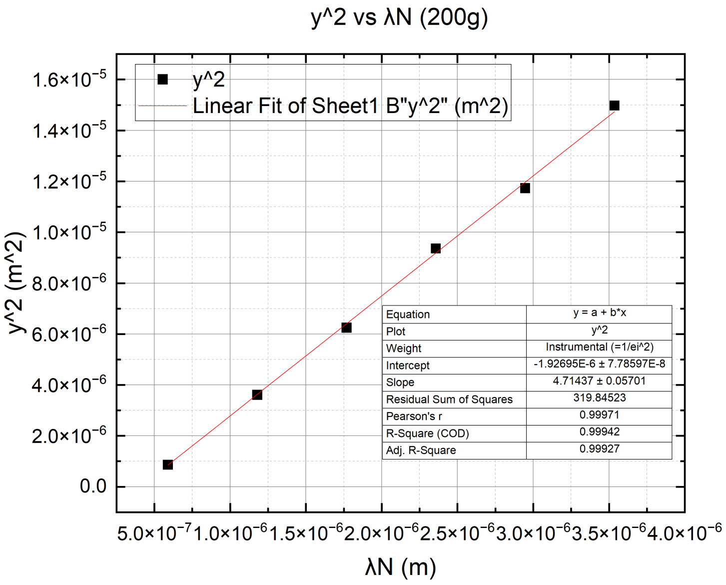

Figure 4.4: Using the slope of this plot we can determine R_2 to be 4.71437 ± 0.05701 m

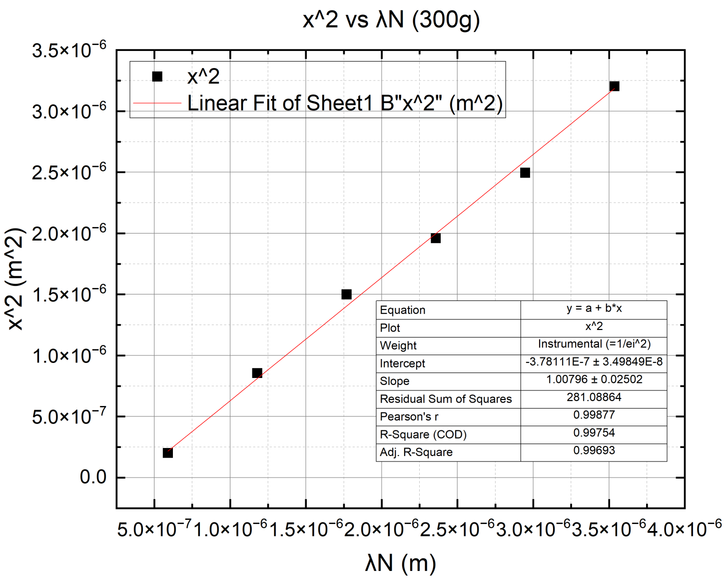

Figure 4.5: Using the slope of this plot we can determine R_1 to be 1.00796 ± 0.02502 m

Comparing the Young's modulus for each mass: $Y_{100g} = (2.7 \pm 0.166127532)\, GPa$ $Y_{200g}\, (2.48 \pm 0.075713184)\, GPa$, $Y_{300g} = (1.78 \pm 0.044583363)\, GPa$.

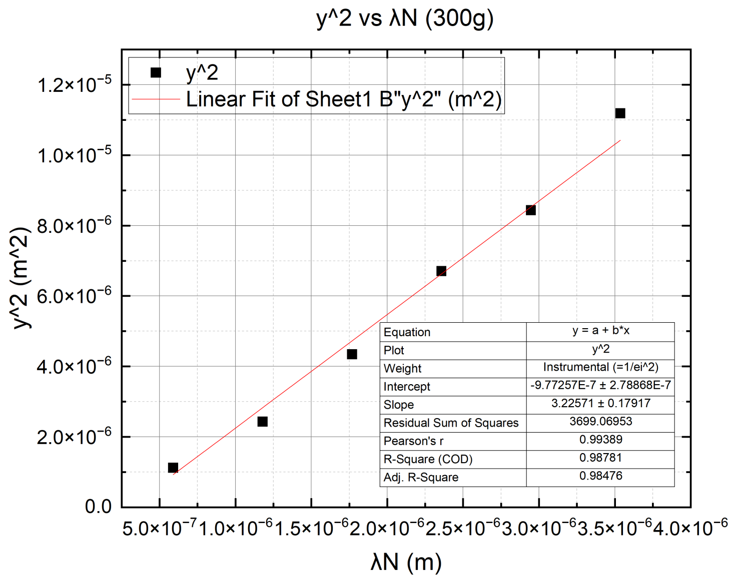

Figure 4.6: Using the slope of this plot we can determine R_2 to be 3.22571 $\pm$ 0.17917 m

The Young's Modulo should be the same however the discrepancy is likely due to the complete inaccuracy when taking measurements for the fringes. It is extremely hard to line up with the fringes making the measurements completely subjective.

5 Conclusion

Overall the experiment was relatively successful. All the Poisson Ratios and Young's Modulo are relatively close. Ideally they would all be identical because obviously the beam doesn't change material once weight is added. Error in this experiment easily comes down to measuring the fringes. It is extremely difficult to get an accurate measurement because the fringes are not completely stationary. This leads to a lot of subjectivity in making the measurements. If I were to do the experiment again (hopefully I never have to) , I would need a more stable microscope and just overall a better way of measuring the fringes. Another possible way to rectify the non-stationary fringes is to have a better platform and to ensure the weights aren't swinging when the measurements are being taken.
Overall it was interesting to see the application of Young's modulus, however I am not sure how useful it is because it requires the material to be translucent allow light to pass through to cause the interference and it is also not very efficient. It is interesting to see the correlation between longitudinal strain and lateral strain and their effects on Poisson's Ratio and Young's Modulus.

6 APPENDIX

REFERENCES

[1] Nagashree on civilengineering.blogspot.com. *LATERAL STRAIN,LONGITUDINAL STRAIN AND POISSON'S RATIO*. 30 Dec 2016. URL: `https://1civilengineering.blogspot.com/2016/12/lateral-strainlongitudinal-strain-and.html`.

[2] H.T. Jessop B.Sc. (1921) LXIV. "On cornu's method of determining the elastic constants of glass". In: ().

[3] *Trinity Lab Manual*. 2024.

6.1 ERROR PROPAGATION

The following error propagation formulas were used:

$$\frac{\partial z}{z} = \sqrt{\left(\frac{\partial x}{x}\right)^2 + \left(\frac{\partial y}{y}\right)^2}$$

When x and y are the same (ie. x^2) we get:

$$\delta x^2 = x^2\sqrt{2}\delta x$$

For Young's Modulus uncertainty:

$$\Delta Y = Y\sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta R_1}{R_1}\right)^2 + \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}$$

6.2 DATA

x							
100g							
L (mm)	R (mm)	x(mm)	N	x (m)	dx	x^2	d(x^2)
7.2	5.3	0.95	1	0.00095	3.5355E-06	9.025E-07	6.72E-09
7.69	5	1.345	2	0.001345	3.5355E-06	1.80903E-06	9.51E-09
8.28	4.35	1.965	3	0.001965	3.5355E-06	3.86123E-06	1.39E-08
8.5	4.03	2.235	4	0.002235	3.5355E-06	4.99523E-06	1.58E-08
8.85	3.52	2.665	5	0.002665	3.5355E-06	7.10223E-06	1.88E-08
9.1	3.5	2.8	6	0.0028	3.5355E-06	0.00000784	1.98E-08

200g							
L (mm)	R (mm)	x (mm)	N	x(m)	dx	x^2 (m^2)	d(x^2)
7	6.71	0.145	1	0.000145	3.5355E-06	2.1025E-08	1.03E-09
8.45	6.29	1.08	2	0.00108	3.5355E-06	1.1664E-06	7.64E-09
8.87	6	1.435	3	0.001435	3.5355E-06	2.05923E-06	1.01E-08
9.1	5.75	1.675	4	0.001675	3.5355E-06	2.80563E-06	1.18E-08
9.39	5.53	1.93	5	0.00193	3.5355E-06	3.7249E-06	1.36E-08
9.6	5.3	2.15	6	0.00215	3.5355E-06	4.6225E-06	1.52E-08

300g							
L (mm)	R (mm)	x (mm)	N	x(m)	dx	x^2 (m^2)	d(x^2)
25.3	24.4	0.45	1	0.00045	3.5355E-06	2.025E-07	3.18E-09
25.95	24.1	0.925	2	0.000925	3.5355E-06	8.55625E-07	6.54E-09
26.15	23.7	1.225	3	0.001225	3.5355E-06	1.50063E-06	8.66E-09
26.3	23.5	1.4	4	0.0014	3.5355E-06	0.00000196	9.9E-09
26.5	23.34	1.58	5	0.00158	3.5355E-06	2.4964E-06	1.12E-08
26.7	23.12	1.79	6	0.00179	3.5355E-06	3.2041E-06	1.27E-08

y							
100g							
L (mm)	R (mm)	y(mm)	N	y (m)	dy	y^2 (m^2)	d(y^2)
59.85	56	1.925	1	0.001925	3.5355E-06	3.70563E-06	1.36E-08
61.11	55.2	2.955	2	0.002955	3.5355E-06	8.73202E-06	2.09E-08
61.8	54.38	3.71	3	0.00371	3.5355E-06	1.37641E-05	2.62E-08
62.45	53.55	4.45	4	0.00445	3.5355E-06	1.98025E-05	3.15E-08
62.82	53	4.91	5	0.00491	3.5355E-06	2.41081E-05	3.47E-08
63.51	52.61	5.45	6	0.00545	3.5355E-06	2.97025E-05	3.85E-08

200g							
L (mm)	R (mm)	y (mm)	N	y(m)	dy	y^2 (m^2)	d(y^2)
60.16	58.3	0.93	1	0.00093	3.5355E-06	8.649E-07	6.58E-09
61	57.2	1.9	2	0.0019	3.5355E-06	0.00000361	1.34E-08
61.62	56.62	2.5	3	0.0025	3.5355E-06	0.00000625	1.77E-08
62.3	56.18	3.06	4	0.00306	3.5355E-06	9.3636E-06	2.16E-08
62.7	55.85	3.425	5	0.003425	3.5355E-06	1.17306E-05	2.42E-08
63.24	55.5	3.87	6	0.00387	3.5355E-06	1.49769E-05	2.74E-08

300g							
L (mm)	R (mm)	y (mm)	N	y(m)	dy	y^2 (m^2)	d(y^2)
60.32	58.2	1.06	1	0.00106	3.5355E-06	1.1236E-06	7.5E-09
60.74	57.62	1.56	2	0.00156	3.5355E-06	2.4336E-06	1.1E-08
61.37	57.2	2.085	3	0.002085	3.5355E-06	4.34722E-06	1.47E-08
61.71	56.53	2.59	4	0.00259	3.5355E-06	6.7081E-06	1.83E-08
62.02	56.21	2.905	5	0.002905	3.5355E-06	8.43903E-06	2.05E-08
62.55	55.86	3.345	6	0.003345	3.5355E-06	1.1189E-05	2.37E-08

Poissons Ratio						
	R1 (m)	**dR1 (m)**	**R2 (m)**	**dR2 (m)**	σ	**d(σ)**
100g	2.45629	0.15058	8.80671	0.1207	0.278911194	0.017520419
200g	1.60676	0.04874	4.71437	0.05701	0.340821785	0.011129844
300g	1.00796	0.02502	3.22571	0.17917	0.312476943	0.019010642

Youngs Modulus		
	Y (Gpa)	**dY (GPa)**
100g	2.704846	0.166128
200g	2.477091	0.075713
300g	1.775931	0.044583

Measurements	
l (m)	0.142 ± 0.0005
b (m)	7.24E-3 ± 5E-6
w (m)	0.04 ± 0.0005
A (m^2)	0.00029 ± 2.5E-7
b^2 (m^2)	5.24E-5 ± 5.1E-8