Susceptibility & Magnetisation

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1 ABSTRACT

This experiment investigates the phenomenon of magnetic susceptibility, magnetic properties of paramagnetic and ferromagnetic materials, and hysteresis. By calibrating the magnetic field, we used the non-linear relationship between magnetic field strength and current to aid us later on in the experiment by allowing us to determine $\vec{B}$ for any known current I.

The field gradient was calibrated using a sample of Mohr's salt of known susceptibility and the calibration constant C was determined to be $C = (14.25 \pm 0.19)m^{-1}$. Using this calibration constant, we determined the susceptibility of paramagnetic salt ($Gd_3Ga_5O_{12}$) to be $\chi = (0.77 \pm 0.015)JT^{-2}kg^{-1}$.

Unfortunately hysteresis for the hematite-αFe_2O_3 sample is not observed. Instead asymptotic behaviour as the magnetic field strength approaches zero is observed as a result of the dependence of σ on B_x^{-1}.

2 INTRODUCTION & THEORY

The aim of this experiment is to gain a deeper understanding of paramagnetic and ferromagnetic materials. We aim to learn how to generate static magnetic fields, to observe hysteresis in a ferromagnetic sample, and to measure magnetic properties of paramagnetic and antiferromagnetic materials.

2.1 FARADAY BALANCE [4]

The magnetic dipole moment ($\vec{m}$) is the measure of a material's tendency to align with an external magnetic field. The energy of a $\vec{m}$ (Joules/Tesla) in a field $\vec{B}$ is defined as $E = -\vec{m}\vec{B}$

From this the force on a dipole in a nonumiform magnetic field is given by equation 2.1 or component-wise by equation 2.2.

$$\vec{F} = \nabla(\vec{m}\vec{B}) \tag{2.1}$$

$$F_z = m_x \partial_z B_x + m_y \partial_z B_y + m_z \partial_z B_z \tag{2.2}$$

However, the magnetic moment in this experiment is aligned with the magnetic field, along the x-direction, but the field gradient produced by the shaped pole pieces in in the z-direction. Hence, 2.2 is reduced to

$$F_z = m_x \partial_z B_x \tag{2.3}$$

In this experiment when the sample is placed between the pole magnets as shown in figure 2.1. It must be placed exactly midway between the poles so that F_x is zero.

The magnetic moment of the sample is can be expressed as the product of the magnetisation per unit mass $\vec{\sigma}(JT^{-1}kg^{-1})$ and the sample mass in kg,m,

$$\vec{m} = m\vec{\sigma}$$

The magnetisation of a **paramagnet** can be expressed as

$$\vec{\sigma} = \chi\vec{B} \tag{2.4}$$

where $\chi(JT^{-2}kg^{-1})$ is the magnetic susceptibility; a measure of how much a material will become magnetized in an applied magnetic field. Hence the force in the z-direction becomes

$$F_z = m\chi B_x \partial_z B_x \tag{2.5}$$

The benefit of the curved pole pieces in this experiment is that they are designed to give a nearly constant value for $B_x \partial_z B_x$.

A **ferromagnet** (described in section 2.2) has a spontaneous magnetisation $\vec{\sigma}_s$ and a magnetic moment of

$$\vec{m} = m\vec{\sigma}_s$$

which is independent of the applied field when all the domains are aligned. Hence, after magnetic saturation,

$$F_z = m\sigma_s \partial_z B_x \tag{2.6}$$

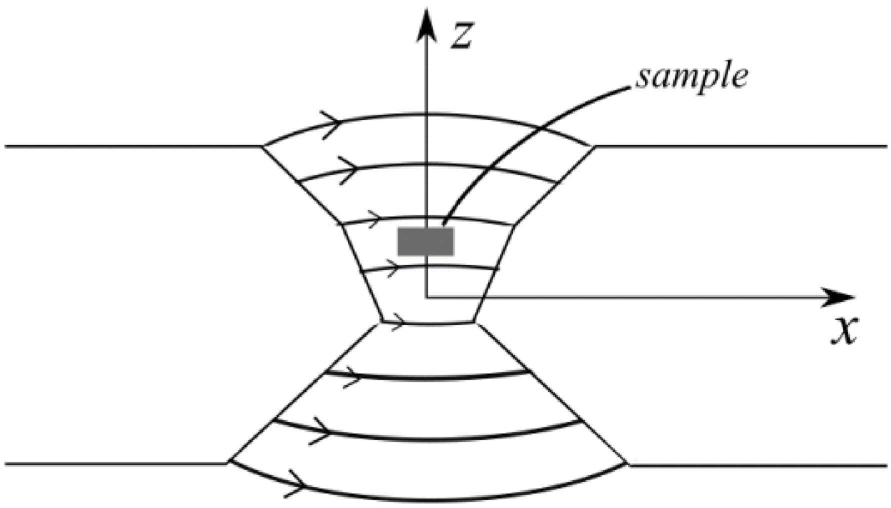

Figure 2.1: The magnetic material suspended between the two pole magnets in the magnetic field $\vec{B}$. [4]

2.2 Types of Magnetism

Paramagnetism: Paramagnetism is a form of magnetism in which the material is weakly attracted to an external magnetic field. The attraction arises due to atomic or molecular dipoles with unpaired electron spins that align with the magnetic field. Unlike a ferromagnet, paramagnets lose their magnetism once the external magnetic field is removed and the dipoles return to their random orientation. Paramagnets also follow **Curie's Law** which states that the magnetic susceptibility is inversely proportional to temperature. In other words, the material's magnetic effect weakens with increase in temperature. Aluminium is an example of a paramagnetic material.

Diamagnetism: Diamagnetic materials are materials with a negative magnetic susceptibility. When under the influence of an external magnetic field, the magnetic moment is induced in the opposite direction of the applied magnetic field. A result of this antiparallel magnetic moment, diamagnetic materials are weakly repelled by magnetic fields. Diamagnetism is a universal property of all materials, however it is so weak that it is only observed in the absence of all other types of magnetism.

Ferromagnetism: Ferromagnetic materials, unlike Paramagnetic materials, retain their magnetism after being under the influence of an external magnetic field. Ferromagnets are made up of magnetic domains which consist of groups of atoms whose magnetic moments are aligned. Once under the effect of an external magnetic field, the magnetic moments align with the field and create larger domains. This results in a strong overall magnetic effect and once the magnetic field is removed, the material retains it's magnetism and the magnetic moments do not return to their original spin direction. Ferromagnetic materials also exhibit **hystersis**, meaning the magnetisation depend on this history of how external magnetic fields were applied and removed. Iron, Cobalt, and Nickel are all examples of ferromagnetic materials.

Antiferromagnetism: Antiferromagnetism is a type of magnetism in which the magnetic moments of adjacent atoms align in opposite directions. This results in no net magnetisation since overall the magnetic moments are balanced without an external magnetic field. Unlike ferromagnets, when an external magnetic field is applied the material is only weakly attracted to the field since opposing magnetic moments cancel each other out. The analogue of the Curie temperature for antiferromagnets is the **Néel temperature**. Above the Néel temperature, the antiparallel alignment is disrupted and the material becomes paramagnetic. Manganese Oxide (MnO), and Iron oxide (FeO) are examples of antiferromagnetic materials.

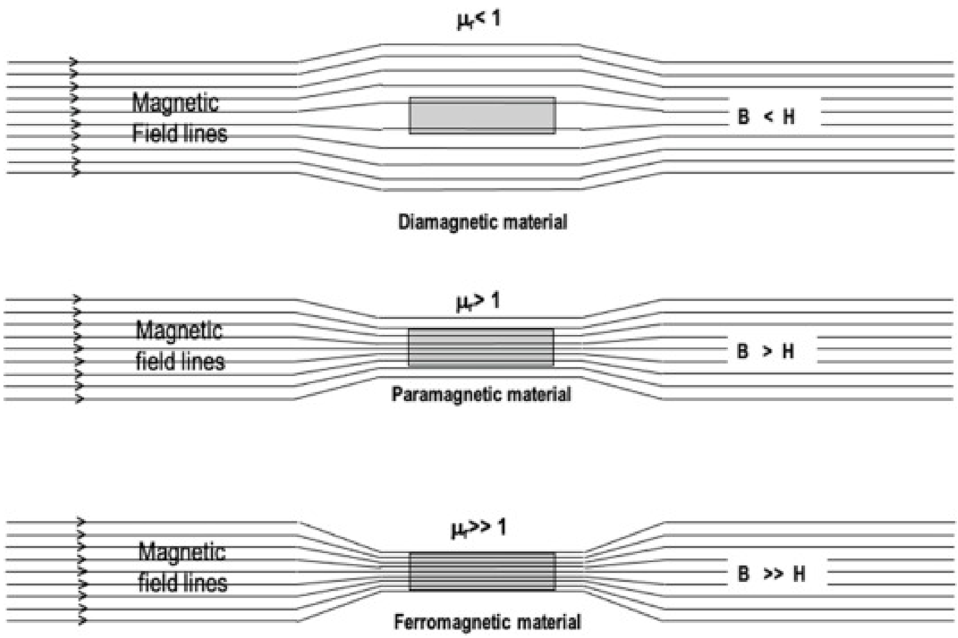

Figure 2.2: The effect of each type of magnetisation on external magnetic fields. μ_r is the relative permeability, which is the susceptibility + 1.[3]

2.3 Hysteresis

Hysteresis is a property of ferromagnetic materials. It is defined as the lagging of the magnetization of a ferromagnetic material (e.g. iron) behind variations of the magnetizing field [4]. This means that when an external magnetic field is applied to a ferromagnetic material, the magnetic moment's align themselves in the direction of the magnetic field and once the applied magnetic field is removed, the magnetic moment of the atoms remain in this orientation until another magnetic field is applied. Figure 2.3 shows a hysteresis loop, where a ferromagnetic material is placed in an alternating magnetic field. When its magnetisation σ is plot as a function of field strength, B, the plot traces a closed loop. Hysteresis has a significant real life application, this is the element of memory in a hard disc drive. A thin ferromagnetic film covers each side of the disc and sequential changes in the direction of magnetisation represent binary data bits.

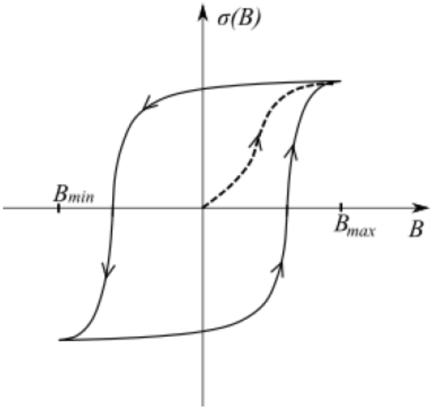

Figure 2.3: Hysteresis loop, the magnetisation of ferromagnetic materials depend on the history of externally applied magnetic fields. [4]

Figure 2.4: Magnetic hysteresis is the method of recording data onto hard disc drive. Sequential changes in the direction of magnetisation represent binary data bits [2]

2.4 CURIE'S LAW

Curie's law states that the magnetic susceptibility (χ) of a paramagnetic material is inversely proportional to it's temperature. As the temperature increases, the material's ability to become magnetised decreases.

$$\chi = \frac{C}{T} \tag{2.7}$$

Where χ is the magnetic susceptibility, C is Curie's constant which is unique to each material, and T is temperature in Kelvin. At higher temperatures, thermal energy disrupts the alignment of the magnetic moments making it harder for the material to become magnetised.

$$\chi = \mu_0 \frac{N g^2 \mu_B^2 J(J+1)}{3kT} \tag{2.8}$$

μ_0 is the vacuum permeability, μ_B is the Bohr magneton, J is the quantum number, N is the number of magnetic ions per kg of sample and g is the Landé g-factor, k is Boltzmann's constant, and T is temperature.

Curie's law typically only applies to paramagnetic materials at high temperatures and low magnetic fields. It breaks down at very low temperatures or high magnetic field strengths where quantum effect and other forms of magnetism dominate.

3 Experimental Setup

The centre of attention in this experiment is given in figure 2.1 above. A sample in a holder is suspended between two curved magnets. Two solenoids are at either side of these curved magnets and a current is sent through these solenoids which produces a magnetic field between the two poles. The current, and hence the magnetic field strength is varied using a potentiometer. A Hall probe is connected to a Gauss meter and placed between the magnetic poles so that the magnetic field strength can be measured.

The sample being investigated is suspended from a string attached to an electronic balance. This balance is very sensitive so it is important that the setup is not near any drafts or open windows as this will affect the mass readings. It is also important that we use the given masses for the samples as too little of the sample will result in inaccurate measurements if the force in the z-direction is negligible. If the mass of the sample is too high then it may be pulled sideways to one of the poles since the force in the x-direction may be of the same magnitude of the force in the z-direction.

There are two main switches in this apparatus. Switch 1 on the power supply allows power to run to the coil. Switch 2 down by the coil has the positions Forward, off, and Reverse, which allows us to decide the polarity of the magnetic field. It is extremely important that whenever any switches are being switched, or when the power supply is being turned off, that the voltage is reduced to zero and no current is flowing. If there is current flowing and the current is switched off to the large coil will result in a very large backvoltage which can be dangerous for the user and for the equipment.

To account for this and also to account for the toxicity of the samples lab goggles/glasses and gloves are used for the duration of the experiment.

4 Methodology

Following the safety precautions given in the last section, the power supply is turned on provided all other switches are off. The apparatus is ensured to be set up as described with the hall probe between the two poles and connected to the Gauss meter. The current limit is then set so that it has a maximum of 10 Amps. Switch 2 is turned to the desired polarity and then switch 1 is turned on to allow current to flow to the coil.

The initial step of this experiment is to calibrate the field and the gradient of the field. It is important that we are calibrating the field where the sample will be placed so it is ensured that the tip of the Hall probe is in this location. To calibrate the field we increase the voltage in a intervals of 5 volts from zero to a maximum of 85V. At each interval the current on the power supply and the magnetic field strength on the Gauss meter is noted. Using the same intervals, we do the same going from 85V to zero. Then reverse the polarity and repeat the whole process again from zero to 85V and back down to zero. Using the values obtained, $\vec{B}$ is plot as a function of I.

Next the field gradient is calibrated. Using a known mass of 50-60 mg of Mohr's salt ($\chi = 0.330JT^{-2}kg^{-1}$ at $20°C$), a similar method to the previous part is used where we increase in 5V intervals from 0 to 85V but we do not need to go back down or reverse polarity. This time we measure the apparent mass of Mohr's salt at each interval and the magnetic field strength. Due to the shape of the pole pieces, $\partial_z B_x = CB_x$ where C is a constant dependent on the pole piece geometry and z. This leaves us with equation 4.1. Using these results we can determine C by plotting F_z vs B_z^2 and using the slope.

$$F_z = Cm\chi B_x^2 \tag{4.1}$$

Our next sample is a paramagnetic salt ($Gd_3Ga_5O_12$), 10-15mg. Using the value for C that we have obtained we now measure the susceptibility of this paramagnetic salt and compare it with what is expected from the Curie Law in equation 2.8.

Finally we examine a sample of 20-25mg of a weakly ferromagnetic material, hematite-α (Fe_2O_3). The benefit of using a weakly ferromagnetic material is that it will not retain it's magnetism permanently so can be used again and again for this experiment and each student will get the same outcome. By noting values of F_z and B_x for changing values of I, we can calculate a series of values of σ using $F_z = m\sigma CB_x$. Using our field calibration graph we obtain values for Bx from I and we can plot σ vs B_x to deduce σ_s for hematite. Similarly to the field calibration we take values for I increased, decreased, and reversed decrease and increase. From this we can plot the hysteresis loop for hematite similar to figure 2.3

5 Results & Discussions

5.1 Field Calibration

By using the Hall probe to determine the magnetic field strength and varying the current we could plot this relationship $B_x(I)$ in figure 5.1. This is useful later on in the experiment such that for any I we can find the corresponding value of B_x. However we found that for consistency and overall convenience it is easier to use these values for the magnetic field for all sections of this experiment since we are keeping the intervals of current/voltage the same. Later on in the experiment it can be seen that the poles retain some of the magnetic field which can lead to inconsistency in the data, when swapping polarity for example, or swapping sample. Ideally we would do one section then leave the apparatus for an hour or so and then some back to proceed to the next section however this is obviously not viable due to time restrictions in the lab.

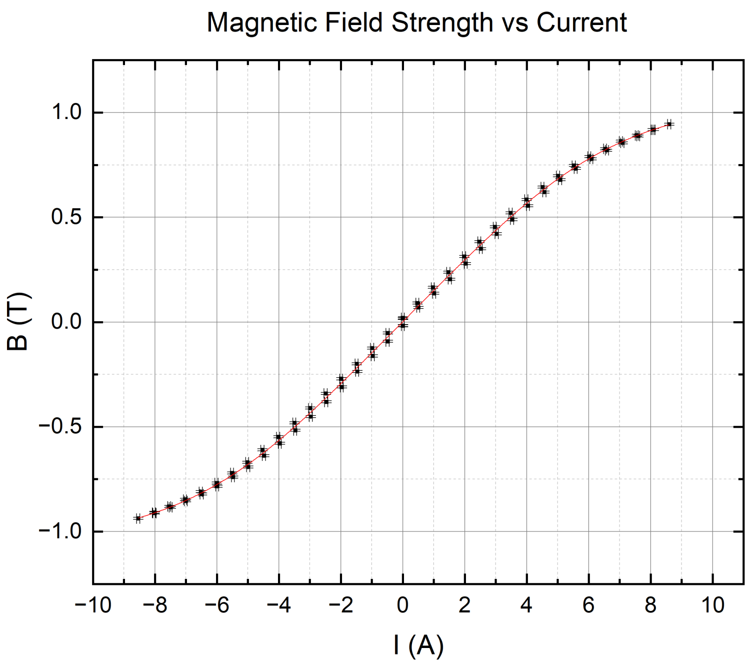

Figure 5.1: Magnetic field strength, B_x, as a function of the current I. Using this calibration, throughout the experiment for any value of I we can find the corresponding value of B_x

5.2 Field Gradient Calibration

Since we know the susceptibility of Mohr's salt and the mass of the sample, by equation 4.1 we can use the slope from figure 5.2 to find C, a constant that depends on the pole piece geometry and also on z.

$$C = \frac{slope}{m\chi_{Mohr'sSalt}} \tag{5.1}$$

From the plot the slope is $2.5385x10^{-4} \pm 3.463x10^{-6} NT^{-2}$. We know the mass of the sample, m, is 54mg or 5.4e-5 kg. $\chi_{Mohr'sSalt} = 0.33JT^{-2}kg^{-1}$.

$$C = (14.25 \pm 0.19)NJ^{-1} = (14.25 \pm 0.19)m^{-1} \tag{5.2}$$

Using this calibration constant we can determine the susceptibility of other magnetic samples. However it is important to place the samples in the same position since C depends on z.

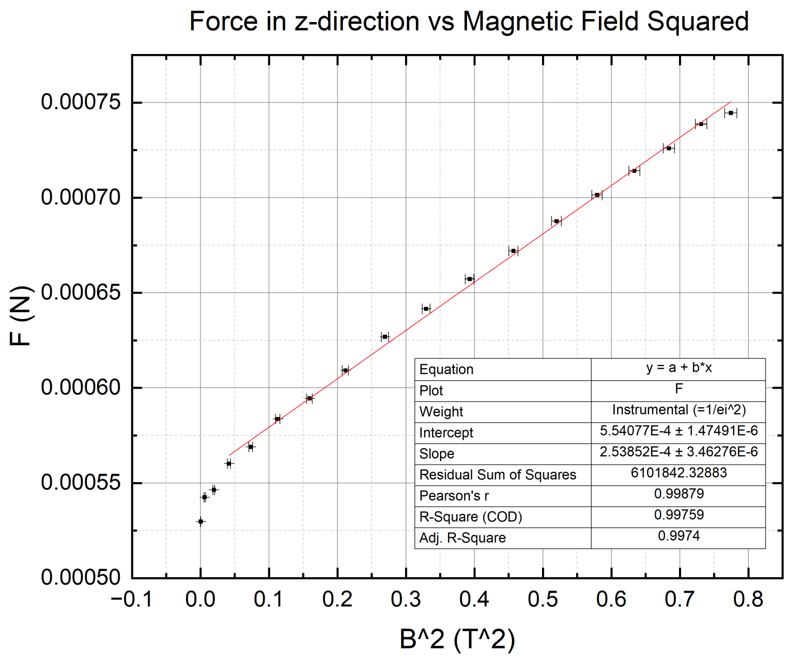

Figure 5.2: Using our knowledge of the susceptibility of the Mohr's salt we find C, the constant in equation 4.1. The force in the z-direction of the sample is plot against the magnetic field strength squared and the slope will allow us to find C.

5.3 Paramagnetic Salt

Using

$$\chi = \frac{slope}{mC}$$

we can work out the susceptibility of the paramagnetic salt.

$$\chi = \frac{(2.6565e - 4 \pm 3.666e - 6)NT^{-2}}{(2.43e - 5)kg(14.25 \pm 0.19)m^{-1}}$$
$$\chi = (0.7672 \pm 0.01472)JT^{-2}kg^{-1}$$
$$\chi \approx (0.77 \pm 0.015)JT^{-2}kg^{-1}$$

From above we concluded that the susceptibility of paramagnetic salt is $\chi = (0.77 \pm 0.015)JT^{-2}kg^{-1}$. We can compare this value with the expected value which we can determine using equation 2.8. Since Gd^{3+} is the only magnetic ion and there are 3 moles, g = 2 and J = $\frac{7}{2}$. Using molar masses we can work out that $N = 1.79x10^{24}$ ions/kg. Subbing into our equation we calculate the expected value to be $\chi \approx 1x10^{-6}$? We understand that our expected value is incorrect. After some research to find an expected value for the susceptibility of $Gd_3Ga_5O_{12}$, we found another report on a similar experiment and their calculated value of $\chi \approx (0.68 \pm 0.0007)JT^{-2}kg^{-1}$ and their expected value is $\chi_{expected} \approx 0.81JT^{-2}kg^{-1}$ [1].

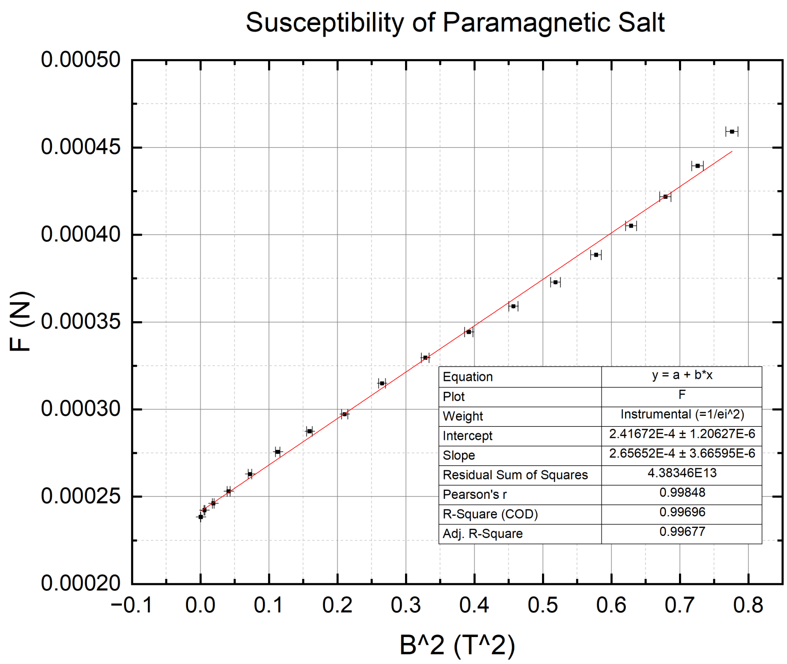

Figure 5.3: To figure out the susceptibility of this paramagnetic salt, we can use C found in the last section and the slope of the plot of force in the z direction vs magnetic field squared for $Gd_3Ga_5O_{12}$

5.4 Ferromagnetic Material

After many unsuccessful attempts at observing the hysteresis loop for hematite-αFe_2O_3, we plot σ vs B_x. Calculating σ from $F_z = B_x \sigma m C$ with $m = 5\mu_B$, C our calibration constant from a series of values for B_x and F_z leads to asymptotic behavior as B approaches zero. This can be observed in figure 5.4.

Below is a sample of the data taken during this section of the experiment.

Voltage (V) \pm 0.05	Flux (T) \pm 0.005	mass g	Fz (N)	\sigma (J/T*kg)
0	-0.017	1.226	0.000196	-1.74662E+19
5	0.054	1.2262	0.000198	5.5536E+18
10	0.124	1.2264	0.0002	2.44245E+18
15	0.196	1.2266	0.000202	1.56037E+18
20	0.266	1.2268	0.000204	1.16091E+18
25	0.335	1.227	0.000206	9.30661E+17
30	0.404	1.2279	0.000215	8.04785E+17
35	0.469	1.228	0.000216	6.96413E+17

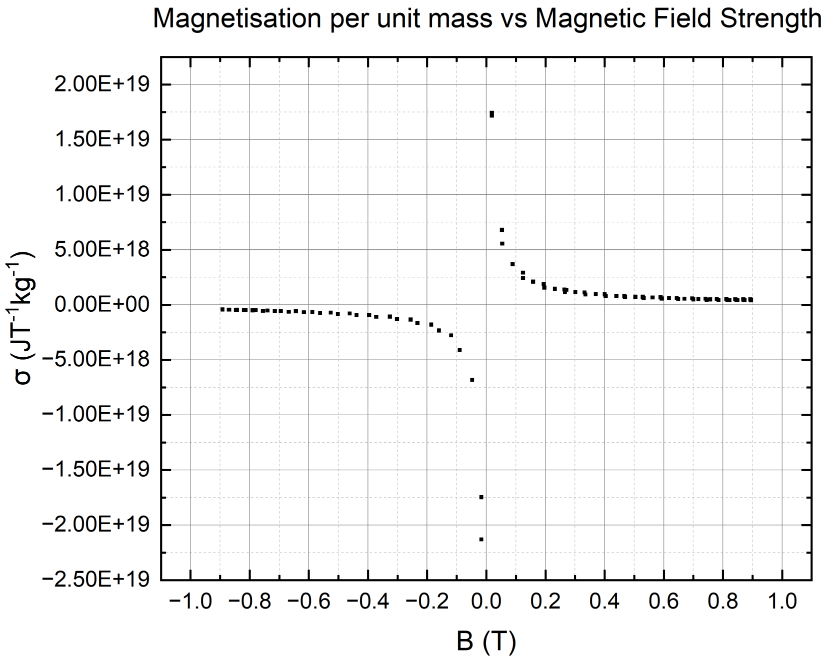

Figure 5.4: In an attempt to plot the hysteresis loop for hematite, we plot σ against magnetic field strength. By calculating $\sigma = \frac{F_z}{B_x C m}$ we can see sigma asymptotes to infinity as B goes to zero.

6 Conclusions

In the first section, we created a magnetic field between the two poles by passing a current through two solenoids. By varying the current we calibrated magnetic field which left us with a non linear relation for $B(I)$. This proved useful in determining the magnetic field strength for certain values of I in later parts of the experiment. However it was important to note that when the current was switched off, the magnetic poles still retained some magnetic field strength (particularly when reversing the polarity). Ideally we would leave the apparatus for some time so that every experiment has the same starting point but this is not viable for a lab like this due to time restraints.

Next, using a sample of Mohr's salt of known susceptibility we calibrated the field gradient. By plotting the force in the z direction vs the magnetic field squared and using the slope of the linear relationship, we could determine the calibration constant C to be $C = (14.25 \pm 0.19)m^{-1}$.

Having obtained a value for C, using a similar method to the last section we determined the susceptibility of paramagnetic salt ($Gd_3Ga_5O_{12}$). Using the slope of force vs magnetic field strength squared, the initial mass, and the calibration constant C, we calculated the susceptibility to be $\chi = (0.77 \pm 0.015)JT^{-2}kg^{-1}$. Using Curie's law we attempted to verify our result. However after obtaining a value of $\chi \approx 1x10^{-6}$, we understand that the expected value is wrong.

Expecting to find hysteresis when plotting $\sigma(B_x)$ we instead find asymptotic behaviour as B approached zero. If we were to repeat the experiment we would take more time to understand how to calculate σ and repeat the experiment multiple times to reduce error.

7 References

References

[1] Danny Bennett and Maeve Madigan. "Susceptibility and Magnetisation Measurements". In: *https://www.maths.tcd.ie/~dbennett/js/mag_susc.pdf* (2015).

[2] Evan-Amos. In: *https://commons.wikimedia.org/w/index.php?curid=27940250* ().

[3] T. Gnanasekaran. "Science and Technology of Liquid Metal Coolants in Nuclear Engineering". In: *https://www.sciencedirect.com/book/9780323951456/science-and-technology-of-liquid-metal-coolants-in-nuclear-engineering* (2022).

[4] *PYU33TP1 Lab Manual*. 2024.

8 Appendix

8.1 Error

To calculate the error for B^2 from B with constant error $\pm 0.005T$,

$$Z = X^2 \tag{8.1}$$
$$\frac{\Delta Z}{Z} = 2\frac{\Delta X}{X} \tag{8.2}$$
$$\tag{8.3}$$

When calculating the error of the susceptibility of paramagnetic salt we need to propagate through division:

$$Z = \frac{X}{Y} \tag{8.4}$$
$$\Delta Z = Z\sqrt{\left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2} \tag{8.5}$$

8.2 Data

Since the attempt to observe hysteresis was unsuccessful I will only attach the raw data for this. There is too much data for this experiment to attach all the raw data.

Forward		
Voltage (V) \pm 0.05	**Flux (T) \pm 0.005**	**mass g \pm 5E-5**
0	-0.017	1.226
5	0.054	1.2262
10	0.124	1.2264
15	0.196	1.2266
20	0.266	1.2268
25	0.335	1.227
30	0.404	1.2279
35	0.469	1.228
40	0.532	1.2284
45	0.593	1.2289
50	0.649	1.2293
55	0.7	1.2294
60	0.745	1.2296
65	0.785	1.2297
70	0.816	1.2299
75	0.846	1.2302
80	0.871	1.2308
85	0.895	1.2311
85	0.895	1.2311
80	0.873	1.231
75	0.849	1.2306
70	0.822	1.2305
65	0.271	1.231
60	0.757	1.2311
55	0.717	1.2311
50	0.67	1.2313
45	0.618	1.2312
40	0.562	1.231
35	0.504	1.2307
30	0.44	1.2302
25	0.37	1.2299
20	0.301	1.2292
15	0.232	1.2287
10	0.158	1.2284
5	0.089	1.2281
0	0.019	1.228

Reverse		
Voltage (V) \pm 0.05	Flux (T) \pm 0.005	mass g \pm 5E-5
0	0.019	1.2283
5	-0.048	1.228
10	-0.119	1.2283
15	-0.186	1.2285
20	-0.256	1.229
25	-0.326	1.2295
30	-0.396	1.2305
35	-0.462	1.2306
40	-0.526	1.2309
45	-0.588	1.2314
50	-0.644	1.2316
55	-0.695	1.2318
60	-0.74	1.2317
65	-0.78	1.2319
70	-0.814	1.2318
75	-0.843	1.2317
80	-0.869	1.2316
85	-0.892	1.2314
85	-0.892	1.2314
80	-0.871	1.232
75	-0.847	1.2322
70	-0.82	1.2325
65	-0.79	1.2331
60	-0.755	1.2336
55	-0.715	1.2338
50	-0.668	1.2339
45	-0.617	1.2342
40	-0.562	1.2342
35	-0.502	1.234
30	-0.439	1.2335
25	-0.372	1.2331
20	-0.302	1.2324
15	-0.233	1.2318
10	-0.16	1.2311
5	-0.09	1.2308
0	-0.017	1.2304

Forward again to see retention of magnetic field		
Voltage (V) \pm 0.05	**Flux (T) \pm 0.005**	**mass g**
0	-0.017	1.2304
5	0.053	1.2303
10	0.124	1.2304
15	0.194	1.2306
20	0.264	1.2307
25	0.332	1.2312
30	0.4	1.2314
35	0.467	1.2317
40	0.529	1.2323
45	0.589	1.2327
50	0.644	1.2328
55	0.696	1.2333
60	0.741	1.2336
65	0.779	1.2339
70	0.812	1.2342
75	0.842	1.2343
80	0.869	1.2347
85	0.893	1.2352