# 1 Conditional Expectation

**Definition.** Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Let $(X_i)_{i \in I}$ be a collection of random variables defined on this space. Then we define $\sigma(X_i : i \in I) \subseteq \mathcal{F}$ to be the smallest $\sigma$-algebra such that all of the $X_i$ are measurable, i.e

$$\sigma(X_i : i \in I) = \sigma(X_i^{-1}(B) : i \in I,\ B \in \mathcal{B}(\mathbb{R})).$$

**Definition.** If $B \in \mathcal{F}$ has $\mathbb{P}(B) > 0$ then we define

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

for any $A \in \mathcal{F}$. Furthermore, if $X$ is an integrable random variable we define

$$\mathbb{E}[X|B] = \frac{\mathbb{E}[X \mathbb{1}(B)]}{\mathbb{P}(B)}.$$