Introduction

The course is split into two parts:

- Logic: syntax and semantics.
- Set theory: what does the universe of sets look like?

Course structure

(I) Propositional logic (logic)

(II) Well-orderings & ordinals (set theory)

(III) Posets & Zorn's lemma (set theory)

(IV) Predicate logic (logic)

(V) Set theory (set theory)

(VI) Cardinals (set theory)

Books:

1. Johnstone, *Notes on Logic & Set Theory*
2. Van Dalen, *Logic & Structure* (Chapter 4 and what 'goes next')
3. Hajnal & Hamburger, *Set Theory* (Chapters 2 and 6)
4. Forster, *Logic, Induction & Sets*

1 Propositional Logic

Let P be a set of *primitive propositions*. Unless otherwise stated, $P = \{p_1, p_2, \ldots\}$. The *language* L or $L(P)$ is defined inductively by

1. If $p \in P$, then $p \in L$
2. $\perp \in L$ ($\perp$ is read 'false')
3. If $p, q \in L$ then $(p \Rightarrow q) \in L$.

e.g $((p_1 \Rightarrow p_2) \Rightarrow (p_1 \Rightarrow p_3)), (p_4 \Rightarrow \perp), (\perp \Rightarrow \perp)$.

Notes.

1. Each proposition (member of L) is a finite string of symbols from language: $(), \Rightarrow, \perp, p_1, p_2, \ldots$ (for clarity often omit outer brackets, use other types of bracket, etc).
2. 'L is defined inductively' means, more precisely, the following

- Put $L_1 = P \cup (\perp)$;
- Having defined L_n, put $L_{n+1} = L_n \cup \{(p \Rightarrow q) : p, q \in L_n\}$;
- Set $L = \bigcup_{n \geq 1} L_n$.

3. Every $p \in L$ is uniquely built up from steps 1,2 using 3. For example, $((p_1 \Rightarrow p_2) \Rightarrow (p_1 \Rightarrow p_3))$ can from $(p_1 \Rightarrow p_2)$ and $(p_1 \Rightarrow p_3)$.

We can now introduce $\neg p$ ('not p') as an abreviation for $(p \Rightarrow \perp)$; $p \vee q$ ('p or q') as an abreviation for $(\neg p) \Rightarrow q$; $p \wedge q$ ('p and q') as an abreviation for $\neg(p \Rightarrow (\neg q))$.