Bisection Method

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1 Introduction

The bisection method is a numerical technique used to find the root of a continuous function within a given interval. Here's a mathematical introduction to the method:

2 Objective

The primary objective of the bisection method is to locate a root of a continuous function $f(x)$ within a specified interval $[a, b]$.

3 Interval Selection

To use the bisection method, you initially need to choose an interval $[a, b]$ such that $f(a).f(b) < 0$, which ensures that the function crosses the x-axis within the interval.

4 Algorithm

- Start with an interval $[a_0, b_o]$ such that $f(a_0)$ and $f(b_0)$ have opposite signs.
- Compute the midpoint of the interval: $c = \frac{a+b}{2}$
- Evaluate the function $f(c)$.
- If $f(c) = 0$ then c is the root.
- if $f(a).f(c) < 0$ set c to b. Else set c to a.
- Iterate until the desired tolerance is reached.

5 Convergence

The bisection method converges linearly, with the interval being halved each iteration. This leads to rapid convergence.

6 Conditions for Convergence

The bisection method guarantees convergence under the following conditions:

- The function $f(x)$ must be continuous on the interval $[a, b]$.
- If $f(a).f(b) < 0$ by the intermediate value theorem, one root must exist with the interval.

7 Accuracy

The accuracy of the bisection method depends on the desired level of precision specified by the user and the number of iterations performed. It provides a guaranteed level of accuracy based on the width of the interval at each iteration. In summary, the bisection method is a simple yet powerful technique for finding roots of continuous functions within a given interval by iteratively narrowing down the interval containing the root. It is widely used in numerical analysis and computational mathematics for solving equations and optimization problems.

8 Functions to Check

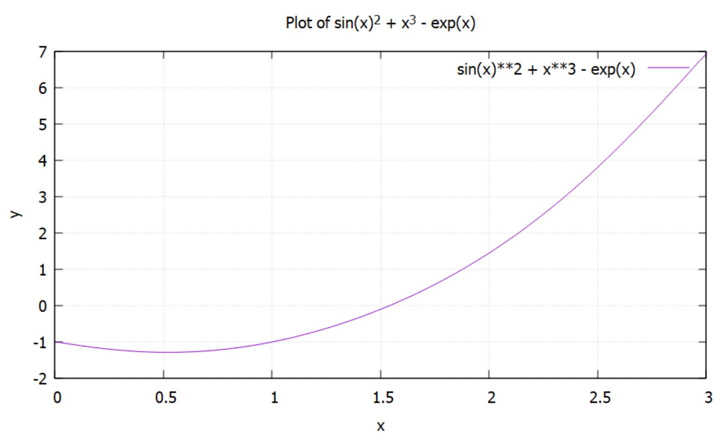

Figure 1: Root = 1.54

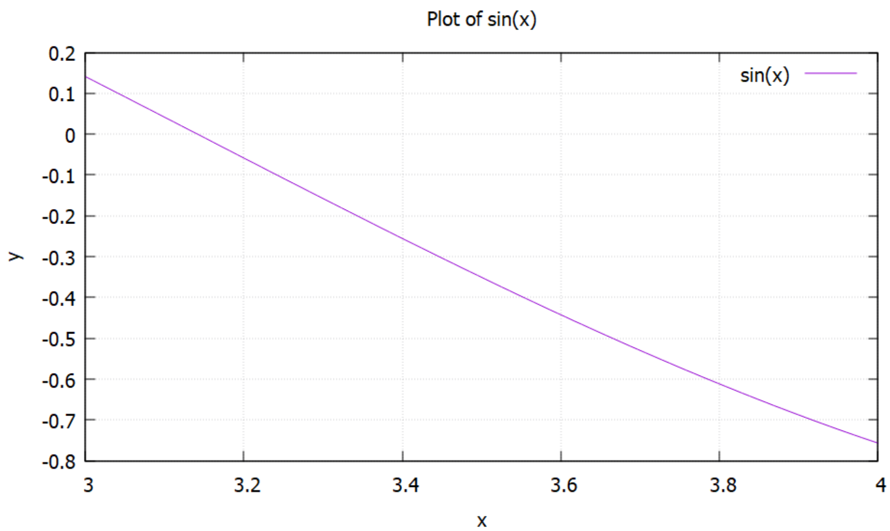

Figure 2: Root = 3.14

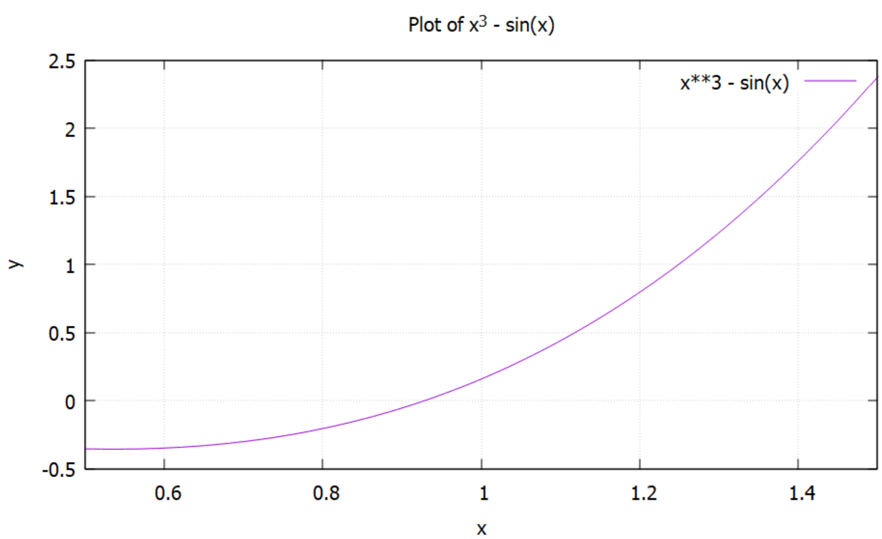

Figure 3: Root = 0.93