Newton Raphson Method

Conor Sheehan

April 2024

1 Introduction

The Newton-Raphson method, also known as Newton's method, is an iterative numerical technique used to find successively better approximations to the roots of a real-valued function.

It was developed independently by Isaac Newton and Joseph Raphson in the 17th century and is one of the most widely used root-finding algorithms.

The method is based on the idea of using local linear approximations to iteratively refine guesses for the roots of a function until a satisfactory approximation is obtained.

2 Objectives

The primary objective of the Newton-Raphson method is to find the roots of a given function $f(x) = 0$ by iteratively improving initial guesses. By repeatedly applying the Newton-Raphson formula, the method converges to a root of the function with increasing accuracy.

The method aims to provide a fast and efficient way to find roots of functions, particularly when analytical solutions are not feasible or practical.

3 Mathematics

Let f(x) be a continuous and differentiable function defined on an interval containing the root $x \in R$.

The Newton-Raphson formula for finding an approximation X_{n+1} to the root r is given by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'_{n+1}(x)} \tag{1}$$

The method converges quadratically near a simple root (a root with multiplicity one), meaning that the number of correct digits roughly doubles with each iteration when close to the root.

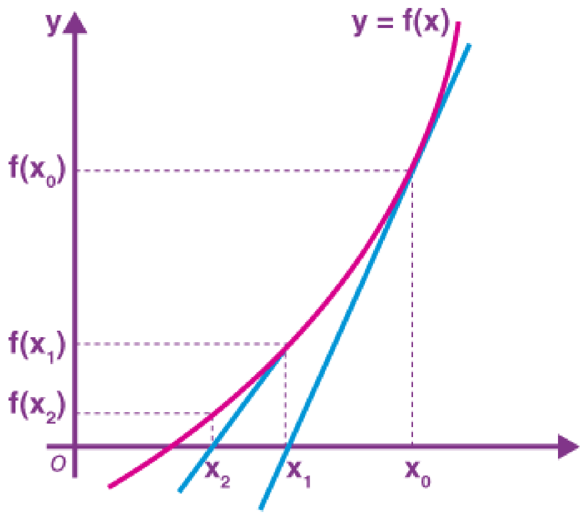

Figure 1: Newton Raphson Method

4 Algorithm

- Start with an initial guess x_0 close to the root of the function.
- Iterate using the Newton-Raphson formula to compute successive approximations $x_0, x_1, x_2, ..., x_n$ until the desired level of accuracy is achieved or until convergence criteria are met.
- The iterations continue until $|x_n - x_{n-1}|$ becomes sufficiently small, indicating convergence to the root.

5 Conditions for Convergence

The accuracy of the Newton-Raphson method depends on the chosen initial guess, the behavior of the function near the root, and the number of iterations performed.

When the method converges, it can provide highly accurate approximations to the root, often achieving machine precision within a small number of iterations.

However, the method may exhibit numerical instability or divergence if the initial guess is poor or if the function behaves unpredictably near the root.

6 Accuracy

The Newton-Raphson method typically converges rapidly when the initial guess is sufficiently close to the root and when the function is well-behaved (continuous and differentiable) near the root.

Convergence may fail or become slow if the initial guess is far from the root, if the function has multiple roots or singularities, or if the function has a flat region near the root where the derivative approaches zero.