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Problem Sheet - 2

A1) $p = 0.2$, $q = 0.8$

From Binomial Distribution,

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= {}^{10}C_0 (0.2)^0 (0.8)^{10} + {}^{10}C_1 (0.2)(0.8)^9 + {}^{10}C_2 (0.2)^2 (0.8)^8 + {}^{10}C_3 (0.2)^3 (0.8)^7$

$= 0.1073 + 0.2684 + 0.3019 + 0.2013$

$= 0.8789$

$\therefore$ No. of investigators $= 0.8789 \times 200$

$= 175.8 \approx 176$

A7) ~~Mean~~ =

Deaths	0	1	2	3	4
Frequencies	122	60	15	2	1

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{60 + 30 + 6 + 4}{200} = \frac{100}{200}$$

$= 0.5$

$$\text{Required Distribution} = \frac{N e^{-\lambda} \lambda^r}{r!}$$

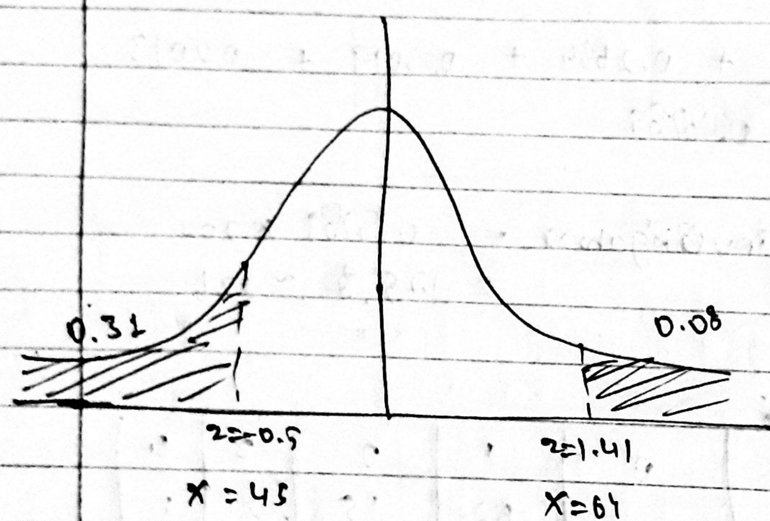
$$= \frac{200\, e^{-0.5} (0.5)^r}{r!}$$

$$= \frac{122 (0.5)^r}{r!}$$

r	N.P(r)
0	122
1	61
2	15.25 ≈ 15
3	2.54 ≈ 3
4	0.317 ≈ 0

A9) 31% of items under 45.

$\therefore$ $x = 45$, Area = 0.31. $\therefore$ $z = -0.5$



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8% of items over 64.

$x =$ 64, Area = 1 - 0.08, $z = 1.41$

$\therefore$ $$-0.5 = \frac{45 - \mu}{\sigma}$$

$\therefore$ $\mu - 0.5(\sigma) = 45$ ——— ①.

$$1.41 = \frac{64 - \mu}{\sigma}$$

$\mu + 1.41(\sigma) = 64$ ——— ②.

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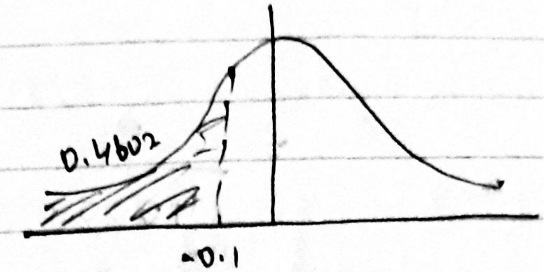
Solving both eq^n.

$\mu = 49.97 \simeq 50$. $\sigma = 9.94 \simeq 10$.

$z = \frac{x-\mu}{\sigma} = \frac{49-50}{10} = \frac{-1}{10} = -0.1$

Area = 0.4602.



$\therefore$ 46% percentage of items are under 49.

A10) Given, mean = 40000 km

$\therefore \frac{1}{\lambda} = 40,000 \Rightarrow \lambda = \frac{1}{40,000}$

$f(x) = \lambda e^{-\lambda x} = \frac{1}{40,000} e^{-x/40000}, \; x>0$

(i) $P(x > 20000) = \int_{20000}^{\infty} f(x)\,dx.$

$= \frac{1}{40000} \int_{20000}^{\infty} e^{\frac{-x}{40000}}\,dx$

$= \frac{1}{40,000} \left[-40000\, e^{\frac{-x}{40000}} \right]_{20000}^{\infty}$

$= -\left[e^{\frac{-x}{40000}} \right]_{20000}^{\infty} = e^{-0.5} = 0.6065$

(ii) $P(x \le 30000) = \int_0^{30000} f(x)\,dx$

$$= \frac{1}{40000}\int_0^{30000} e^{\frac{-x}{40000}}\,dx$$

$$= \frac{1}{40000}\left[-40000\, e^{\frac{-x}{40000}}\right]_0^{30000}$$

$$= -\left[e^{\frac{-x}{40000}}\right]_0^{30000}$$

$$= 1 - e^{-0.75} = 0.5270$$

A(4) The density function of x is given by

$$f(x) = \beta \alpha x\, e^{-\alpha x^2}, \quad x > 0$$

$\because \beta = 2$

$\therefore f(x) = 2\alpha x\, e^{-\alpha x^2}, \quad x > 0$

$$P(x > 5) = \int_5^{\infty} 2\alpha x\, e^{-\alpha x^2}\,dx$$

$$= \left(-e^{-\alpha x^2}\right)_5^{\infty} = e^{-25\alpha}$$

Given that $P(x > 5) = e^{-0.25}$

$$\therefore e^{-25\alpha} = e^{-0.25}$$

$$\boxed{\alpha = \frac{1}{100}}$$

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Problem - Sheet - 3

1. ($H_0 = 60\%$) → 60% of the shoppers Entering the store leave without making a purchase

($H_1 > 60\%$) => $\frac{35}{50} = 0.7 \Rightarrow 70\%$

No. of people = 50
No. of people left without purchase = 35

$$z = \frac{|p - P|}{\sqrt{\frac{PQ}{n}}}$$

$$\Rightarrow \frac{0.7 - 0.6}{\sqrt{\frac{0.6 \times 0.4}{50}}} = \frac{0.1 \sqrt{50}}{\sqrt{24}\,(0.1)}$$

$$= \sqrt{\frac{50}{24}} = 1.441$$

($1.441 < 1.645$) (1 - tailed) right.

4. First mean ($\bar{x}_1$) = 67.5 inches.
Second mean ($\bar{x}_2$) = 68.0 inches

And standard deviation σ = 2.5 inches.

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \Rightarrow z = \frac{|67.5 - 68.0|}{\sqrt{\frac{(2.5)^2}{1000} + \frac{(2.5)^2}{2000}}}$$

For the Weibull distribution with parameters α and β,

$$\text{Mean} = \alpha^{-1/\beta}\,\Gamma\left(\frac{1}{\beta}+1\right)$$

$$\therefore \quad \text{Mean} = \left(\frac{1}{100}\right)^{-1/2}\Gamma\left(\frac{3}{2}\right)$$

$$= 10 \times \frac{1}{2}\,\Gamma\left(\frac{1}{2}\right)$$

$$= 5\sqrt{\pi}$$

$$\text{Var}(x) = \alpha^{-\frac{2}{\beta}}\left[\Gamma\left(\frac{2}{\beta}+1\right) - \left\{\Gamma\left(\frac{1}{\beta}+1\right)\right\}^2\right]$$

$$= \left[\frac{1}{100}\right)^{-1}\left[\Gamma(2) - \left\{\Gamma\frac{3}{2}\right\}^2\right]$$

$$= 100\left[1 - \left(\frac{1}{2}\sqrt{\pi}\right)^2\right] = 100\left(1 - \frac{\pi}{4}\right)$$

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$z = |3.158| > 1.96$

A5) $\bar{x} = 4 \quad \mu = 4.2 \quad \sigma^2 = 0.6$

$\therefore \quad z = \left| \frac{\bar{x} - \mu}{\sqrt{\sigma^2/n}} \right|$

$\therefore \quad z = \left| \frac{4 - 4.2}{\sqrt{\frac{0.6}{100}}} \right| = 3.33 > 1.96$

A7) $\bar{x}_1 = 47 \quad \bar{x}_2 = 49 \quad \sigma_1 = 28 \quad \sigma_2 = 40$

$n_1 = 1000 \quad n_2 = 1500$

$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{2}{\sqrt{\frac{28^2}{1000} + \frac{40^2}{1500}}} = \frac{2}{\sqrt{0.784 + 1.06}}$

$\therefore \quad \boxed{z = 1.47}$

A9) $n_1 = 500 \quad x_1 = 16 \quad n_2 = 100 \quad x_2 = 3$

$p_1 = \frac{16}{500} = 0.032 \qquad p_2 = \frac{3}{100} = 0.03$

$q_1 = 0.968 \qquad q_2 = 0.97$

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$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{500 \times 0.32 + 100 \times 0.03}{600}$$

$$= 0.0316$$

$$Q = 1 - P = 0.968$$

$$z = \frac{P_1 - P_2}{\sqrt{\frac{PQ}{n_1} + \frac{PQ}{n_2}}} = \frac{0.032 - 0.03}{\sqrt{0.0309\left(\frac{1}{500} + \frac{1}{100}\right)}}$$

$$\boxed{z = 1.045}$$