# Empirical Asset Pricing: Problem Set 2

March 18, 2024

1. Principal Components Analysis

   (a) Download the daily and monthly returns on the Fama/French 5 factors from Ken French's website and download also the momentum factor (daily and monthly) from July 1963 until March 2024.

   (b) Use the daily returns in a given month to compute a measure of monthly realized variance by simply squaring returns,

   $$\widehat{\sigma}_t^2 = \sum_{d=1}^{D} R_{d,t}^2,$$

   where $R_{d,t}$ is the excess return on day $d$ in month $t$.

   (c) Compute scaled factor returns as

   $$R_t^{Scaled} = \frac{\widehat{\sigma}^2}{\widehat{\sigma}_{t-1}^2} R_t,$$

   where $\widehat{\sigma}^2$ is the full-sample average variance of the factor.

   (d) Report the average returns and Sharpe ratios for $R_t$ and $R_t^{scaled}$ for each of the 5 FF + momentum factors.

   (e) Regress the scaled returns on the original factor and compute the alphas and their t-statistics (univariately).

   (f) Compute the principal component decomposition of the variance series. In particular, report the weights of the principal components and the fraction of variance explained by the first, second, ..., sixth principal component.

   (g) Write $\widehat{\sigma}_{it}^2 = \phi_i PC_{1t} + u_{it}$ where $i$ corresponds to anomaly $i$ and $PC_{1t}$ the first principal component (note, you get $\phi_i$ from the principal component decomposition directly). Repeat (d) and (e) using $\phi_i PC_{1t}$. Interpret the results.

2. Risk Premium - PCA (Lettau and Pelger, 2020)

   (a) Get the monthly 48 Fama-French industry portfolios and the risk free rate from Ken French's Data library

   (b) From APT, the $T \times N$ excess return matrix R can be decomposed into K principal components F ($T \times K$) and factor loadings $\Lambda$ ($K \times N$).

$$R = F\Lambda' + \epsilon$$

   First find the Principal Components of the sample covariance matrix $\frac{1}{T}R'R - \bar{R}\bar{R}'$ by minimizing:

$$\widehat{F}_{PCA}, \widehat{\Lambda}_{PCA} = \min_{\Lambda, F} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left( (R_{i,t} - \bar{R}_i) - (F_t - \bar{F})\Lambda_i' \right)^2$$

   Follow the usual steps, i.e. obtain an eigenvalue-decomposition of the covariance matrix of Excess Return vector R (with N = 48 portfolios) and get N orthogonal factors. Sort them by variances (corresponding eigenvalues). Select the top K = 5 factors as estimates of F ($\widehat{F}$).

   (c) Recall that the usual PCA ignores the first moment (mean returns) and we typically scale the returns to make them mean-zero before calculating the principal components. This might be problematic if there are big pricing errors. RP penalizes them. Calculate these RP-PCA factors:

$$\widehat{F}_{RP}, \widehat{\Lambda}_{RP} = \min_{\Lambda, F} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left( R_{i,t} - F_t\Lambda_i' \right)^2 + \zeta \frac{1}{N} \sum_{i=1}^{N} \left( \bar{R}_i - \bar{F}\Lambda_i' \right)^2$$

where $\zeta$ a risk premium parameter.

(d) Check your work by testing $\widehat{F}_{PCA} = \widehat{F}_{RP}$ when $\zeta = -1$.

(e) Now, once you have the RP factors $\widehat{F}_{RP}$ compute the maximal Sharpe Ratio from the MV frontier spanned by $\widehat{F}_{RP}$ and the corresponding implied Stochastic Discount Factor ($M_t$) for $\zeta$ = 0, 1, 2, 5, and 10:

$$SR = \sqrt{\mu_F \Sigma_F^{-1} \mu_F}$$

$$\widehat{b}_{MV,RP} = \Sigma_F^{-1} \mu_F$$

$$M_t = 1 - \widehat{b}'_{MV,RP}(\widehat{F}_{RP,t} - E[\widehat{F}_{RP,t}])$$

where $\mu_F$ and $\Sigma_F$ are the mean and covariance of $\hat{F}_{RP}$