LUO'S QES RESEARCH

Quantitative Research
ESG
Portfolio Strategy

MULTI-DIMENSIONAL ALPHA

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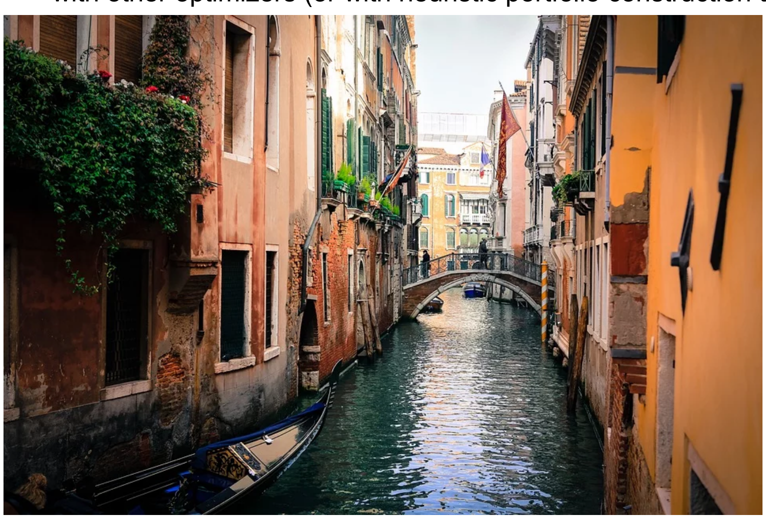
COMBINING VIEWS FROM MULTIPLE SOURCES

Practical Implementation of the Black-Litterman Framework via the Venice API

- **Active Managers Face a Challenge when Combining Risk and Return Forecasts.** One of the most important challenges facing active managers is how to best combine investment ideas from multiple sources. Alpha ideas often have very different coverages and structures with different investment horizons. When constructing portfolios, however, estimates from the various models must be brought together. The Black-Litterman (BL) approach is an elegant and systematic way to address this practical question in the portfolio construction process. Compared to traditional approaches, the BL framework often leads to higher Sharpe ratio/IR, lower noise, and lower tracking error, with more intuitive and less portfolios. While it is theoretically sound, the BL model is complex with many input parameters, which seriously limits its widespread usage. This research is designed to address these practical issues.
- **Designed Specifically for Fundamental and Quantamental Portfolios.** Fundamental portfolio managers incorporate information from disparate sources such as investment recommendations from their internal research analysts, discussions with company management and industry experts, suggestions from the sell-side, ideas from alpha capture programs, quantamental models, and data science explorations. In theory, infusing the depth of a fundamental manager with the breadth of quantitative models should at least provide a diversification benefit. In practice, however, it can be extremely difficult to implement successfully. Using a suite of real-life examples, we demonstrate how fundamental managers can take advantage of the BL framework to incorporate multiple views.
- **Quantitative Managers Attempting to Optimally Mix Signals.** Quantitative managers spend much of their time examining factors (i.e., systematic ways to rank stocks and other assets) to construct multifactor models. Many interesting new datasets (e.g., sector-specific data, event-driven strategies), however, tend to have limited history and breadth. The struggle is how to mix these heterogenous signals together in the most efficient fashion.
- **Introducing the Venice (View ENsemble Information Collaboration Enhancement).** Our Venice tool allows users to implement the powerful Black-Litterman method, without the confusing math and coding behind the scenes. It can be used to easily combine predictions from multiple sources. The Venice is a Python API, fully integrated with our Wolfe QES suite of risk models and optimizer. However, users can easily import their own risk models and implement with other optimizers (or with heuristic portfolio construction techniques).



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A Letter to our Readers

QES Handbook of Portfolio Construction Series

One of the most important challenges for active managers is how to best combine views or investment ideas from multiple sources. Alpha ideas from different sources often have very different coverages and are available in different formats with different investment horizons.

- For asset management firms with multiple investment teams, presumably each team has its own unique alpha-generation ability, albeit with different investment styles. Traditionally, each PM operates on his/her own, while only risk is monitored at the firm level. In recent years, central risk book has gained tremendous popularity. CIOs and central risk books at such firms could pool together the best ideas from each PM team and build a central investment portfolio, i.e., central risk book. Because the central investment team can source uncorrelated investment ideas and insights from both internal teams and external contributors (such as sell-side alpha capture participants), this team in theory could generate even better performance than the underlying PMs. The central risk model is widely adopted among sell-side investment banks and buy-side multi-strategy hedge funds.
- The primary goal of fundamental and discretionary portfolio managers has always been to consolidate investment recommendations from their internal research analysts, discussions with company management and industry experts, suggestions from the sell-side, and potentially ideas from quantamental models and data science explorations.
- Quantitative managers always must deal with mixing multiple factors into integrated models. However, as the availability of alternative data starts to explode, managers rush to study such datasets. Many interesting new datasets, however, tend to have limited coverage and breadth. Systematic portfolio managers typically emphasize breadth over skill[1]; and therefore may turn down such data. Quantitative managers traditionally view each alpha source as another factor in a multifactor model context. In this research, we introduce a novel and elegant approach to incorporate such high alpha/low coverage signals.

In the third part of this series, we address a very important and practical question in the portfolio construction process – how to combine different views or models into an integrated framework, all in a systematic and consistent manner.

At investment firms using the fundamental approach, research analysts are typically organized by industries and sectors. Analysts recommend buy/sell ideas from their respective coverage universe to portfolio managers. Managers then form portfolios based on the analysts' best ideas and their own judgment calls.

Firms employing the alpha capture program collect signals and trading ideas from external sources. The alpha capture program encourages sell-side investment banks and boutique firms to submit "trading ideas" to buy-side investment management clients, in a timely manner. As an exchange, the buy-side investment management firms typically reward the service providers with direct commission

[1] Based on the fundamental law of active management, $Performance = Skill \times \sqrt{Breadth}$. A simplified interpretation (albeit not exactly accurate) suggests that $IR = IC \times \sqrt{Breadth}$, where $Breadth$ can be proxied by number of stocks with data coverage. Because the predictive power of most data and signals (i.e., IC) is typically in the range of 1% to 6%, the impact of data coverage (i.e., $Breadth$) is normally more significant than IC.

payments, based on the performance of those “trading ideas”. Systematically incorporating the alpha capture trading signals into a portfolio is not a trivial task.

- Quantitative managers spend much of their time examining factors (i.e., systematic ways to rank stocks and other assets) to construct multi-factor models. They often have multiple different factors or models. Again, the struggle is how to mix these factors together in the most efficient fashion.
- Quantamentals. In recent years, we have seen the practice of combining fundamental and quantitative models become quite popular. Traditional fundamental managers are adding quantitative overlays, while quantitative managers are developing models using more fundamental techniques. In theory, since fundamental managers focus on depth (i.e., deep understanding of the companies they cover) and quantitative managers excel at breadth (i.e., great coverage of many stocks), infusing one into the other should at least have a diversification benefit. In practice, however, because the philosophies behind the two approaches are so different, we find it extremely difficult to implement.

There are also many complications when we attempt to pool views/models together.

- We may not have a model for every single asset in our investment universe. In GTAA or global macro strategies, it is almost impossible to have one all-encompassing model that can simultaneously estimate the returns of all asset classes. Managers typically have one model for major asset classes (e.g., equity, fixed income, commodities, etc.), one for countries (e.g., country or regional equity indices), one model for rates (e.g., various durations), one for credit (e.g., investment grade versus high yield), and so on. Similarly, fundamental stock pickers are also unlikely to have the time and resources to conduct detailed company-level analyses on thousands of stocks. Rather, they tend to focus on a subset of stocks fitted to their specialties. When we construct portfolios, however, we need to put all these estimates from various models together.
- One asset might be modeled separately in two or more models. For example, we may have US equity in our asset class model (along with fixed income and commodities) and at the same time, US equity is also one investment instrument in our country rotation model. Reconciling the two views is also a challenge.
- The various models come in different flavors. Some models rank stocks. Some models use continuous scores. Yet still, some analysts only have buy/sell recommendations.
- The views can be either absolute or relative. An example of an absolute view is – we expect the return of Stock A to be 15% over the next year. An example of a relative review is – we believe equities will outperform bonds by 5% in the next quarter.

We argue that the Black-Litterman (BL) model is an elegant way to combine views. While it is theoretically sound, the BL model is not easy to implement in practice, which seriously limits its widespread usage. In this paper, we attempt to give a practical guide to implement the BL model using a series of real-life examples:

- How to incorporate fundamental views such as buy/sell recommendations, quartile rankings, etc.;

- How to incorporate quantitative model scores, and what are the benefits of using the BL model, compared to using the model scores directly;
- How to incorporate corporate events, pairs trading, and relative value strategies; and
- How to incorporate views on risk into the portfolio construction process.

The BL model is also far from the only way to blend views. In this research, we also compare the BL model with other approaches, such as naïve combination, risk-based allocation, etc.

Regards,

Yin, David, and Luo's QES team

Combining Views

In actual investing, predicting asset returns is probably the most important task. It is certainly the most difficult. In fact, the Efficient Market Hypothesis (EMH) suggests that asset returns are unpredictable. In practice, all models are wrong, while some are useful. There are several practical issues that are closely related:

- **Multifactor Models and Signal Weighting.** In this case, researchers have a collection of factors (or features in machine learning jargon). Factors can be used to rank assets, because they have some predictive power of future asset returns. In this setting, the underlying factors usually have similar coverage. More importantly, each factor should represent one unique dimension of information and be relatively consistent over time. Selecting the relevant factors and combining them into a multifactor model can better capture the diversification benefit (because the underlying factors may not be highly correlated), and therefore deliver stronger performance than the underlying factors alone. How to select factors is usually called factor (or feature) selection, while how to weight the chosen factors into an integrated multifactor model is part of the signal weighting problem. Factor selection and signal weighting can be done separately or in one uniformed way. Traditional signal weighting is discussed in detail in *Signal Research and Multifactor Models* (see Luo, et al [2017b]), while dynamic factor weighting with macroeconomic regimes is addressed in *Style Rotation, Machine Learning, and the Quantum LEAP* (see Luo, et al [2017c]). In this case, each factor is normally not directly investable. Researchers need to develop multifactor models, which can be used to construct active portfolios.
- **Model Averaging and Ensemble.** In the second case, researchers have multiple models. Each model may be constructed using different datasets and/or methodologies. The challenge is whether we should pick one model or attempt to further mix multiple models together. Empirical studies usually suggest that model averaging is more accurate and performs better than a single model (e.g., Moral-Benito [2013]). In machine learning, this step is called ensemble learning, i.e., using multiple learning algorithms to achieve better predictive performance than could be obtained from any of the constituent models alone. In our research, we find that model averaging and ensemble tend to achieve better success when there is a significant diversity among the models (see *Man Versus Machine – MALTA*, Wang, et al [2018] and *Machine Learning Real Estate*, Luo, et al [2019]). In this case, each underlying model can be used to construct active portfolios, but the combined model typically performs better than the components.
- **Asset Allocation.** In the third setup, we have multiple portfolios. Each portfolio can be a standalone asset or more likely, a composition of assets (e.g., a portfolio of stocks). A portfolio can be either passive (e.g., a market-capitalization weighted index) or active (e.g., a long/short market neutral portfolio using a multifactor model). The key difference of this scenario with the above two is that each asset is directly investable. Asset allocation is about how to find the optimal weight for each asset.

In this research, we propose a practical implementation of the Black-Litterman framework, which can be implemented in all the above three settings.

THE BLACK-LITTERMAN APPROACH – A PRIMER

In this section, we provide an introduction of the Black-Litterman (BL) framework. This is the most technical section in this paper. It is useful to have some basic understanding of the theory before we show you the application. In the following sections, we will discuss practical implementations of the BL model. Specifically, we have developed a tool called the Venice (View ENsemble Information Collaboration Enhancement) that investors can implement the BL model with their own data directly, without the need to understand the complex mathematics and computer programming behind the model. The Venice tool is an API (Application Programming Interface) based on the Python programming language. With minimal need for coding, portfolio managers can easily combine multiple different views (e.g., fundamental analysis, quantitative models, event-driven strategies, alpha capture programs) into one fully integrated framework.

THE BASICS

The original Black-Litterman (BL) model (see Black and Litterman [1990, 1991, 1992], He and Litterman [2002]) allows portfolio managers to overlay their views (i.e., return expectations) onto the market consensus (i.e., market implied returns). The final combined predictions (called posterior predictions) therefore, incorporate both investors' active views and market equilibrium expectations. Portfolios based on the combined predictions tend to be more robust and realistic with less extreme positions than equivalent strategies based solely on managers' predictions.

Market Equilibrium Expectations (Bayesian Prior)

In the classic BL model, we start from the market equilibrium. Once we assume asset returns follow joint multivariate normal distribution[2], we can derive our baseline return expectation. If we do not have any active views, our baseline return expectation should lead to the passive benchmark, with no active positions. There are six fundamental building blocks in this step:

- The Capital Asset Pricing Model (CAPM)
- Reverse optimization
- Bayesian estimation
- The concept of a universal hedge ratio
- Mean-variance optimization
- Market equilibrium return

First, we consider a market with N assets, whose returns are normally distributed:

[E1] $$r \sim N(\mu, \Sigma)$$

Where, r is a $(N \times 1)$ vector of asset returns,

μ is a $(N \times 1)$ vector of mean returns,

Σ is a $(N \times N)$ variance-covariance matrix

[2] Although the assumption of joint multivariate normal distribution is often invalid in finance, as detailed in *Risk, Portfolio Construction, and Performance Attribution* (see Luo, et al [2017d]), the main tools used in this research remain robust.

The asset-by-asset variance-covariance matrix Σ can be estimated in multiple different ways, e.g., based on historical return data[3], by applying some sort of shrinkage, or by structured risk models. In our Port@ framework, we use a factor model to estimate the covariance matrix (see *Port@ble Ownership*, Alvarez, et al [2018] for details). Compared to return prediction, risk (i.e., covariance matrix Σ) can be estimated with reasonable accuracy.

On the other hand, expected asset returns μ are far more difficult to predict. Passive investors (e.g., index funds) view asset returns as unpredictable. Fundamental investors use a combination of valuation techniques and subjective knowledge to indirectly infer their return expectations. Systematic managers apply multifactor models to rank securities.

In Bayesian theory, the μ vector itself is modeled as a random variable with estimation errors. In the BL model, it is assumed to follow a normal distribution:

[E2] $$\mu \sim N(\pi, \tau\Sigma)$$

Where, π is a $(N \times 1)$ vector containing the predicted return of each asset. The parameter τ is where some of the controversies about the BL model arise. The BL theory does not provide a number for τ; therefore, researchers must come up with their own. He and Litterman [2002] use a value of 0.025. Lee [2000] sets a value between 0.01 and 0.05, and then calibrates the model based on a target level of tracking error. Satchell and Scowcroft [2000] suggest a value of 1 for τ. In the classical BL model, $\tau\Sigma$ is the standard error matrix of the prior estimate. As suggested by Walters [2013], τ is typically set as a number much less than 1, as it reflects the fact that the uncertainty in the mean of the distribution is much smaller than the variance in the returns.

To estimate π, the BL model starts from the Markowitz [1952] model, where investors want to maximize risk-adjusted return:

[E3] $$argmax_{\omega}\left(\omega'\pi - \frac{\lambda}{2}\omega'\Sigma\omega\right)$$

Where,

ω is a vector $(N \times 1)$ containing the weight of each asset (to be solved by the optimizer),

λ is the risk aversion parameter[4],

Rational investors want to maximize their expected returns for a given level of risk tolerance (or alternatively, minimize risk for a given level of return).

The optimal (equilibrium) weight vector ω can be solved using a closed-form formula[5]:

[E4] $$\widetilde{\omega} = (\lambda\Sigma)^{-1}\pi$$

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[3] Estimating variance-covariance matrix using sample data is considered highly problematic by practitioners. However, this approach is widely used in academia. Our empirical analysis suggests that portfolios based on the sample covariance matrix do not necessarily underperform strategies based on commercial risk models. However, due to multiple data issues, the sample covariance matrix is not guaranteed to be positive definite; therefore, optimizers may not always be able to find feasible solutions.

[4] The risk aversion parameter, λ, depends on investors' utility function, i.e., their preference of the risk and return tradeoff. We will discuss how to estimate the λ parameter in a later section.

[5] The closed-form solution is available only if there are no other constraints in the portfolio optimizer. In practice, active managers typically face multiple constraints, e.g., turnover, maximum single position weight, etc. With constraints, a numerical optimizer is required.

Since we try to understand the market equilibrium (or average) expectation of asset returns, the aggregated or average investor holds the market portfolio. The optimal weight vector $\widetilde{\omega}$ coincides with the market weight vector ω_{Market}, only if the expected return π is equal to the return implied by each asset's relative market capitalization. By reverse optimization, we can obtain the implied return vector π as:

[E5] $$\pi = \lambda\Sigma\omega_{Market}$$

In the original BL model, λ was set exogenously as $\lambda = 1.2$. Alternatively, λ can also be estimated empirically:

[E6] $$\lambda = \frac{E(r_{Market}) - r_f}{\sigma^2_{Market}}$$

Where,

$E(r_{Market})$ is the estimated return of the market,

r_f is the risk free rate, and

σ^2_{Market} is the estimated variance of the market.

In practice, researchers still must estimate $E(r_{Market})$ and σ^2_{Market}. The risk free rate, r_f, is typically proxied by short-term or long-term government bond yields. As discussed earlier, the covariance matrix Σ can be estimated in multiple different approaches.

To summarize, the market consensus return vector, μ, (i.e., the Bayesian prior) follows a multivariate normal distribution:

[E7] $$\mu \sim N(\pi, \tau\Sigma) = N(\lambda\Sigma\omega_{Market}, \tau\Sigma)$$

The underlying parameters can all be estimated with reasonable accuracy. We will discuss how to estimate these parameters in the next section.

An Investor's Views

An investor's views are essentially active predictions of future asset returns. A view can be expressed in multiple ways, e.g.,:

- **Absolute views** are set for each asset individually. For example, our research analyst has a 12-month target price of $105 for Stock A, which is currently trading at $100. Assuming the stock does not pay any dividend, then the implied return is 5% over the next year.
- **Relative views** reflect investors' preferences for one asset versus another. For instance, our pairs trading model may suggest that Stock B should outperform Stock C by 2% in the next 12 months. Consequently, the return differential between B and C is 2% over the next year.

Both absolute and relative views can be incorporated easily by a $(K \times N)$ pick matrix (P) and an expected return vector q of size $(K \times 1)$, where K is the number of views that we want to express. For example, if we have five stocks and two views (as specified in the above two examples), the pick matrix can be written as:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}$$

and,

$$q = \binom{5\%}{2\%}$$

Therefore, in this example, we expect the first stock (A) to have a return of 5% (absolute view); and the second stock (B) to outperform the third one (C) by 2% (relative view).

Nobody can predict asset returns with certainty. Therefore, the BL model assumes that our views face uncertainty, with an error term, $q + \varepsilon$. The error term is further assumed to follow a normal distribution:

[E8] $\varepsilon \sim N(0, \Omega)$

Under the BL model, the error term (called view uncertainty) is normally distributed with a mean of zero and a variance-covariance matrix of Ω. The confidence of our views is expressed by taking the inverse of the Ω matrix, Ω^{-1}. Many researchers further suggest that the view uncertainty can be expressed as[6]:

[E9] $\Omega = \kappa P \Sigma P'$

The original BL model suggests that investors' view uncertainty is proportional to each asset's own variance, scaled by the parameter κ. It is somewhat intuitive, as our predictions for more volatile assets are probably less precise than stocks with lower volatilities. However, it may be too restrictive to have all uncertainties scaled by a single parameter κ. In later sections, we will relax the assumption and study other ways to specify the uncertainty matrix Ω.

Now, we can see that investors' views also follow a multivariate distribution:

[E10] $P \cdot \mu \sim N(q, \Omega)$

Black-Litterman Posterior Prediction

Finally, after applying Bayes' theorem and some mathematical manipulations, the posterior probability of the expected return vector $r_{BL} = \mu | \pi$ follows a multivariate normal distribution with mean and variance:

[E11] $E(r_{BL}) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}q]$

[E12] $Var(r_{BL}) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}$

Essentially, the expected return vector r_{BL} is a risk-weighted average of the market equilibrium returns π and the view returns q , weighted by the inverse of the covariance matrix $(\tau\Sigma)^{-1}$ and the view uncertainty matrix $P'\Omega^{-1}$, respectively. The relative importance of the prior return (i.e., market equilibrium return) and the view return in the kth row of P is determined by the ratio of Ω_{kk} and $\tau p_k' \Sigma p_k$. In this case, $p_k' \Sigma p_k$ is the variance of the view portfolio, while Ω_{kk} is the variance of the kth asset. The higher the confidence of our views (i.e., the smaller Ω or the larger Ω^{-1}), the closer our eventual return expectation (i.e., r_{BL}) is to our views q. In the extreme example where our view is completely certain with no uncertainty (or highly uncertain with a very large variance), the point estimates of r_{BL} are the same as our views (or market equilibrium returns).

[6] As we will elaborate in later sections, this is certainly not the only way to specify view uncertainty. Some researchers use the same scaling parameter for both τ and κ, i.e., $\tau = \kappa$. In this case, it greatly simplifies the model specification and computation. However, it may also lead to counterintuitive results.

Assuming the uncertainty in our views is independent of the covariance of asset returns, the posterior covariance matrix is:

[E13] $$\Sigma_{BL} = Var(r|\pi) = \Sigma + Var(r_{BL}) = \Sigma + [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}$$

In the original form of the BL model, in the case of non-constrained optimization, an asset's weight deviates from its market capitalization weight, only if an investor's view on the asset is different from the CAPM implied return.

In practice, even if an investor only has one view on one single asset, it could change the return expectation of other assets, since stocks are correlated (via the covariance matrix).

The choice of the τ parameter has a significant impact on the final covariance matrix Σ_{BL}. Many researchers typically choose a small value for τ to minimize potential adverse effect. We will discuss how to calibrate the τ shortly.

EXAMPLE 1

Let us start from a simple example, assuming that we have only two *uncorrelated* assets: A and B, with the following prior and views:

Figure 1 A Two-Asset Example

	Prior View	Prior Volatility	View Return	View Uncertainty
A	10%	20%	30%	15%
B	6%	10%	NA	NA

Sources: Wolfe Research Luo's QES

The market equilibrium return vector is given by:

$$\pi = \begin{bmatrix} 10\% \\ 6\% \end{bmatrix}$$

Since the two assets are uncorrelated, the covariance matrix can be computed as:

$$\Sigma = \begin{bmatrix} (20\%)^2 & 0 \\ 0 & (10\%)^2 \end{bmatrix} = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.01 \end{bmatrix}$$

In this case, we have only one (bullish) absolute view[7] on Asset A of 30%, which is much higher than the prior return of 20%. We understand that predicting asset returns is extremely challenging; therefore, our view is highly uncertain with a standard deviation of 15%[8]. Our pick matrix, view return, and view uncertainty matrices are therefore:

$$P = [1 \quad 0]$$

$$q = [30\%]$$

$$\Omega = [(15\%)^2] = [0.0225]$$

[7] This corresponds to a typical high conviction buy idea from a fundamental analyst.

[8] Assuming normal distribution, the expected return of Asset A is in the range of 15% and 45%, with a 70% probability (i.e., ± 1 standard deviation).

Following convention, let us further set $\tau = 1$. We can then derive our BL return expectation as:

$$E(r_{BL}) = \begin{bmatrix} 22.8\% \\ 6\% \end{bmatrix}$$

We can see that, after expressing a very positive but also highly uncertain view on Asset A, our return expectation for Asset A increases from 10% to 22.8%. We can further see that the new return expectation is essentially a weighted average of our prior of 10% and our view of 30%, weighted by the inverse of the prior variance of 0.04 (the squared volatility of 20%) and view uncertainty of 0.0225 (the squared standard deviation of 15%).

$$\frac{(0.04)^{-1} \times 10\% + (0.0225)^{-1} \times 30\%}{(0.04)^{-1} + (0.0225)^{-1}} = 22.8\%$$

Because Assets A and B are uncorrelated, our view on Asset A has no impact on the return expectation for B. We can also see the influence of the τ parameter. For example, if we set $\tau = 0.025$, our BL expected return for Asset A becomes 10.9% – almost the same as our prior expectation of 10%, because a small τ means a high confidence on our prior return expectation (or low confidence on our views).

Furthermore, we can see the impact of our view on the covariance matrix.

$$\mathrm{Var}(r_{BL}) = \begin{bmatrix} 0.0144 & 0 \\ 0 & 0.01 \end{bmatrix}$$

Therefore, the final covariance matrix is:

$$\Sigma_{BL} = \Sigma + \mathrm{Var}(r_{BL}) = \begin{bmatrix} 0.0544 & 0 \\ 0 & 0.02 \end{bmatrix}$$

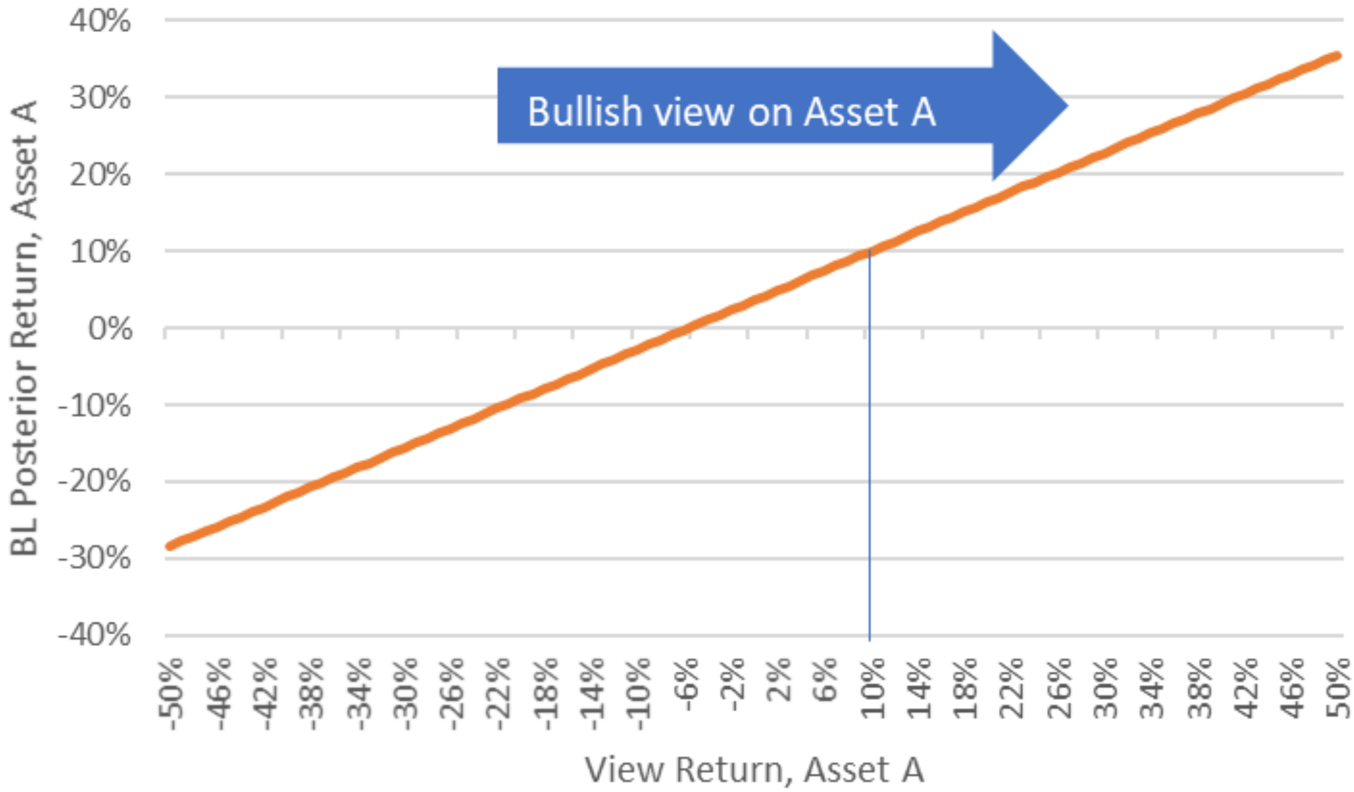
The BL volatility of Asset A rises to 23.3% (from 20%), while the volatility of Asset B increases to 14.1% (from 10%). Surprisingly, even though our view on Asset A does not impact the posterior return of Asset B (because the two assets are uncorrelated), it does change the volatility (and variance) estimate of Asset B.

Moreover, setting a smaller value of the τ parameter would shrink the final covariance matrix towards the prior covariance matrix. For example, if we set $\tau = 0.025$, the final volatilities of Assets A and B would be 20.2% and 10.1%, respectively.

The Bullishness/Bearishness of Views

Intuitively, the more bullish our view is, the higher BL posterior return is. For instance, if we vary our view return expectation of Asset A from -50% to 50%, the BL posterior return prediction goes from -28.4% to 35.6% for Asset A (see Figure 2). When our view return is the same as our prior at 10%, the posterior return is also 10%, i.e., our view is the same as the market consensus.

Figure 2 The Relationship between the Bullishness/Bearishness of View and Return of Asset A

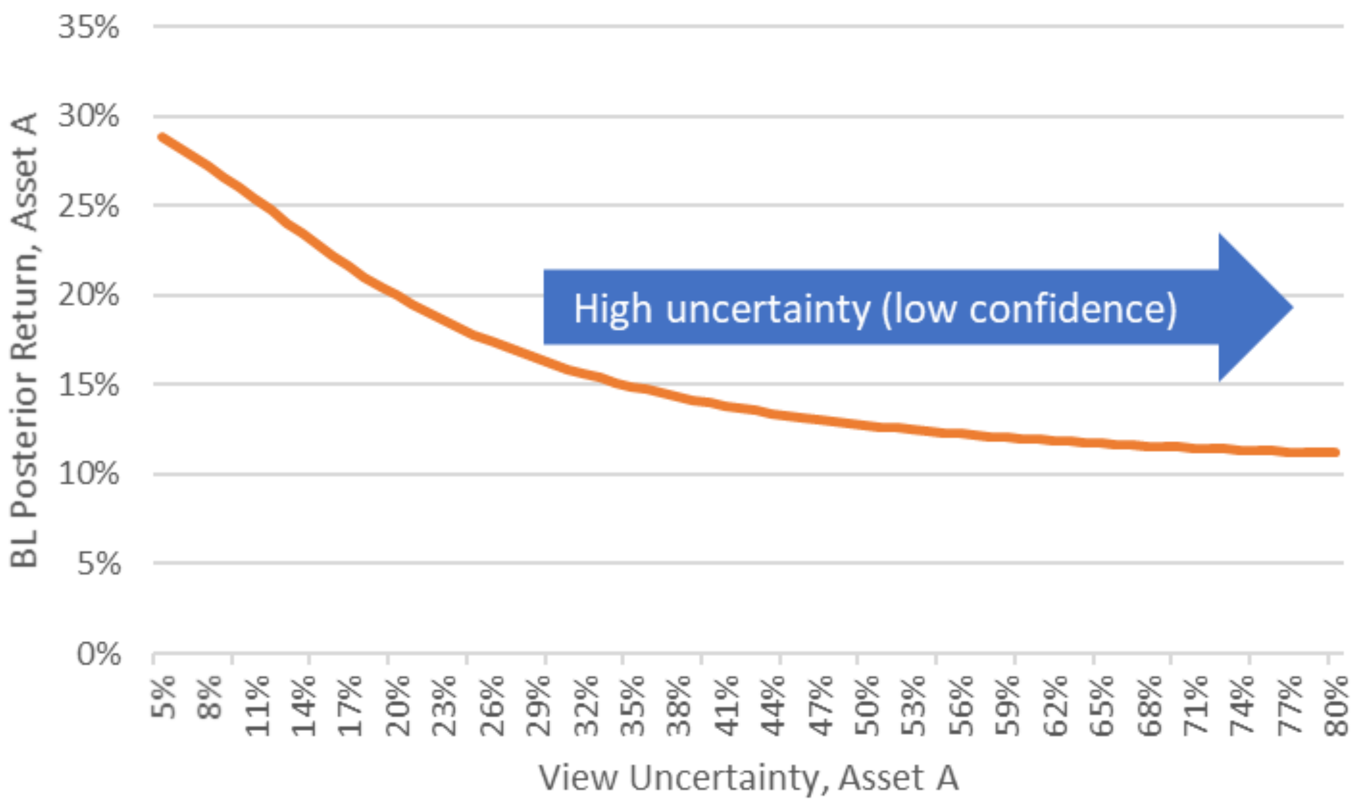


Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

View Uncertainty/Confidence

Next, we want to understand how our view uncertainty (or confidence) influence the final BL posterior return expectation. As we change the uncertainty level of our view on Asset A from 5% (low uncertainty/high confidence) to 80% (high uncertainty/low confidence), the BL posterior return moves from close to 30% (our view) to 10% (prior), as shown in Figure 3. The impact of view uncertainty on the posterior return expectation appears to be nonlinear, at a declining rate (i.e., a convex curve).

Figure 3 The Relationship between View Uncertainty and Return of Asset A



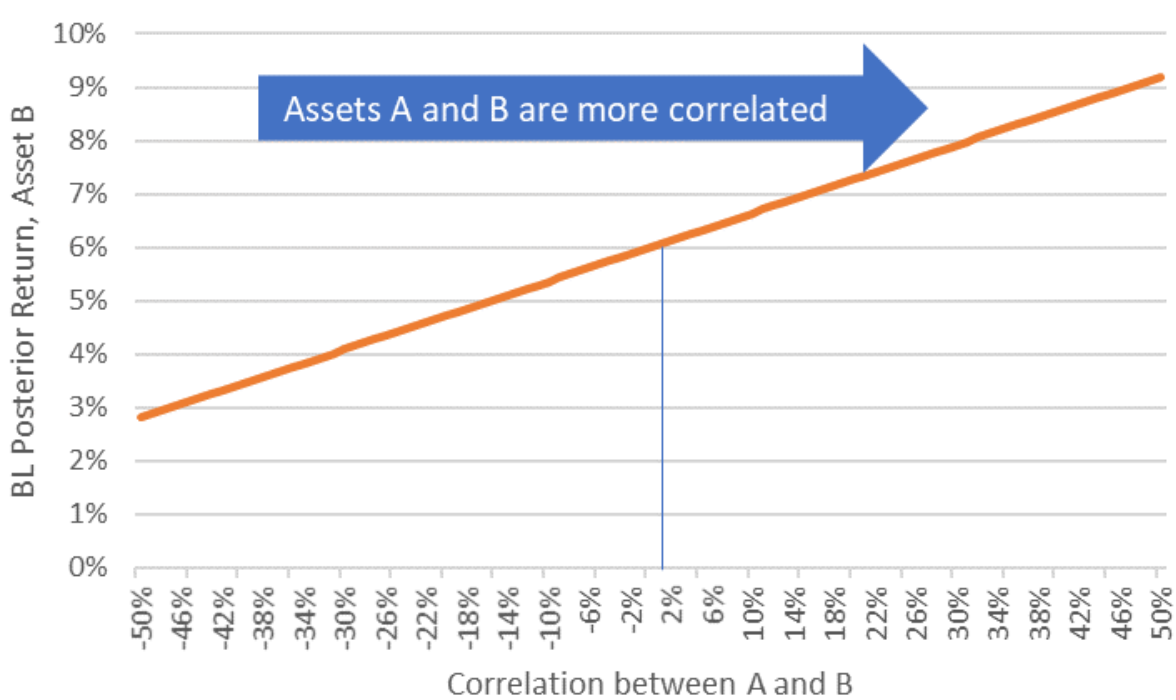
Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

The Impact on Correlated Assets

So far, we have been assuming that the two assets are uncorrelated. If Assets A and B are correlated, a view on Asset A would also change the BL posterior return expectation for Asset B. By varying the correlation between Assets A and B, from -50% to 50%, the posterior return for Asset B increases from 2.8% to 9.2% (see Figure 4). Obviously, when two assets are positively (negatively) correlated, a positive view on one asset should raise (lower) the return expectation of the other asset as well. The higher the correlation between the two assets, the higher the impact on posterior return expectation.

The impact of correlation is important. Even for those assets that we do not have explicit views, their posterior return predictions are likely to change, so long as we express our views on correlated assets.

Figure 4 The Relationship between Correlation and Return of Asset B



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Introducing the Venice – QES Implementation of the BL Model

There are many ways to implement the BL model (see Walters [2013] for a summary). In this section, we introduce our framework to combine multiple views into one integrated prediction, via a modified BL model – Venice (View ENsemble Information Collaboration Enhancement).

To implement the BL model, we need to estimate the following key parameters:

- **The scaling parameter $\boldsymbol{\tau}$.** Unlike other parameters (which can be estimated empirically), τ is typically set *a priori*.
- **Risk aversion parameter $\boldsymbol{\lambda}$.** λ can either be set arbitrarily as a fixed number or be estimated empirically (see [E6]) based on the equity risk premium (i.e., $E(r_{Market}) - r_f$) and equity market risk (i.e., σ_{Market}^2).
- **Asset-by-asset covariance matrix $\boldsymbol{\Sigma}$.** Academic research mostly uses the sample covariance matrix (and occasionally applies shrinkage). Practitioners often use a multifactor commercial risk model.
- **View return vector $\boldsymbol{q}$.** View return may not always be explicit and straightforward; therefore, data transformation is often required. If analysts use scores (e.g., z-scores, quartiles, or rankings), we need to translate scores into expected return predictions. Fundamental analysts may not have explicit return forecasts. Instead, they may use a buy/hold/sell rating system. For alpha capture programs, we may only have managers' holding data.
- **Uncertainty (or confidence) of views $\boldsymbol{\Omega}$.** This is probably the most difficult part of the BL model implementation. Most managers typically do not have explicit data on view uncertainty/confidence. Therefore, practitioners often assign some arbitrary numbers.

The biggest hurdle to implement the BL model is how to estimate the above parameters, especially view returns and the associated view uncertainty.

Basic Parameters

First, we set the scene by specifying one of the most confusing parameters in the original BL model, the scaling parameter, τ. Then, we will use a classic example to further simplify our demonstration. Next, we will discuss how to determine the risk aversion parameter $\boldsymbol{\lambda}$. We leave the view return and uncertainty to the next section.

Setting the Scaling Parameter τ

As shown in the examples in the previous section, we argue that setting $\tau = 1$ makes the practical implementation of the BL model much more intuitive than the conventional wisdom of specifying $\tau \ll 1$. The τ parameter also often interacts with our view uncertainty matrix, Ω (via the κ parameter), which makes the task of specifying both parameters confusing. If we always set $\tau = 1$, we can focus our efforts on Ω. Our approach is consistent with Meucci [2010].

In the case of $\tau = 1$, Equations [E11] and [E12] can be rewritten as:

[E14] $$E(r) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}q] = [\Sigma^{-1} + P'\Omega^{-1}P]^{-1}[\Sigma^{-1}\pi + P'\Omega^{-1}q]$$

[E15] $$Var(r) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1} = [\Sigma^{-1} + P'\Omega^{-1}P]^{-1}$$

Example 2

Now, let us introduce the second simple yet realistic example. Assuming that we have an estimated return for every stock in our investment universe (which is true for most multifactor quantitative models), the pick matrix P is simply the identify matrix I (i.e., $P = I$). Furthermore, if we follow the common practice of setting $\Omega = \kappa P'\Sigma P$, then $\Omega = \kappa P'\Sigma P = \kappa I'\Sigma I = \kappa\Sigma$

After substituting $\Omega = \kappa\Sigma$ into the original BL model (i.e., Equations [E11] and [E12]), we can simplify the posterior expected return to:

[E16] $$E(r) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}q] = \frac{\left(\frac{\pi}{\tau}+\frac{q}{\kappa}\right)}{\left(\frac{1}{\tau}+\frac{1}{\kappa}\right)}$$

[E17] $$Var(r) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1} = \frac{\tau\kappa}{\tau+\kappa}\Sigma$$

Therefore, our expected return is a weighted average of the market equilibrium return, π, and our view, q, weighted by the inverse of two scaling parameters τ and κ.

When we set $\tau = 1$, Equations [E16] and [E17] can be further simplified as:

[E18] $$E(r) = \frac{\left(\frac{\pi}{\tau}+\frac{q}{\kappa}\right)}{\left(\frac{1}{\tau}+\frac{1}{\kappa}\right)} = \frac{\left(\pi+\frac{q}{\kappa}\right)}{\left(\pi+\frac{1}{\kappa}\right)} = \frac{\kappa}{1+\kappa}\pi + \frac{1}{1+\kappa}q$$

[E19] $$Var(r) = \frac{\tau\kappa}{\tau+\kappa}\Sigma = \frac{\kappa}{1+\kappa}\Sigma$$

Alternatively, if we follow another common practice in the industry and set $\tau = \kappa$, then:

[E20] $$E(r) = \frac{\left(\frac{\pi}{\tau}+\frac{q}{\kappa}\right)}{\left(\frac{1}{\tau}+\frac{1}{\kappa}\right)} = \frac{1}{2}(\pi + q)$$

[E21] $$Var(r) = \frac{\tau\kappa}{\tau+\kappa}\Sigma = \frac{\tau}{2}\Sigma$$

In this case, the expected return is the simple average of market implied return and our model predicted return. In addition, the new covariance matrix is simply a scaled version of the original covariance matrix Σ. Therefore, in the case of using a traditional multifactor model, the BL model is simply the equally-weighted average of the CAPM implied return and our model scores.

If we decide not to use the κ approach for our view uncertainty and assume $P = I$, the mean and covariance matrix can be simplified as:

[E22] $$E(r) = [\Sigma^{-1} + \Omega^{-1}]^{-1}[\Sigma^{-1}\pi + \Omega^{-1}q]$$

[E23] $$Var(r) = [\Sigma^{-1} + \Omega^{-1}]^{-1}$$

It should be now immediately clear that the BL model simply weights our prior (e.g., market equilibrium return expectation of π) and our view q. The weights applied to our prior and view are the inverse of their respective covariance matrices Σ and Ω. In particular, in the two extreme examples below where we are completely certain about our view or we have no confidence about our view at all, we can see:

- Extreme 1 (no uncertainty in our views): $\Omega \to 0$, $E(r) = q$
- Extreme 2 (no confidence in our views): $\Omega \to \infty$, $E(r) = \pi$

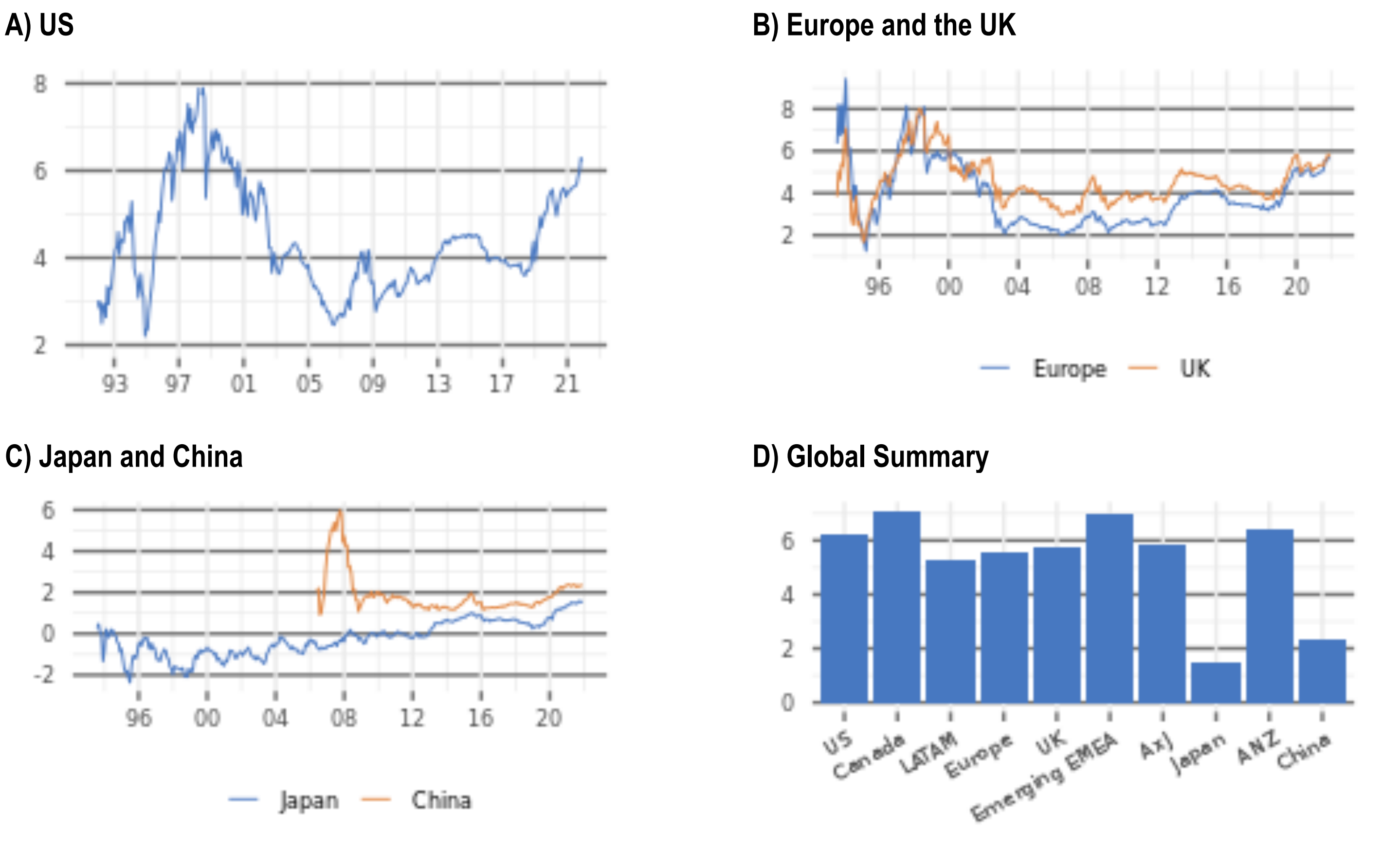
Risk Aversion Parameter λ

Based on [E6], the risk aversion parameter, λ, can be estimated empirically based on the equity risk premium (i.e., $E(r_{Market}) - r_f$) and equity market risk (i.e., σ^2_{Market}).

As a demonstration, we use an expanding window of monthly returns of the Russell 3000 total return index to computer the average return of the market $r_{Market,t}$ and a 10-year rolling window to estimate $\sigma_{Market,t}$. We further use the three-month US treasury bill yield as the risk free rate, $r_{f,t}$. As shown in Figure 5(A), the estimated λ_t parameter appears to be positively correlated to realized market returns and negatively correlated to market volatility. For example, the steep decline of λ_t from the late 1990s to the mid-2000s is mainly due to the plunge of the equity market in the early 2000s. The market volatility during late-2008/early-2009 causes the risk aversion parameter to dip again. Similarly, the gradual increase of λ_t since 2009 can be attributed to the long bull market cycle since the global financial crisis.

The risk aversion parameters in Developed Europe and the UK mirror each other closely (see Figure 5B). As we shift to Japan, it is surprising to see a negative estimated risk aversion parameter λ_t, due to a prolonged period of weak equity market (see Figure 5C). A negative λ_t parameter is problematic for many models. In such cases, we recommend to manually override the parameter and set λ_t as an arbitrary small number (e.g., $\lambda_t = 1.2$ as recommended in Black and Litterman [1990, 1991, and 1992]). With enough data history, equity markets typically deliver higher returns than the risk free rate; and therefore, λ_t should stay positive. Currently, λ_t is positive in all 10 regions of the world (see Figure 5D).

Figure 5 Estimated Risk Aversion Parameter



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Stock-by-stock Covariance matrix Σ_t

The Venice model is flexible enough to allow users to import any risk model. In all simulations in this research, we use our own QES risk models[9] (see *Port@ble Ownership*, Alvarez, et al [2018] for details).

Stock Weighting Vector ω_t

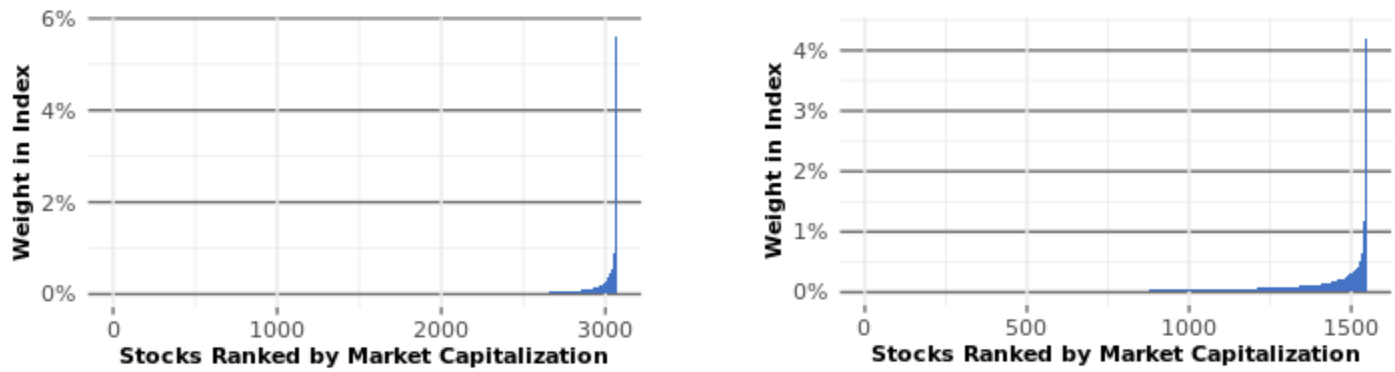
In the standard BL implementation, we need each stock's weight in a broad (market capitalization weighted) index, such as the Russell 3000 index for US equities or MSCI World for global developed market. Please note that it is critical to have the point-in-time index constituents and weights for each index we want to use. As detailed in *The Big and the Small Sides of Big Data* (see Luo, et al [2017a]), extrapolating current index constituents backwards introduces considerable look-ahead and survivorship biases, causing potentially misleading results.

As shown in Figure 6, both the Russell 3000 and MSCI World indices are top heavy. The largest 10 companies account for 25% and 19% in these two indices, respectively. There are many stocks in the Russell 3000 index with weights below 1bps. Similarly, most stocks in the MSCI World index have weights between 1bps and 10bps.

[9] Please contact us or your Wolfe Research sales representative if you are interested in testing our suite of global equity risk models.

Figure 6 Index Weight Distribution, December 31, 2021

A) Russell 3000 **B) MSCI World**



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Specifying View Expected Returns

After discussing some of the easier steps, now let us address the first major challenge to implement the BL framework – how to specify the expected return vector q. Active investors may use fundamental/discretionary, systematic/quantitative, or a hybrid quantamental approach to form their views on future asset returns. Some approaches – such as multifactor models and analyst target prices – can be translated into return expectations easily. On the other hand, many analysts express their views using buy/hold/sell ratings, a buy/watch list, and other categorical systems – we need to translate these into numerical returns with a meaningful statistical distribution that matches actual stock returns.

Standard Approach

Grinold and Kahn [2000] provide a guiding principle of translating forecasts (i.e., score) to expected returns, which is commonly used in the investment industry:

[E24] $$q_{Excess} = IC \times \sigma_q \times Score$$

Where,

q_{Excess} is a $(K \times 1)$ vector of expected returns, i.e., views to be added to the Venice model,

IC (or Information Coefficient) is the correlation coefficient between our forecasts and realized returns, measuring our skill,

σ_q is a $(K \times K)$ diagonal matrix of the asset-by-asset volatility matrix of the K assets for which we have views, and

$Score$ is a $(K \times 1)$ vector of model scores.

For quantitative models, computing historical realized IC is relatively straightforward (see *Signal Research and Multifactor Models*, Luo, et al [2017b]). The challenge, however, is that future model performance can be very different from historical skill level. As detailed in *The Future of Active Management* (Luo, et al [2019a]), as more and more market anomalies are recognized by investors, the market is getting more efficient. Consequently, the performance of most active strategies has declined significantly in recent years and is likely to further deteriorate.

Fundamental managers do not typically preserve a history of their past predictions. Therefore, we may have to use some simple rule of thumb, e.g., an IC of 10% for an exceptional manager; an IC of 5% for a good analyst; and an IC of 0%-1% for an average researcher.

For σ_q, Grinold and Kahn [2000] suggest that we should use idiosyncratic volatility, assuming forecasts are orthogonal to common risk factors. However, as we demonstrated in our previous research (see *Port@ble Ownership*, Alvarez, et al [2018]), most active managers (even fundamental managers) do take significant factor risk. Therefore, there are also arguments for the usage of total volatility of each stock in σ_q.

For quantitative models, the $Score$ vector is typically expressed as some kind of normalized scores (see *Signal Research and Multifactor Models*, Luo, et al [2017b] for the choices of data normalization techniques and their impact on model performance). As we will discuss later, it is generally easier for quantitative models to specify scores. Most model outputs are typically normalized z-scores and roughly follow a standard normal distribution. Furthermore, factor models typically provide forecasts of excess returns rather than raw returns; therefore, we may need to add back market equilibrium returns to obtain our view vector:

[E25] $$q = q_{Excess} + \pi$$

Fundamental Research with Target Price

Unlike quantitative models, fundamental analysts/PMs do not typically provide explicit return prediction. However, many fundamental analysts indeed use valuation models, which typically produce target prices. Converting from target prices to return expectations is straightforward:

[E26] $$q = \frac{TargetPrice - CurrentPrice + ExpectedDividend}{CurrentPrice}$$

Dividends are generally much easier to predict than target price because most companies do not dramatically alter their dividend policies. In practice, researchers often simply extrapolate future dividends by annualizing the most recent dividend payments. More sophisticated models can be employed to forecast dividends (see *In Yield We Trust*, Wang, et al [2020a] and *Cutting Dividend*, Wang, et al [2020c] for details).

Given that return prediction (i.e., forecasting target price) is extremely difficult and highly inaccurate, the estimated capital appreciation component (i.e., $\frac{TargetPrice - CurrentPrice}{CurrentPrice}$) typically dominates the dividend yield part (i.e., $\frac{ExpectedDividend}{CurrentPrice}$). Therefore, many analysts ignore dividend from the computation:

[E27] $$q = \frac{TargetPrice - CurrentPrice}{CurrentPrice}$$

Fundamental Research Using Buy/Sell Recommendations

Instead of expected returns or target prices, some analysts only produce lists of stocks to buy and sell. One simple way to translate a buy and sell list into views is to assign a score of $+1.0$ to the buys and a score of -1.0 to the sells.

If a manager has a more refined fractile rating, say deciles or quintiles, we can simply compute the z-score of the fractiles. For example, for quintile ranking (e.g., Strong Buy, Buy, Hold, Sell, and Strong

Sell), we can use scores as computed in Figure 7. Alternatively, Meucci [2010] sets scores of -2, -1, 1, and 2 for very bearish, bearish, bullish, and very bullish views.

The most noticeable problem with the simple scoring system is that the distribution of the scores (e.g., level, mean, standard deviation, and possibly higher moments) is very different from actual stock returns. For example, if we have five stocks that are rated as Strong Buy, Buy, Hold, Sell, and Strong Sell, respectively and we transform them into z-scores as specified in Figure 7, the resulting scores certainly do not correspond to actual stock returns. Furthermore, because analysts assign ratings on a list of stocks, the percentage of stocks receiving each rating may also change over time. Therefore, the distribution of the transformed scores is also time varying.

One possible solution is to further normalize the above scoring system, using the methods presented in the previous section (see Equation [E24]) to convert our normalized scores into the return-scale.

Figure 7 Transforming Analyst Ratings to Scores

Analyst Rating	Quintile	Z-Score	Alternative Score
Strong Buy	1.00	1.26	2
Buy	2.00	0.63	1
Hold	3.00	0.00	0
Sell	4.00	-0.63	-1
Strong Sell	5.00	-1.26	-2

Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Relative Value Views

Relative value or pairs trading is a classic example of how relative views can be implemented. In the case of a pairs trading strategy, as detailed in *Innovations in Paris Trading Strategies* (see Li, et al [2021]), we essentially recommend buying one stock and shorting another. While most pairs trading models do not directly predict the expected return differential, it can be estimated by assuming the spread will converge to its historical average:

$$\widehat{r_A} - \widehat{r_B} = q$$

For instance, let us assume that we have five assets and our pairs trading model suggests that the second stock is likely to outperform the fourth one; then the pick matrix P becomes:

$$P = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

How to specify the pick matrix P for relative views involving more than two assets can be complicated. The easiest and most intuitive way is to use an equal weighting scheme (see Satchell and Scowcroft [2000]). For example, in the same five-asset example, if we expect the first and second assets to outperform the third asset, we can use the first row of the matrix below. Under Satchell and Scowcroft [2000] specification, the relative size (market capitalization) of involved assets is ignored. The first two assets are equally weighted and each receives half of the weight.

Idzorek [2002] suggests an alternative approach, in which each asset is weighted by its relative market capitalization. In the same example as above, if the second stock's market capitalization is three times as large as the first one, we can specify the weights as in the second row of the P matrix above. There are other ways in the literature, e.g., Litterman, et al [2003] use percentage values.

$$P = \begin{bmatrix} 0.5 & 0.5 & -1 & 0 & 0 \\ 0.25 & 0.75 & -1 & 0 & 0 \end{bmatrix}$$

Models without Full Coverage

One of the common challenges in utilizing portfolio optimization techniques in portfolio construction is that our models have limited breadth, i.e., only covering a small subset of our investment universe. This is particularly problematic when we manage our portfolio against a specific benchmark. The incomplete coverage issue is precisely one of the design purposes of the BL framework. Managers can express their views on as many or as few assets as they desire, and the Venice model will provide an integrated forecast for every asset in our investment universe.

Event-Driven Strategies

As we show in our previous research on takeover prediction (see *Machine Learning Takeovers*, Wang, et al [2017]), risk arbitrage (see *Systematic Alpha from Risk Arbitrage*, Wang, et al [2018a]) and insider transactions (see *Seeking Alpha from Insider Transactions*, Jussa, et al [2018a]), corporate events have significant impact on share performance and company fundamentals. However, corporate events are not typically included in most quantitative and fundamental portfolios, because events have limited coverage (e.g., few companies are taken over, have significant insider transactions at any given point of time) and arrive irregularly (e.g., we do not have SEC investigation news every day). In a later section, we will use a few corporate events as an example to show how BL model can be used.

Industry-Specific Models

As discussed in our previous research (see *The Future of Active Management*, Luo, et al [2019a]), we find that factors based on data that is unique to a specific industry (e.g., banks, oil & gas exploration & production companies, REITs) tend to have strong and uncorrelated alphas compared to traditional factors based on financial statement data. However, coverage is generally a concern, as these models are limited to only certain industries. We could use this as another example in a later section.

SPECIFYING VIEW UNCERTAINTY AND CONFIDENCE

Once we specify view returns, the next step is to express the uncertainty (or confidence) around our views. For view uncertainty, we first conduct a comprehensive academic literature review. In this section, we elaborate the standard approach and then introduce a few other alternatives.

The Standard Approach

As discussed in previous sections, many authors directly link the uncertainty matrix Ω to the asset-by-asset covariance matrix Σ:

[E28] $$\Omega = \kappa P \Sigma P'$$

The standard approach is intuitive and theoretically appealing. Naturally, we have higher (lower) confidence on stocks that are less (more) volatile. Therefore, we can assume our uncertainty (or confidence) is proportional to each stock's volatility (or variance). In this setup, users only need to set the κ parameter – a higher value of κ corresponds to a higher uncertainty (lower confidence).

The simplicity of the standard approach does come with a cost. As shown in Equations [E18] and [E19], the BL posterior return is a weighted average of our prior (i.e., market implied returns π) and our predictions (q), depending solely on κ. The assumption behind the standard approach (i.e., view confidence only depends on each stock's risk) may be unrealistic. For example, an analyst might understand one company far better than another stock in the same industry, with no relation to each stock's volatility.

Despite its shortcomings, this is our default method of setting the uncertainty. Investors may want to set a different κ parameter for each view. Intuitively, that confidence level depends on an analyst/model's historical accuracy/performance.

Models with Sufficient History

If we have sufficient historical data of our past return predictions, we can compute the realized accuracy (estimation errors) as:

[E29] $$q_{Error,t} = q_{Actual,t} - q_t$$

Where,

$q_{Error,t}$ is a $(N \times 1)$ vector of return estimation error at time t,

$q_{Actual,t}$ is a $(N \times 1)$ vector of actual realized return at time t, and

q_t is a $(N \times 1)$ vector of return forecast formed at time t.

If we further assume our estimation error vector $q_{Error,t}$ is a covariance stationary random walk time series, i.e., our estimation errors are uncorrelated over time, following a normal distribution:

[E30] $$q_{Error,t} \sim N(\theta, \Omega)$$

We can estimate the uncertainty matrix Ω empirically:

[E31] $$\widehat{\Omega} = \frac{1}{T-1} \sum_{t=1}^{T} \left(q_{Error,t} - \overline{q_{Error}}\right)\left(q_{Error,t} - \overline{q_{Error}}\right)'$$

We normally expect strategies or models with higher skill (measured by IC) to have smaller uncertainty. In this case, we make the implicit assumption that our view uncertainty is consistent over time.

Analyst Estimate Data

Chen, et al [2015] uses the consensus sell-side analyst target price as a proxy for fundamental views. Then, they compute target price implied return in Equation [E27] as views. They further use the standard deviation of target prices divided by the consensus target price to measure view uncertainty.

A Generalized Approach

Idzorek [2002] suggests a systematic approach of how to set the confidence level for each view, based on investors' targeted overweight/underweight of each asset.

THE VENICE TOOLBOX

The Venice tool is available to Wolfe Research's clients as an API. We have two versions for the Venice API – one in R and one in Python. The Venice API allows users to set each of the key parameters presented in this section with ease. In the next few sections, we will use a series of realistic examples to demonstrate how investors can use the Venice tool to combine views.

VENICE FOR QUANTITATIVE MANAGERS

On the surface, the BL framework appears to be more suitable for fundamental managers. Quantitative models typically have broad coverage. As a result, combining multiple models is mostly done via signal weighting (see *Signal Research and Multifactor Models*, Luo, et al [2017b]) or machine learning ensemble approach (e.g., *Man versus Machine*, Wang, et al [2018c]). However, as we will demonstrate in this section, even for systematic managers with a single source of return forecast driven from a multifactor model, there is still potential benefit of incorporating the BL model.

We all know that predicting asset returns is extremely difficult and our return forecasts tend to suffer from large estimation errors. One of the common critiques of quantitative investing is that models often lead to extreme and counterintuitive portfolios with high turnover.

In addition, we may have multiple models, with different coverages. Optimally combining models and signals is not a trivial task. Moreover, a popular theme that many firms are starting to investigate – the alpha capture program – explicitly requires managers to incorporate external signals, which may have rather different methodologies and coverages.

QUANTITATIVE VIEWS

In this section, we conduct an empirical backtest, using the US equity market (i.e., the Russell 3000) as our investment universe. Please note that the Venice tool can be implemented on any equity market in the world – examples using international stocks will be presented in the next few sections. For demonstration purpose, we will use two quantitative models as our views – one is based on a standard multifactor model (called Benchmark Model or BM) and the other is based on machine learning algorithms called the MALTA[10]. Again, users can import any model (or models) as views.

- **BM (Benchmark Model).** The first model is a fairly simple Benchmark Model (BM) that equally weights eight well-known stock-selection factors (see *Signal Research and Multifactor Models*, Luo, et al [2017b] for details):
  - **Value – Defensive:** Trailing earnings yield – we prefer companies with high earnings yield
  - **Value – Cyclical:** Book-to-market – we buy companies with high book-to-market, i.e., cheap stocks on valuation
  - **Growth:** Consensus FY1/FY0 EPS Growth – we prefer companies with high earnings growth
  - **Price Momentum:** 12M total return excluding the most recent month – we prefer companies with positive price momentum
  - **Analyst Sentiment:** 3M EPS revision – we buy companies with positive earnings revisions
  - **Quality – Profitability:** Return on equity – we like firms with high ROEs
  - **Quality – Leverage:** Debt/Equity ratio – we prefer companies with low financial leverage

[10] See *Man Versus Machine – MALTA*, Wang, et al [2018c] for detailed methodology behind the MALTA model.

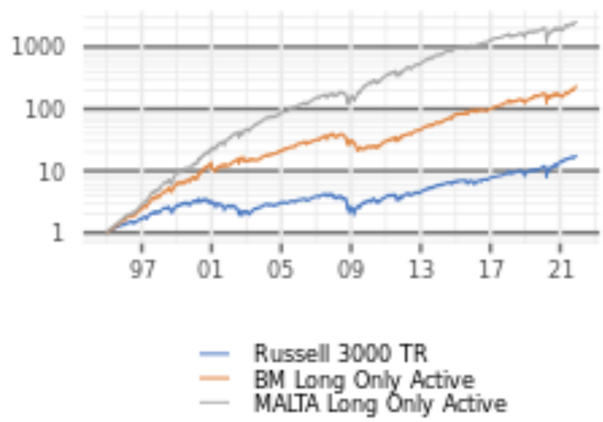
- Quality – Earnings Quality: Sloan's accruals – we buy companies with low accruals

- **A Sophisticated Machine Learning Model – MALTA**. The MALTA model uses a time series boosting meta algorithm that samples data from multiple time horizons to ensure robustness throughout different market regimes. Within each time sample, we apply our proprietary MBBT (Multi-Branch Boosted Tree) algorithm to dynamically select signals from our global factor library (with >300 factors). The MBBT is a nonlinear classification algorithm, identifying and selecting both linear and nonlinear payoff patterns.

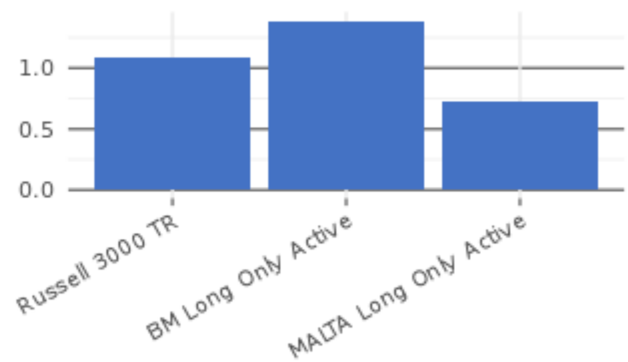
Active long-only strategies based on both the BM and MALTA models have outperformed the passive Russell 3000 index in the long run, with the MALTA being the clear winner (see Figure 8A). In recent years, the BM strategy has outperformed the market and the MALTA model (see Figure 8B).

Figure 8 Two Quantitative Models, Long/Short Quintile Portfolios

A) Cumulative Performance, US



B) Sharpe Ratio, January 2017 to Present



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

MODEL SETUP

For illustration purposes, we specify the key parameters in the Venice as follows:

- The **risk aversion parameter** λ_t is assumed to be time varying and is estimated empirically, following the procedure described in Equation [E6].
- Stock-by-stock **covariance matrix** Σ_t . At each month end, we use the Wolfe QES Standard Risk Model to generate the (3000×3000) covariance matrix Σ_t.
- **Stock weighting vector** $\omega_{Market,t}$ is based on each stock's weight in the Russell 3000 index.
- **Pick matrix** P. Since our MALTA model has predicted returns of almost all stocks in the Russell 3000 index, the pick matrix should be close to the identity matrix, i.e., $P \cong I$. We begin with an identity matrix and remove the rows corresponding to stocks for which we do not have a view return.
- **View return vector** q_t is computed as follows:

- Excess return vector $q_{Excess,t} = IC_t \times \sigma_{q,t} \times Score_t$, where IC_t is a scalar (estimated using a rolling average monthly IC of the BM/MALTA model at time t), $\sigma_{q,t}$ is a $(K \times 1)$ vector (each element corresponds to the volatility of each stock, taken from the square root of the diagonal elements from the covariance matrix Σ_t), and $Score_t$ is a $(K \times 1)$ vector of the BM/MALTA model normalized scores.
- The final overall view return vector q_t is then computed as $q_t = q_{Excess,t} + \pi_t$.

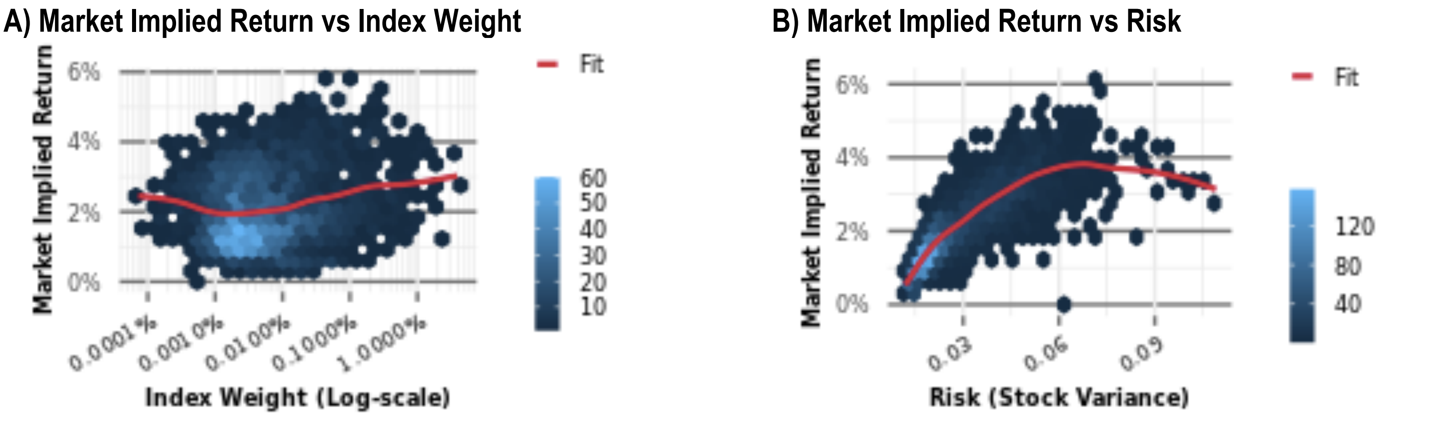
- **View uncertainty matrix** Ω_t is estimated as follows:
  - Compute the model return estimation error vector $q_{Error,t} = q_{Actual,t} - q_t$, where $q_{Actual,t}$ is a $(K \times 1)$ vector of actual returns and q_t is a $(K \times 1)$ vector of our model predicted return as defined above.
  - We estimate the uncertainty based on $\Omega_t = \kappa_t P \Sigma_t P'$. The κ_t is computed as $\kappa_t = \frac{\sum_{i=1}^{M_t} \sigma^2_{q(error,i,t)}}{\sum_{i=1}^{M_t} \sigma^2_{i,t}}$, where, M_t is the number of stocks with more than two years of history on date t, $\sigma^2_{q(error,i,t)}$ is the variance of estimate error for stock i on date t (estimated using a six-month half-life), and $\sigma^2_{i,t}$ is the variance of stock i on date t from our risk model.

THE BL MODIFIED VIEW

As argued by Walters [2013], the BL model is essentially a shrinkage estimator that pushes our model prediction towards the market consensus (prior), so our final forecasts are not as extreme. In this section, we show how the BL combined views are related to market implied returns, risk, and the uncertainty of our views (i.e., model predictions).

Equation [E5] suggests that the market equilibrium return is proportional to each stock's risk (variance) and weight in the benchmark index. As shown in Figure 9(A), the relationship between market equilibrium return and market capitalization is upward-sloping but nonlinear, due to correlation/covariance effect. If we exclude stocks with tiny weights in the index, we do see that market-implied return is positively correlated to each stock's weight in the index. Similarly, the relationship between market-implied return and risk (variance) is also nonlinear, possibly due to the correlation terms. In addition, stock variance is heavily skewed, while return is more symmetric.

Figure 9 Market Implied Return

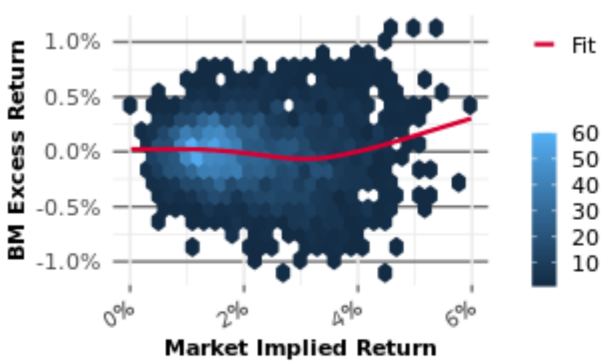


Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

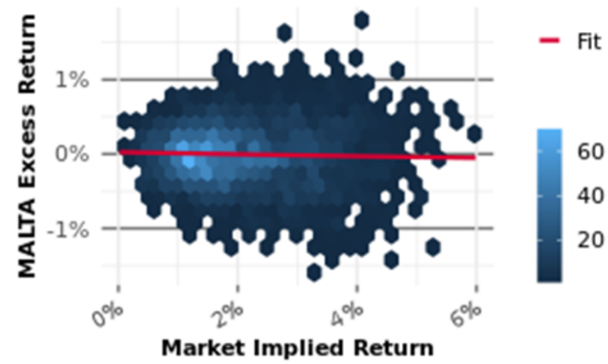
Because the BM and MALTA models make explicit forecasts on future stock returns, the model predicted excess returns deviate from the market consensus view considerably (see Figure 10A and B). The MALTA model is completely uncorrelated to the prior market view. The forecast returns, however, are a mixture of the market implied returns added to the excess view returns of each model (see [E32]). As such, the forecast returns align closely with the market implied returns with noticeable differences when the excess view returns are large. Compared to the BM, the MALTA forecast returns show larger deviations from the market implied returns, since the MALTA is a nonlinear machine learning model (see Figure 10C and Figure 10D).

Figure 10 Model Excess Return and Forecast Return

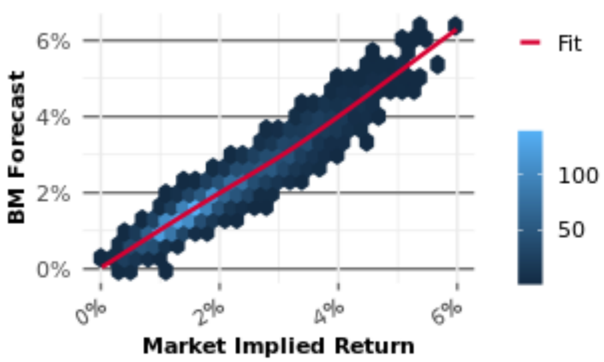
A) BM Excess Returns vs Market Implied Return



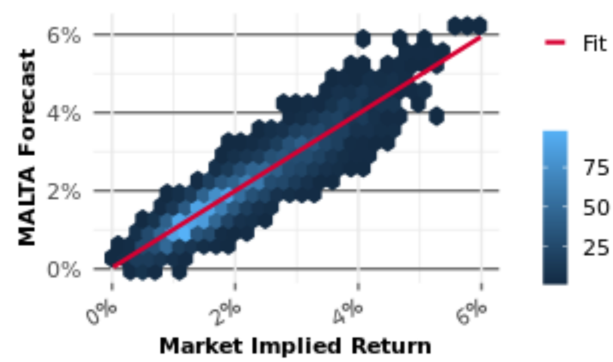
B) MALTA Excess Returns vs Market Implied Return



C) BM-Forecast Return vs Market Implied Return



D) MALTA-Forecast Return vs Market Implied Return

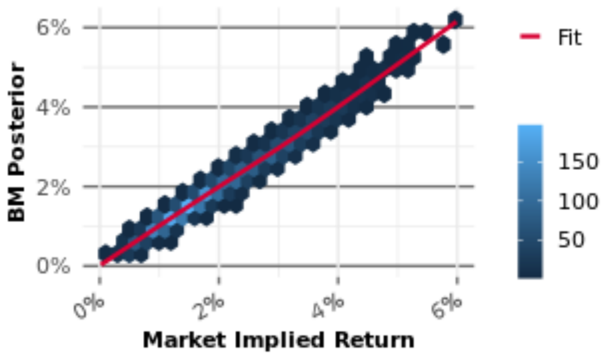


Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

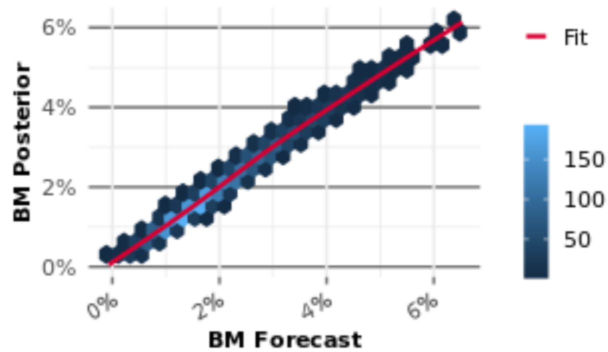
For both the BM and MALTA models, the posterior return aligns closely with both the market implied return and the model forecast return (see Figure 11).

Figure 11 Black-Litterman Posterior Return

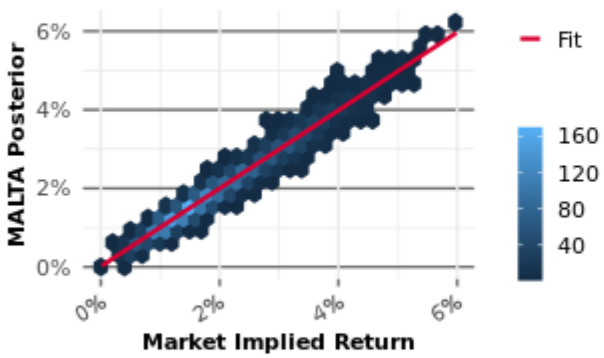
A) BM BL Posterior Return vs Market Implied Return



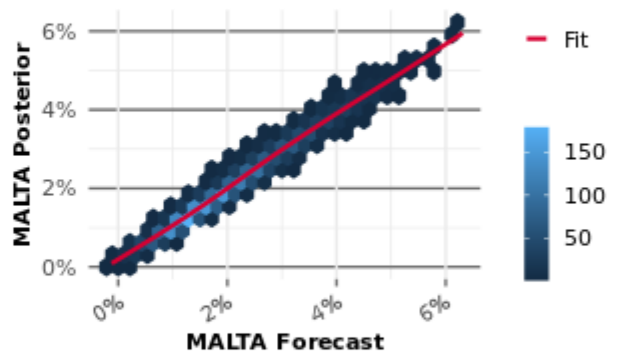
B) BM-BL Posterior Return vs BM Forecast



C) MALTA BL Posterior Return vs Market Implied Return



D) MALTA-BL Posterior Return vs MALTA Forecast

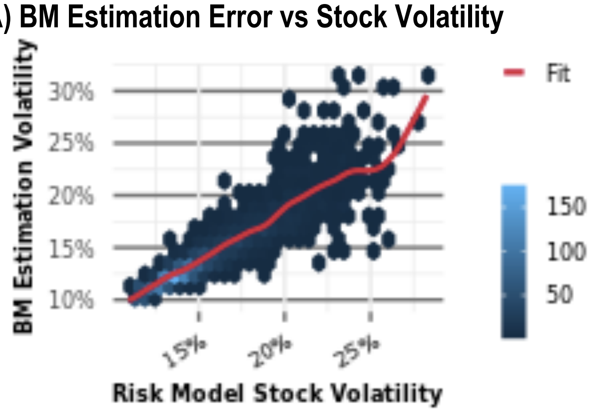


Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

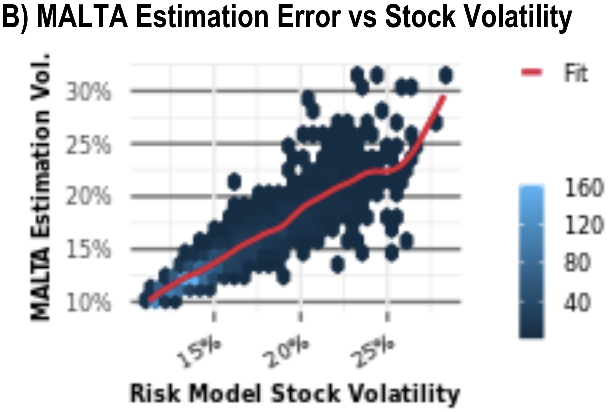
As shown in Figure 12, the model estimation error variance $\sigma^2_{q(error,i,t)}$ can be reasonably proxied by each stock's variance $\sigma^2_{i,t}$ from our risk model. Therefore, there is some supporting evidence of specifying view uncertainty with $\Omega_t = \kappa_t P \Sigma_t P'$.

Figure 12 Model Forecast Estimation Error

A) BM Estimation Error vs Stock Volatility



B) MALTA Estimation Error vs Stock Volatility



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Finally, as shown in Figure 13, the MALTA-BL posterior return is 16% correlated with the MALTA excess return whereas the BM-BL Posterior return is only 11% correlated with the BM Excess Return. The MALTA model is more correlated with its posterior return due to its strong performance.

Figure 13 Correlation Matrix – Market-Implied Return, MALTA Prediction, and BL Combined Views

	Market-Implied Return	BM Excess Return	BM-BL Posterior	MALTA Excess Return	MALTA-BL Posterior
Market-Implied Return	100%				
BM Excess Return	-4%	100%			
BM-BL Posterior	99%	11%	100%		
MALTA Excess Return	-4%	55%	5%	100%	
MALTA-BL Posterior	98%	7%	99%	16%	100%

Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

PORTFOLIO SIMULATION

Now, we study the impact of the BL model on portfolio performance. We use the Russell 3000 index as our benchmark and construct four long-only active portfolios:

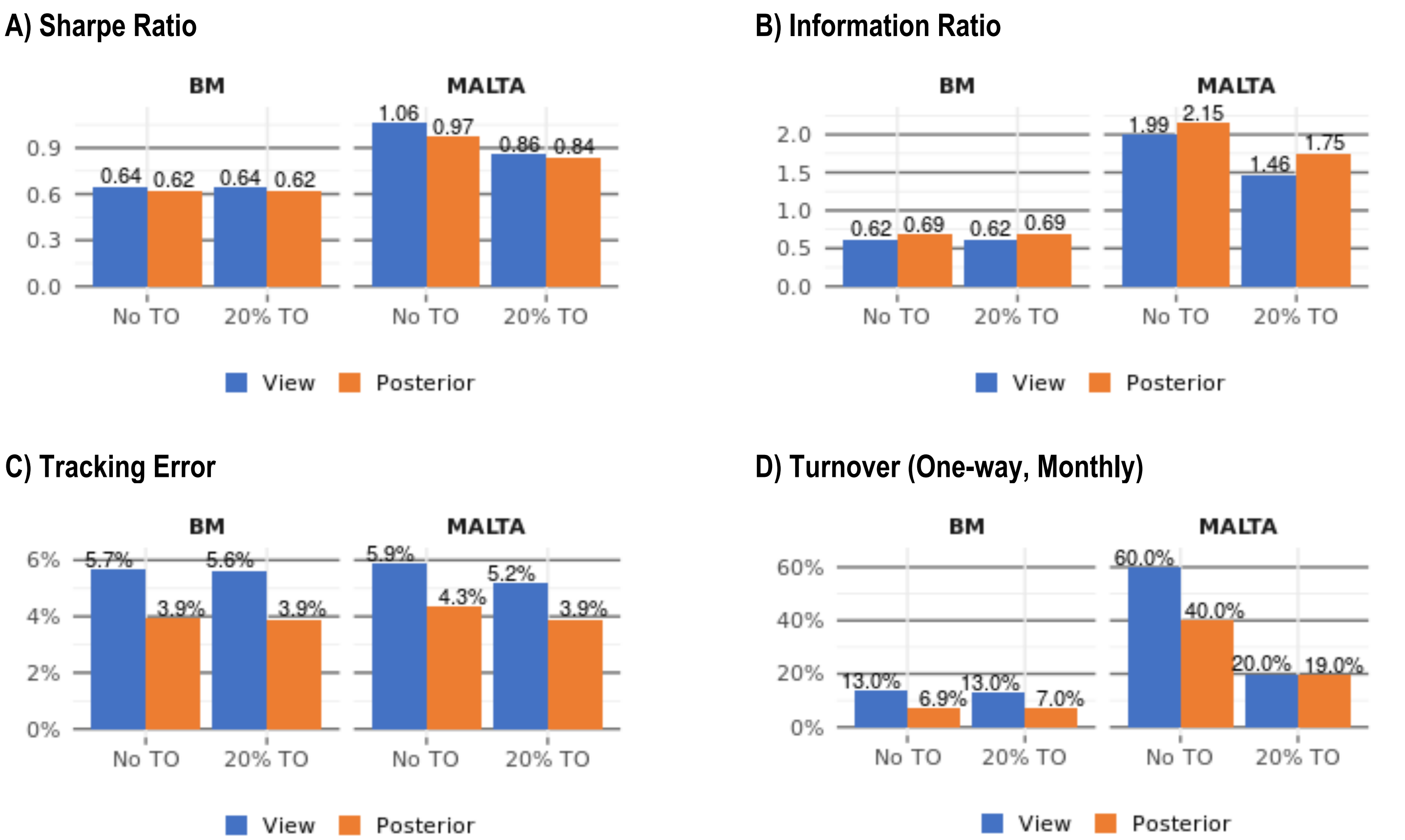
- BM portfolio without turnover constraint ("BM-View, no TO")
- BM portfolio with a maximum 20% turnover per month constraint ("BM-View, 20% TO")
- MALTA portfolio without turnover constraint ("MALTA-View, no TO")
- MALTA portfolio with a maximum 20% turnover per month constraint ("MALTA-View, 20% TO")
- BM-BL portfolio without turnover constraint ("BM-BL, no TO")
- BM-BL portfolio with a maximum 20% turnover per month constraint ("BM-BL, 20% TO")
- MALTA-BL portfolio without turnover constraint ("MALTA-BL, no TO")
- MALTA-BL portfolio with a maximum 20% turnover per month constraint ("MALTA-BL, 20% TO")

The above portfolios otherwise have the same setup:

- Objective function (maximizing expected return): $argmax_{\omega}\left(\omega'\pi - \frac{\lambda}{2}\omega'\Sigma\omega\right)$
- Constraint:
  - Long only, i.e., $\omega \geq 0$
  - Fully invested with no leverage $\sum \omega = 1$
  - Maximum 1.5% weight for any single stock, i.e., $\omega \leq 1.5\%$
  - Assuming an average transaction cost of 20bps per trade
  - Monthly rebalance, from December 1996 to present (~25 years)

As shown in Figure 14(A), all eight active portfolios have higher Sharpe ratios than the Russell 3000 index (at 0.49x). Although the BL framework does not appear to improve Sharpe ratio, it boosts relative performance (i.e., IR or Information Ratio) meaningfully. Compared to using the BM (or MALTA) model alone, combining views with the BL lifts IR, regardless of the turnover constraints (see Figure 14B). The improvement in performance is particularly strong for the MALTA portfolio with turnover constraint. Lastly, since the BL model shrinks our views toward market consensus, the BL portfolios have lower tracking errors than our original strategies (see Figure 14C). Furthermore, because the BL posterior views are less extreme than the original models, it also reduces turnover (see Figure 14D)

Figure 14 Portfolio Performance



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

A Note on Scaling Excess Returns

During our analysis, we find that the excess returns of our models can be of a much larger scale than the market implied return. Therefore, the excess return can dominate the final return expectation via Equation [E25]. For instance, the *ex ante* IR of the MALTA signal can be as high as over 20x (see Figure 15). Although the MALTA signal has very strong predictive power, an IR of 10x-20x is unrealistic.

Figure 15 MALTA Excess Return *Ex Ante* Information Ratio

Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

The oversized excess return expectations from the MALTA model lead to extreme stocks weights for unconstrained portfolios. Because of our portfolio constraints (especially the long-only and turnover constraints), we end up with a very low TC (Transfer Coefficient). As explained in *Risk, Portfolio Construction, and Performance Attribution* (see Luo, et al [2017d]), the TC measures the extent to which the optimized portfolio captures the original signal. A TC of 100% means the signal is unaffected by the constraints of the optimization. A low TC indicates that our constraints are too strong, and the optimization alters the signal substantially. Without properly scaling the signal, the average TC of the MALTA-BL posterior portfolio is only 16%, which is very small[11].

One way to scale our signal is to ensure that the volatility of the unconstrained optimized portfolio is similar to that of the benchmark (see *GEM Portfolio Construction*, Wang, et al [2021]) and Pederson, et al [2021]).

Given a set of excess returns, the unconstrained mean-variance optimized portfolio based on those returns is:

[E33] $$\omega_q = \frac{1}{\lambda}\Sigma^{-1} q_{Excess}$$

And its volatility is:

[E34] $$\sigma_q = \sqrt{\omega_q' \Sigma \omega_q} = \frac{1}{\lambda}\sqrt{q_{Excess}' \Sigma^{-1} q_{Excess}}$$

The volatility of the benchmark can be calculated as:

[E35] $$\sigma_b = \sqrt{\omega_b' \Sigma \omega_b}$$

[11] TCs for typical long-only active portfolios should be around 40% to 60%.

For our MALTA model, the average volatility of the unconstrained portfolio is 387%, whereas the average volatility of the benchmark during the backtest is 17%. Therefore, if we want to scale the MALTA model to have a volatility equal to the benchmark, we would need to multiply it by $\frac{17\%}{387\%} = 0.04$.

The above approach, however, does not account for the predictive power of the MALTA signal. Therefore, we need to find a balance of model predictive power and volatility, which can be accomplished by scaling relative to a reference IC.

[E36] $$\eta = \frac{E(IC)}{Reference\ IC} \times \frac{\sigma_b}{\sigma_q}$$

Where,

$E(IC)$ is our estimated model IC,

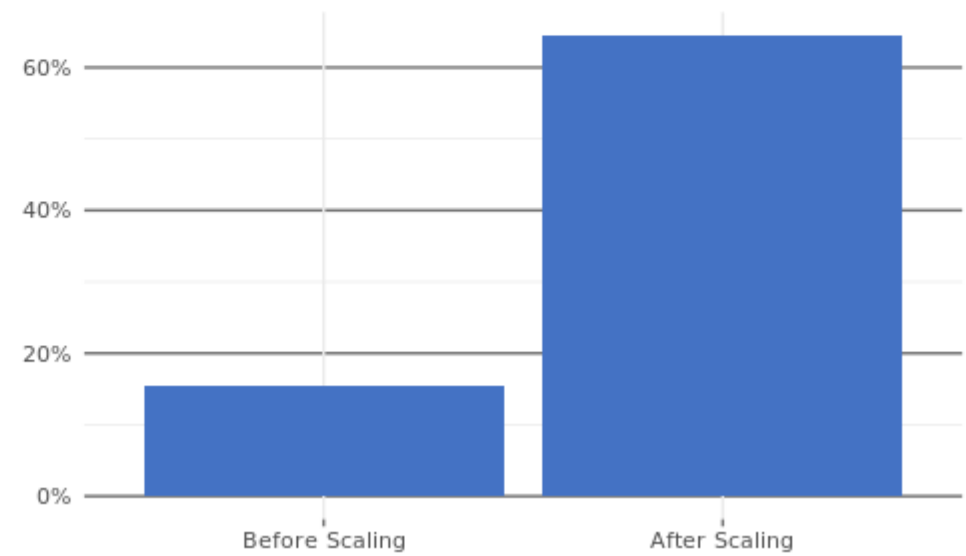
$Reference\ IC$ is our reference IC, and

η is our excess return scalar.

For all optimized portfolios in this paper, we estimate the expected IC using an exponentially-weighted rolling average with a half-life of two years and a reference IC of 3%.

After scaling the excess returns in this way, we significantly increase the TC for the MALTA-BL posterior optimization, from 16% to 65% (see Figure 16).

Figure 16 BL MALTA Posterior Transfer Coefficient



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

VENICE FOR QUANTAMENTAL MANAGERS

As addressed in *The Future of Active Management* (see Luo, et al [2019a]), there has been a strong and growing trend towards combining quantitative and fundamental analysis in the investment world. It makes intuitive sense. Quantitative models and fundamental analysis specialize in different dimensions – models tend to have high breadth (i.e., they cover many stocks), but fundamental analysis excels at depth (i.e., stronger predictive power on a few stocks). More importantly, these two investment strategies tend to be uncorrelated, which provides diversification benefit.

Quantamental investing sounds great in theory, but often faces tremendous challenges in practice. Quantitative analysts and fundamentals analysts tend to see the world differently. The BL framework provides an elegant solution. However, there are few academic research papers on this topic. Chen, et al [2015] is a rare exception. Even more problematically, there is not readily available tools that practitioners can use to implement the BL model. Our Venice tool is designed specifically for the purpose of combining multiple predictions. In this section, we use a few practical examples to demonstrate how the Venice tool and the BL model can help managers mix quantitative and fundamental predictions.

One of the biggest challenges for fundamental portfolio managers is to gain coverage. Fundamental analysts are mostly industry specialists. Each analyst covers around 20-50 stocks and conducts in-depth analysis. To have a complete coverage of the entire Russell 3000 index, one would need 60-150 analysts – probably too expensive for many asset management firms. Most investment management firms employ fewer analysts and choose their own specialties. Portfolio managers then form concentrated portfolios with around 20-50 stocks. This poses a particular challenge for long-only portfolios that need to track a benchmark.

BACKGROUND

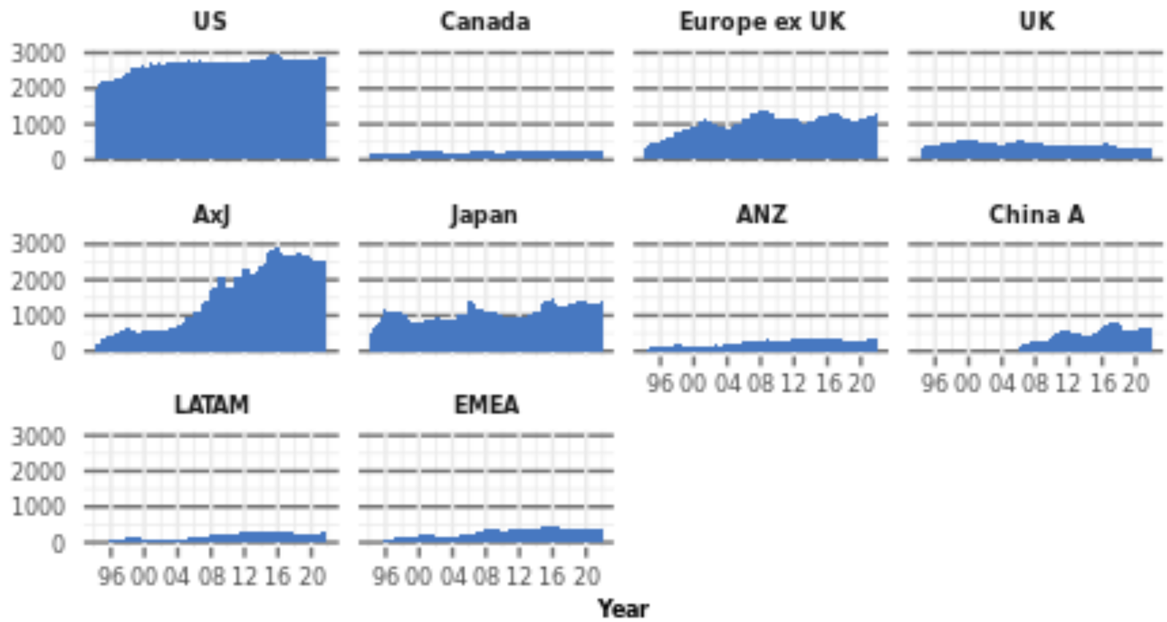
In this section, we use sell-side analyst recommendations as a proxy for the fundamental view and our MALTA model as an example for quantitative models.

Investment Universe

Since we study how to combine quantitative and fundamental analysis, it is intuitive to form our universe as those stocks that we have both quantitative and fundamental views.

Consistent with our standard regional definition (e.g., *Signal Research and Multifactor Models*, Luo, et al [2017b] and *Man Versus Machine – MALTA*, Wang, et al [2018c]), we divide the world into 10 regions: US, Canada, LATAM, Europe ex UK, UK, emerging EMEA, Asia ex Japan, Japan, Australia and New Zealand, and China (onshore). We use the Russell 3000 as our investment universe for the US, the S&P/TSX Composite Index for Canada, MSCI China Onshore Index for China, and the S&P BMI indices for other regions (see Figure 17).

Figure 17 Investment Universe



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

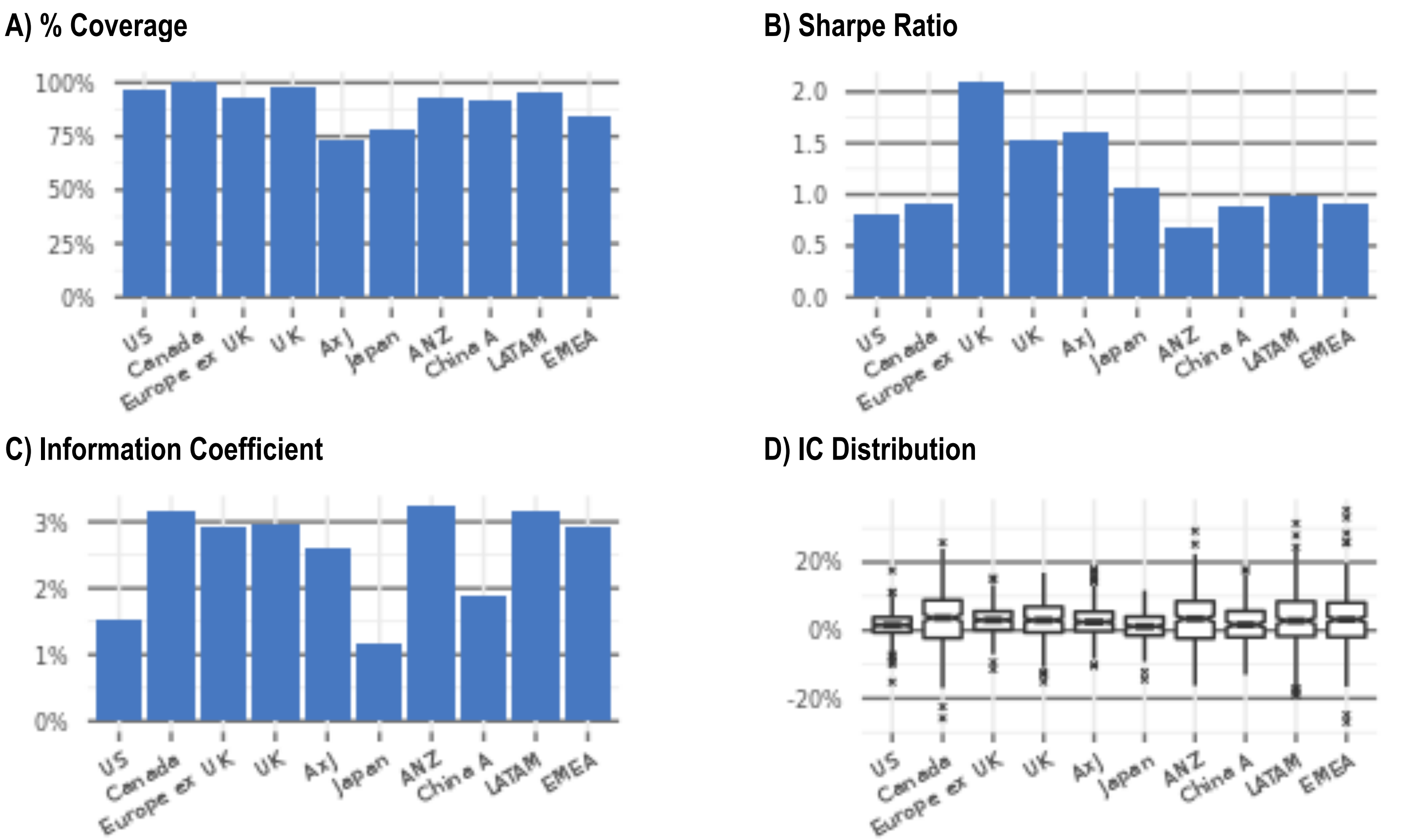
Fundamental Analysis

We use the IBES consensus recommendation as a proxy for fundamental views. Third-party data vendors such as IBES, Bloomberg, Factset, and Capital IQ collect sell-side analyst recommendation data in their databases. In IBES, for example, consensus recommendations – both mean and median – are available for many stocks globally, on a daily basis. Sell-side analysts are predominantly fundamental analysts. They rate the stocks they cover as "Strong Buy", "Buy", "Hold", "Sell", and "Strong Sell". Sell-side analysts are often organized by industries; therefore, each analyst typically only covers stocks in the same industry. Interested readers should refer to [Network of Economically Linked Firms] (see Rohal, et al [2021b]) for more details on sell-side analyst data.

Instead of using the absolute levels of recommendations, we use three-month recommendation revisions as our fundamental view, i.e., $(ConsensusRating_t - ConsensusRating_{t-3M})/CConsensusRating_{t-3M}$. It is well documented in academic literature that the level of analyst recommendations is highly biased and has little predictive power of future stock performance. What really matters is whether the sell-side is revising their views upwards or downwards for a given stock.

As shown in Figure 18(A), the coverage of sell-side analyst recommendations is over 70% of our research universe for all regions. For major markets such as the US, Canada, Europe, and the UK, the coverage is above 90%. We also find that fundamental analysts have modest stock-selection skill. As shown in Figure 18(B), a long/short portfolio where we buy the top 20% of stocks favored by the sell-side analysts and short the bottom 20% of stocks with the most negative views, has generated consistent alpha in all 10 regions. Another way to measure analysts' predictive power of future stock returns is Spearman's rank IC (i.e., the rank correlation between analyst recommendations and forward one-month stock returns). As shown in Figure 18(C), although on average, IC is positive in all 10 regions, there are noticeable periods when fundamental analysis does not perform well (see Figure 18D). Fundamental stock picking appears to be especially difficult in the US, Japan, and China, where analysts are able to add alpha only slightly more than half of the time.

Figure 18 Fundamental Prediction



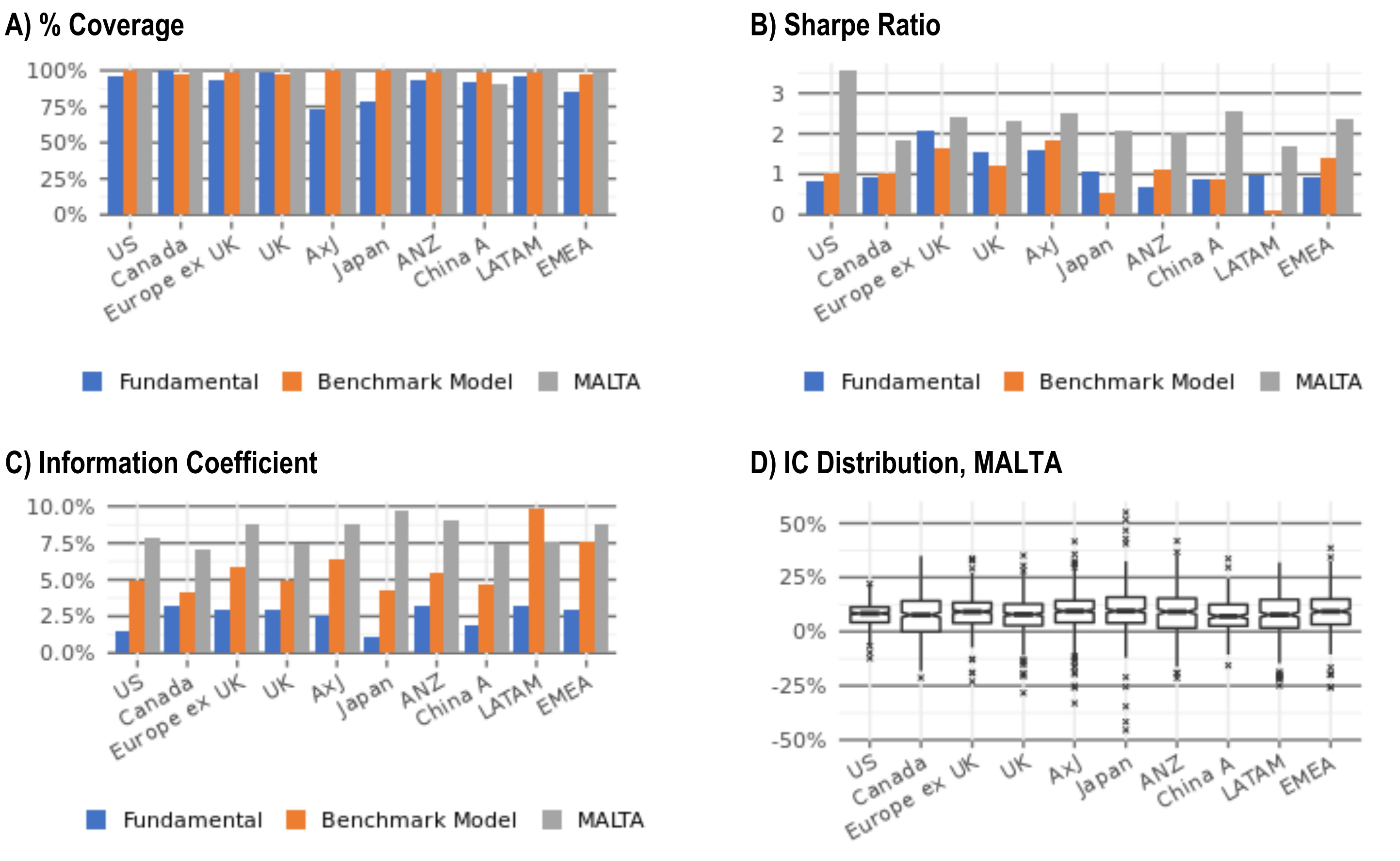
Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Quantitative Versus Fundamental Views

By construction, most standard multi-factor models have great coverage (see Figure 19A). Both BM and MALTA have performed well in the long term. Our MALTA model has clearly outperformed the BM and fundamental strategies, with a much higher Sharpe ratio in every single region (see Figure 19B). Given the broad investment universe, both quantitative models deliver more consistent performance compared to fundamental analysis (see Figure 19C). In fact, the MALTA model's top three quartile ICs stay positive in every region[12] (see Figure 19D). To save space, in this section, we primarily use the MALTA model as a proxy for quantitative models and show how to combine quantitative and fundamental views. The Venice API can handle most common quantitative and fundamental inputs.

[12] Meaning that the MALTA model delivers positive predictive power of future stock returns more than 75% of the time.

Figure 19 Fundamental versus Quantitative Models



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Correlation between Fundamental and Quantitative Strategies

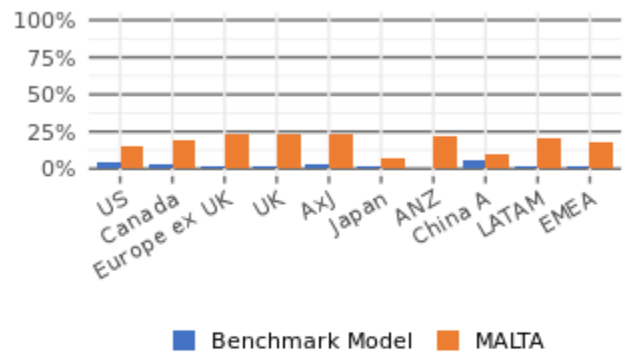
In theory, fundamental analysts conduct in-depth analysis on the companies they cover (typically in the same industry), by studying quantitative (e.g., financial statement ratios, valuation multiples, and discounted cash flows) and qualitative (e.g., management team, industry trend) information. However, in practice, fundamental analysis is heavily influenced by the same factors used in quantitative models, e.g., valuation, price momentum, quality (see our previous research, Port@ble Ownership, Alvarez, et al [2018]).

Similarly, quantitative models also examine similar information used by fundamental analysts, e.g., valuation ratios, analyst revisions. Therefore, we do expect fundamental and quantitative strategies to be somewhat correlated.

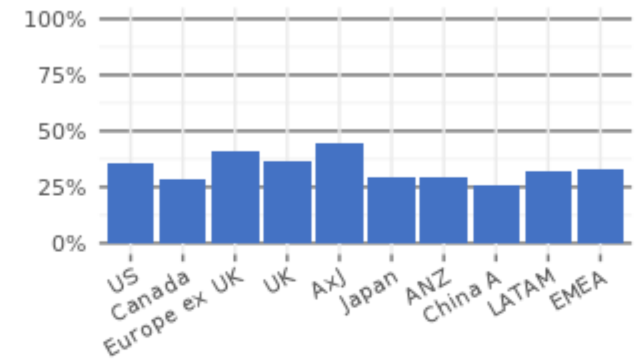
As shown in Figure 20(A), neither the BM nor our MALTA model is highly correlated with fundamental analysis – all correlations are below 25%. In addition, despite the common perception that almost all quantitative models are highly correlated, our MALTA model is only modestly correlated to the BM (see Figure 20B).

Figure 20 Rank IC Correlation between Quantitative Models and Fundamental Analysis

A) Quantitative Models vs Fundamental Analysis



B) MALTA vs BM



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

BASELINE APPLICATION

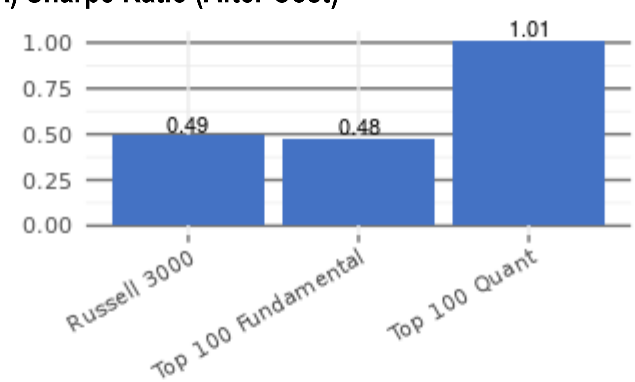
To set the scene, let us start from a few simple yet realistic portfolios. We use the broad US equity market (i.e., the Russell 3000 index) as an example, with monthly rebalance, no turnover constraint, and a 20bps transaction cost assumption.

- Benchmark: The Russell 3000 total return index
- Fundamental Portfolio: We buy the top 100 stocks ranked by analyst recommendations (revisions) and stocks are equally weighted in the portfolio.
- Quantitative Portfolio: We buy the top 100 stocks ranked by the MALTA model, equally weighted.

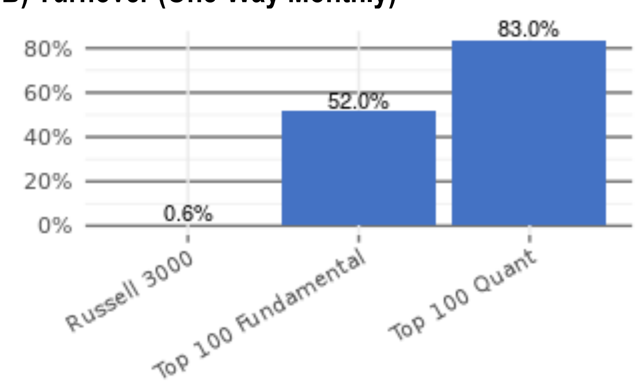
As shown in Figure 21A, the fundamental portfolio has similar performance to the market, but the MALTA portfolio beats the benchmark considerably. As expected, both active strategies have much higher turnover than the passive index (see Figure 21B).

Figure 21 The Baseline

A) Sharpe Ratio (After Cost)



B) Turnover (One-Way Monthly)



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

NAÏVE COMBINATION

Before diving into BL view combination, let us examine a few simple and intuitive ways to mix predictions:

- **Approach I**. In the first approach, we take a simple average of a quantitative (e.g., our MALTA model) and fundamental model (e.g., analyst recommendations) by mixing their normalized z-scores. We invest in the top ranked 100 stocks by the combined scores. We then equally weight these stocks and form a long-only portfolio.
- **Approach II**. In the second approach, we start from our fundamental analysis, by taking the top quintile of stocks based on analyst recommendations. Then, we overlay with our quantitative model by taking the top 100 stocks[13] ranked by our MALTA model. This is essentially a sequential sort, and therefore, the order of sorting matters. Fundamental analysis takes a higher priority.
- **Approach III**. The third approach is similar to Approach II, but we reverse the order of our sequential sort – we first sort all stocks based on our MALTA model and take the top 20% of stocks. Then, we pick the best 100 stocks[14] with the most positive analyst recommendation. In this approach, the quantitative view is assigned a higher priority.
- **Approach IV**. In the fourth approach, we perform an independent sort. We rank all stocks based on our quantitative and fundamental models separately. Then, we take the top quartile of stocks ranked by both models. Please note that in this approach, we cannot control the number of stocks in our portfolio, but the approach above roughly ends up with 100 stocks.

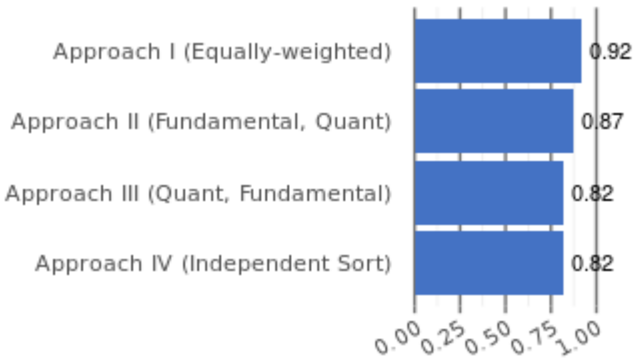
There is no theory to suggest which one of the above four approaches should perform the best. Therefore, it is mostly an empirical question. The results are likely to be different if we change either the quantitative or fundamental models in our backtesting.

As shown in Figure 22(A), all four naïve approaches of combining fundamental and quantitative models outperform the benchmark. In this particular example, the first approach of equally weighting the two models produces the highest Sharpe ratio. The turnover profiles of the four approaches are all very high, but comparable to each other (see Figure 22B).

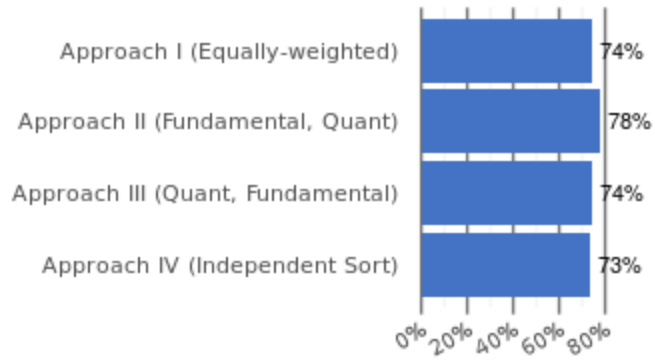
[13] Top 100 is roughly the top quintile.
[14] Top 100 is roughly the top quintile.

Figure 22 Naïve Combination

A) Sharpe Ratio (After Cost)



B) Turnover (One-Way Monthly)



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

QUANTITATIVE PORTFOLIO CONSTRUCTION TECHNIQUES

In addition to screening, another common approach of incorporating quantitative techniques at a fundamental firm is to use the fundamental approach to select stocks to buy and sell, while using quantitative models to construct portfolios, e.g., determining the weight of each stock.

Fundamental portfolio managers typically use heuristic approaches to weight those stocks in their portfolio, with equally weighting and market capitalization weight being the two most common techniques. On other hand, there are many sophisticated quantitative portfolio construction techniques that managers can deploy.

We continue to use analyst recommendations as a proxy for fundamental analysis. We further assume that we want to invest in the top 100 stocks based on analyst recommendations, as our buy list. We then apply each of the following quantitative portfolio construction techniques on these 100 stocks to determine their weights in our portfolio. Technical details about these portfolio construction techniques can be found in *Signal Research and Multifactor Models* (see Luo, et al [2017b]).

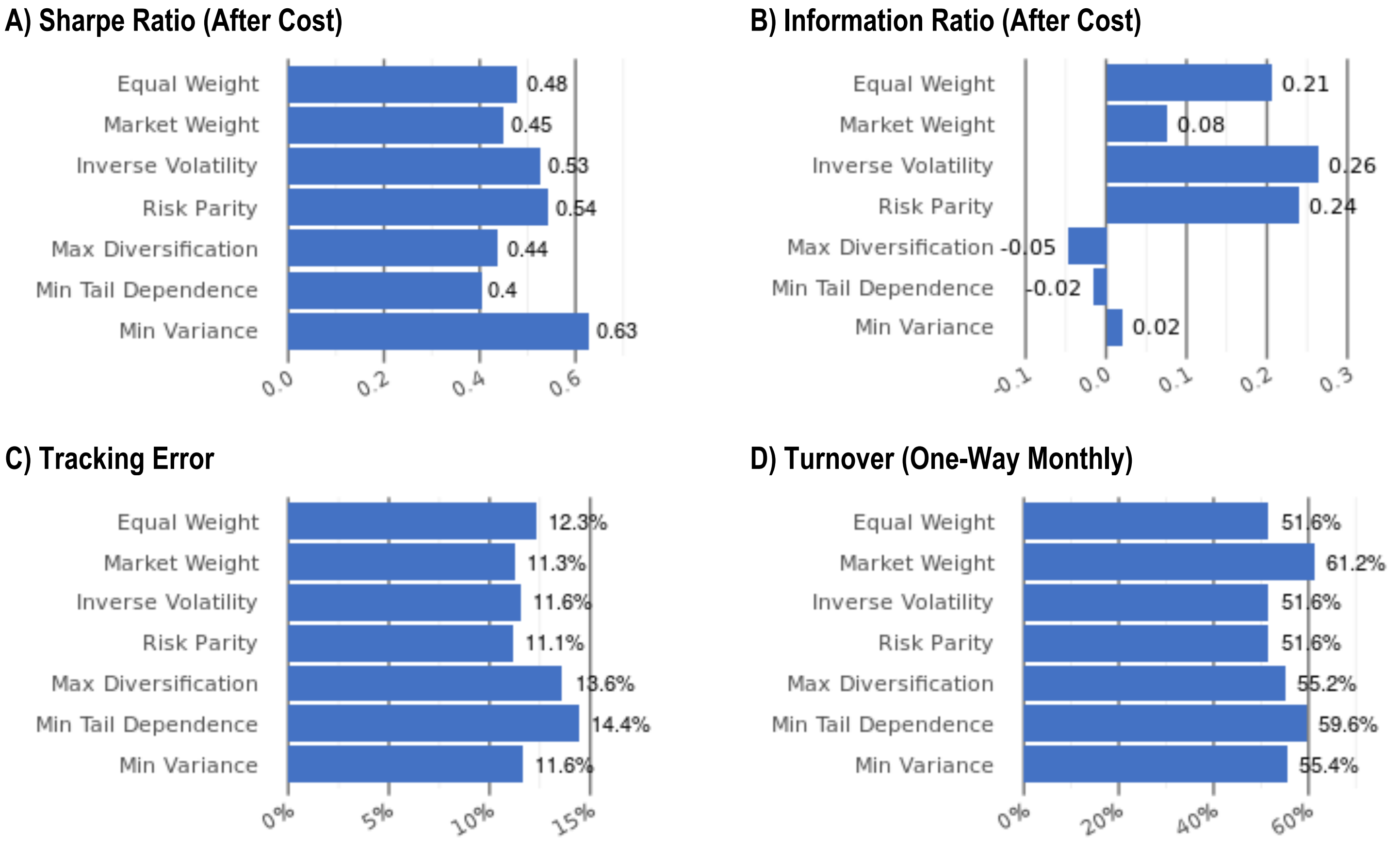
- **Equal weight (EQW).** We equally weight the top 100 fundamental stocks; therefore, each stock receives a weight of 1% in the portfolio, i.e., $\omega_i = 1\%$.
- **Market weight (MKW).** We weight each stock by its market capitalization, i.e., $\omega_i = \frac{MarketCap_i}{\sum_{k=1}^{100} MarketCap_k}$.
- **Inverse volatility (IV).** We weight each stock by the inverse of its volatility, i.e., $\omega_i = \frac{\sigma_i^{-1}}{\sum_{k=1}^{100} \sigma_k^{-1}}$.
- **Risk parity (ERC).** As explained in *Signal Research and Multifactor Models* (see Luo, et al [2017b]), the risk parity algorithm allocates equal risk budget to every stock in the portfolio. When pairwise correlations are the same or when the number of assets is large, risk parity portfolio converges to the inverse volatility portfolio.

- **Maximum diversification (MDP).** Maximum diversification algorithm (see Choueifaty and Coignard [2008]) produces the most diversified portfolio, by overweighting the least correlated assets.
- **Minimum tail dependence (MTP).** Our minimum tail dependence portfolio overweighs assets that are the least correlated to other assets at the tail level. It is closely related to the maximum diversification portfolio, by replacing the correlation matrix with tail dependence matrix[15].
- **Minimum variance (GMV).** The GMV portfolio minimizes the weighted average risk (i.e., variance) of the portfolio. It generally picks stocks with lower risk.

As shown in Figure 23(A), three of the risk-based allocations easily beat the two naïve weighting schemes (equal weight and market capitalization weight) based on Sharpe ratio. The minimum variance portfolio delivers the highest Sharpe ratio, and the inverse volatility portfolio delivers the highest information ratio (see Figure 23B). The maximum diversification and minimum tail dependence portfolios deviated greatly from the benchmark, with the highest tracking error leading to low information ratios, see Figure 23C), and the market weight portfolio, surprisingly, has higher turnover than all other portfolios (see Figure 23D).

[15] As shown in *Signal Research and Multifactor Models* (see Luo, et al [2017b]), tail dependence matrix is typically estimated using a copula model.

Figure 23 Quantitative Portfolio Construction Techniques



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

INCORPORATING FUNDAMENTAL VIEWS USING THE BLACK-LITTERMAN APPROACH

Now, we want to examine how fundamental managers can blend their views into market priors using our Venice tool. Please note that we only incorporate managers' fundamental views in this section, without using quantitative models yet.

Model Design

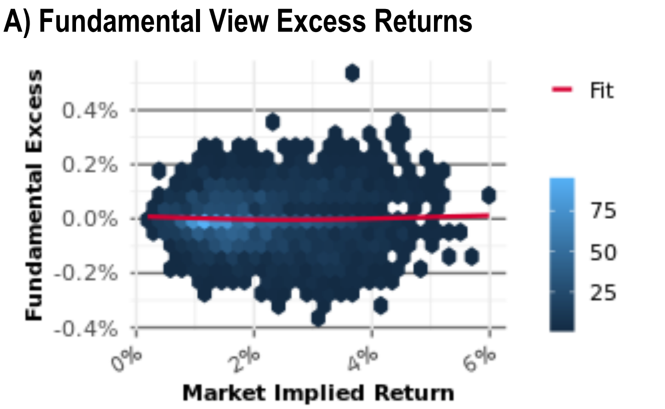
We need to specify the many parameters to perform the BL analysis. Much of the model setup is the same as the quantitative demonstration section above. Here we highlight a few key differences:

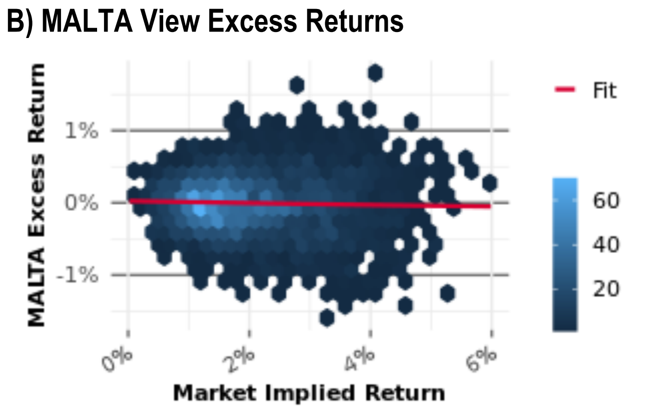
- The View return vector, q, depends on a manager's skill (IC_t). For simplicity, we arbitrarily set it as $IC_t = 5\%$ for fundamental analysis. The 5% IC corresponds to a good manager. The actual average IC of the analyst recommendation signal is only around 1% (see Figure 18C).

Correlation to the Market

The excess returns are uncorrelated with the market implied return for both the Fundamental and MALTA models (see Figure 24). Since the MALTA model has higher predictive power than our proxy of fundamental analysis, the excess view returns are larger.

Figure 24 Correlation to the Market-Implied Return





Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Portfolio Simulation

To measure the impact of the BL model on portfolio performance, we use the Russell 3000 index as our benchmark and construct four portfolios:

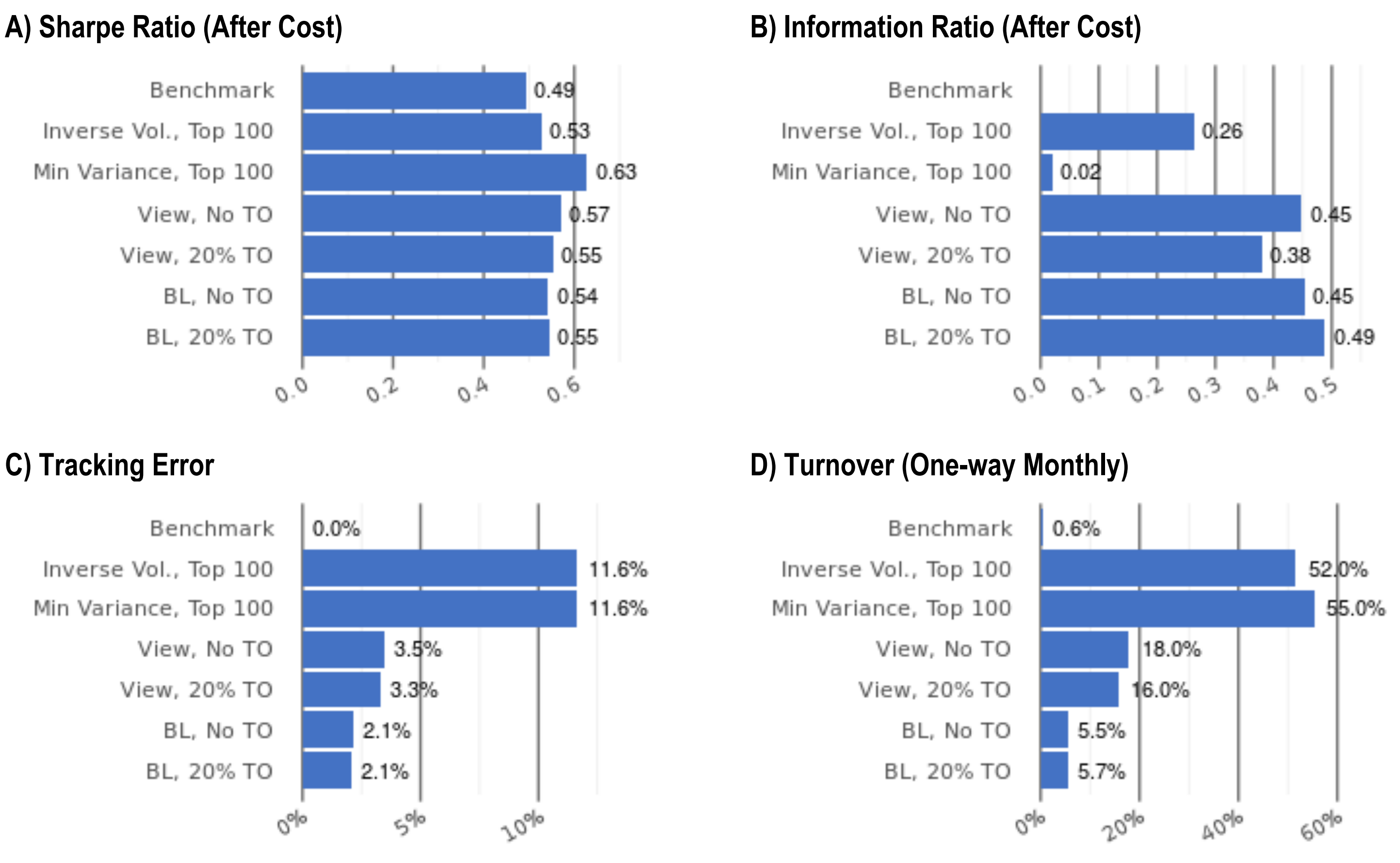
- A long-only active portfolio using only fundamental input (analyst recommendations), without turnover constraint ("FUN View, no TO")
- A long-only active portfolio using only fundamental input (analyst recommendations), with a maximum 20% turnover per month constraint ("FUN View, 20% TO")
- Using the BL approach, with fundamental view, without turnover constraint ("FUN-BL, no TO")
- Using the BL approach, with fundamental view, with a maximum 20% turnover per month constraint ("FUN-BL, 20% TO")

Otherwise, the four portfolios have the same setup:

- Objective function (maximize expected return): $argmax_{\omega}\left(\omega'\pi - \frac{\lambda}{2}\omega'\Sigma\omega\right)$
- Constraint:
  - Long only, i.e., $\omega \geq 0$
  - Fully invested and no leverage $\sum \omega = 1$
  - Maximum 1.5% weight for any single stock, i.e., $\omega \leq 1.5\%$
  - Assuming an average transaction cost of 20bps per trade
  - Monthly rebalance, from December 1996 to present (~25 years)

We compare the BL technique to the two best performing simpler techniques – the inverse volatility weighted portfolio and the minimum variance portfolio. Based on Sharpe ratio, the minimum variance portfolio is still the best performing strategy (see Figure 25A), though its tracking error is quite large (Figure 25C). The four BL portfolios clearly shine from an IR perspective (Figure 25B). The inverse volatility portfolio has a similar Sharpe ratio to the BL portfolios, but its tracking error is also much larger and its information ratio much lower than the BL portfolios. The turnovers of the simpler methods are also much larger than the BL technique (see Figure 25D). Therefore, for fundamental managers who care about relative performance to their benchmarks, the BL technique offers a superior alternative to other naïve portfolio construction techniques, with high IR, low tracking error, and low turnover.

Figure 25 Combining Fundamental Views via BL



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Combining Fundamental and Quantitative Views via the Venice

In this section, we combine fundamental (proxied by analyst recommendations) and quantitative (as measured by the BM and MALTA models) views together, using the BL framework. This is the first area in which we start to see the major benefit of our Venice platform.

We can stack up together our fundamental views and quantitative models to form the pick matrix, view return vector, and view uncertainty matrix. In this case, we assume that the uncertainties between fundamental and quantitative views are uncorrelated.

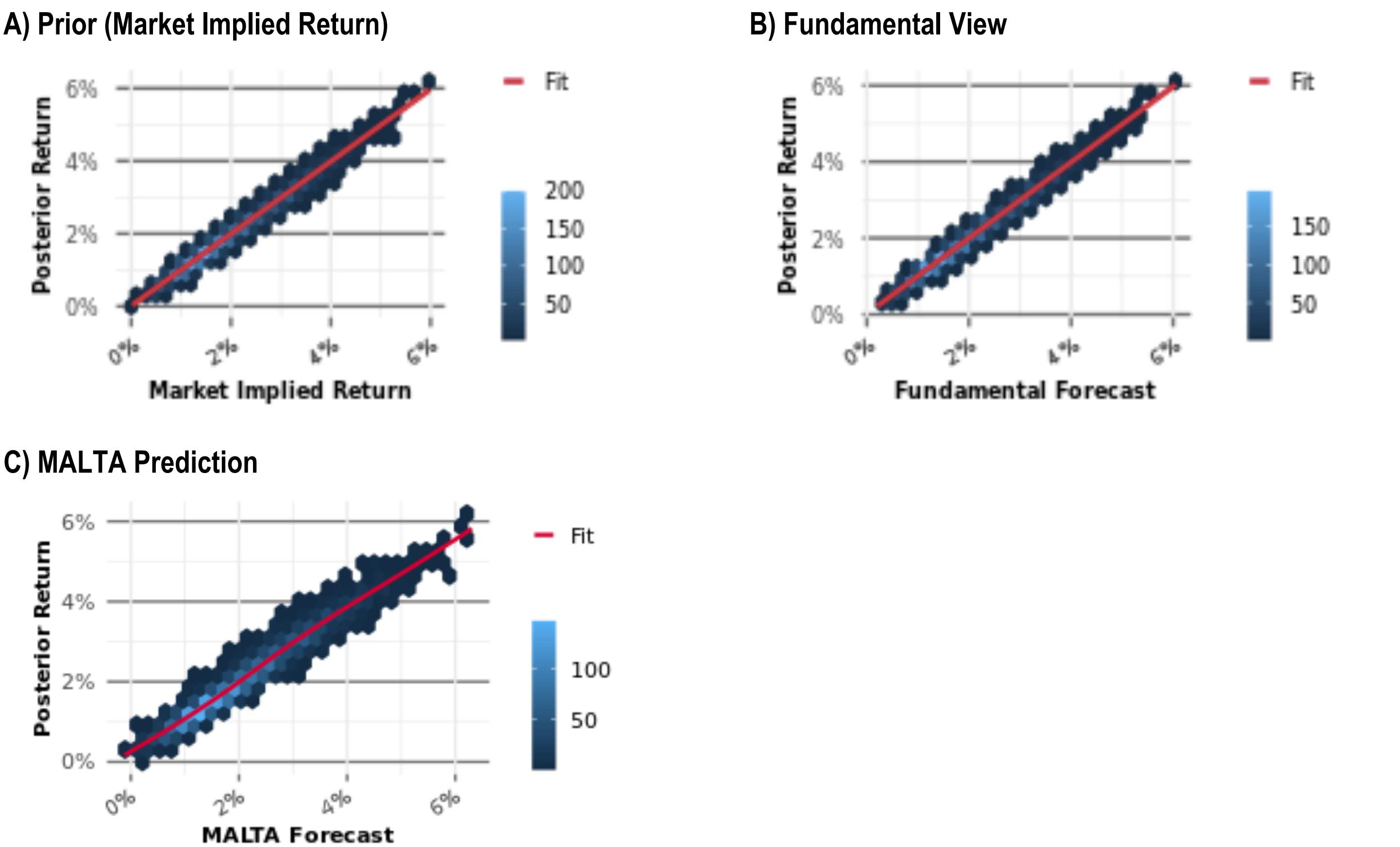
$$P = \begin{pmatrix} P_{Fundamental} \\ P_{Quantitative} \end{pmatrix}$$

$$q = \begin{pmatrix} q_{Fundamental} \\ q_{Quantitative} \end{pmatrix}$$

$$\Omega = \begin{pmatrix} \Omega_{Fundamental} & 0 \\ 0 & \Omega_{Quantitative} \end{pmatrix}$$

Figure 26 demonstrates how the three inputs (market equilibrium returns, fundamental views, and quantitative views) impact the final BL posterior return predictions. The posterior return shrinks the forecast returns toward our prior (i.e., the market implied returns). Since the fundamental forecast has smaller deviations from the market consensus, the shrinkage is not as noticeable as it is for the MALTA predictions.

Figure 26 Model Predicted Return vs BL Posterior Return



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Portfolio Simulation

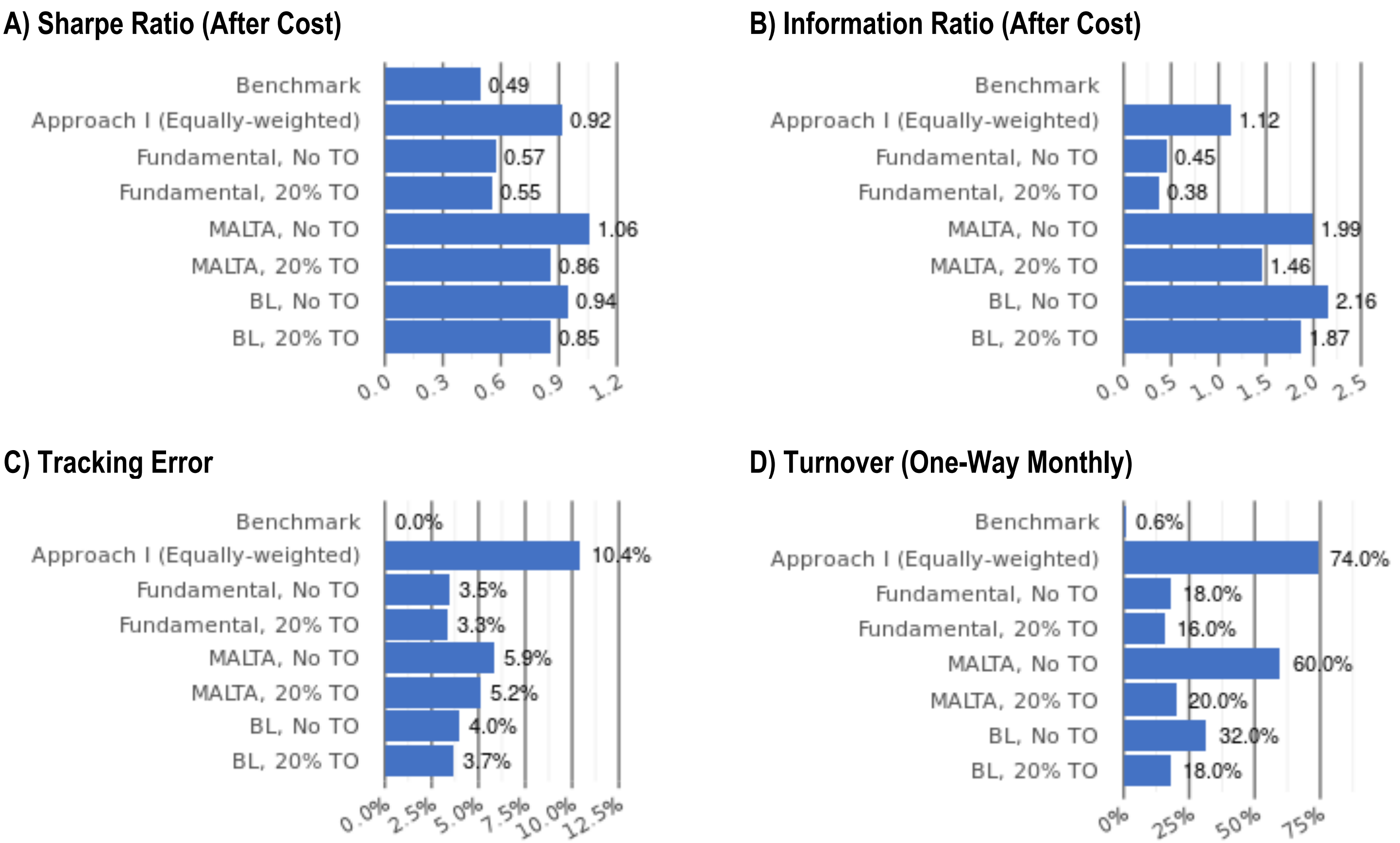
The posterior return prediction is then used to construct the following long-only active portfolios, using our optimizer with the same constraints as specified in the previous sections:

- Fundamental + MALTA portfolio without turnover constraint ("FUN-MALTA-BL, no TO")
- Fundamental + MALTA portfolio with a maximum 20% turnover per month constraint ("FUN-MALTA-BL, 20% TO")

We compare the BL portfolios to the simplistic equally weighted approach from the previous section. Although Sharpe ratios of the BL portfolios is on par with our equally weighted approach (see Figure

27A), the IRs are far superior (see Figure 27B), mostly due to the substantial reduction in tracking error (see Figure 27C). Furthermore, we also observe a contraction in turnover, even for our non-turnover constrained portfolios (see Figure 27D). Clearly, as advertised, the BL approach shrinks our predictions towards market consensus views; and therefore provides less extreme portfolios by reducing tracking error and turnover.

Figure 27 Combining Views via BL



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

How Are These Portfolio Construction Techniques Related?

In this section, we try to put all the different ways of mixing views together. In total, we have studied 18 different approaches (see Figure 28). Please note that the "TopFundamental" and "EquallyWgt" approaches below are essentially the same.

Figure 28 Summary of Strategies

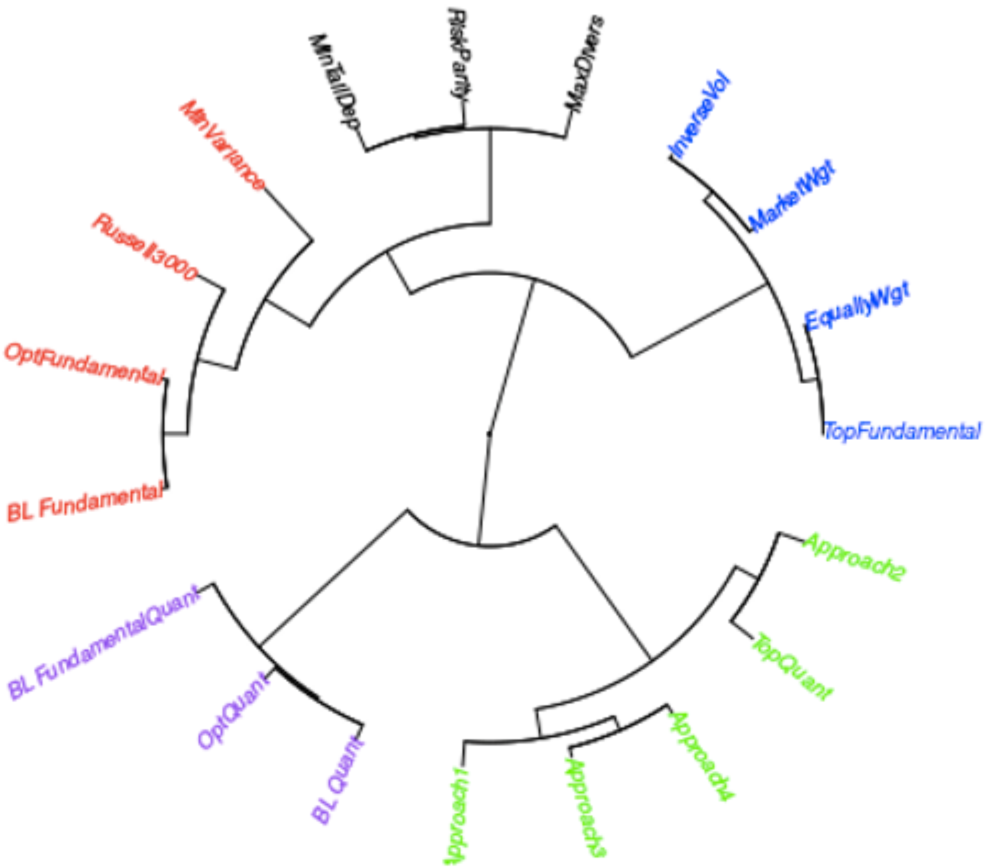
#	Strategy Description	Display Name
1	Index (Russell 3000)	Russell3000
2	Top 100 stocks based on the fundamental model, equally weighted	TopFundamental/EquallyWgt
3	Top 100 stocks based on the quant model, equally weighted	TopQuant
4	Top 100 stocks, by equally weighting the fundamental and quant models	Approach1
5	Top 100 stocks, first by fundamental (top 20%), and then quant (top 20%)	Approach2
6	Top 100 stocks, first by quant (top 20%), and then fundamental (top 20%)	Approach3
7	Top 100 stocks, ranked by both fundamental and quant models	Approach4
8	Top 100 fundamental stocks, equally weighted	TopFundamental/EquallyWgt
9	Top 100 fundamental stocks, market cap weighted	MarketWgt
10	Top 100 fundamental stocks, inverse volatility weighted	InverseVol
11	Top 100 fundamental stocks, risk parity weighted	RiskParity
12	Top 100 fundamental stocks, maximum diversification weighted	MaxDivers
13	Top 100 fundamental stocks, minimum tail dependence weighted	MinTailDep
14	Top 100 fundamental stocks, minimum variance weighted	MinVariance
15	Optimized portfolio, using the fundamental model	OptFundamental
16	Optimized portfolio, using the quant model	OptQuant
17	Black-Litterman portfolio, overlaying the fundamental model	BLFundamental
18	Black-Litterman portfolio, overlaying the quant model	BLQuant
19	Black-Litterman portfolio, mixing the fundamental & quant views	BLFundamentalQuant

Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

The cluster analysis (see Figure 29) suggests that these portfolio construction techniques fall into four main categories:

- **Optimized quantitative strategies (purple).** Due to the exceptional performance and relatively low estimation errors of our quantitative MALTA model, all optimized strategies with a quantitative overlay fall into this cluster. The BLFundamentalQuant is grouped into this cluster due to the superior performance of the MALTA model over the fundamental views.
- **Optimized fundamental strategies (red and black).** The OptFundamental and BLFundamental portfolios form a group of their own and are closely related to the Russell 3000 benchmark due to the relatively weak performance of the analyst revisions signal. There is also a group which includes optimizations performed over the top 100 stocks ranked by the fundamental views.
- **Non-optimized fundamental strategies (blue).** This group contains several naïve fundamental portfolios using the top 100 ranked stocks according to the fundamental views.
- **Blended methods (green).** This group contains the naïve methods for combining the quantitative model with the fundamental model. It also contains the TopQuant portfolio, indicating the strong influence of the top 100 quantitative stocks in the blended models.

Figure 29 Cluster Analysis



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Event Overlay with the Venice

Traditionally, most active asset managers do not proactively use corporate events as an alpha generation tool. Likewise, events are not typically incorporated in most quantitative factor models. There are numerous challenges in incorporating events in the investment process:

- At any given point-in-time, there are only a few companies involved in a given event (e.g., mergers and acquisitions, shareholder activism, stock repurchases). As a result, pure event portfolios tend to be highly concentrated. This is particularly problematic for long-only strategies, as event portfolios would normally lead to huge tracking errors compared to market capitalization weighted benchmarks.
- Event-driven strategies are typically managed by dedicated managers, mostly in the hedge fund space, requiring a different skillset from traditional fundamental and quantitative analysis.
- Events come at an irregular frequency; while traditional stock-selection factors are regularly spaced (e.g., daily, weekly, monthly, or quarterly).
- Event databases can also be expensive; therefore, investors are unsure whether the benefit outweighs the cost.

We have seen some strong interest in event-driven strategies from the investment community in recent years, possibly due to the following reasons:

- Our research suggests that event-driven strategies not only have significant alpha, but also appear to be uncorrelated with traditional quantitative factors (see *The Future of Active Management*, Luo, et al [2019a]).
- We have seen rapid growth in event-driven funds and strategies in recent years, partially driven by the popularity of ESG and SPACs[16].
- Asset owners (e.g., pension funds, foundations, endowments, family offices and sovereign wealth funds), funds of funds, and asset allocation funds are starting to show increased interest in event alpha. Traditionally, when a pension fund wants to capture risk arbitrage premia, it would have to either invest in a risk arb hedge fund or build out an internal team to trade M&A deals internally. Both can be quite expensive. There are systematic approaches that can capture the return stream of such investment styles (see *Systematic Alpha from Risk Arbitrage*, Wang, et al [2018a]).

In this section, we use three event-driven strategies to demonstrate how to incorporate corporate events in active management:

- **Takeover Prediction.** In *Machine Learning Takeover* (see Wang, et al 2017), we explore a large number of both traditional and alternative data (e.g., SEC filings, legal proceedings) to identify potential takeover targets. We then apply a suite of sophisticated machine learning techniques and develop the SMAP (Systematic Merger and Acquisition Prediction) model. The SMAP model provides a daily takeover probability estimate for every stock in the Russell 3000 index. In this section, we only focus on the top 10% (i.e., top decile) of stocks ranked by the SMAP model.

[16] SPACs stand for Special Purpose Acquisition Companies.

- **Systematic Risk Arbitrage.** Risk arbitrage (also known as merger arbitrage or M&A arbitrage) typically involves purchasing the takeover target company's stock (and sometimes also shorting the acquirer's stock at the same time). Traditionally, risk arbitrage strategies are mostly managed by discretionary hedge funds. In *Systematic Alpha from Risk Arbitrage* (see Wang, et al [2018a]) and *Global Systematic Risk Arbitrage – SARA Global* (see Wang, et al [2018b]), we introduce our quantitative event-driven model that trades risk arbitrage deals.
- **Insider Transaction.** In *Seeking Alpha from Insider Transactions* (see Jussa, et al [2018a]) and *Enhanced SIAS* (Jussa, et al [2018b]), based on the asymmetric information hypothesis, corporate insiders (senior management, directors of the board, and controlling shareholders) know more about their own companies than outside investors. Therefore, insider transactions potentially reveal management's confidence about the firm's future outlook. In the SIAS (Systematic Insider Alpha Strategy), we use insider transaction data, combined with fundamental and behavioral overlays, to identify those "high conviction" transactions that are predictive of future firm performance. The SIAS is a highly concentrated portfolio with around 13 stocks on average.

Figure 30 Summary of Event Strategies

#	Strategy Description	Start Date	Avg # of Stocks	Sharpe Ratio
1	SMAP Takeover Prediction	1996:1	296	0.61x
2	SARA Risk Arbitrage Strategy	1999:2	86	0.69
3	SIAS Insider Transaction	2008:7	13	0.69x

Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Incorporating Events

There are multiple ways to incorporate corporate events into an active portfolio. In the following, we will compare three approaches. The first two are more commonly used by practitioners. The third one takes advantage of the BL/Venice framework; and therefore is more complex but more elegant.

Separate Event Portfolios

The simplest way is to have separate event portfolios. For example, we could construct an independent portfolio solely based on shareholder activism. Then we can proceed to combine this event portfolio with other active portfolios. Eventually, it becomes an asset allocation exercise. We essentially treat each event portfolio as an asset class. On the positive side, the event portfolios can be managed separately by different managers. On the negative side, this process adds operational costs and can be inefficient[17]. More importantly, since many types of corporate events are rather infrequent, event portfolios tend to be highly concentrated with potentially high risk.

In this simulation, we develop three separate event portfolios – based on the SMAP, SARA and SIAS, respectively. Stocks in all portfolios are equally weighted. Please note that all three event portfolios are long only. We then combine the three event portfolios with a long-only active strategy based on the BM model as introduced in the previous section (i.e., the "BM-View, no TO" portfolio). Obviously, the stock

[17] For example, one stock could be on the long side based on one event strategy while falling on the short side in a different event strategy. Therefore, we essentially trade the same stock twice, in the opposite direction.

weighting scheme in the event portfolios, as well as the asset allocation decision are somewhat arbitrary.

Event Signals

Alternatively, we could construct factors or signals based on corporate events. Then, we can combine the event factors with our main investment strategy (either fundamental or quantitative). We could add event signals in a naïve way, with an arbitrary weight.

In this example, for all stocks in the SMAP top decile portfolio, we assign a score of 0.5, which roughly corresponds to a $+0.5$ sigma in the benchmark quant model (BM). Similarly, we give scores of +1 and +2 (i.e., 1x and 2x standard deviations of the BM) for stocks in the SARA and SIAS portfolios, respectively. The scores (i.e., 0.5, 1, and 2) for the three events are somewhat arbitrary, but roughly reflect the confidence (performance) and coverage of the three strategies.

Based on the combined BM and event models, the final score is then used to construct a long-only active portfolio, following the same portfolio optimization scheme in the previous sections.

BL View Combination via the Venice

Lastly, this is another classic example that the BL model is designed for. The BL framework is certainly more elegant and intuitive than the first two approaches. However, it is also far more complex – not all managers have the resources to implement the BL model. Our Venice platform greatly eases the BL implementation.

To implement the BL framework, we make the following assumptions:

- **View return.** We use the market-implied return as our prior. Then we add four sets of views – three expressed by the three event models (SMAP, SARA and SIAS) and one by our general-purpose benchmark model BM.

[E37] $$q = \begin{pmatrix} q_{SMAP} \\ q_{SARA} \\ q_{SIAS} \\ q_{BM} \end{pmatrix}$$

- **Pick matrix.** Since both the event and BM models express absolute views, the pick matrix is fairly simple. It is created by stacking the pick matrices from the event models with the pick matrix from the benchmark model.

[E38] $$P = \begin{pmatrix} P_{SMAP} \\ P_{SARA} \\ P_{SIAS} \\ P_{BM} \end{pmatrix}$$

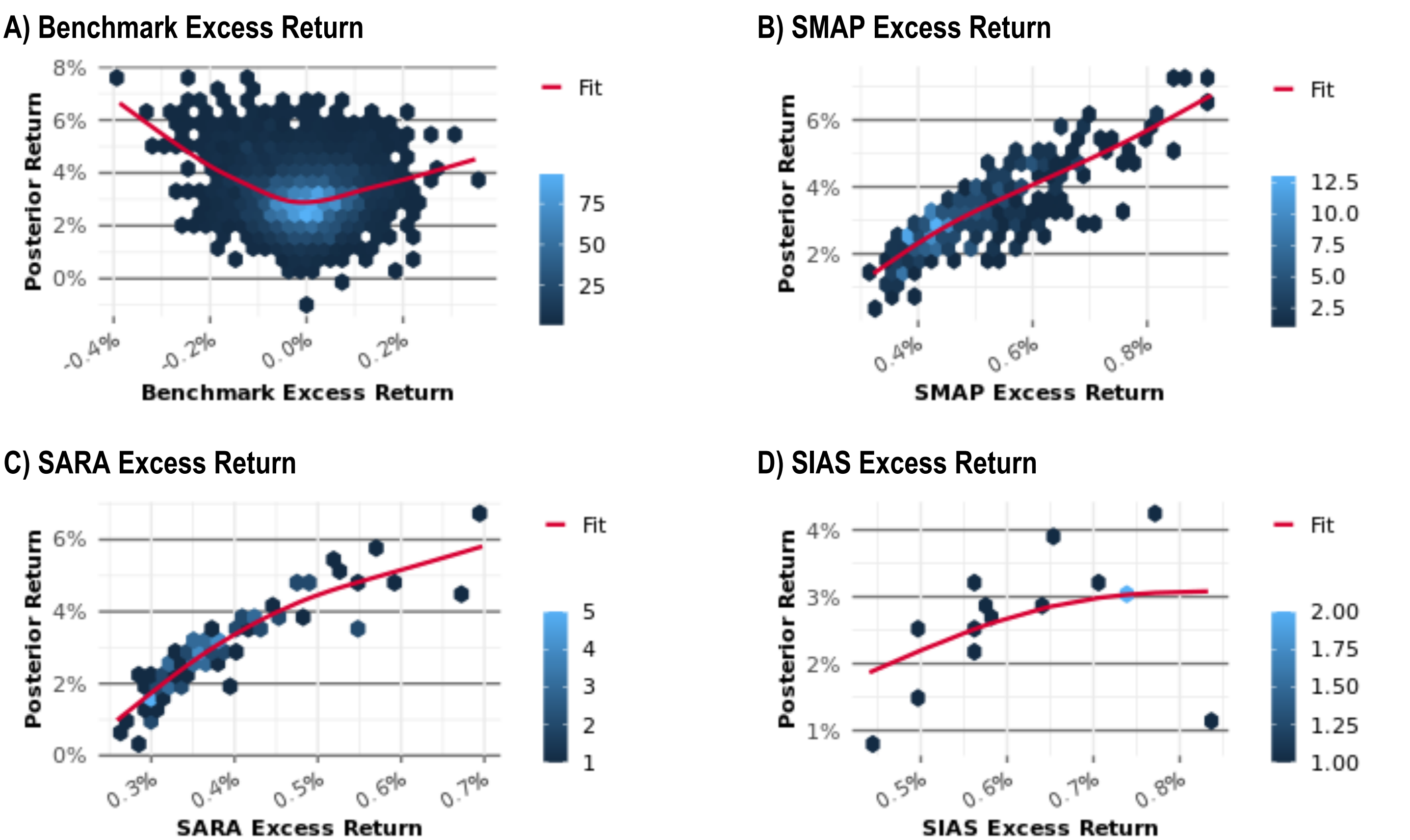
- **View uncertainty.** We define the view uncertainty matrix as $\Omega_{Event,i} = \kappa_{Event,i} P_t' \Sigma_t P_t$. In this case, $\kappa_{Event,i}$ is a constant number, proportional to the Sharpe ratio of event strategy i as shown in Figure 30. The uncertainty matrices from the event models are then combined with that of the benchmark model as shown in Equation [E39].

[E39]
$$\Omega = \begin{pmatrix} \Omega_{SMAP} & 0 & 0 & 0 \\ 0 & \Omega_{SARA} & 0 & 0 \\ 0 & 0 & \Omega_{SIAS} & 0 \\ 0 & 0 & 0 & \Omega_{BM} \end{pmatrix}$$

- **Portfolio Simulation.** The final portfolio is constructed using a mean-variance optimization, following the same process as the "BM-View, no TO" portfolio. Building a portfolio using corporate events can be challenging, due to the small coverage. With the BL framework, we have an expected return for each stock in our investment universe, regardless if there is an event for the stock or not. Constructing a portfolio becomes much easier and the portfolio also tends to be more intuitive.

As shown in Figure 31(A), despite the BM's broad coverage, it has only limited impact on the final BL posterior return predictions. On the other hand, the three event signals appear to be more aligned with the final combined views (see Figure 31B, C, and D). As discussed in *The Future of Active Management* (see Luo, et al [2019a]), the excess returns from most corporate events tend to be uncorrelated to common risk factors, including the overall equity market. Consequently, they exert more significant influence in forming the BL posterior returns than a plain vanilla multifactor model such as the BM.

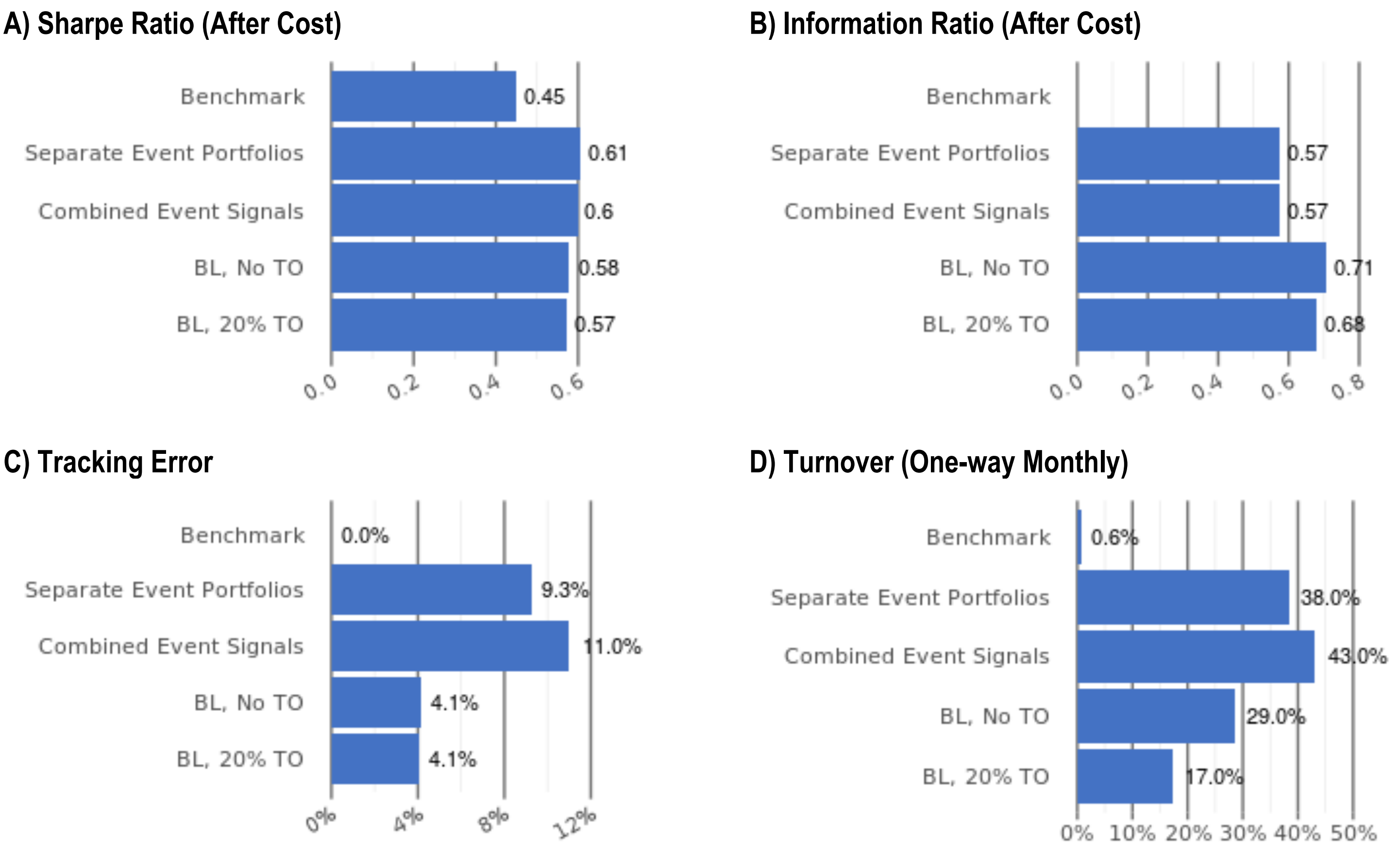
Figure 31 Model Excess Return vs BL Posterior Return



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Although the BL portfolios have similar Sharpe ratios as the two naïve combinations, IRs are much higher, due to the lower tracking error and turnover (see Figure 32).

Figure 32 Event Strategies – Performance Comparison



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

INCORPORATING INDUSTRY-SPECIFIC MODELS

Similar to alternative data and machine learning, the word "quantamental" has been heavily overused. Nonetheless, we strongly believe in the benefit of blending fundamental insights with quantitative techniques. However, it is easier said than done. Given the tremendous difference in investment philosophy, we have seen more failures than successes in bringing the two teams together at many buy-side organizations over the years.

In our opinion, one of the best approaches to blend the depth from fundamental research and the computing power from quantitative analysts is via industry-specific modeling. For each industry, we want to incorporate a few alternative data sources – some are widely used by fundamental analysts (but typically not in a systematic way) and some are not. More importantly, we want to have a thorough understanding of the key drivers and performance metrics used by fundamental analysts for each industry, i.e., the so-called "domain knowledge" in machine learning. Lastly, we borrow the tools from our quantitative library to systematically backtest and implement our industry-specific patterns – this step is called "feature engineering" in machine learning jargon. Given the high barriers of entry, we believe sector-specific models tend to be less crowded than traditional multifactor models.

In the past few years, we have launched six industry-specific models:

- **Global Banking Industry.** In *Banking on the Banks – Welcome to BALI* (see Luo, et al [2018]), we showcase our bank industry-specific model, covering more than 700 banks globally. We take advantage of banking industry data sources such as Compustat's Bank & Thrift package, S&P Capital IQ's and Worldscope's bank templates, and IBES' bank estimates.
- **Global Public Real Estate Sector.** In a series of research (see Luo, et al [2019b, 2019c, 2019d]), we introduce our Gressa (Global Real Estate Stock Selection Alpha) model that covers around 800 real estate stocks – both REITs (Real Estate Investment Trusts) and property stocks globally, using many alternative data sources.
- **Global TMT Sector.** Our TALIA (Technology Adaptive LEAP Insight Alpha) model produces a daily return forecast of nearly 2,000 stocks in the information technology and communication services sectors (see *Global TMT Stock Selection Models – Introducing TALIA*, Rohal, et al [2018]). In the TALIA model, we introduce several alternative data sources, e.g., IDC Trackers, Google Trends, S&P Capital IQ's cable and wireless data, news and social media from Ravenpack.
- **Global Energy Sector.** In *Active Global Energy Stock Investing Guide* (see Wang, et al [2019a], we introduce our Malessa (Machine Adaptive Learning Energy Stock Selection Alpha) model that covers around 450 energy stocks globally, with a total market cap over $3.5 trillion. We have an in-depth discussion on how to properly adjust macro risk (e.g., crude oil, natural gas, interest rates) in energy stock modeling.
- **Global Oil & Gas Exploration & Production Industry.** In *Energy 3.0* (see Wang, et al [2019b]), we developed 150+ true fundamental drivers of Oil & Gas E&P companies, using S&P Capital IQ's industry-specific data. Our DEPTH (Dynamic Exploration and Production Tactical Hedge) model covers around 130 E&P companies globally.

- **Global Health Care Sector.** In *Alternative Data and Machine Learning for the Health Care Sector* (see Rohal, et al [2021a]), we launched our CARE (Clinical trials, Analytical Regulatory NLP, and Event) model that covers around 1,300 health care stocks globally.

As shown in Figure 33, the coverage of the six industry-specific model varies – from Talia's 596 stocks to DEPTH's 71 companies. All six sector models outperform the BM considerably, with much higher Sharpe ratios. The correlation between sector models and BM is modest, ranging from 31% to 64%. Please note that although in this research, we conduct the simulation using US equities, all six sector-specific models are global in nature, covering stocks in the US and 44 other countries.

Figure 33 Summary of Sector-Specific Models in the US

#	Strategy Description	Start Date	Avg # of Stocks	Sector Model Sharpe Ratio	BM Sharpe Ratio	Correlation between Sector Model and BM
1	BALI (Banks)	1997:1	269	3.2x	1.1x	42%
2	Gressa (Real Estate)	2003:2	145	1.2x	0.1x	31%
3	Talia (TMT)	2006:2	596	1.4x	0.3x	41%
4	Malessa (Energy)	1995:2	153	1.7x	0.8x	53%
5	DEPTH (Oil & Gas Exploration & Production)	2007:2	71	1.1x	0.5x	52%
6	CARE (Health Care)	2005:1	432	1.1x	0.4	64%

Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Combining Sector-Specific and General-Purpose Models

Industry-specific models, by design, only cover one specific industry; and therefore tend to have much lower coverage than general purpose multifactor models. This is particularly problematic for long-only strategies, as an individual industry portfolio will normally have a large tracking error to market capitalization weighted benchmarks.

In this section, we discuss and compare three common approaches of mixing sector-specific and general-purpose models.

Separate Sector Portfolios

The simplest way is to form separate sector-specific portfolios. For example, we could construct an independent portfolio solely based on our energy sector-specific model. Then we can combine this energy portfolio with our general portfolio (formed using our general-purpose models). Eventually, it becomes an asset allocation exercise. We essentially treat each sector portfolio as an asset class. On the positive side, sector portfolios can be managed separately. However, it also adds operational complexity and the resulting portfolios can be inefficient[18].

In this simulation, we develop six separate sector portfolios – based on each of the six sector-specific models (BALI, Gressa, Talia, Malessa, DEPTH, and CARE). The six sector portfolios are all long-only and constructed using mean-variance optimization. We then combine the six sector portfolios with a long-only active strategy based on the BM model as introduced in the previous section (i.e., the "BM-View, no TO" portfolio). The six sector portfolios are equally weighted and form half of the total weight.

[18] For example, one stock could be on the long side based on one industry-specific strategy, while falling on the short side in the general-purpose strategy. Therefore, we essentially trade the same stock twice in opposite directions.

The general-purpose portfolio ("BM-View, no TO") forms the other half of the total weight. Essentially, each of the six sector portfolios receive 1/6 of half of the total weight, while the general-purpose portfolio is given a 50% weight. Obviously, the weights allocated to the sector-specific and general-purpose portfolios are somewhat arbitrary.

Naïve Signal Replacement

Alternatively, we could combine sector-specific model prediction (score) and general-purpose model score. The challenge is how to decide the weights between the two sets of models.

In our simulation, we simply replace those stocks that are covered by our sector-specific models with our sector model scores. For those stocks in the sectors without such models, we continue to use the BM model scores. In this case, we essentially assign a 0% weight to the BM and a 100% weight to our sector models for the six sectors that we have sector-specific models (i.e., energy, banks, TMT, real estate, and health care). Because the DEPTH model (upstream oil & gas exploration and production sub-industry) coverage is a subset of the Malessa (energy sector), we only use the Malessa model in this exercise.

Next, based on the Sector + BM model, we construct a long-only active portfolio via the same mean-variance optimization framework.

BL View Combination via the Venice

This is a classic example for which the BL model can be implemented. The BL framework is certainly more elegant and intuitive than the first two approaches. However, it is also far more complex – not all managers have the resources to implement the BL model. Our Venice platform greatly eases the BL implementation.

To implement the BL framework, we make the following assumptions:

- **View return.** We use the market-implied return as our prior. Then we add seven sets of views – six expressed by the six sector-specific models (BALI, Gressa, Talia, Malessa, DEPTH, and CARE) and one by our general-purpose benchmark model BM.

[E40] $$q = \begin{pmatrix} q_{Banks} \\ q_{TMT} \\ q_{RealEstate} \\ q_{Energy} \\ q_{Energy\ (E\&P)} \\ q_{HealthCare} \\ q_{BM} \end{pmatrix}$$

- **Pick matrix.** Since all sector and general-purpose models express absolute views, the pick matrix P can be specified as in Equation [E41].

[E41] $$P = \begin{pmatrix} P_{Banks} \\ P_{TMT} \\ P_{RealEstate} \\ P_{Energy} \\ P_{Energy\ (E\&P)} \\ P_{HealthCare} \\ P_{BM} \end{pmatrix}$$

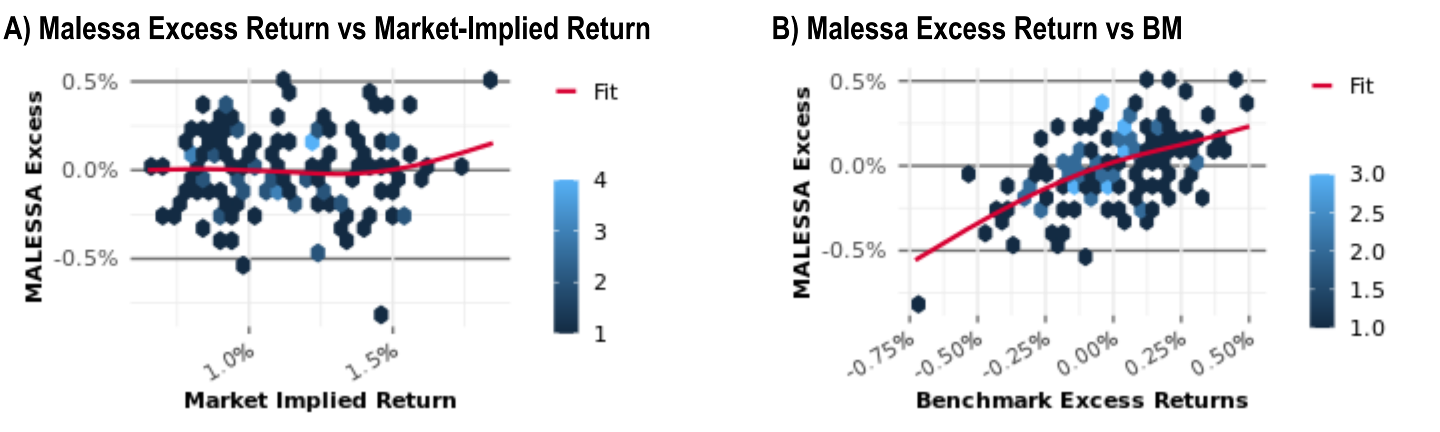
- **View uncertainty.** We define the view uncertainty matrix as $\Omega_{Sector,i} = \kappa_{Sector,i} P_t' \Sigma_t P_t$. In this case, $\kappa_{Sector,i}$ is a constant number, proportional to the Sharpe ratio of sector model i as shown in Figure 33. The uncertainty matrices from the sector models are then combined with that of the benchmark model as shown in Equation [E42].

[E42]
$$\Omega = \begin{pmatrix} \Omega_{Banks} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Omega_{TMT} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_{RealEstate} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega_{Energy} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Omega_{Energy\,(E\&P)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Omega_{HealthCare} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{BM} \end{pmatrix}$$

- **Portfolio Simulation.** The final portfolio is constructed using a mean-variance optimization, following the same process as the "BM-View, no TO" portfolio.

Sector-specific models tend to be very different from market consensus (e.g., the Malessa signal in Figure 34A) and only modestly correlated to the BM (see Figure 34B for the Malessa model as an example).

Figure 34 Sector-Specific Models

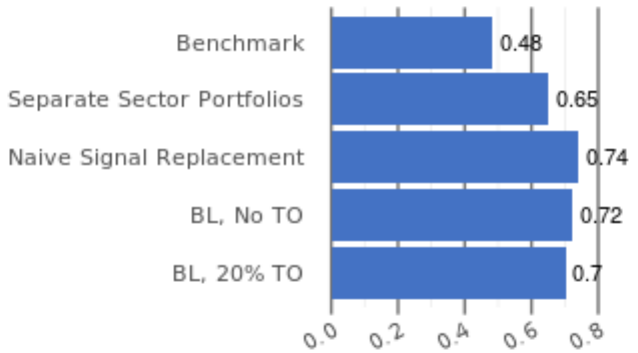


Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

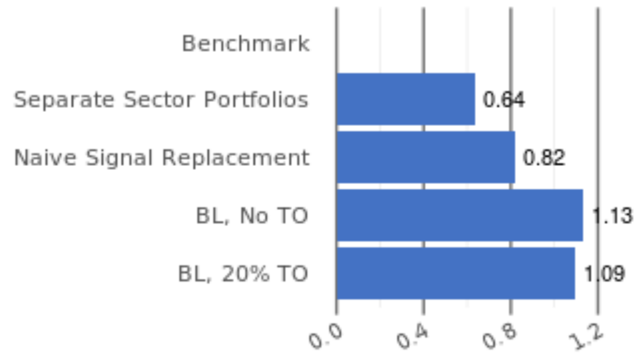
We compare the BL optimized portfolios with the naïve methods described above. The BL portfolios have similar Sharpe ratios to the naïve signal replacement portfolio, but they clearly outperform the separate sector portfolios method (see Figure 35A). The BL method really shines as measured by IR (see Figure 35B). IRs of the BL portfolios are about 0.3x larger than the naïve signal replacement and nearly 0.5x larger than the separate sector portfolios method. This outperformance is achieved with significantly lower tracking error (see Figure 35C) as well as lower turnover (see Figure 35D).

Figure 35 Performance Comparison

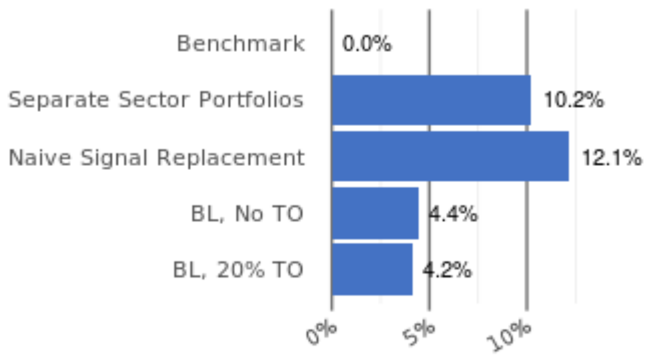
A) Sharpe Ratio (After Cost)



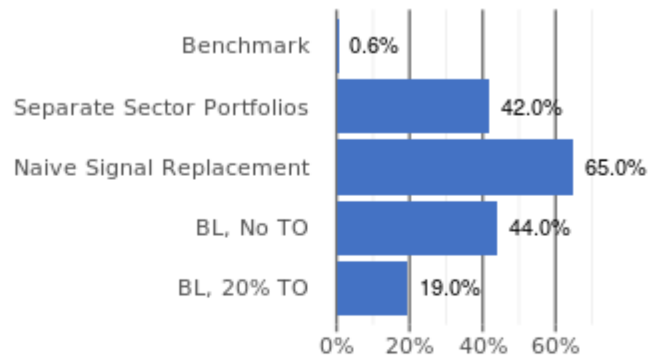
B) Information Ratio (After Cost)



C) Tracking Error



D) Turnover (One-way Monthly)



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

Improved Risk Estimation

So far, we have used the Wolfe QES Standard Risk Model to generate the stock-by-stock covariance matrix for all portfolio optimizations. The Venice tool allows users to import any other risk model they would like to use (e.g., Axioma, MSCI Barra). Our optimizer is then deployed to construct all the optimized portfolios in this research.

In most cases, the BL model is used to combine views in order to have a more accurate and intuitive return prediction. However, another benefit of the BL framework is that it also generates a posterior covariance matrix, which may improve our risk estimation. Here we re-state equation [E13]:

[E13] $$\Sigma_{BL} = \Sigma + Var(\hat{r}) = \Sigma + [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}$$

The BL posterior covariance matrix is the sum of our prior risk estimates (e.g., our standard risk model) and our view uncertainty. The larger our view uncertainty (Ω), the greater the posterior risk (variance) Σ_{BL} tends to be.

Minimum Variance Portfolio Simulation

As detailed in *GEM Portfolio Construction* (see Wang, et al [2021]) and *Thinking (Risk Models), Fast and Slow* (see Wang, et al [2020b]), one of the best ways to measure the accuracy of risk models is via portfolio simulation, especially by constructing Global Minimum Variance (GMV) portfolios. The GMV portfolio does not rely on any return prediction. Rather, it is constructed purely based on risk estimates. Therefore, comparing the two GMV portfolios, one using our original risk model and the other using the new Event-BL posterior risk model, can help us assess the effectiveness of our new risk estimation.

In this section, we use a simple example to show the impact of our BL posterior covariance matrix on portfolio performance. We use our event models as our view and then compute the BL posterior covariance matrix Σ_{BL} as our new risk model. Next, we simulate a GMV portfolio, using the Wolfe QES Standard Risk Model and then our Event-BL posterior covariance matrix, respectively.

- Baseline **GMV-Original Risk Model**. GMV portfolio using the Wolfe QES Standard Risk Model, using the Russell 3000 investment universe
- **GMV-Event-BL Risk Model.** GMV portfolio using the BL posterior covariance matrix as our risk model (our suite of event models serving as the view)

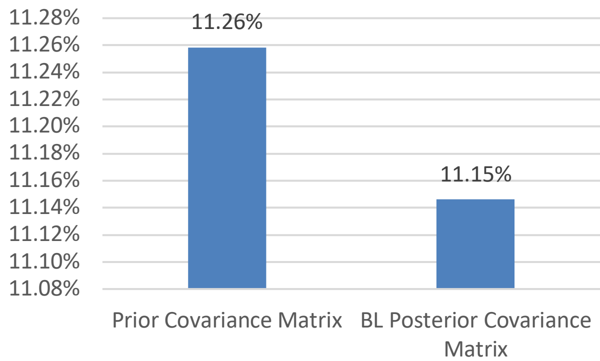
The two GMV portfolios are both constructed in the same way, as defined below:

- Objective function (minimize expected portfolio variance): $argmin_{\omega}\left(\frac{1}{2}\omega'\Sigma\omega\right)$
- Constraint:
  - Long only, i.e., $\omega \geq 0$
  - Fully invested and no leverage $\sum\omega = 1$
  - Assume an average transaction cost of 20bps per trade
  - Monthly rebalance, from December 1996 to present (~25 years)

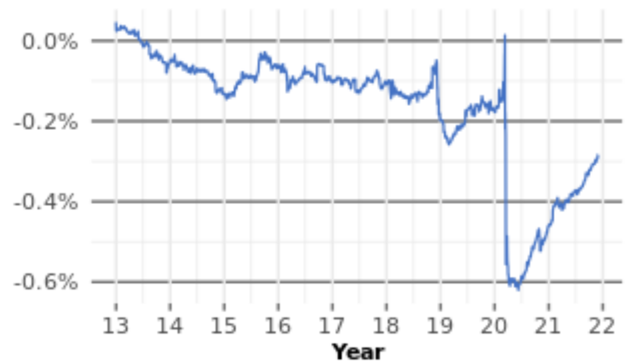
As shown in Figure 36(A), the BL posterior covariance matrix reduces the ex post portfolio risk modestly and consistently (see Figure 36B). The risk reduction is particularly noticeable during the COVID pandemic period (see Figure 36B). This is likely due to the exceptional performance of the event models in recent years, not only in predicting return but also in predicting risk.

Figure 36 Minimum Variance (GMV) Portfolio Realized Volatility

A) Realized Volatility



B) Difference in Realized Volatility



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

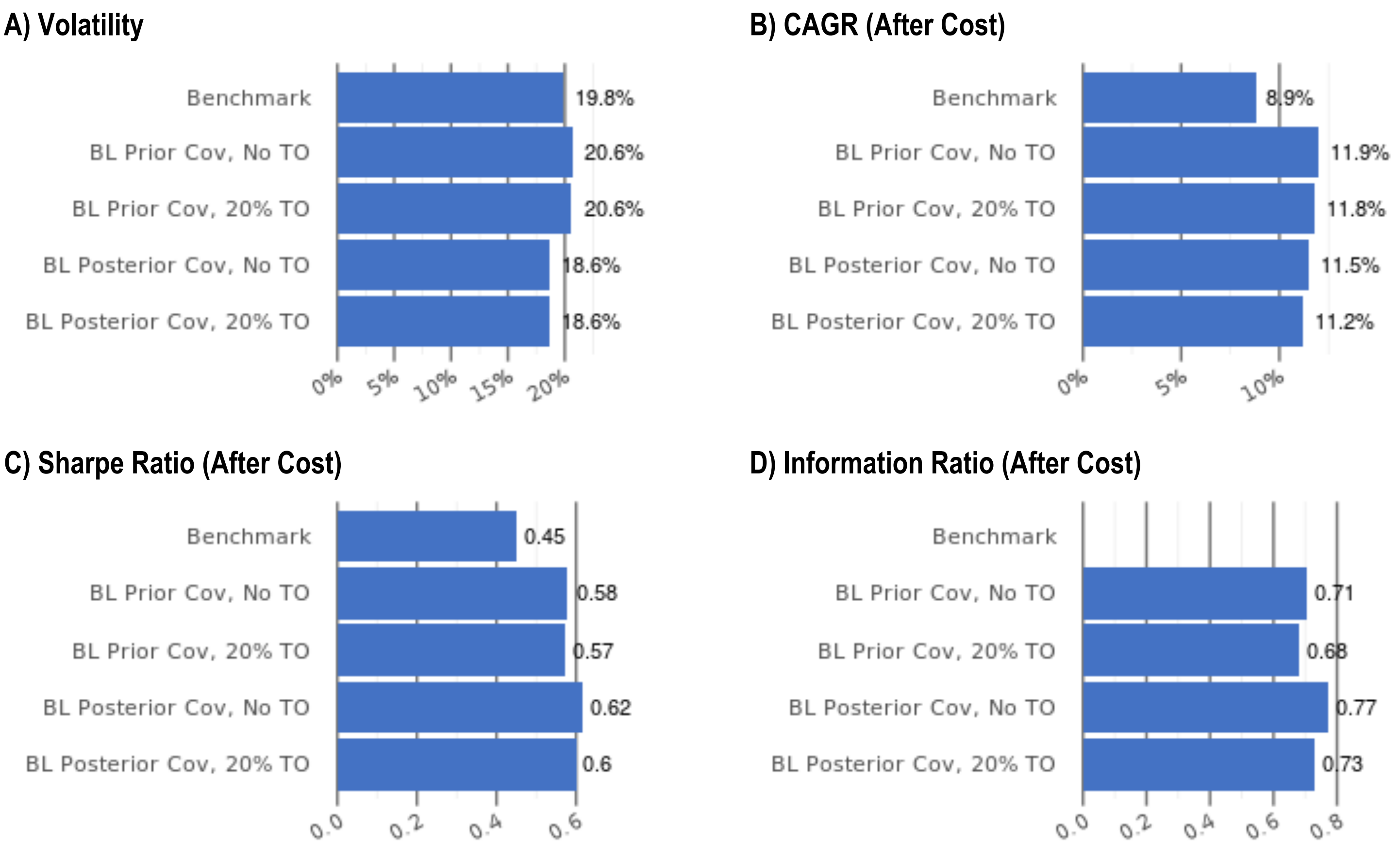
Active Portfolio Simulation

A better risk model is supposed to provide more accurate risk estimates. However, better risk prediction does not necessarily lead to higher Sharpe ratio or IR. Now, let us see how the BL posterior covariance matrix impacts the bottom line of our portfolio.

- **Event-BL Risk Model, no TO.** In this case, we use our Event models as our view (return forecast) to form a long-only active portfolio, with the same constraints as "BM-View, no TO". More importantly, we use our Event-BL posterior covariance matrix as our risk model in the portfolio construction process.
- **Event-BL Risk Model, 20% TO.** In this case, we use our Event models as our view (return forecast) to form a long-only active portfolio, with the same constraints as "BM-View, 20% TO". More importantly, we use our Event-BL posterior covariance matrix as our risk model in the portfolio construction process.

Using the Event-BL posterior covariance matrix meaningfully improves risk-adjusted performance, with a reduction in volatility of 2% (see Figure 37A), and a slightly higher Sharpe ratio (see Figure 37C) and IR (see Figure 37D).

Figure 37 Performance Comparison



Sources: Bloomberg Finance LLP, FTSE Russell, Haver, S&P Capital IQ, Refinitiv, Wolfe Research Luo's QES

CONCLUSION

In conclusion, we first provide a primer on the general theory behind the Black-Litterman framework. Then we present the practical details on how to implement the BL model, using a series of real-life examples, ranging from fundamental analysis, quantitative models, event-driven and pairs trading strategies, to alpha capture programs. All examples in this paper are implemented using our Venice tool, which is an easy-to-use API. The Venice is available as a Python API and fully integrated with our Wolfe QES suite of risk models and portfolio optimizer.

Compared to traditional signal weighting, portfolio allocation, and other naïve combinations, the BL framework is both systematic and elegant. However, specifying all input parameters behind the BL model can be confusing. In particular, investors need to specify their views following the rigid BL set-up – the return expectation and the uncertainty of their opinions.

Through a series of real-life portfolio simulations, we find that the BL framework often leads to stronger risk-adjusted performance (especially as measured by IR), higher transfer coefficients[19], lower turnover, and less extreme and more intuitive portfolios.

- For fundamental managers using both fundamental analysis and quantitative inputs, the BL framework can optimally weight various views based on the uncertainty of each view (e.g., past performance). The BL portfolios tend to have stronger performance, lower turnover, and be more diversified.
- For quantitative managers, we find that the BL framework adds significant incremental value, lifting the Sharpe ratio and IR, while reducing noise and tracking error, even for models with an exceptional track record such as the MALTA model. The BL framework is particularly useful to incorporate models with low coverage (e.g., alpha capture programs, sector-specific models) and unorthodox structures (e.g., event-driven strategies).
- For managers using event-driven strategies, pairs trading, StatArb models, and alpha capture programs, the BL framework can effectively mix the often more sporadic signals into a broader investment universe, in a consistent and efficient manner.

Arguably, the BL model is far from the only approach to combine views. In this paper, we compare the performance of the BL model with other more naïve approaches:

- Linearly weighting the predictions from multiple models
- Stock screens based on sequential sort
- Stock screens based on independent sort
- Weighting stocks using risk-based allocation techniques (e.g., risk parity, minimum variance, maximum diversification, etc.)
- Constructing separate portfolios using each of the return prediction sources

There are many other potential benefits using the BL model and there are many areas of potential improvements. In forthcoming research, we plan to relax the many overly restrictive assumptions

[19] Meaning that our portfolios are better aligned with our views.

underlying the classic BL model, in particular, the multivariate normal distribution assumption of asset return and view distribution, and show the trade-offs of doing so.

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