# Consumption and Asset Returns*

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**Abstract**

The stochastic process for consumption is hard to pin down accurately in the data, yet it is the crucial ingredient of the most widespread class of models in macrofinance. We propose a new way of identifying the joint dynamics of stocks, bonds and consumption, that stems from the central insight of consumption-based asset pricing models but does not rely on strong parametric restrictions. We show that aggregate consumption growth reacts slowly to the common innovations in bond and stock returns. The persistent (but not exceedingly so) moving average component accounts for over 25% of the total quarterly variation in consumption. These common shocks, spanned by both asset classes, explain a large part of the time series variation of stock returns, and a significant (but small) fraction of the time series variation of bond returns, while being able to simultaneously price *both* cross-sections, i.e. capturing not only the standard size, value or industry-sorted portfolios of stocks, but maturity-sorted Tresuries too.

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# I Introduction

Consumption-based asset pricing models have provided many important insights for our understanding of financial markets. However, despite numerous applications, there are still considerable doubts regarding the actual mechanism driving the results. It is also well known that, using consumption data alone, it is hard to identify the properties of the underlying stochastic process of consumption. Nevertheless, assumptions about the consumption persistence and volatility have a profound impact on the ability of macro-finance models to rationalized observed asset prices. As a result, researchers often have to rely on a set of beliefs that are difficult, if not impossible, to validate outside the framework under consideration. Chen, Dou, and Kogan (2015) refer to this source of model fragility as 'dark matter' in asset pricing, – something hard to independently test or detect, yet inevitable to rely on to make sense of the data, given the currently available models.

This paper tries to fill the gap in the understanding and modeling of the consumption process. The intuition behind our identification strategy is rooted in the central insight of the intertemporal Euler equation of models that have consumption as one of the state variables entering the utility function: a variety of shocks affecting the household force it to adjust *both* investment and consumption plans. Therefore, one can use the cross-section of returns to extract the innovations that are reflected in *both* consumption and financial assets. Importantly, we do not take a stand on the origin of these shocks, as they could come from various sources. In fact, returns that reveal information about the state variables of the economy are a feature of almost *any* consumption-based asset pricing model. Our approach, therefore, allows the data to 'speak for itself,' and helps us establish a new set of facts regarding the joint dynamics of stocks, bonds, and consumption growth, without relying on strong parametric restrictions.

Our first result is summarized in Figure 1. We find that consumption growth reacts slowly, but significantly, to the common innovations spanned by both bond and stock returns. Furthermore, these shocks have a clear business cycle pattern and explain most of the time series variation of stock returns (on average about 79%), and a significant, but small, share of the time series variation in bond returns. Importantly, they generate substantial (but not excessive) predictability for future consumption growth, which is significant both statistically and economically, as it accounts for over a quarter of the nondurable consumption growth time variation.

One of the main implications of this slow adjustment in the consumption growth is that the contemporaneous covariance of consumption and returns understates and mismeasures the true risk of an asset. This renders empirically measured risk premia implausible in both their sheer magnitude and in their cross-sectional dispersion. On the contrary, the

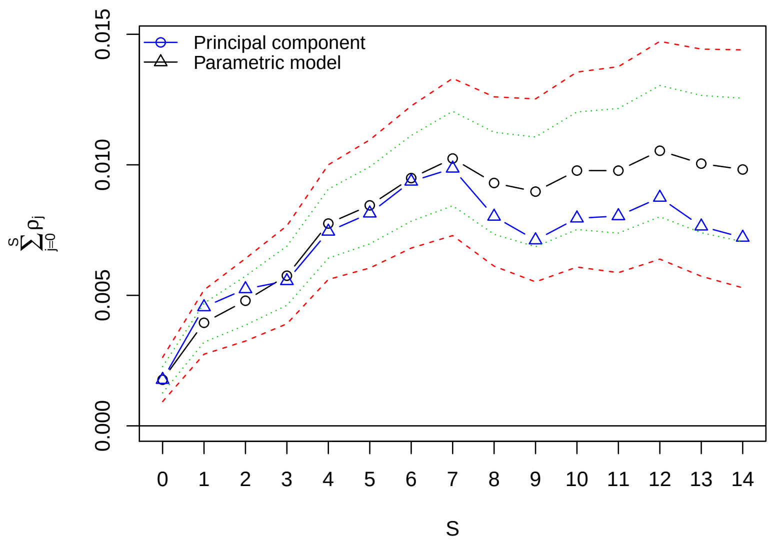



**Figure 1:** Cumulative response function of consumption to $f_t$ shocks.

Posterior means (line with circles) and centered posterior 90% (dashed line) and 68% (dotted line) coverage regions. Triangles denote the first principal component of $cov(r_{i,t}^{ex}, \Delta c_{t,t+S})$. Quarterly frequency.

cumulated response of consumption growth to the common shocks, can *jointly* explain not only the broad cross-section of stock returns (including industry portfolios), but also the average term structure of the interest rates, since the asset exposures increase dramatically in both their *magnitude* and *spread*.

To uncover, identify, and test, the existence of the persistent component within the consumption growth, we employ three different empirical strategies.

First, we set up a flexible (state-space) parametric model to extract common shocks from the financial markets and estimate their propagation patterns within the time series of consumption growth. The timing is crucial here. There is a lot of empirical and theoretical evidence suggesting that consumption could be slow to adjust to changing economic conditions, and therefore we allow consumption growth to react (potentially) with lags to these shocks. We model consumption growth as the sum of two independent processes: a (potentially, since parameters are estimated) long memory moving average component that (potentially) co-moves with asset returns, and a transitory component orthogonal to financial assets. Innovations to asset return are in turn modeled as depending (potentially) on the long memory component of consumption plus an orthogonal source. The joint system allows us to test and identify the presence of the persistent component in consumption, and verify whether it is spanned by the shocks affecting financial markets. To draw the analogy with the time series decomposition, instead of postulating, say, the existence of an AR(1) process for the consumption growth ex ante, we estimate its moving average representation and identify its innovations with the help of asset returns. In other words, the key identification restriction is that these innovations should be reflected in the financial markets *at the same time* as the innovations occur.[1]

The model has important implications not only for the *conditional mean* of the consumption growth, but for its volatility process as well. We find that once the slow-moving component of the consumption growth is properly accounted for, there is no evidence for volatility clustering. This in turn implies that either consumption volatility shocks are only weakly supported by the data, or the class of models we are rely on needs to incorporate a leverage effect in the consumption growth process, i.e. a contemporaneous correlation between a shock to the mean, and a shock to the volatility (which, to the best of our knowledge, has never been implemented in the literature).

Our second empirical approach relies on directly testing the main empirical prediction of the model: a particular form of term structure in the covariances between asset returns and consumption growth. We therefore measure it directly in the data and test whether it implies a similar persistence level to our parametric setting. In fact, Figure 1 confirms that the results are almost identical.

Finally, we confirm these results through a cross-sectional Empirical Likelihood -based estimation of a very broad class of consumption-based asset pricing models that allows for a multiplicative SDF representation as a function of consumption growth and a possibly latent additional component.

To summarize, we find that: *a*) consumption reacts very slowly (i.e. over a period of two to four years), but significantly, to the shocks spanned by the asset returns, and this slow-moving component accounts for about 27% of the time series variation of the consumption process; *b*) conditional on the slow-moving component in consumption growth, models with stochastic volatility in consumption should include a contemporaneous leverage effect; *c*) returns on portfolios of stocks load significantly on these common shocks, with a pattern that closely mimics the value and size pricing anomalies, and this component tends to explain between 36% and 95% of their time series variation; *d*) returns on the US treasury bonds load significantly on the same innovations, with loadings increasing with the time to maturity, but this component explains no more than 3.5% of their time series variation; *e*) most of the variation in bond returns is captured by a single non-spanned factor (i.e. that does not seem to require a risk premium), that is independent from both consumption and stock returns; *f*) total exposure to consumption risk, captured by the factor model and estimated loadings,

[1] Note that we also provide a broad set of supportive evidence for this identification assumption.

can explain between 57% and 90% of the joint cross-section of stocks and bond returns.[2]

## I.1 Related literature

Our paper is related to several large areas in the literature. Naturally, consumption growth, being at the core of one of the most widespread classes of models, has been the focus of many empirical and theoretical studies, and it would probably be impossible to list all the important contributions to this field. Therefore, in this review we focus only those closest to our paper.

The study of the underlying structure of consumption growth, and whether innovations to its mean or volatility should be priced in the cross-section of asset returns has been a subject of a long debate in the literature. In his seminar paper, Hall (1978) examined the random walk hypothesis for the consumption growth, and also documented substantial autocorrelation within the data, that would be inconsistent with this assumption. Hansen and Singleton (1982) were among the first to document the failure of the traditional asset pricing models to match many stylized empirical facts of the returns. This in turn led to the development of many alternative frameworks that suggest alternative mechanisms for the shocks origin and propagation. What makes it particularly hard to accurately test or infer the underlying structure of the consumption process, is that the data is measured with a substantial error and is subject to time agregation, both of which making statistical inference a lot more difficult[3].

In the core of our identification strategy, lies the notion that equilibrium prices of financial assets should be determined by their equilibrium risk to households' marginal utilities and, in particular, current and future marginal utilities of consumption: agents are expected to demand a premium for holding assets that are more likely to yield low returns when the marginal utility of consumption is high i.e. when consumption (current and expected) is low. Our findings are therefore broadly consistent (both qualitatively and quantitatively) with the consumption dynamics postulated by the Long Run Risk (LRR) literature (see e.g. Bansal and Yaron (2004), Hansen, Heaton, Lee, and Roussanov (2007), Bansal, Kiku, and Yaron (2016)), but are also supportive of a broader class of consumption based asset pricing models.

While there is ample empirical evidence suggesting that that either growth or volatility shocks are priced in the cross-section of asset returns (Lettau and Ludvigson 2001, Bansal,

[2]In our baseline specification we consider a cross section of 46 asset given by 12 industry portfolios, 25 size and book-to-market portfolios, and 9 bond portfolio, but the results appear robust to alternative specifications.

[3]See, e.g. Breeden, Gibbons, and Litzenberger (1989), Wilcox (1992), Bansal, Kiku, and Yaron (2016), Kroencke (2017)

Dittmar, and Lundblad (2005), Malloy, Moskowitz, and Vissing-Jorgensen (2009), Savov (2011), and Boguth and Kuehn (2013)), there is much less agreement on what are the relative contributions of these components, the frequency of the shocks, and even the sign of their price of risk. Bansal, Kiku, Shaliastovich, and Yaron (2014) and Campbell, Giglio, Polk, and Turley (2014) argue that volatility shocks are priced, even conditional on those on the mean. Jacobs and Wang (2004), and Balduzzi and Yao (2007) use survey data to estimate the variability of idiosyncratic consumption risk across households, and find that it is priced in the cross-section of portfolios sorted by size and value. Tedongap (2007) estimates conditional volatility of consumption through a GARCH model and finds that the value stocks are more exposed to these innovations, leading in turn to the corresponding risk premium. Bandi and Tamoni (2015) and Boons and Tamoni (2015) go a step further and decompose the process for consumption growth into different frequencies components. They find that only the shock with a half-life of about 4 years time, i.e. business cycle frequency, plays a significant role in explaining the cross-section of returns through the asset exposure to it, thus supporting the importance of the business cycle fluctuations in determining the risk premium. Dew-Becker and Giglio (2016), in contrast, argue that the shocks that are most important for explaining the joint dynamics of the macroeconomic fundamentals and asset returns are those of an extremely low frequency.

Our framework allows us to identify the horizon of the shocks spanned by asset returns and consumption growth, as well as the speed of their propagation. Contrary to most empirical work, we do not take a stand ex ante on the speed of the consumption adjustment and instead work directly with the MA decomposition that pins down the underlying shocks and response to them through the span of the asset returns. This approach of leveraging the information contained in the broad cross-section of financial assets to provide inights about the underlying stochastic process of a state variable, is also similar in spirit to a recent work by Jagannathan and Marakani (2016). They show that the price-dividend ratios of a broad cross-section of asset returns could be used to obtain reliable estimates of the long-run risk factors, implied by the model of Bansal and Yaron (2004). Since both consumption growth and the real risk-free rate are measured with considerable error, it is hard to rely on the market-wide indicators to infer the degree of preidictability within by the model (e.g. whether the log market price-dividend ratio predicts dividend or consmption growth), as those estimates could be sustantially bised. The power of a broad cross-section of the assets, in turn, provides a natural remedie to this problem. Another study closely related to ours that shares the perspective of studying the process for consumption through the lens of asset returns, is Schorfheide, Song, and Yaron (2017) that proposes a bayesian strategy for identifying the long-run component in the mixed frequency setting reconciling the financial and macroeconomic sectors of the economy.

Although the main focus of our paper is on identifying the stochastic process for the consumption growth, our paper is also naturally connected to the large literature on the co-pricing of stocks and bonds.[4] In particular, our focus on the role of macroeconomic risk is related to a series of works that combine the affine asset pricing framework with a parsimonious mix of macro variables and bond factors for the joint pricing of bonds and stocks. In particular: Bekaert and Grenadier (1999) and Bekaert, Engstrom, and Grenadier (2010), that presents a linear model for the simultaneous pricing of stock and bond returns that jointly accommodate the mean and volatility of equity and long term bond risk premia; Brennan, Wang, and Xia (2004), that assumes that the investment opportunity set is completely described by two state variables given by the real interest rate and the maximum Sharpe ratio, and the state variables (estimated using US Treasury bond yields and inflation data) are shown to be related to the equity premium, the dividend yield, and the Fama-French size and book-to-market portfolios; Lettau and Wachter (2011), that focus on matching an upward sloping bond yield term structure and a downward sloping equity yield curve via an affine model that incorporates persistent shocks to the aggregate dividend, inflation, risk-free rate, and price of risk processes; Koijen, Lustig, and Nieuwerburgh (2010), that develops an affine model in which three factors –the level of interest rates, the Cochrane and Piazzesi (2005) factor,[5] and the dividend-price ratio– have explanatory power for the cross-section of bonds and stock returns, while the latter two factors have explanatory power for the time series of these assets; Ang and Ulrich (2012), that considers an affine model in which returns to bonds (real and nominal) and stocks, are decomposed into five components meant to capture the real short rate dynamics as well as term premium, inflation related components (a nominal premium, an expected inflation as well as an inflation risk component) as well as a real cash flow risk element. Finally, our identification of the unspanned latent factor in the bond returns is consistent with a similar finding of Chernov and Mueller (2012). Our analysis also explains the finding of Parker and Julliard (2005) that consumption risk measured by the covariance of asset returns and cumulated consumption growth can explain a large fraction of the variation in average returns across the 25 Fama-French portfolios and, more broadly, on the empirical evidence linking slow movements in consump-

[4]E.g.: Fama and French (1993) expands the original set of Fama and French (1992) stock market factors (meant to capture the overall market return, as well as the value and the size premia), with two bond factors (the excess return on a long bond and a default spread), meant to capture term and default premia; Mamaysky (2002) built upon the affine term structure framework canonically used in term structure modelling (see, e.g., Duffie and Kan (1996)) by adding affine dividend yields to help pricing jointly bonds and stocks.

[5]Cochrane and Piazzesi (2005) find that a single factor (a single tent-shaped linear combination of forward rates), predicts excess returns on one- to five-year maturity bonds. This factor tends to be high in recessions, but forecasts future expansion, i.e. this factor seems to incorporate good news about future consumption.

tion and asset returns (see, e.g., Daniel and Marshall (1997), Bansal, Dittmar, and Lundblad (2005), Jagannathan and Wang (2007), Hansen, Heaton, and Li (2008), Malloy, Moskowitz, and Vissing-Jorgensen (2009)).

The paper is organized as follows. Section II formally defines the concept of slow consumption adjustment risk in a broad class of consumption based asset pricing models. Sections III presents the econometric methodology, while a description of the data is reported in Section IV. Our empirical findings are reported in Section V, we dsicuss the robustness of these results to various assumptions in Section VI, and finally conclude in Section VII. Additional methodological details, as well as robustness checks and additional empirical evidence, are reported in the Appendix.

# II The Model

To model parametrically the reaction of consumption to the same shocks that are spanning asset returns, we postulate that the consumption growth process can be decomposed in two terms: a white noise disturbance, $w_c$ with variance $\sigma_c^2$, that is independent from financial market shocks, plus a (covariance stationary) autocorrelated process–the slow consumption adjustment component–that depends on the current and past stocks to asset returns. To avoid taking an ex ante stand on the particular time series structure of the slow adjustment component (or its absence), we work with its (potentially infinite) moving average representation. That is we model the (log) consumption growth process as:

$$\Delta c_{t-1,t} = \mu_c + \sum_{j=0}^{\bar{S}} \rho_j f_{t-j} + w_t^c; \tag{1}$$

where $\bar{S}$ is a positive integer (potentially equal to $+\infty$), the $\rho_j$ coefficients are square summable, and most importantly $f_t$, a white noise process normalised to have unit variance, is the fundamental innovation upon which all asset returns loads contemporaneously i.e. given a vector of log excess returns, $\mathbf{r}^e$, we have

$$\underset{N\times 1}{\mathbf{r}_t^e} = \underset{N\times 1}{\boldsymbol{\mu}_r} + \underset{N\times 1}{\boldsymbol{\rho}^r} f_t + \underset{N\times 1}{\mathbf{w}_t^r} \tag{2}$$

where $\boldsymbol{\mu}_r$ is a vector of expected values, $\boldsymbol{\rho}^r$ contains the asset specific loadings on the common risk factor, $\mathbf{w}_t^r$ is a vector of white noise shocks with diagonal covariance matrix $\Sigma_r$ (the diagonality assumption can be relaxed as explained below and in Appendix A.1), that are meant to capture asset specific idiosyncratic shocks.

But why measure risk, and price returns, using the slow consumption adjustment framework as in equations (1)-(2) instead of simply contemporaneous risk – i.e. setting $\rho_j = 0 \ \ \forall j > 0$? First, it is a well-known fact that consumption displays excessive smoothness in response to the wealth shocks (Flavin (1981), Hall and Mishkin (1982)), which can be caused by various adjustment costs (Gabaix and Laibson (2001)) and asynchronous consumption/investment decisions (Lynch (1996)). Moreover, the problem could be further exacerbated if the agent has a nonseparable utility function, potentially including labour or other state variables that are also costly to adjust, and hence leading to further staggering in the consumption adjustment in response to wealth innovations. Second, if there is measurement error in consumption, then using a one-period growth rate does not reflect the true pricing impact of consumption shocks in the stochastic discount factor. Indeed, in a recent paper Kroencke (2017) demonstrates that one of the reasons for the failure of the standard consumption-based model to solve the equity premium and risk-free rate puzzles, is that NIPA consumption data is filtered to eliminate the impact of the measurement error. The unfiltered data, in turn, produces substantially better results. The fourth quarter to fourth quarter consumption growth of Jagannathan and Wang (2007), as well as the ultimate consumption risk of Parker and Julliard (2005), are related to the reconstructed unfiltered time series of consumption growth, and therefore provide a better measure for the overall consumption risk.

The dynamic system in equations (1)-(2) can be reformulated as a state-space model, and Bayesian posterior inference can be conducted to estimate both the unknown parameters $(\mu_c,\ \boldsymbol{\mu}_r,\ \{\rho_j\}_{j=0}^{\bar{S}},\ \boldsymbol{\rho}^r,\ \sigma_c^2,\ \Sigma_r)$ and the time series of the unobservable common factor of consumption and asset returns $(\{f_t\}_{t=1}^{T})$. This estimation procedure is described below and additional details are presented in Appendix A.1.

A crucial point that allows us to achieve identification of the shocks, is the lead-lag structure of the consumption process and its (potential) link to asset returns. Without equation (1), the shocks would be underidentified, making it difficult to give any particular rotation a structural interpretation. Another way to think of this estimation approach, is to uncover the shocks that drive financial returns through their impulse response function on consumption, in the spirit of Uhlig (2005) identification in Structural-VARs.

Note that, since $\Delta c_{t-1,t+S} \equiv \sum_{j=0}^{S} \Delta c_{t-1+j,t+j} \equiv \ln\left(C_{t+S}/C_{t-1}\right)$, from the one period consumption growth process in equation (1) we can recover the dynamic of cumulated consumption growth with a simple rotation since

$$[\Delta c_{t-1,t}, \Delta c_{t-1,t+1}, ..., \Delta c_{t-1,t+S}]' \equiv \Gamma\, [\Delta c_{t-1,t}, \Delta c_{t,t+1}, ..., \Delta c_{t-1+S,t+S}]'$$

where $\Gamma$ is a lower triangular square matrix of ones (of dimension S). From this last expression it is easy to see that the $\rho_j$ coefficients identify the impulse response function of consumption to the shock $f_t$, spanned by the asset markets, as

$$\frac{\partial \mathbb{E}\left[\Delta c_{t-1,t+S}\right]}{\partial f_t} = \sum_{j=0}^{S} \rho_j \tag{3}$$

where $\rho_{j>\bar{S}} := 0$.

Equations (1)-(2) rely *only* on the time series properties of asset returns and consumption. And in fact, nothing in the formulation of the joint system directly requires the shocks to be priced in the cross-section of returns, or expected asset returns to align with their exposure to consumption (albeit this is what we would expect in a consumption based asset pricing model). However, this becomes a testable implication, since the covariance between asset returns and consumption growth over one or several periods is fully characterised by the loadings of the dynamic system on the factor $f_t$:

$$Cov\left(\Delta c_{t-1,t+S}; \mathbf{r}_t^e\right) \equiv \sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r. \tag{4}$$

That is, the time series estimates of the latent factor loadings ($\hat{\rho}_j$ and $\hat{\boldsymbol{\rho}}^r$) can be used to assess whether the slow consumption adjustment component has explanatory power for the cross-section of risk premia (via, for instance, simple cross-sectional regressions of returns on these estimated covariances).

The particular one factor structure in the pattern of exposures of asset returns to consumption growth over different time periods in equation (4) also provides us with a natural alternative recovery method of the moving average component in consumption, that does not rely on the parametric likelihood and directly stems from the empirically measured covariances. We discuss this further in Section VI.1.

Finally, note that the formulation in equations (1)-(2) can be generalized to allow for a bonds specific latent factor ($g_t$) to which consumption, potentially, reacts slowly over time. This is an appealing extension since the factor $f_t$, as shown in the empirical section, explains most of the time series variability of stocks, more than a quarter of the variance of consumption growth, but a small share of the time series variation of bonds. The dynamic system in this case becomes:

$$\Delta c_{t-1,t} = \mu_c + \sum_{j=0}^{\bar{S}} \rho_j f_{t-j} + \sum_{j=0}^{\bar{S}} \theta_j g_{t-j} + w_t^c; \tag{5}$$

$$\underset{N\times 1}{\mathbf{r}_t^e} = \underset{N\times 1}{\boldsymbol{\mu}_r} + \underset{N\times 1}{\boldsymbol{\rho}^r} f_t + \left[ \underset{N_b\times 1}{\boldsymbol{\theta}'^b}, \quad \mathbf{0}'_{N-N_b} \right]' g_t + \underset{N\times 1}{\mathbf{w}_t^r}; \tag{6}$$

where $N_b$ is the number of bonds and they are ordered first in the vector $\mathbf{r}_t^e$, $\boldsymbol{\theta}^b \in \mathbb{R}^{N_b}$ contains the bond loadings on the factor $g_t$ –a white noise process with variance normalized to one. Note that in this case the implied covariance of consumption and returns becomes:

$$Cov\left(\Delta c_{t-1,t+S}; \mathbf{r}_t^e\right) \equiv \sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r + \left[ \boldsymbol{\theta}'^b, \quad \mathbf{0}'_{N-N_b} \right]' \sum_{j=0}^{S} \theta_j. \tag{7}$$

Several observation regarding our parametric formulation of the joint dynamics of consumption and asset returns are in order. First, note that both the one-factor (equations (1)-(2)) and two-factor (equations (5)-(6) models are strongly overidentified. Second, the identification of the MA component in the consumption process is basically via the impulse response function of consumption growth, in the spirit of Uhlig (2005) identification in Structural-VARs. Third, estimation of the model would generally remain consistent even in the presence of time varying volatility in the true processes (with the partial exception of the 'leverage effect' case), hence our formulation is robust along this dimension.

# III Estimation

Our empirical analysis is based on both parametric and nonparametric inference, ensuring the results are robust to the methodology employed. The main approach (Section III.1) consists in rewriting the model in equations (1)-(2) in state-space form and emploing standard Bayesian filtering techniques to recover the unobservable latent consumption factor ($f_t$) and other model parameters. Since the model is tightly parametrised, with the factor loadings driving not only the time series, but also the cross-sectional relationships between asset returns, this in turn allows us to assess model performance in both time series and cross-sectional dimensions, using variance decomposition and Fama-MacBeth (1973) cross-sectional regressions.

In Section VI we also test directly the term structure implications of the asset exposure to consumption growth over different horizons to recover the moving average parameters within the consumption growth. In addition, we rely on the standard semi-parametric techniques (e.g. GMM and Empirical Likelihood estimation) to demonstrate that the power of the cross-sectional asset pricing results does not depend on the choice of the methodology. Section VI provides further details on the moment constructions, parameter estimation and tests used for inference.

## III.1 Parametric Inference

We can rewrite the dynamic model in equations (1)-(2) in state-space form, assuming Gaussian innovations, as

$$\mathbf{z}_t = \mathbf{F}\mathbf{z}_{t-1} + \mathbf{v}_t, \quad \mathbf{v}_t \sim \mathcal{N}\left(\mathbf{0}_{\bar{S}+1}; \Psi\right); \tag{8}$$

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{H}\mathbf{z}_t + \mathbf{w}_t, \quad \mathbf{w}_t \sim \mathcal{N}\left(\mathbf{0}_{N+1}; \Sigma\right). \tag{9}$$

where $\mathbf{y}_t := [\Delta c_t, \mathbf{r}_t^{e\prime}]$, $\mathbf{z}_t := [f_t, ..., f_{t-\bar{S}}]'$, $\boldsymbol{\mu} := [\mu_c, \boldsymbol{\mu}_r']'$, $\mathbf{v}_t := \left[f_t, \mathbf{0}_{\bar{S}}'\right]'$, $\mathbf{w}_t := [w_t^c, \mathbf{w}_t'^r]'$,

$$\Psi := \underbrace{\begin{bmatrix} 1 & \mathbf{0}_{\bar{S}}' \\ \mathbf{0}_{\bar{S}} & \mathbf{0}_{\bar{S}\times\bar{S}} \end{bmatrix}}_{(\bar{S}+1)\times(\bar{S}+1)}, \quad \mathbf{F} := \underbrace{\begin{bmatrix} \mathbf{0}_{\bar{S}}' & 0 \\ I_{\bar{S}} & \mathbf{0}_{\bar{S}} \end{bmatrix}}_{(\bar{S}+1)\times(\bar{S}+1)}, \tag{10}$$

$$\Sigma := \underbrace{\begin{bmatrix} \sigma_c^2 & \mathbf{0}_N' \\ \mathbf{0}_N & \Sigma_r \end{bmatrix}}_{(N+1)\times(N+1)}, \quad \mathbf{H} := \underbrace{\begin{bmatrix} \rho_0 & \rho_1 & ... & \rho_{\bar{S}} \\ \boldsymbol{\rho}^r & \mathbf{0}_N & ... & \mathbf{0}_N \end{bmatrix}}_{(N+1)\times(\bar{S}+1)}. \tag{11}$$

and $I_{\bar{S}}$ and $\mathbf{0}_{\bar{S}\times\bar{S}}$ denote, respectively, an identity matrix and a matrix of zeros of dimension $\bar{S}$.

Similarly, the dynamic system in equations (5)-(6) can be represented in the state-space form (8)-(9) with: $\mathbf{z}_t := [f_t, ..., f_{t-\bar{S}}, g_t, ..., g_{t-\bar{S}}]'$; $\mathbf{v}_t := \left[f_t, \mathbf{0}_{\bar{S}}', g_t, \mathbf{0}_{\bar{S}}',\right]' \sim \mathcal{N}\left(\mathbf{0}_{\bar{S}+1}; \Psi\right)$; $\Psi$ and $F$ block diagonal with blocks repeated twice and given, respectively, by the two matrices in equation (10); and with space equation coefficients given by

$$\mathbf{H} := \underbrace{\begin{bmatrix} \rho_0 & ... & ... & \rho_{\bar{S}} & \theta_0 & ... & ... & \theta_{\bar{S}} \\ \rho_1^r & 0 & ... & 0 & \theta_1^b & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ \rho_{N_b}^r & 0 & ... & 0 & \theta_{N_b}^b & 0 & ... & 0 \\ ... & 0 & ... & 0 & 0 & 0 & ... & 0 \\ \rho_N^r & 0 & ... & 0 & ... & 0 & ... & 0 \end{bmatrix}}_{(N+1)\times 2(\bar{S}+1)}. \tag{12}$$

The above state-space system implies the following conditional likelihood for the data:

$$\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\mu}, \mathbf{H}, \Psi, \Sigma, \mathbf{z}_t \sim \mathcal{N}\left(\boldsymbol{\mu} + \mathbf{H}\mathbf{z}_t; \Sigma\right) \tag{13}$$

where $\mathcal{I}_{t-1}$ denotes the history of the state and space variables until time $t-1$. Hence, under a diffuse (Jeffreys') prior and conditional on the history of $\mathbf{z}_t$ and $\mathbf{y}_t$, and given the diagonal structure of $\Sigma$, we have the standard Normal-inverse-Gamma posterior distribution for the parameters of the model (see e.g. Bauwens, Lubrano, and Richard (1999)). Moreover, the posterior distribution of the unobservable factors $\mathbf{z}_t$ conditional on the data and the parameters, can be constructed using a standard Kalman filter and smoother approach (see, e.g., Primiceri (2005)).

When combined with a log linearized consumption Euler equation for a very broad class of asset pricing models,[6] the above specification for the dynamics of consumption and asset returns implies, in the presence of only one latent factor ($f_t$) common to both assets and consumption

$$\mathbb{E}\left[\mathbf{R}_t^e\right] = \alpha + \left(\sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r\right) \lambda_f \tag{14}$$

where $\lambda_f$ is a positive scalar variable that captures the price of risk associated with the slow consumption adjustment risk, and $\alpha \in \mathbb{R}^N$. If consumption fully captures the risk of asset returns, the above expression should hold with $\alpha = \mathbf{0}_N$, otherwise $\alpha$ should capture the covariance between the omitted risk factors and asset returns.

Similarly, if we also allow for a bond specific latent factor ($g_t$), the implied cross-sectional model of returns is

$$\mathbb{E}\left[\mathbf{R}_t^e\right] = \alpha + \left(\sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r\right) \lambda_f + \left[\begin{array}{cc} \boldsymbol{\theta}'^b, & \mathbf{0}'_{N-N_b} \end{array}\right]' \sum_{j=0}^{S} \theta_j \lambda_g \tag{15}$$

with the additional testable restriction $\lambda_f = \lambda_g$.

Equation (14) (and similarly equation (15)), conditional on the data and the parameters of the state-space model, defines a standard cross-sectional regression, hence the parameters $\alpha$, $\lambda_f$ and $\lambda_g$ can be estimated via standard Fama and MacBeth (1973) cross-sectional regressions. This implies that, not only we can compute posterior means and confidence bands for both the coefficients of the state space model and for the unobservable factor's time series, but we can also compute means and confidence bands for the Fama and MacBeth (1973) estimates of the cross sectional regressions defined in equations (14) and (15). That is, we can jointly test the ability of the slow consumption adjustment risk of explaining both

[6] See equation (23) and discussion therein.

the time series and the cross-section of asset returns with a simple Gibbs sampling approach described in detail in Appendix A.1.

# IV Data Description

Bond holding returns are calculated on a quarterly basis using the zero coupon yield data constructed by Gurkaynak and Wright (2007)[7] from fitting the Nelson-Siegel-Svensson curves daily since June 1961, and excess returns are computed subtracting the return on a three-month Treasury bill. We consider the set of the following maturities: 6 months, 1, 2, 3, 4, 5, 6, 7, and 10 years, which gives us a set of 9 bond portfolios.

We consider several portfolios of stock returns. The baseline specification relies, in addition to the bond portfolios, on the 25 size and book-to-market Fama-French portfolios (Fama and French (1992)), and 12 industry portfolios, available from Kenneth French data library. We consider monthly returns from July, 1961 to December, 2017, and accumulate them to form quarterly returns, matching the frequency of consumption data. Excess returns are then formed by subtracting the correponding return on the three-month Treasury bill.

Consumption flow is measured as real (chain-weighted) consumption expenditure on non-durable goods per capita available from the National Income and Product Accounts (NIPA). We use the end-of-period timing convention and assume that all of the expenditure occurs at the end of the period between $t$ and $t+1$. We make this (common) choice because under this convention the entire period covered by time $t$ consumption is part of the information set of the representative agent before time $t+1$ returns are realised. All the returns are made real using the corresponding consumption deflator.

Overall, this gives us consumption growth and matching real excess quarterly holding returns on 46 portfolios, from the forth quarter of 1961 to the end of 2017.

# V Empirical Evidence

We start with presenting preliminary empirical evidence for the power of returns to reveal information about the time-varying mean of the consumption growth process. Section V.1 discusses this point further with regards to the autocorrelation patters, consumption predictability, and long-run response to market-wide shocks within a simple S-VAR. We then turn to the empirical findings obtained via the parametric model presented in Section II.

[7]The data is regularly updated and available at:
http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html

## V.1 Preliminary evidence

Before delving into the estimation of the consumption process discussed in the previous sections, we present preliminary evidence of *a*) substantial autocorrelation in the consumption growth process, *b*) a strong link between consumption predictability and asset return, and *c*) a slow adjustment of consumption to financial market innovations. We establish these results, that motivate our formulation of the moving average process in equations (1)-(2), via three, model-free, empirical observations.

First, Figure 2 plots the autocorrelation function (left panel, with 95% and 99% confidence bands), and the *p*-values of the Ljung and Box (1978) and Box and Pierce (1970) tests of joint significance of the autocorellations, of the one quarter log consumption growth ($\Delta c_{t,t+1}$). The figure clearly shows that the individual autocorrelations are individually statistically significant up to the one year horizon (left panel), and jointly statistically significant (right panel) at the 1% level even after about 15 quarters (and significant at lower confidence levels at even longer horizons). That is, there is substantial persistence in the time series of consumption growth.

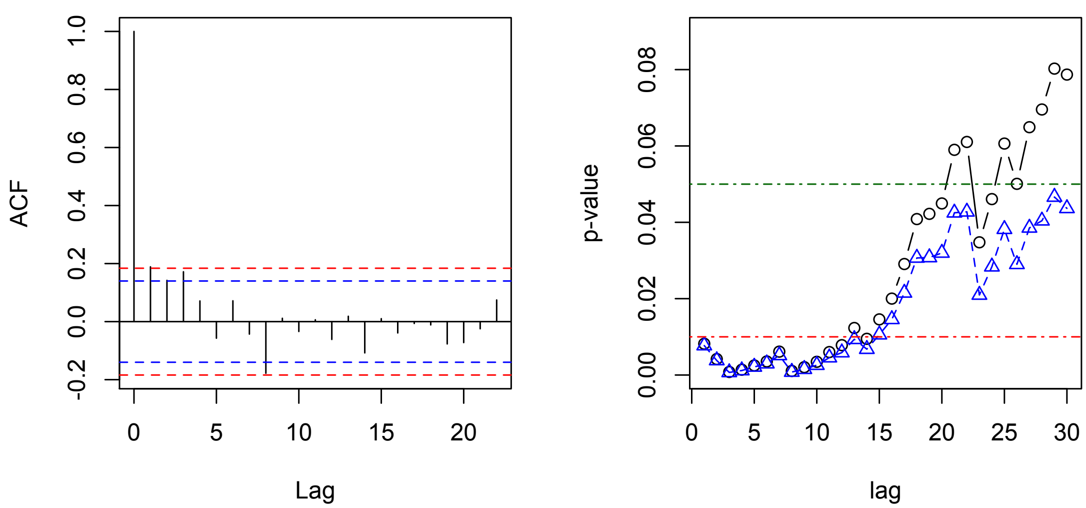



**Figure 2:** Autocorrelation structure of consumption growth.

Left panel: autocorrelation function of consumption growth ($\Delta c_{t,t+1}$) with 95% and 99% confidence bands. Right panel: $p-$values of Ljung and Box (1978) (triangles) and Box and Pierce (1970) (circles) tests.

Second, we run multivariate linear predictive regressions of cumulated log consumption growth $\Delta c_{t,t+1+S}$, for several values of $S$, on the first eight principal components of time $t$ asset returns.[8] Figure 3 depicts summary statistics for these predictive regressions at

[8] We use the first eight principal components of the 25 size and book-to-market Fama-French portfolios, 12 industry portfolios, and 9 bond portfolios, because they explain about 95% of the asset returns variance.

different horizons ($S$). In particular, the left panel plots the time series adjusted $R^2$ of these regressions, and the right panel the $p$-value of the $F$-test of joint significance of the regressors for this and some additional specifications.

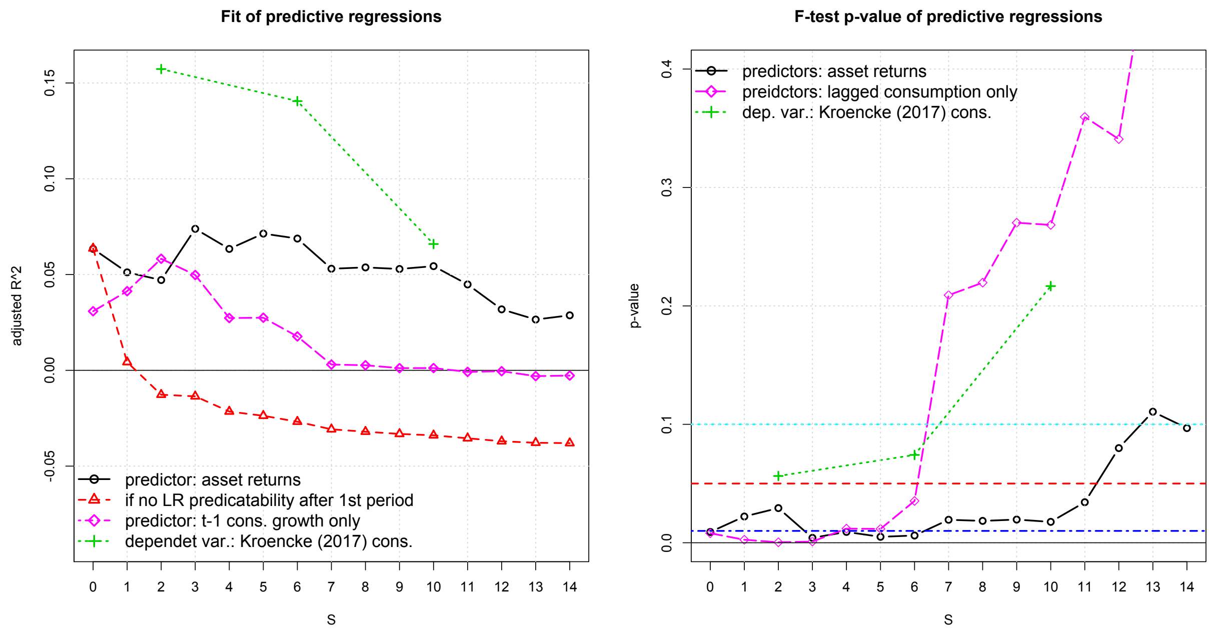



**Figure 3:** Predictive regressions of $\Delta c_{t,t+1+S}$ on time $t$ asset returns.

Predictive regressions of $\Delta c_{t,t+1+S}$ on the first eight principal components of asset returns between time $t-1$ and $t$ for different values of $S$. Left panel: adjusted $R^2$ (black solid line with circles) and theoretical adjusted $R^2$ (red dashed line with triangles) if all the predictability was driven by the first period, and purple dashed line with rhombi stands for the adjusted $R^2$ when only $t-1$ consumption growth is used as a predictor. Dotted line with pluses corresponds to using asset returns to predict the unfiltered consumption growth of Kroencke (2017). Right Panel: $p$-value of the $F$-test of joint significance of the covariates, as well as the 10% (dotted line), 5% (dashed line), and 1% (dot-dashed line) significance thresholds.

Several observations are in order. At $S = 0$ the time series adjusted $R^2$ is quite large being about 6.3%. Moreover, the regressors are jointly statistically significant (the $p$-value of the $F$-test is less than 1%). For $S > 0$, since $\Delta c_{t,t+1+S} \equiv \Delta c_{t,t+1} + \Delta c_{t+1,t+1+S}$, if asset returns did not predict the autocorrelated component of future consumption growth, both the $R^2$ and adjusted $R^2$ should (asymptotically) decrease monotonically in $S$, as depicted by the red dashed lines with triangles in the left panel of Figure 3. Instead, for $S > 0$, the figure shows no such decrease in the data (black line with circles) – actually, predictability increases at intermediate horizons. Moreover, the regressors are jointly statistically significant for basically any horizon up to 15 quarters following the returns.

Could one achieve the same level of predictability by using just consumption data, either due to an independent persistent component propagating through the actual consumption

Using fewer, or more, principal components, or even directly the asset returns series, we have obtained very similar results to the ones reported.

growth, or through accumulated non-classical measurement errors that display a certain degree of persistence?[9] This is unlikely. The purple lines in Figure 3 depict the degree of predictability obtained using just the previous period consumption growth, $\Delta c_{t-1,t}$. While highly significant at the horizon for up to six quarters, using lagged consumption as a predictor is inferior to extracting information from asset returns: not only this variable fails to capture the long range of true predictability, but even at the short horizon it is almost always underperforming stock and bond returns.

Measurement errors in consumption are unlikely to yield such a persistent level of predictability either. While non-classical errors could possibly contribute to a wide range of statistical artifacts, most of their impact should either disappear within a horizon of about one year (should it be related to seasonal smoothing), or be much smaller in magnitude. In order to test this, we repeat the same predictive exercise, using unfiltered consumption data of Kroencke (2017)[10] (green lines on Figure 3). Should the initial result be an accidental by-product of a countercyclical measurement error, coming from the inherent smoothing of consumption levels, once the prediction is run on the unfiltered data, it must go away. If anything, the power of asset returns to forecast consumption growth rates becomes even more apparent. Unfortunately, since only yearly data is available for unfiltered consumption growth, this naturally shortens the sample size, increases standard errors, and leads to the feasible use of only three predictive horizons. However, even taking these limitations into account, asset returns still remain significant predictors of future consumption growth.

The above results highlight that not only there is substantial predictability in consumption growth, but also that asset returns are robust predictors of this growth.

Third, the state-space representation of the slow consumption adjustment process in equations (1)-(2) postulates the presence of long-run shocks in the consumption growth process to which asset returns react only contemporaneously. To verify this conjecture, we recover the long-run impact of common innovations to financial market returns and nondurable consumption using a simple bivariate structural vector autoregression (S-VAR) model for a broad market excess return index and consumption growth.[11]

We achieve identification via long-run restrictions á la Blanchard and Quah (1989). That is, we distinguish a fundamental long-run shock, that can have a long-run impact on both market return and consumption, and a transitory shock that is restricted not to have a

[9] Seasonal smooting of consumption levels often leads to countercyclical measurement errors in the growth rates.

[10] We are grateful to Tim Kroenke for making the data available on his website.

[11] We construct the excess return index as the first principal component of a cross-section that includes excess returns (with respect to the three-month Treasury bill) of the 25 Fama-French size and book-to-market portfolio, 12 industry portfolio, and Treasury securities with maturities of 6 months, 1, 2, 3, 4, 5, 6, 7, and 10 years.

long-run impact on the former. The details of this identification strategy are presented in Appendix A.2.

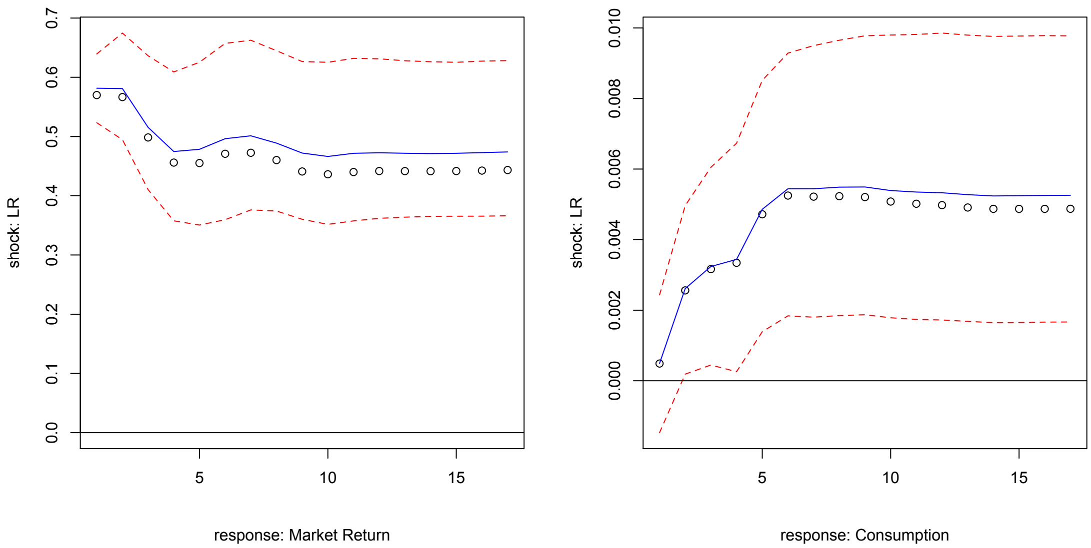



**Figure 4:** Cumulated response functions to a long-run shock.

Left panel depicts the cumulated response function for the excess return index from the composite cross-section of stock and bond returns, while the right panel plots the cumulated impulse response function of consumption growth. The graphs include posterior median (continuous line), mean (circles), and centred 95% coverage region (dashed lines).

Figure 4 displays the cumulated impulse response functions to a one standard deviation long-run S-VAR shock, and highlights a very heterogeneous response of asset returns and consumption growth to this shock. Asset returns (left panel) react fully to the shock contemporaneously, with no statistically significant additional reaction in the periods following the initial shock. Instead, consumption growth (right panel) reacts gradually to the shock over several quarters, with the cumulated effect reaching its peak only 5-7 quarters after the initial shock. These patterns are consistent with the moving average process postulated in equations (1)-(2). Moreover, as shown in the next section, the reaction of consumption to the long-run shocks is extremely similar to the one implied by the estimated loadings of the persistent consumption component on the common latent factor $f$.

## V.2 Parametric evidence

While our model in equations (1)-(2) allows for a potentially infinite number of lags for the consumption process, in order to proceed with the actual estimation one has to choose a particular value of $\bar{S}$. For the rest of the section we use $S = 15$ for a number of reasons.

First, equation (1) implies a certain autocorrelation structure of the nondurable consumption growth through the combination of the common factor lags and its loadings. Hence, the value of $S$ should be high enough to capture most of the time series autocorrelation in the consumption growth. Figures 2-4 of the previous section suggest that most of the dependence is captured by the first 15 lags, hence this value is a natural choice for the lag truncation.

Second, intuitively most of the pricing impact from the consumption adjustment is probably taking place within the business cycle frequency, consistent with a number of recent empirical studies (e.g. Bandi and Tamoni (2015)). Therefore, $S = 15$ is a rather conservative choice, since it provides a 4 year window to capture most of the interaction between the consumption and returns.

Finally, our results remain robust to including additional lags.

We start our analysis by examining the time-series properties of a one (common) factor model implied by equations (1)–(2). We then turn to the 2-factor specification described by equations (5)–(6). Finally, we present the cross-sectional properties of the model and demonstrate that the slow consumption adjustment risk is a priced factor, explaining a significant proportion of the cross-sectional variation in returns.

### V.2.1 The consumption mean process

The consumption growth process in equations (1) and (5) is similar to the moving average decomposition, and allows us to model the dynamics of the slow consumption adjustment ($\Delta c_{t,t+1+S}$) in response to a common and/or a bond-specific shock. Figure 5 depicts the loadings of consumption on the latent factor $f$ as a function of the horizon $S$. If $S = 0$, the case of a standard consumption-based asset pricing model, the moving average component of the consumption virtually does not load on the common factor. This is expected, since the factor manifests itself at a lower frequency. Indeed, as $S$ increases, the impact of the common factor becomes more and more pronounced, levelling off at around $S = 11$. Interestingly, the pattern of the loadings observed in our two-factor model, is similar (at least at the horizons we consider) to the one implied by the moving average representation of the consumption process in Bansal and Yaron (2004)[12]. In short, our setting reveals a similar degree of persistency and response rates, as their consumption process. The pricing of stocks and bonds, however, differs, because we consider a more flexible, reduced form model that nevertheless uncovers a very similar consumption-related pattern in the data as the one implied by the long-run risk model. Note that allowing for a bond specific latent factor leaves the consumption loadings on the $f_t$ shocks virtually unchanged and consumption does not load significantly on the bond specific factor $g_t$ (see, respectively, figures A1a and A1b in the Appendix).

[12]For more details on the construction of the MA representation for the Bansal and Yaron (2004) framework see Appendix A.3

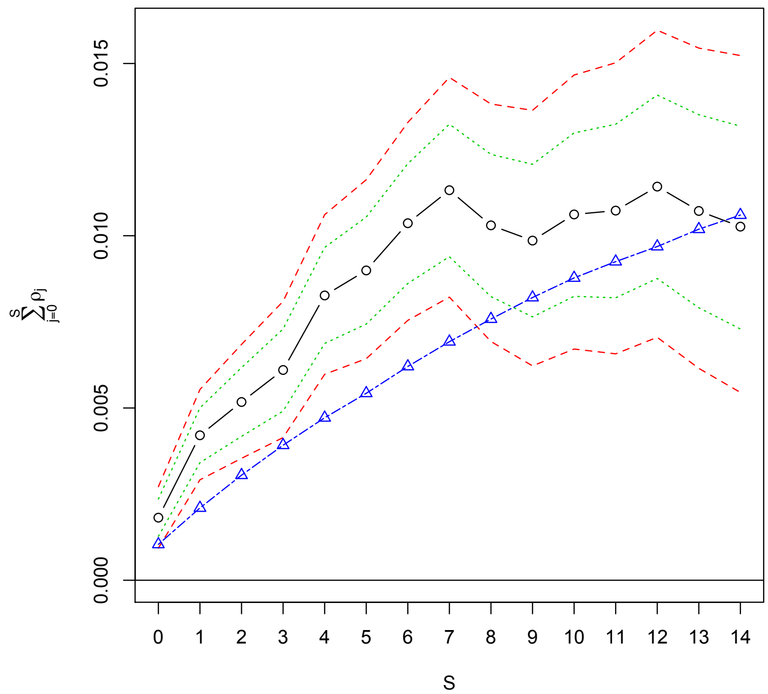



**Figure 5:** Cumulated consumption adjustment response to the common factor ($f_t$) shock.

Posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions. Estimation based on the one-factor model in equations (1) and (2). Triangles denote Bansal and Yaron (2004) implied values.

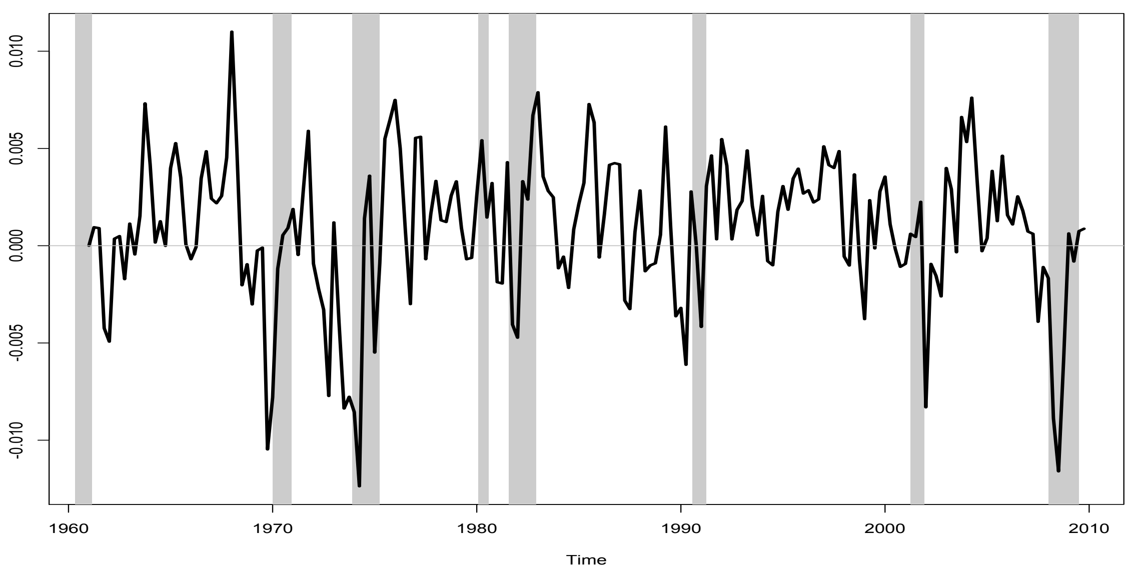



**Figure 6:** Moving average of $f_t$ component of consumption growth

The graph presents the moving average of $f_t$ component of consumption growth, based on the posterior median estimates for the $f_t$ shocks. Grey areas denote NBER recessions.

Figure 6 shows the median filtered MA component of consumption, based on our estimates of $f_t$ innovations. The slow moving component within consumption aptly captures the business cycle fluctuations and has a pronounced exposure to recession risk. How big is the long-term impact of these shocks? Slow consumption adjustment explains a significant proportion of the time series variation in consumption growth. As Figure 7 demonstrates, the common factor is responsible for roughly 27% of the variation in the one-period nondurable consumption growth, 33% of the two-period consumption growth, and so on, followed by a slow decline towards just above 5% for the 15-period growth. Interestingly, the model retains significant predictive power (albeit, much lower) even for the one-period consumption that will occur nearly 4 years from now. As shown in Figure A2 of the Appendix, adding a bond specific factor has a minimum impact on the explanatory power of the model for future consumption growth.

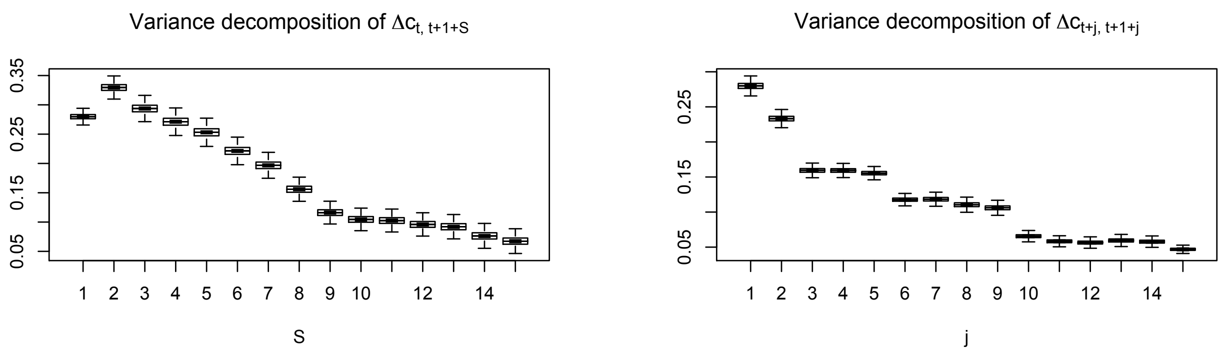



**Figure 7:** Shore of consumption growth variance explained by the MA component $f$.

Box-plots (posterior 95% coverage area) of the percentage of time series variances of consumption growth explained by the MA component $f$. Left panel: cumulated consumption growth $\Delta c_{t,t+1+S}$. Right panel: one period consumption growth $\Delta c_{t+j,t+1+j}$.

### V.2.2 Clustering and predictability of consumption volatility

Since the estimation of the conditional mean of the consumption growth process in equations (1) and (5) would be consistent even in the presence of time varying conditional consumption volatility, the presence of volatility clustering can be assessed by analyzing the serial correlation of the squared one-step ahead forecast errors (see, e.g., Engle (1982)) of the consumption process. That is, by examining the autocorrelation and predictability of $\widehat{Var}_t\left(\Delta c_{t,t+1}\right) := \left(\Delta c_{t,t+1} - \widehat{\mathbb{E}}_t\left[\Delta c_{t,t+1}\right]\right)^2$, where the conditional mean is computed at each $t$ using the estimated $\rho_j$ and $\theta_j$ coefficients and latent state variables $f_{\tau\leq t}$ and $g_{\tau\leq t}$.[13]

[13] That is, $\widehat{\mathbb{E}}_t\left[\Delta c_{t,t+1}\right]$ is the posterior mean of $\mu_c + \sum_{j=1}^{\bar{S}} \rho_j f_{t+1-j} + \sum_{j=1}^{\bar{S}} \theta_j g_{t+1-j}$ at each $t$.

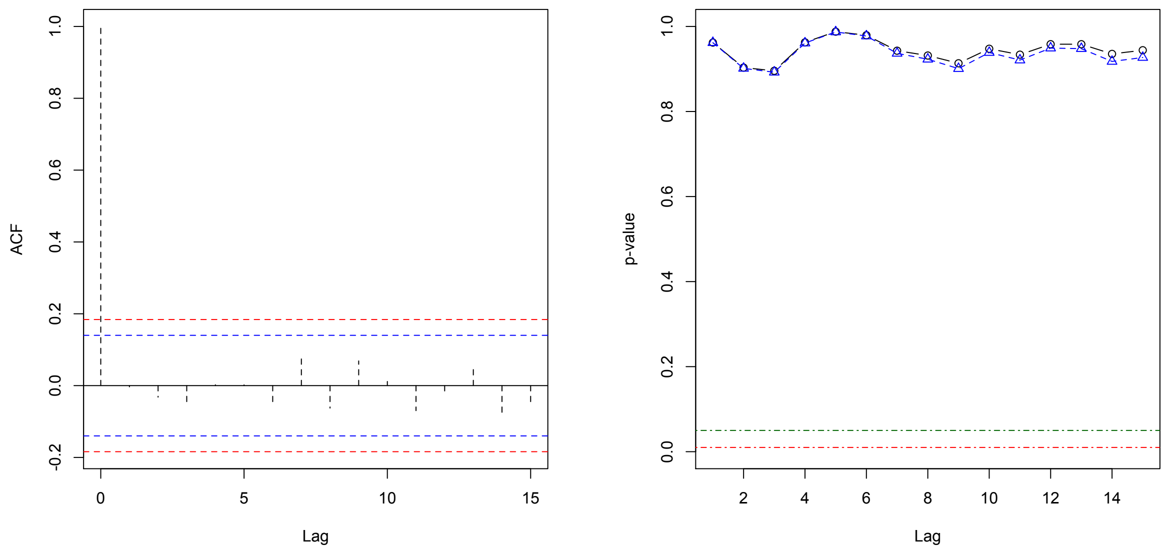



**Figure 8:** Autocorrelation structure of consumption growth squared forecast errors.

Left panel: autocorrelation function of $\widehat{Var}_t\left(\Delta c_{t,t+1}\right)$ with 95% and 99% confidence bands. Right panel: $p$-values of Ljung and Box (1978) (triangles) and Box and Pierce (1970) (circles) tests.

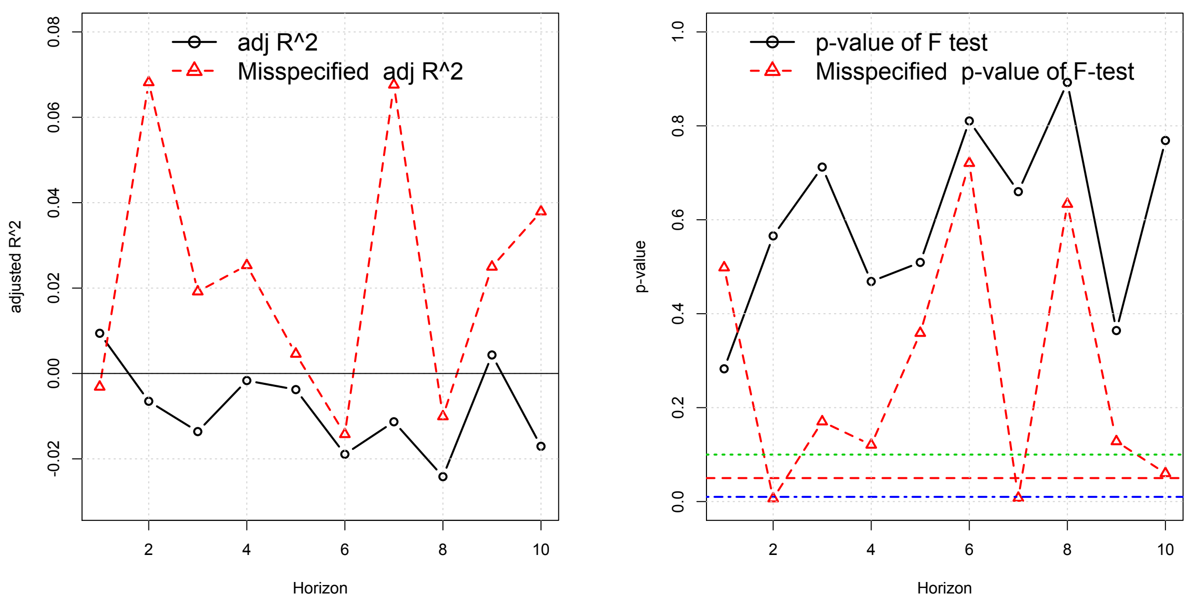



**Figure 9:** Predictability of consumption squared forecast errors.

Predictive regressions of $\widehat{Var}_{t+h}\left(\Delta c_{t+h,t+h+1}\right)$ on the time $t$ first eight principal components of asset returns at several horizons $h$. Left panel: adjusted $R^2$. Right Panel: $p$-value of the $F$-test of joint significance of the covariates as well as the 10%, 5%, and 1% significance thresholds (respectively, horizontal dotted, dashed and dot-dashed lines). Continuous lines with circles denote statistics for the correctly specified conditional mean for the consumption growth process, while dashed lines with triangles depict statistics under the assumption of a constant conditional mean.

Figure 8 reports the autocorellation function (left panel), as well as the $p$-values of the Ljung and Box (1978) and Box and Pierce (1970) tests (right panel) of joint significance of the autocorrelations of $\widehat{Var}_t(\Delta c_{t,t+1})$ and shows no evidence of volatility clustering in the consumption growth process. Nevertheless, conditional consumption volatility might still, in principle, be correlated with financial asset returns. We test this hypothesis by running linear predictive regressions of $\widehat{Var}_{t+h}(\Delta c_{t+h,t+h+1})$, at several horizons $h$, on the time $t$ first eight principal components of stock and bond returns (the same test used to establish predictability of the first conditional moment of consumption growth in Section V.1). Summary statistics for these predictability regressions are reported in figure 9. The continuous lines with circles depict the adjusted $R^2$ (left panel) and the the $p$-value of the joint significance $F$-test of the regressors (right panel). At any horizon, the adjusted $R^2$ is either negative or extremely close to zero, and the $p$-values (that range from 0.2826 to 0.922) show that asset returns are not significant predictors of future consumption volatility.[14]

It is important to notice that, given our finding of a common latent factor driving both asset returns and consumption growth, if one were to erroneously model the conditional mean of consumption growth, one would be likely to find spurious evidence of volatility clustering in the consumption process. For instance, erroneously modeling the conditional mean as being constant, the autocorrelation of $\widehat{Var}_t(\Delta c_{t,t+1})$ would be different from zero.[15] This in turns would make it more likely to find spurious predictability of the consumption volatility process. We assess this possibility empirically by running predictability regressions for the misspecified $\widehat{Var}_t(\Delta c_{t,t+1})$ on the first eight principal components of asset returns. Summary statistics for these regressions are depicted by the dashed lines with triangles in figure 9. The figure shows that, at several horizons, the misspecification of the mean process generates substantial adjusted $R^2$, and makes the regressors appear jointly statistically significant.

### V.2.3 Time series properties of stocks and bonds

We now turn to the time series properties of stocks and bonds implied by our model in equations (1)–(2) (and (5)–(6)).

[14]In order to verify that our results are not driven by a misattribution of a latent priced time varying volatility process to the mean process, we performed the same series of tests estimating the consumption process in equation (**??**) *without* imposing the structure in equation (2). This robustness check delivers results in line with the ones in figures (8) and (9).

[15]The $k$-th autocorrelation of $(\Delta c_{t,t+1} - \mu_c)^2$, for $k \leq \bar{S}$, is proportional to

$$Cov\left(\left(\sum_{j=k}^{\bar{S}} \rho_j f_{t-j} + \theta_j g_{t-j}\right)^2 ; \left(\sum_{j=k}^{\bar{S}} \rho_{j-k} f_{t-j} + \theta_{j-k} g_{t-j}\right)^2\right) \neq 0.$$

Figure 10 depicts the stock loadings on the latent factor $f_t$. The loadings are displayed with the 25 Fama and French (1992) size and book-to-market sorted portfolios ordered first (e.g. portfolio 2 is the smallest decile of size and the second smaller decile of book-to-market ratio) and the 12 industry portfolios order second (portfolios 26-39 in the graph).

All the stock portfolios display a significant and positive exposures to the common factor. Even more interestingly, there is a widely recognisable pattern in the factor loadings: decreasing from the smallest to the largest decile on size, and with a similar effect along the book-to-market sorting dimension. This is in line with the size and value anomalies and, in addition, provides some preliminary evidence that these shocks play an important role in explaining the cross-sectional dispersion of stocks returns. These findings also remain unchanged when a second, bond-specific factor is added to the model as in equations (5)–(6) (see figure A3 in the Appendix).

Furthermore, these loadings are not only statistically, but also economically significant as show by figure 11: the common factor $f$ explains on average 79% of the time series variation in the stock returns, ranging from 36% to nearly 95% for individual portfolios. Moreover, this explanatory power in our model is produced by a single consumption-based factor, as opposed to some of the alternative successful specifications that typically rely on 3 or more explanatory variables. As shown in figure A4 in the Appendix, adding a bond specific factor leaves the variance decomposition of stock returns virtually unaffected.

The loadings of the bond portfolios on the common consumption factor $f_t$ are reported in figures 12a and 12b for, respectively, the one and two latent factor specifications. Both sets of estimates show an upward sloping term structure of the loadings, and the point estimates are very similar in the two specifications, with the main difference being that, allowing for a bond specific factor ($g_t$) delivers much sharper estimates of the loadings on the common factor $f_t$. The magnitude of these loadings is considerably smaller than that of stocks – a feature that, as shown later, will allow us to price jointly the cross-sections of stocks and bonds. While these numbers may not seem as impressive as those for the cross-section of stocks, the pattern is highly persistent and significant, confirming a common factor structure between nondurable consumption growth and asset returns.

The loadings on the bond specific factor $g_t$ are reported in figure 13. These loadings are highly statistically significant, and increase steeply and monotonically with maturity, revealing a very pronounced maturity-driven pattern.

Finally, figure 14 report the share of time series variation of bond returns explained by the $f_t$ shocks (left panel), and the $f_t$ and $g_t$ shocks (right panel), and highlights the importance of allowing for a bond specific factor to characterize the time series of bond returns. The common factor $f_t$ accounts for a small (about 1.5%), but statistically significant, proportion

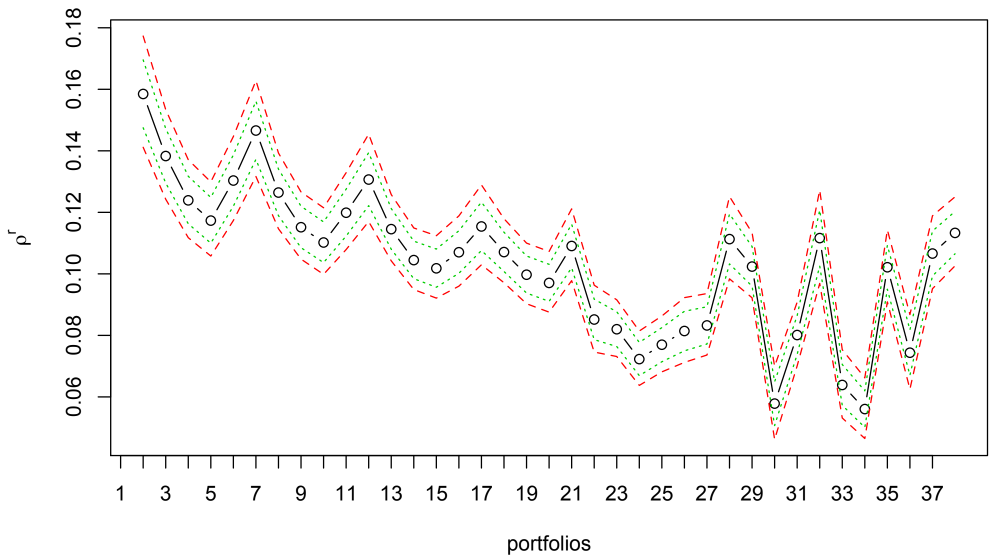



**Figure 10:** Common factor loadings ($\boldsymbol{\rho}^r$) of the stock portfolios in the one-factor model.

Posterior means of the stocks factor loadings on $f_t$ (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions in the one latent factor model. Ordering of portfolios: 25 Fama and French (1992) size and book-to-market sorted portfolios (e.g. portfolio 2 is the smallest decile of size and the second smaller decile of book-to-market ratio), and 12 industry portfolios.

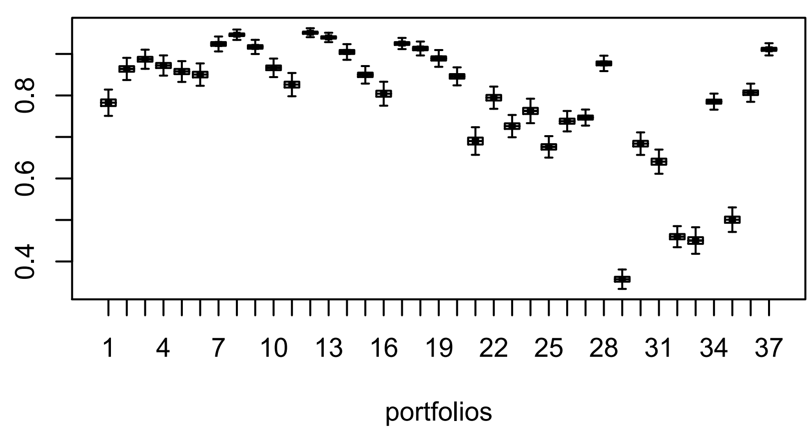
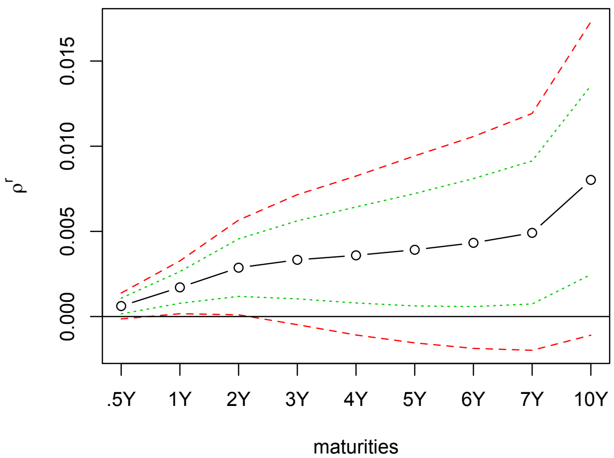
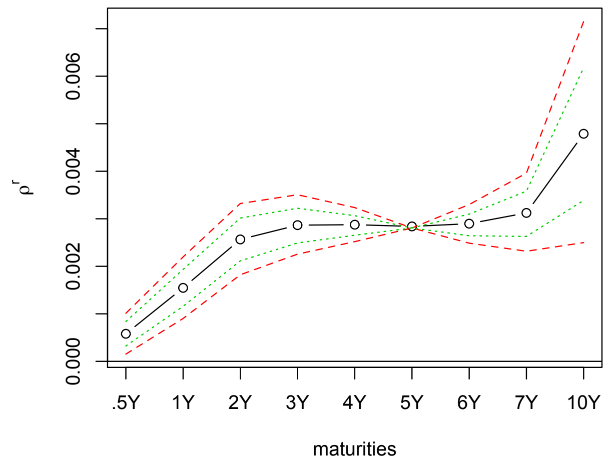



**Figure 11:** Share of stock portfolios' return variance explained by the $f$ component.

Box-plots (posterior 95% coverage area) of the percentage of time series variances of individual stock portfolio returns explained by the $f$ component in the one factor model. Ordering of portfolios: 25 Fama and French (1992) size and book-to-market sorted portfolios (e.g. portfolio 2 is the smallest decile of size and the second smaller decile of book-to-market ratio), and 12 industry portfolios.




**(a)** Bond loadings on common factor, $f_t$, one-factor model.




**(b)** Bond loadings on common factor, $f_t$, two-factor model.

**Figure 12:** Bond loadings on common factor $f_t$.

Posterior means of the bonds factor loadings on $f_t$ (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions in the one latent factor model. Estimation based on the one-factor model in equations (5) and (6), left panel, and two-factor model in equations (5), right panel.

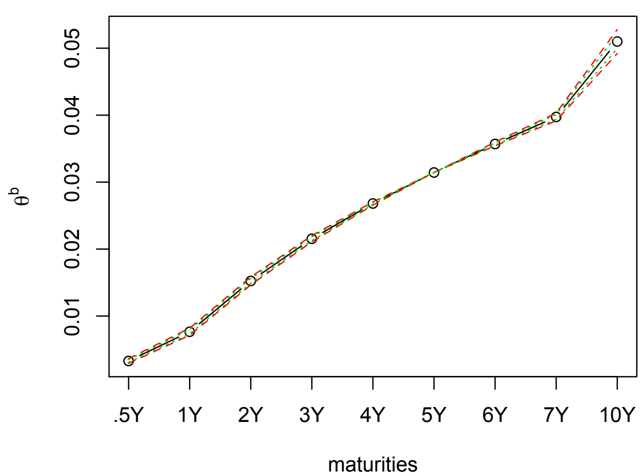
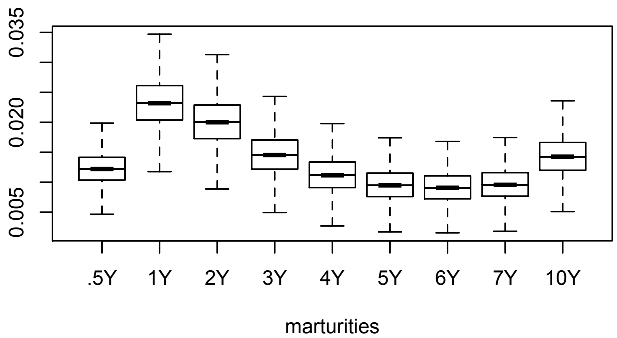
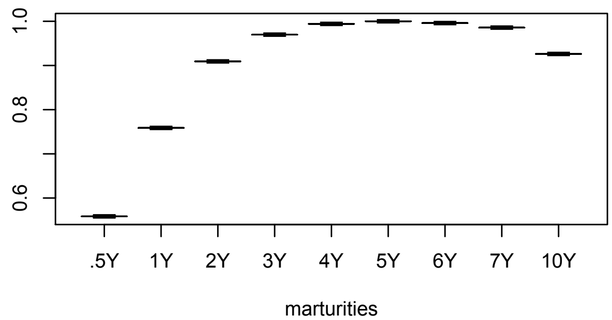



**Figure 13:** Bond loadings ($\boldsymbol{\theta}^b$) on the bond-specific factor ($g_t$).

Posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions, of the bond loadings on the bond specific factor $g_t$.




**(a)** Percentage of time series variances of bond returns explained by the common component $f_t$, one-factor specification.




**(b)** Percentage of time series variances of bond returns jointly explained by the common, $f_t$, and bond specific, $g_t$, components.

**Figure 14:** Variance decomposition box-plots of bond returns.

Box-plots (posterior 95% coverage area) of the percentage of time series variances of bond returns explained by the $f_t$ (left panel), and $f_t$ and $g_t$ (right panel) shocks.

of the time series variation in bond returns. The bond-specific factor, in turn, manages to capture most of the residual time series in variation in returns. While the model captures just about 55% of the variation in the 6-month bond returns, its performance rapidly improves with maturity and results in a nearly perfect fit for the time horizon of about five years.

#### V.2.4 What drives the slow consumption adjustment shock?

Figure 6 revealed a clear business cycle fluctuation within the moving average component of consumption growth, but what exactly is being captured by those shocks? Figure 15 looks at the exposure of the common innovations, $f_t$, to traditional asset pricing factors and principal components of bond returns. The single common factor identified from the composite cross-section of assets returns, has a complicated mix of exposure to several popular sources of risk.

Probably, the most prominent feature of the common financial shock, is its exposure to the value and momentum factors. While the former is not surprising, given the very pronounced pattern of loadings on this factor for the 25 Fama-French in figure 10, the latter is an interesting result, suggesting that momentum's origin is at least in part due to the same shocks that drive consumption growth, as well as other investment strategies and asset classes.

The innovations, spanned by financial assets, are clearly loading not only on the major sources of the variation and risk premia in stock returns, but those of bonds as well. They are substantially exposed to the first principal component of bond returns and its third. The yield curve slope has long been found to be a significant predictor of various macroeconomic

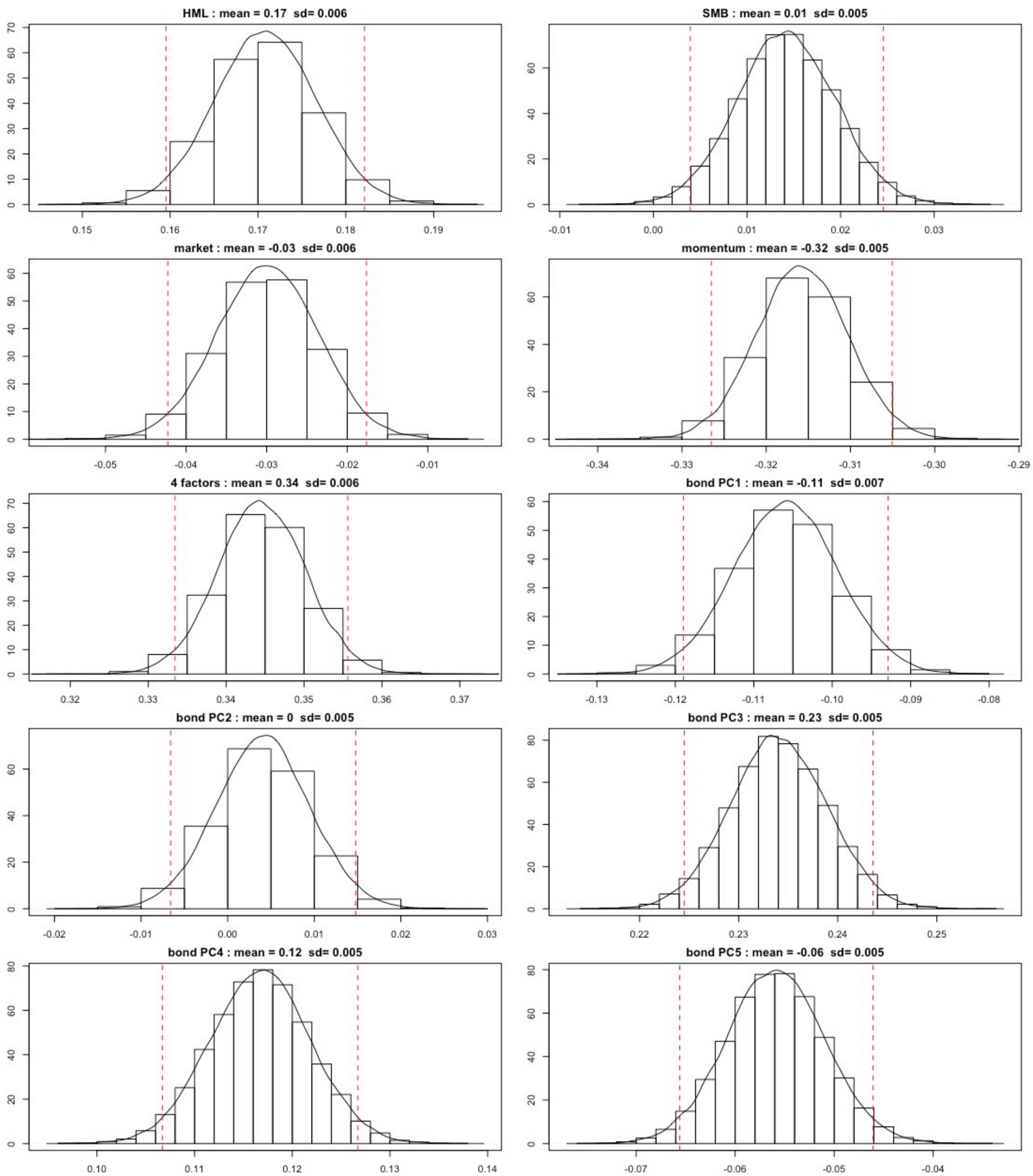



**Figure 15:** Common innovations, popular risk factor, and principal components of bond returns.

Posterior distributions of the correlation coefficients between the MA innovations $f_t$ and: the HML and SMB factors (first row); market excess returns and momentum factor (second row); a linear combination of HML, SMB, market, and momentum factors (left panel on third row); the first five principal components (PC1 to PC5) of bond excess returns (remaining panels). Dashed red lines indicate centered 90% posterior coverage areas. Posterior means and standard deviations of the correlation are reported at top of each sub-graph.

indicators, most notably the GDP growth and recessions[16]. It is therefore not surprising that it is largely the slope that is captured by the common component between asset returns and forward-looking propagation of these shocks in consumption. Another bond factor that these common shocks strongly load on, is the $4^{th}$ principal component of the term structure of bond risk premia, that has been recently found to be a significant determinant for the cross-section of bond returns in Adrian, Crump, and Moench (2013).

Next, we investigate whether this risk is actually *priced* in the cross-section of assets.

### V.2.5 The price of consumption risk

Recovering factor loadings in equations (5)–(6) also produces a cross-section of average returns on the set of portfolios. Figure 16 displays the scatterplot of the average vs. fitted excess returns for the baseline mixed cross-section of 46 assets. While the subset of bond returns demonstrates an almost perfect fit (lower left corner of the plot), the variation in the cross-section of stocks is also well-captured.

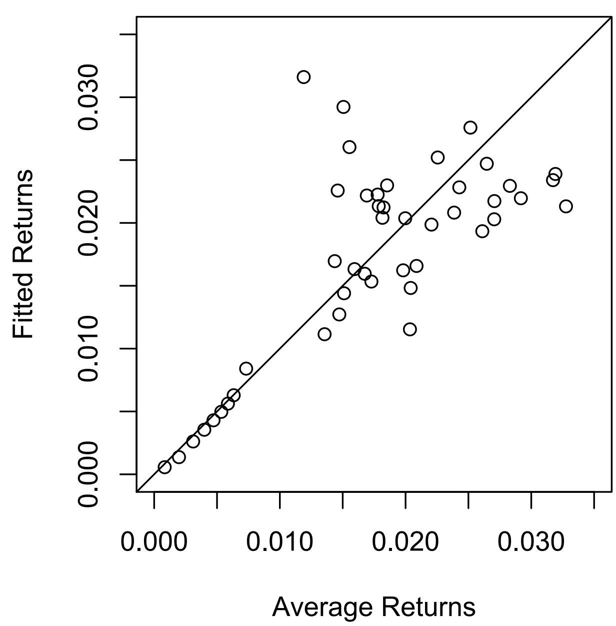



**Figure 16:** Consumption-based risk: average and fitted excess returns.

Fitted versus average returns using the consumption betas implied by the latent factor specification in equations (5)–(6).

[16]See, among others, Hamilton and Kim (2002), Stock and Watson (1989), Harvey (1989), and Harvey (1991)

**Table 1:** Cross-Sectional Regressions with State-Space Loadings

| Row: | $\alpha$ | $\lambda_f$ | $\lambda_g$ | $\lambda_f = \lambda_g$ | $\bar{R}^2$ |
|---|---|---|---|---|---|
| | | One latent factor specification | | | |
| (1) | .0056 | 14.77 | | | .57 |
| | [0.0051, .0062] | [8.89, 26.01] | | | [.54, .60] |
| (2) | | 20.00 | | | .90 |
| | | [12.05, 35.16] | | | [.89, .91] |
| | | Two latent factor specification | | | |
| (3) | .0057 | 14.97 | | | .57 |
| | [.0052, .0061] | [8.72, 27.45] | | | [.54, .60] |
| (4) | | 20.30 | | | 0.90 |
| | | [11.85, 37.18] | | | [0.89, 0.91] |
| (5) | .0069 | 13.79 | −1.44 | | .56 |
| | [−539.5, 497.7] | [7.96, 25.49] | [−539.5, 497.7] | | [0.53, 0.59] |
| (6) | | 20.27 | 19.57 | | .91 |
| | | [11.83, 37.12] | [−1140, 1218] | | [.90, .92] |
| (7) | .0053 | | | 15.24 | .57 |
| | [.0042, 0.0064] | | | [8.80, 28.40] | [.53, .60] |
| (8) | | | | 20.29 | .90 |
| | | | | [11.85, 37.19] | [.89, .91] |

The table presents posterior means and centred 95% posterior coverage (in square brackets) of the Fama and MacBeth (1973) cross-sectional regression of excess returns on $\sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r$ (with associated coefficient $\lambda_f$) and $\left[\ \boldsymbol{\theta}'^b,\ \ \mathbf{0}'_{N-N_b}\ \right]' \sum_{j=0}^{S} \theta_j$ (with associated coefficient $\lambda_g$). The column labeled $\lambda_f = \lambda_g$ reports restricted estimates. Cross-section of assets: 25 Fama and French (1992) size and book-to-market portfolio; 12 industry sorted portfolios; 9 bond portfolios.

Further, as equation (15) demonstrates, model-implied factor loadings of the asset returns determine their full exposure of the SCA risk and thus allow not only to assess the cross-sectional fit of the model, but also to test whether the slow consumption adjustment is indeed a *priced* risk factor, and whether the common and bond factors share the same value of the risk premium.

Following the critique of Lewellen, Nagel, and Shanken (2010), we are using a mixed cross-section of assets to ensure that there is no dominating implied factor structure of the returns. Indeed, if that was the case, it could lead to spuriously high significance levels, quality of fit, and significantly complicate the overall model assessment. However, as Figure 15 indicated, the slow consumption adjustment factor does not heavily load on any of the main principal components of the returns. Further, we provide confidence bounds for the cross-sectional measure of fit to ensure the point estimates reflect the actual pricing ability of the model. Finally, since both stocks and bonds have significant loadings on the common factor (and in the case of bonds, also on the bond-specific one), we do not face the problem of *irrelevant*, or *spurious* factors (Kan and Zhang (1999)), that could also lead to the unjustifiably high significance levels.

Table 1 summarizes the cross-sectional pricing performance of our parametric model of consumption on a mixed cross-section of 9 bond portfolios, 25 Fama-French portfolios sorted by size and book-to-market, and 12 industry portfolios. For each of the specifications, we recover the full posterior distribution of the factor loadings, and estimate the associated risk premia using Fama-MacBeth (1973) cross-sectional regressions. Regardless of the specification, there is strong support in favour of the slow consumption adjustment being a priced risk in the composite cross-section of assets with the risk premia of about 14-20%.

The average pricing error is about 0.005% per quarter, and the cross-sectional $R^2$ varies from 57% to 91%, depending on whether the intercept is included in the model. While allowing for a common intercept in the estimation substantially lowers cross-sectional fit, 95% posterior coverage bounds remain very tight, providing a reliable indication about the model performance.

While the risk premium on the common factor is strongly identified and seems to play an important role in explaining the cross-section of both stock and bond returns, the bond factor loadings do not provide an equally large spread for recovering its pricing impact with the same degree of accuracy. As a result, the risk premium appears to be insignificant, unless its value is restricted to that of the common factor. To summarise, the bond-specific factor is *unspanned*, in the sense that while it is essential for explaining most of the time series variation in bond returns and producing a correct slope of the yield curve, it does not have any cross-sectional impact on bond returns.

# VI Robustness and Discussion

The relatively tight restrictions on the parametric model in equations (1)-(2) allow us not only to pin down the underlying parameters of the joint consumption-returns process, but also do it with with a high degree of precision. This finding, of course, comes at a price of imposing additional constraints on the data generating process, some of which may not hold in practice. In this section we aim to test the strongest prediction of our parametric setting: the term structure of asset exposure to consumption risk. In order to address this issue, we consider two additional semi-parametric approaches.

First, we focus on the term structure of the assets consumption exposure, which provides an alternative way to test for the presence of a slow-moving component within the consumption growth. Second, we turn to the standard approach of the cross-sectional asset pricing literature (cross-sectional pricing tests) to on the pricing performance of a broad class of consumption-based asset pricing models that admit a multiplicative SDF representation. We show that, consistent with the results from Section V.2.5, multiperiod SDFs that rely on forward-looking aggregated consumption growth have a substantial success at pricing the cross-sections of both stocks and bonds. Furthermore, we show that this success stems form the increase in both the average, and the *spread*, of the asset exposure to common shocks, that the to slow propagation mechanism of shocks to consumption generates.

## VI.1 The term structure of consumption exposure

One of the key implications of equations (1)–(2) is a very particular pattern in the covariances of asset returns with multiperiod forward-looking consumption growth, i.e. for $i \in (1, \ldots, n)$

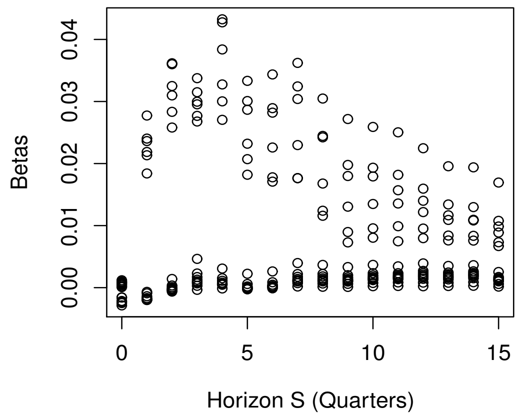
$$\begin{aligned} cov(r^{ex}_{i,t+1}, \Delta c_{t,t+1}) &= \rho^r_i \rho_0 \\ cov(r^{ex}_{i,t+1}, \Delta c_{t,t+2}) &= \rho^r_i (\rho_0 + \rho_1) \\ &\ldots \\ cov(r^{ex}_{i,t+1}, \Delta c_{t,t+k}) &= \rho^r_i \left( \sum_{j=1}^{k} \rho_{k-1} \right) \end{aligned} \tag{16}$$

This implies that asset exposures to consumption risk at different horizons have a very particular factor structure, driven by a single common component, which is exactly equal to $(\rho_0, \rho_0 + \rho_1, \ldots, \sum_{j=1}^{k} \rho_{k-1})'$ – i.e. the cumulative response function of consumption to an $f_t$ shock. This property is not affected by the potential presence of cross-sectional correlations between stocks and bonds, or additional factors driving stocks and bonds that are orthogonal to consumption. Therefore, if the time-varying dynamics of consumption growth from eq. 1 describes well the data generating process, we should be able to recover the same pattern of loadings by simply extracting the first uncentered principal component of $cov(r^{ex}_{i,t+1}, \Delta c_{t,t+k})$.

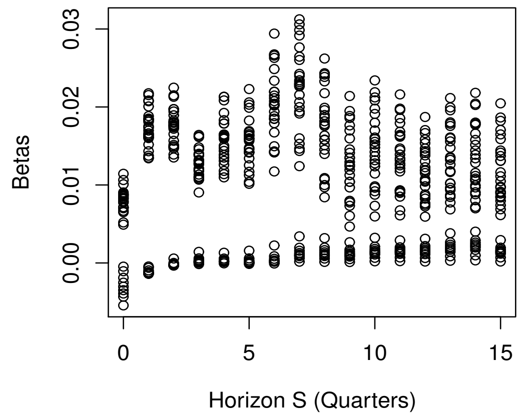
Fig. 1 in the Introduction illustrates our findings. We normalize the first PCA so that it has the same origin as the $\rho_0$ filtered from the data. Remarkably, it almost perfectly matches the filtered cumulated response function, identifying the same persistent time-varying mean of consumption growth.

One possible concern regarding this approach could be the fact that consumption growth alone is rather persistent, and hence, one would mechanically expect an increase in the covariance over longer horizons, even when there is no additional common exposure in the later periods. Indeed, if the returns were correlated only with the contemporaneous innovation in consumption growth, and the latter was, say, an AR(1) process, you would still see an increase in the covariance over longer horizons. However, that would also imply a monotonic decline in the *betas* of the asset returns with respect to the long-term consumption growth. But such a pattern is not present in the data. Not only does the cumulated consumption growth increase the average asset exposure, it also improves the *spread* of the latter. While the standard one-period consumption growth does not perform well in either dimensions, leading to the equity premium puzzle and a relatively poor cross-sectional fit, the moving average component in the consumption growth seems to achieve both objectives: it increases the amount of measured risk as well as the cross-sectional dispersion.

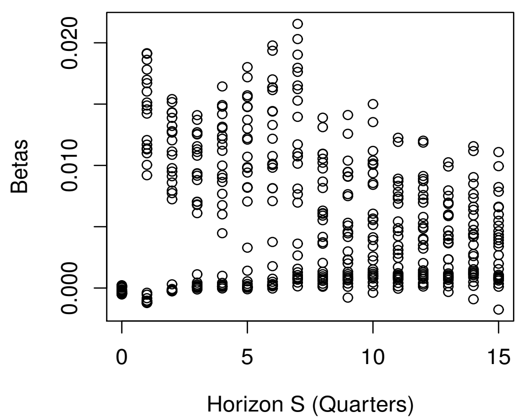
**Figure 17:** Cross-sectional spread of exposure to slow consumption adjustment risk



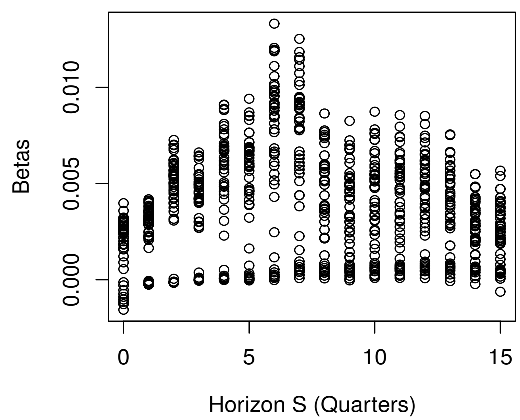

**(a)**




**(b)**




**(c)**




**(d)**

Panels present the spread of normalised betas for the various sets of assets and horizon S (0-15): (a) 9 bonds and 6 Fama-French portfolios, (b) 9 bonds and 25 Fama-French portfolios, (c) 9 bonds, 12 Industry and 6 Fama-French portfolios, (d) 9 bonds, 12 Industry and 25 Fama-French portfolios. All the parameters were estimated by Empirical Likelihood.

Figure 17 displays the dispersion of normalized the normalized betas,[17] associated with

[17]We define betas as the ratio between the asset covariance with the model-implied scaled SDF and its

the consumption growth over different horizon values and for different cross-sections of assets. As we move away from the standard case of $S = 0$, two observations immediately arise. First, there is a substantial improvement in the average asset exposure to consumption growth, which leads to lower and more accurate estimates of the risk aversion. However, it is the increase in the spread of betas, with a particular contribution from the stocks, which is most striking. The 'fanning out' effect, observed for the higher values of the consumption horizon S, further supports the hypothesis that the fundamental source of risk in the asset returns is related to the aggregate consumption growth, and should take into account its slow speed of adjustments to shocks.

Finally, the fact that there is a significant correlation between asset returns and consumption growth over several periods (both in terms of its level and spread), also serves as an additional robustness check against a potential problem of the *spurious factors* type (Kan and Zhang (1999)), i.e. factors that are only weakly related to the asset returns and thus only *appear* to be driving the cross-section of asset returns.

## VI.2 Semi-parametric inference

Representative agent based consumption asset pricing models with either CRRA, Epstein and Zin (1989), or habit based preferences, as well as several models of complementary in the utility function, and models with either departures from rational expectations, or robust control, or ambiguity aversion, and even some models with solvency constraints,[18] all imply a consumption Euler equation of the form

$$C_t^{-\phi} = \mathbb{E}_t \left[ C_{t+1}^{-\phi} \tilde{\psi}_{t+1} R_{j,t+1} \right] \tag{17}$$

for any gross asset return $j$ including the risk free rate $R_{t+1}^f$, and where $\mathbb{E}_t$ is the rational expectation operator conditional on information up to time $t$, $C_t$ denotes flow consumption, $\tilde{\psi}_{t+1}$ depends on the particular form of preferences (and expectation formation mechanism) considered, and the $\phi$ parameter is a function of the underlying preference parameters.[19] Rearranging terms, moving to unconditional expectations, and using the definition of covari-

variance.

[18]See, e.g.: Bansal and Yaron (2004); Abel (1990), Campbell and Cochrane (1999), Constantinides (1990), Menzly, Santos, and Veronesi (2004); Piazzesi, Schneider, and Tuzel (2007), Yogo (2006); Basak and Yan (2010), Hansen and Sargent (2010); Chetty and Szeidl (2015); Ulrich (2010); Lustig and Nieuwerburgh (2005).

[19]E.g., $\phi$ would denote relative risk aversion in the CRRA framework, while it would be a function of both risk aversion and elasticity of intertemporal substitution with Epstein and Zin (1989) recursive utility.

ance, we can rewrite the above equation as a model of expected returns

$$\mathbb{E}\left[\mathbf{R}_{t+1}^{e}\right]=-\frac{Cov\left(M_{t+1};\mathbf{R}_{t+1}^{e}\right)}{\mathbb{E}\left[M_{t+1}\right]}. \tag{18}$$

where $M_{t+1} := \left(C_{t+1}/C_t\right)^{-\phi}\tilde{\psi}_{t+1}$ represents the stochastic discount factor between time $t$ and $t+1$ and $\mathbf{R}^e \in \mathbb{R}^N$ denotes a vector of excess returns. Log-linearizing the above relationship, expected returns can be expressed as

$$\mathbb{E}\left[\mathbf{R}_{t+1}^{e}\right]=\left[\phi Cov\left(\Delta c_{t,t+1};\mathbf{r}_{t+1}^{e}\right)-Cov\left(\log\tilde{\psi}_{t+1};\mathbf{r}_{t+1}^{e}\right)\right]\lambda \tag{19}$$

where $\Delta c_{t,t+1} := \ln\left(C_{t+1}/C_t\right)$, $\mathbf{r}^e \in \mathbb{R}^N$ denotes log excess returns, and $\lambda$ is a positive scalar. Since, in the data, the covariance between one period consumption growth and asset returns is small and has a much smaller cross-sectional dispersion than average excess returns, the first term of the above equation is not sufficient for pricing a cross-section of asset returns, and most of the modelling effort in the literature has been devoted to identifying a $\tilde{\psi}$ component that can help rationalise observed returns.

Note that equation (17) above implies that

$$C_t^{-\phi}=\mathbb{E}_t\left[C_{t+1+S}^{-\phi}\psi_{t+1+S}\right]$$

where $\psi_{t+1+S} := R_{t,t+1+S}^{f}\prod_{j=0}^{S}\tilde{\psi}_{t+1+j}$. Hence, the Euler equation

$$\mathbf{0}_N=\mathbb{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\phi}\tilde{\psi}_{t+1}\mathbf{R}_{t+1}^{e}\right] \tag{20}$$

where $\mathbf{0}_N$ denotes and $N$-dimensional vector of zeros, can be equivalently rewritten as

$$\mathbf{0}_N=\mathbb{E}\left[\left(\frac{C_{t+1+S}}{C_t}\right)^{-\phi}\psi_{t+1+S}\mathbf{R}_{t+1}^{e}\right]. \tag{21}$$

Once again, using the definition of covariance, we can rewrite the above equation as a model of expected returns

$$\mathbb{E}\left[\mathbf{R}_{t+1}^{e}\right]=-\frac{Cov\left(M_{t+1}^{S};\mathbf{R}_{t+1}^{e}\right)}{\mathbb{E}\left[M_{t+1}^{S}\right]}. \tag{22}$$

where $M_{t+1}^{S} := \left(C_{t+1+S}/C_t\right)^{-\phi}\psi_{t+1+S}$. That is, under the null of the model being correctly specified, there is an entire family of SDFs that can be equivalently used for asset pricing:

$M^S_{t+1}$ for every $S \geq 0$. Log-linearizing the above expression, we have the linear factor model

$$\mathbb{E}\left[\mathbf{R}^e_{t+1}\right] = \left[\phi Cov\left(\Delta c_{t,t+1+S}; \mathbf{r}^e_{t+1}\right) - Cov\left(\log \psi_{t+1+S}; \mathbf{r}^e_{t+1}\right)\right] \lambda_S \tag{23}$$

where $\Delta c_{t,t+1+S} := \ln\left(C_{t+1+S}/C_t\right)$ and $\lambda_S$ is a positive scalar. The above pricing restriction is *exactly* the one we have tested in section V.2.5.

We now want to avoid testing a linearized relationship and therefore tackle directly the pricing restriction in Euler equation (21):

$$\mathbf{0} = \mathbb{E}\left[M^S_{t+1}\mathbf{R}^e_{t+1}\right]$$

where $M^S_{t+1} := \left(C_{t+1+S}/C_t\right)^{-\phi}\psi_{t+1+S}$ and $S \geq 0$.

The fact that the stochastic discount factor can be decomposed into the product of the consumption growth over several consecutive periods $(C_{t+1+S}/C_t)$ and an additional, potentially unobservable, component, makes the above setting particularly appealing for the application of Empirical Likelihood -based techniques (for an excellent overview, see Kitamura (2006)) as discussed in Ghosh, Julliard, and Taylor (2013). Intuitively, the EL-based inference allows to separate the unobservable part of the SDF from that related to consumption growth, and provides a nonparametric estimate of the former, treating it like a nuisance parameter, and $\phi$ - as a parameter of interest. Section A.4 describes the estimation procedure in detail.

We can also capture the average pricing error of the model implied by equation (21) simply by introducing additional parameters in the following way:

$$\mathbf{0} = \mathbb{E}\left[M^S_{t+1}\left(\mathbf{R}^e_{t+1} - \alpha\right)\right], \tag{24}$$

where $\alpha$ stands for the average rate of return that is not cross-sectionally captured through the covariance between $M^S_{t+1}$ and $\mathbf{R}^e_{t+1}$, since equation (24) implies

$$\mathbb{E}\left[\mathbf{R}^e_{t+1}\right] = \alpha - \frac{Cov\left(M^S_{t+1}, \mathbf{R}^e_{t+1}\right)}{\mathbb{E}[M^S_{t+1}]}. \tag{25}$$

Parameter estimation proceeds following the procedure outlined in Appendix A.4. We consider several versions of equation (24): $\alpha = 0$ (correct model specification); average pricing errors; error specific to a particular asset class ($\alpha_b \neq \alpha_s$); and a common level of mispricing for both stocks and bonds ($\alpha_b = \alpha_s$).

For each model we also report the cross-sectional adjusted R-squared

$$R^2_{adj} = 1 - \frac{n-2}{n-1}\widehat{Var}_c\left(\frac{1}{T}R_{i,t+1} - \hat{\alpha} - \frac{\widehat{Cov}\left((C_{t+1+S}/C_t)^{-\hat{\phi}}, \mathbf{R}^e_{t+1}\right)}{\widehat{\mathbb{E}}\left[(C_{t+1+S}/C_t)^{-\hat{\phi}}\right]}\right) \Big/ \widehat{Var}_c\left(\frac{1}{T}R_{i,t+1}\right) \tag{26}$$

where $\widehat{Var}_c$ is the sample cross-sectional variance and $\widehat{Cov}$ is the sample time series covariance.

Finally, for the sake of completeness we also use two-stage Generalised Method of Moments (GMM, Hansen (1982)) to estimate consumption-based asset pricing models on the cross-section of stock and bond returns, and report its results alongside those for EL. While the estimator-implied probabilities no longer have the convenient nonparametric maximum likelihood interpretation (see the discussion in Section A.4), if one restricts the class of admissible SDF to the external habit models, asset pricing implications and inference based on the *ultimate consumption risk* only, remain valid. Under fairly general conditions, this result is a direct consequence of Proposition 1 in Parker and Julliard (2003), implying that GMM estimates of risk aversion retain consistency and asymptotical normality, and do not require an explicit knowledge of the habit function, if one relies on the multiple period consumption in the estimation.

## VI.3 Semi-parametric evidence

Since the relevance of the multiple period consumption for the cross-section of stocks has already been highlighted by Parker and Julliard (2005), we first focus on the cross-section of bonds only, and provide empirical evidence that the slow moving consumption exposure is important for explaining their cross-section of returns. We then turn to analysing the model performance for pricing a composite set of bonds and stocks.

Table 2 summarizes the performance of the consumption-based asset pricing model on the cross-section of bond returns for various values of $S$ of the ultimate consumption measure of Parker and Julliard (2005). While EL estimation remains valid in the presence of the multiplicative unobservable part of the stochastic discount factor, evaluating GMM output requires a certain degree of caution, since in this case, to the best of our knowledge, the same robustness is achieved only within the class of external habit models (see Proposition 1 of Parker and Julliard (2003)). Nevertheless, for the sake of completeness we report both sets of results.

The $S = 0$ case corresponds to the standard consumption-based asset pricing model, where the spread of the returns is driven only by their contemporaneous correlation with consumption growth. Both EL and GMM output reflect the well-known failure of the classical model to capture the cross-section of bond returns: according to the J-test, the model is rejected in the data, and the cross-sectional adjusted R-squared is negative. Increasing the span of consumption growth to 2 or more quarters drastically changes the picture: the J-test no longer rejects the model, and the level of cross-sectional fit increases up to 85% for $S = 12$, for example.

Further, the estimates of the power coefficient $\phi$ (which in the case of additively separable CRRA utility corresponds to the Arrow-Pratt relative risk-aversion coefficient) not only appear to be much smaller (hence more in line with economic theory), but also more precisely estimated. The large standard error associated with this parameter for the standard consumption-based model ($S = 0$) is due to the fact that the level and spread of the contemporaneous correlation between asset returns and consumption growth is rather low. This in turn leads to substantial uncertainty in parameter estimation. As the number of quarters used to measure consumption risk increase, the link between bond returns and the slow moving component of the consumption becomes more pronounced, resulting in lower standard errors, better quality of fit, and the overall ability of the model to match the cross-section of bond returns. In fact, model-implied average excess returns are very close to the actual ones, in drastic contrast to the standard consumption-based asset pricing model. This is shown in Figure 18 which presents fitted and actual average excess returns on the cross-section of 9 bond portfolios for several values of the consumption horizon S. The contemporaneous correlation between bond returns and consumption growth (Panel (a), $S = 0$) is so low that not only it results is rather poor fit, but actually reverses the order of the portfolios: i.e. the fitted average return from holding long-term bonds is smaller than that of the short term ones. And again, once the horizon used to measure consumption risk is increased, the quality of fit substantially improves, leading to an R-squared of 85% for $S = 12$ (see Panel (b)).

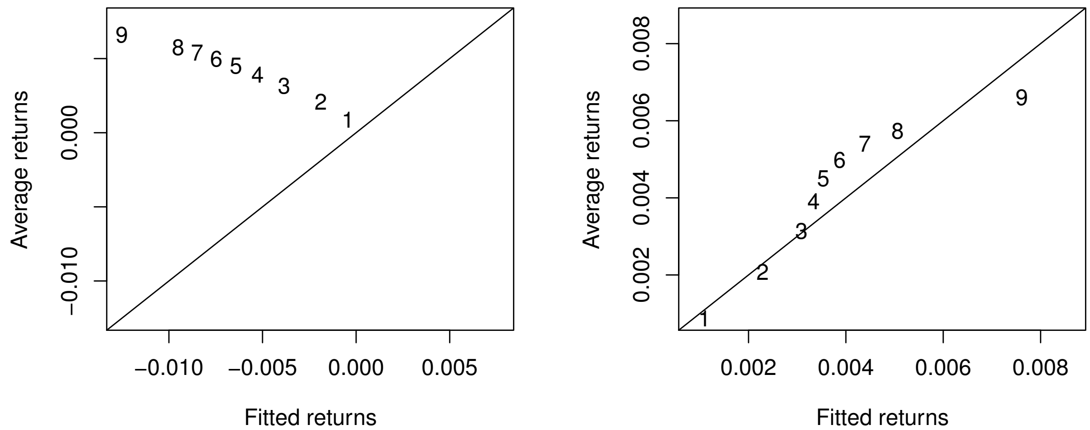
The ability of slow consumption adjustment risk to capture a large proportion of the cross-sectional variation in returns is not a feature of the bond market alone: it works equally well on the joint cross-section of stocks and bonds, providing a simple and parsimonious one factor model for co-pricing securities in both asset classes.

Table 3 summarises the model performance with various joint cross-sections of stocks and bonds for different consumption horizons S. Compared to the standard case of $S = 0$, slow consumption adjustment substantially improves model performance in a number of ways. While a simple consumption-based asset pricing model is rejected by the J-test on all the cross-section of stocks, the test values are dramatically improved over the range of $S = 10 - 12$: in fact, based on Empirical Likelihood Estimation, the model is no longer rejected in any of the cross-sections. Combined with the improved values of the power parameter ($\phi$), the accuracy of its estimation (lower standard errors), and a substantial

**Table 2:** Cross-Section of Bond Returns and Ultimate Consumption Risk

| Horizon S | Empirical Likelihood | | | | Generalised Method of Moments | | | |
|---|---|---|---|---|---|---|---|---|
| Horizon S (Quarters) | $R^2_{adj}(\%)$ (1) | $\alpha$ (2) | $\phi$ (3) | $J$-test (4) | $R^2_{adj}(\%)$ (5) | $\alpha$ (6) | $\phi$ (7) | $J$-test (8) |
| 0 | -837 | 0.0007 | 100 | 13.0888 | -10 | 0.0000 | 4 | 19.5597 |
| | | (0.0003) | (28.5) | [0.0700] | | (0.0002) | (73.5) | [0.0066] |
| 1 | -167 | 0.0009 | 88 | 7.6457 | -35 | 0.0005 | 42 | 11.5448 |
| | | (0.0004) | (24.8) | [0.3649] | | (0.0005) | (47.3) | [0.1166] |
| 2 | 70 | 0.0030 | 120 | 2.8778 | 43 | 0.0009 | 50 | 4.6351 |
| | | (0.0005) | (21.8) | [0.8961] | | (0.0011) | (52.6) | [0.7044] |
| 3 | 39 | 0.0010 | 70 | 4.5187 | 61 | 0.0006 | 35 | 5.1968 |
| | | (0.0004) | (16.2) | [0.7185] | | (0.0005) | (20.5) | [0.6360] |
| 4 | 69 | 0.0008 | 55 | 3.4531 | 48 | 0.0004 | 33 | 3.2207 |
| | | (0.0003) | (13.4) | [0.8402] | | (0.0004) | (16.1) | [0.8639] |
| 5 | 5 | 0.0008 | 45 | 6.8134 | 38 | 0.0004 | 27 | 6.0294 |
| | | (0.0003) | (10.5) | [0.4486] | | (0.0003) | (13.0) | [0.5363] |
| 6 | 3 | 0.0008 | 42 | 8.9256 | 42 | 0.0002 | 23 | 6.8397 |
| | | (0.0003) | (10.0) | [0.2580] | | (0.0003) | (11.5) | [0.4458] |
| 7 | 64 | 0.0004 | 33 | 9.8236 | 64 | 0.0001 | 22 | 6.4740 |
| | | (0.0003) | (9.9) | [0.1988] | | (0.0003) | (10.7) | [0.4856] |
| 8 | 70 | 0.0006 | 35 | 9.6027 | 69 | 0.0003 | 24 | 6.5862 |
| | | (0.0003) | (10.1) | [0.2122] | | (0.0003) | (12.3) | [0.4732] |
| 9 | 53 | 0.0008 | 55 | 8.2778 | 67 | 0.0004 | 26 | 6.8314 |
| | | (0.0003) | (10.5) | [0.3087] | | (0.0003) | (14.7) | [0.4466] |
| 10 | 77 | 0.0008 | 38 | 10.2472 | 73 | 0.0004 | 25 | 6.8649 |
| | | (0.0002) | (12.3) | [0.1750] | | (0.0003) | (18.4) | [0.4431] |
| 11 | 77 | 0.0008 | 44 | 8.2683 | 72 | 0.0006 | 26 | 7.7110 |
| | | (0.0002) | (14.3) | [0.3095] | | (0.0003) | (23.7) | [0.3588] |
| 12 | 85 | 0.0008 | 78 | 6.1561 | 88 | 0.0008 | 34 | 6.8054 |
| | | (0.0002) | (16.3) | [0.5216] | | (0.0003) | (26.5) | [0.4494] |
| 13 | 69 | 0.0007 | 85 | 5.8494 | 89 | 0.0007 | 37 | 6.0817 |
| | | (0.0002) | (17.5) | [0.5574] | | (0.0003) | (28.7) | [0.5302] |
| 14 | 88 | 0.0006 | 72 | 8.0283 | 90 | 0.0007 | 41 | 6.7445 |
| | | (0.0002) | (19.6) | [0.3301] | | (0.0004) | (30.3) | [0.4560] |
| 15 | 77 | 0.0006 | 70 | 7.3656 | 69 | 0.0008 | 46 | 7.2723 |
| | | (0.0002) | (22.1) | [0.3918] | | (0.0005) | (36.4) | [0.4011] |

The table reports the pricing of 9 excess bond holding returns for various values of the horizon S, and allowing for an intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and two-stage GMM.




**(a)** S=0 **(b)** S=12

**Figure 18:** Slow consumption adjustment factor and the cross-section of bond returns

The figures show average and fitted returns on the cross-section of 9 bond portfolios (1961Q1-2013Q4), sorted by maturity. The model is estimated by Empirical Likelihood for various values of consumption horizon S. $S = 0$ corresponds to the standard consumption-based asset pricing model; $S = 12$ corresponds to the use of ultimate consumption risk, where the cross-section of returns is driven by the their correlation with the consumption growth over 13 quarters, starting from the contemporaneous one.

increase in the cross-sectional quality of fit, measured by the $R^2$, Table 3 presents compelling evidence in favour of the slow consumption adjustment risk being an important driver for the cross-sections of both stocks and bonds. Appendix A.6 provides similar empirical evidence for the alternative model specifications that also include a common or asset class-specific intercept as a proxy for model misspecification.

# VII Conclusions

We show that a flexible parametric model with common factors driving asset dynamics and consumption identifies a slow varying component of consumption that responds to financial shocks. We find that the slow consumption adjustment component stands for more than a quarter of the time series variation of consumption growth, indicating that the shocks spanned by financial market are a first order drivers of consumption risk.

Furthermore, both stocks and bonds load significantly on the innovations to this slow moving component, generating sizeable risk premia and dispersion in returns, consistent with the size and value anomalies, as well as with the positive slope of the term structure of bond excess returns. As a result, our model explains between 36% and 95% of the time series variation in returns and between 57% and 90% of the joint cross-sectional variation in

**Table 3:** Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

| | Empirical Likelihood | | | Generalised Method of Moments | | |
|---|---|---|---|---|---|---|
| Horizon S (Quarters) | $R^2_{adj}(\%)$ (1) | $\phi$ (2) | $J$-test (3) | $R^2_{adj}(\%)$ (4) | $\phi$ (5) | $J$-test (6) |
| *Panel A: 9 Bonds and Fama-French 6 portfolios* | | | | | | |
| 0 | -13 | -7 | 36.8568 | 70 | 60 | 36.3730 |
| | | (26.3) | [0.0013] | | (27.7) | [0.0016] |
| 10 | 95 | 23 | 7.275274 | 89 | 30 | 28.3589 |
| | | (6.0) | [0.9495] | | (6.8) | [0.0194] |
| 11 | 94 | 23 | 6.389318 | 94 | 32 | 29.3379 |
| | | (6.5) | [0.9724] | | (8.5) | [0.0145] |
| 12 | 91 | 22 | 5.864083 | 96 | 35 | 30.5354 |
| | | (6.3) | [0.9819] | | (9.4) | [0.0101] |
| *Panel B: 9 Bonds and Fama-French 25 portfolios* | | | | | | |
| 0 | 46 | 41 | 56.7788 | 64 | 73 | 157.2452 |
| | | (17.8) | [0.0084] | | (15.0) | [0.0000] |
| 10 | 75 | 20 | 24.3141 | 24 | 41 | 31.8799 |
| | | (3.8) | [0.8899] | | (6.1) | [0.5719] |
| 11 | 76 | 20 | 21.2727 | 45 | 21 | 26.3571 |
| | | (3.7) | [0.9563] | | (6.3) | [0.8224] |
| 12 | 70 | 18 | 20.9430 | 49 | 22 | 22.3989 |
| | | (3.4) | [0.9612] | | (7.7) | [0.9364] |
| *Panel C: 9 Bonds, Fama-French 6, and Industry 12 portfolios* | | | | | | |
| 0 | -6 | -3 | 59.7497 | 59 | 68 | 156.2215 |
| | | (21.2) | [0.0003] | | (15.0) | [0.0000] |
| 10 | 54 | 13 | 24.2148 | -68 | 40 | 22.9235 |
| | | (3.9) | [0.6184] | | (7.0) | [0.6891] |
| 11 | 51 | 12 | 24.2189 | -38 | 42 | 22.0777 |
| | | (3.7) | [0.6181] | | (7.0) | [0.7334] |
| 12 | 52 | 12 | 22.1532 | -3 | 45 | 22.0186 |
| | | (3.5) | [0.7295] | | (6.4) | [0.7364] |
| *Panel D: 9 Bonds, Fama-French 25, and Industry 12 portfolios* | | | | | | |
| 0 | 22 | 19 | 82.6606 | 50 | 86 | 213.7053 |
| | | (15.1) | [0.0007] | | (14.2) | [0.0000] |
| 10 | 37 | 8 | 52.2543 | -48 | 42 | 48.9612 |
| | | (2.5) | [0.2440] | | (5.3) | [0.3551] |
| 11 | 37 | 8 | 49.6145 | -7 | 44 | 47.8821 |
| | | (2.3) | [0.3312] | | (5.6) | [0.3963] |
| 12 | 36 | 8 | 47.4384 | 16 | 48 | 41.7552 |
| | | (2.2) | [0.4138] | | (6.3) | [0.6506] |

The table reports the pricing of excess returns of stocks and bonds, allowing for no intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.

stocks and bonds.

We perform a substantial robustness analysis of the uncovered mechanisms, and we provide a rich set of evidence that, while not relying on our parametric formulation, strongly supports our time series and cross-sectional findings.

While generally consistent with the consumption dynamics postulated in the long run risk framework, these empirical findings nevertheless pose several important questions. Can the results be applied to other asset classes, such as currencies or commodities? What is the nature of the unspanned factor, driving most of the time series variation in bonds?

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# A Appendix

## A.1 State Space Estimation and Generalisations

Let $\Pi' := [\boldsymbol{\mu}, \mathbf{H}]$, $x_t' := [1, \mathbf{z}_t']$. Under a (diffuse) Jeffreys' prior the likelihood of the data in equation (13) implies the posterior distribution

$$\Pi' \big| \, \Sigma, \{\mathbf{z}_t\}_{t=1}^{T}, \{\mathbf{y}_t\}_{t=1}^{T} \sim \mathcal{N}\left(\hat{\Pi}'_{OLS}; \Sigma \otimes (x'x)^{-1}\right)$$

where $x$ contains the stacked regressors, and the posterior distribution of each element on the main diagonal of $\Sigma$ is given by

$$\sigma_j^2 \big| \, \{\mathbf{z}_t\}_{t=1} \sim \text{Inv-}\Gamma\left((T - m_j - 1)/2, T\hat{\sigma}^2_{j,OLS}/2\right)$$

where $m_j$ is the number of estimated coefficients in the $j$-th equation. That is, the conditional posterior has a Normal-inverse-$\Gamma$ structure. Moreover, $\mathbf{F}$ and $\Psi$ have a Dirac posterior distribution at the points defined in equation (10). Therefore, the missing part necessary for taking draws via MCMC using a Gibbs sampler, is the conditional distributions of $\mathbf{z}_t$. Since

$$\left.\begin{array}{c} \mathbf{y}_t \\ \mathbf{z}_t \end{array}\right| \mathcal{I}_{t-1}, \mathbf{H}, \Psi, \Sigma \sim \mathcal{N}\left(\left[\begin{array}{c} \boldsymbol{\mu} \\ \mathbf{F}\mathbf{z}_{t-1} \end{array}\right]; \left[\begin{array}{cc} \Omega & \mathbf{H} \\ \mathbf{H}' & \Psi \end{array}\right]\right),$$

where $\Omega := Var_{t-1}(\mathbf{y}_t) = \mathbf{H}\Psi\mathbf{H}' + \boldsymbol{\Sigma}$, this can be constructed, and values can be drawn, using a standard Kalman filter and smoother approach. Let

$$\mathbf{z}_{t|\tau} := E\left[\mathbf{z}_t | \mathbf{y}^{\tau}, \mathbf{H}, \Psi, \Sigma\right]; \quad \mathbf{V}_{t|\tau} := Var\left(\mathbf{z}_t | \mathbf{y}^{\tau}, \mathbf{H}, \Psi, \Sigma\right).$$

where $\mathbf{y}^{\tau}$ denotes the history of $\mathbf{y}_t$ until $\tau$. Then, given $\mathbf{z}_{0|0}$ and $\mathbf{V}_{0|0}$, the Kalman filer delivers:

$$\mathbf{z}_{t|t-1} = \mathbf{F}z'_{t-1|t-1}; \;\; \mathbf{V}_{t|t-1} = \mathbf{F}\mathbf{V}_{t-1|t-1}\mathbf{F}' + \Psi; \;\; \mathbf{K}_t = \mathbf{V}_{t|t-1}\mathbf{H}'\left(\mathbf{H}\mathbf{V}_{\mathbf{t|t-1}}\mathbf{H}' + \Sigma\right)^{-1}$$
$$\mathbf{z}_{t|t} = \mathbf{z}_{t|t-1} + \mathbf{K}_t\left(\mathbf{y}_t - \boldsymbol{\mu} - \mathbf{H}\mathbf{z}_{t|t-1}\right); \;\; \mathbf{V}_{t|t} = \mathbf{V}_{t|t-1} - \mathbf{K}_t\mathbf{H}\mathbf{V}_{t|t-1}.$$

The last elements of the recursion, $\mathbf{z}_{T|T}$ and $\mathbf{V}_{T|T}$, are the mean and variance of the normal distribution used to draw $\mathbf{z}_T$. The draw of $\mathbf{z}_T$ and the output of the filter can then be used for the first step of the backward recursion, which delivers the $\mathbf{z}_{T-1|T}$ and $\mathbf{V}_{T-1|T}$ values necessary to make a draw for $\mathbf{z}_{T-1}$ from a gaussian distribution. The backward recursion can be continued until time zero, drawing each value of $\mathbf{z}_t$ in the process, with the following updating formulae for a generic time $t$ recursion:

$$\mathbf{z}_{t|t+1} = \mathbf{z}_{t|t} + \mathbf{V}_{t|t}\mathbf{F}'\mathbf{V}_{t+1|t}^{-1}\left(\mathbf{z}_{t+1} - \mathbf{F}\mathbf{z}_{t|t}\right); \;\; \mathbf{V}_{t|t+1} = \mathbf{V}_{t|t} - \mathbf{V}_{t|t}\mathbf{F}'\mathbf{V}_{t+1|t}^{-1}\mathbf{F}\mathbf{V}_{t|t}.$$

Hence parameters and states can be drawn via the Gibbs sampler using the following algorithm:

1. Take a guess $\tilde{\Pi}'$ and $\tilde{\Sigma}^{-1}$ (e.g. the frequentist maximum likelihood estimates), and use it to construct initial draws for $\boldsymbol{\mu}$ and $\mathbf{H}$. Using also $\mathbf{F}$ and $\Psi$, draw the $\mathbf{z}_t$ history using the Kalman recursion above with (Kalman step)

$$\mathbf{z}_t \sim \mathcal{N}\left(\mathbf{z}_{t|t+1}; \mathbf{V}_{t|t+1}\right).$$

2. Conditioning on $\{\mathbf{z}_t\}_{t=1}^{T}$ (drawn at the previous step) and $\{\mathbf{y}_t\}_{t=1}^{T}$ run $OLS$ imposing the zero restrictions and get $\hat{\Pi}'_{OLS}$ and $\hat{\Sigma}_{OLS}$, and draw $\tilde{\Pi}'$ and $\tilde{\Sigma}^{-1}$ from the Normal-inverse-$\Gamma$ (N-i-$\Gamma$ step). Use these draws as the initial guess for the previous point of the algorithm, and repeat.

Computing posterior confidence intervals for the cross-sectional performance of the model, conditional on the data, is relatively simple since, conditional on a draw of the time series parameters, estimates of the risk premia ($\lambda$'s in equations (14) and (15)) are just a mapping obtainable via the linear projection of average returns on the asset loadings in $\mathbf{H}$. Hence, to compute posterior confidence intervals for the cross-sectional analysis, we repeat the cross-sectional estimation for each posterior draw of the time series parameters, and report the posterior distribution of the cross-sectional statistics across these draws..

## A.2 S-VAR Identification via Long-Run Restrictions

Consider the structural vector autoregression of order $p$ for the vector of variables $X_t$ (given by the quarterly consumption growth and market returns):

$$\Gamma_0 X_t + \Gamma(L) X_{t-1} = c + \varepsilon_t, \;\; \varepsilon_t \sim iid\mathcal{N}(0, I),$$

where $c$ is a vector of constants, $L$ denotes the lag operator, $\Gamma(L) \equiv \Gamma_1 + \Gamma_2 L + ... + \Gamma_p L^{p-1}$, and each $\Gamma_j$ is a two by two matrix. In order to identify the S-VAR using long-run restrictions we follow Blanchard and Quah (1989) and work with the moving average representation

$$X_t = \kappa + A(L)\begin{bmatrix} \varepsilon_t^{sr} \\ \varepsilon_t^{lr} \end{bmatrix} \tag{27}$$

where $\kappa$ is a vector of constants, $A(L) \equiv A_0 + A_1 L + A_2 L^2 + ... A_\infty L^\infty \equiv [\Gamma_0 + L\Gamma(L)]^{-1}$, $\varepsilon_t^{sr}$ and $\varepsilon_t^{lr}$ denote, respectively, short- and long-run Gaussian shocks with covariance matrix normalized to be equal to the identity matrix.

The two types of shocks are identified imposing the restriction $\sum_{j=0}^{\infty} \{A_j\}_{1,2} = 0$ where $\{.\}_{1,2}$ returns the $(1,2)$ element of the matrix. That is, the short-run shock has no long-run effect on at least one of the elements of the vector Y. This restriction also implies that $A(1) \equiv \sum_{j=0}^{\infty} A_j$ should be a lower triangular matrix.

The S-VAR coefficient can be easily recovered from the reduced form VAR

$$X_t = \gamma + B(L) X_{t-1} + v_t, \;\; v_t \sim \mathcal{N}(0, \Omega)$$

where $B(L) = B_0 + B_1 L + ... + B_p L^p$ and $\Omega = \Gamma_0^{-1} \left(\Gamma_0^{-1}\right)'$ can be estimated via OLS.

Given the restrictions $\sum_{j=0}^{\infty} \{A_j\}_{1,2} = 0$, it follows that $D := [I - B(1)]^{-1} \Gamma_0^{-1}$ should be a lower triangular matrix. Note also that $DD' = [I - B(1)]^{-1} \Omega [I - B(1)]^{-1\prime}$. Hence, an estimate of the $DD'$ matrix, $\widehat{DD'}$, can be constructed from the reduced form OLS estimates $\hat{B}(L)$ and $\hat{\Omega}$, and imposing the lower triangular structure on $D$, we can estimate $\hat{D}$ from the Choleski decomposition of $\widehat{DD'}$. Finally, we can recover the S-VAR parameters from $\hat{\Gamma}_0^{-1} = \left[I - \hat{B}(1)\right] \hat{D}$, $\hat{\Gamma}(L) = -\hat{\Gamma}_0 \hat{B}(L)$ and $\hat{c} = \hat{\Gamma}_0 \hat{\gamma}$. Impulse response functions and their confidence regions can then be constructed following Sims and Zha (1999).

## A.3 The Moving Average Representation of The Long Run Risk Process

We we assume the same data generating process as in Bansal and Yaron (2004), with the only exception that we introduce a square-root process for the variance, as in Hansen, Heaton, Lee, and Roussanov (2007), that is:

$$\Delta c_{t,t+1} = \mu + x_t + \sigma_t \eta_{t+1}; \;\; x_{t+1} = \rho x_t + \phi_e \sigma_t e_{t+1}; \;\; \sigma_{t+1}^2 = \sigma^2 (1 - \nu_1) + \nu_1 \sigma_t^2 + \sigma_w \sigma_t w_{t+1},$$

where $\eta_t$, $e_t$, $w_t$, $\sim$ iid$\mathcal{N}(0,1)$. The calibrated *monthly* parameter values are: $\mu = 0.0015$, $\rho = 0.979$, $\phi_e = 0.044$, $\sigma = 0.0078$, $\nu_1 = 0.987$, $\sigma_w = 0.00029487$. To extract the quarterly frequency moving average representation of the process, we proceed in two steps. First, we simulate a long sample (five million observations) from the above system treating the given parameter values as the truth. Second, we aggregate the simulated data into quarterly observation and we use them to estimate, via MLE, the moving average representation of consumption growth in equation (1).

## A.4 Empirical Likelihood Estimation

Consider the following transformation of the Euler equation:

$$\mathbf{0} = \mathbb{E}\left[M_t^S \mathbf{R}_t^e\right] \equiv \int \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \psi_{t+S} \mathbf{R}_t^e dP = \int \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \frac{\psi_{t+S}}{\bar{\psi}} \mathbf{R}_t^e d\mathbf{P}$$
$$= \int \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e d\Psi = \mathbb{E}^\Psi\left[\left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right] \tag{28}$$

where $P$ is the unconditional physical probability measure, $\bar{\psi} = \mathbb{E}[\psi_{t+S}]$, $\Psi$ is another probability measure, related to the physical one through the Radon-Nikodym derivative[20] $\frac{d\Psi}{dP} = \frac{\psi_{t+S}}{\bar{\psi}}$.

Empirical Likelihood provides a natural framework for recovering parameter estimates and probability measure $\Psi$ defined by equation (28), by minimising Kullback-Leibler Information Criterion (KLIC):

$$(\hat{\Psi}, \hat{\phi}) = \arg\min_{\Psi,\phi} D(P||\Psi) \equiv \arg\min_{\Psi} \int \ln\frac{dP}{d\Psi} dP \quad \text{s.t.} \quad \mathbf{0} = \mathbb{E}^\Psi\left[\left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right] \tag{29}$$

Equation (29) provides a nonparametric maximum likelihood estimation of the probability measure, induced by the unobservable components of the SDF, and has been used in various applications, including the recovery of the risk-neutral probability density (Stutzer (1995)). For more information on the rationale behind this change of measure, see Ghosh, Julliard, and Taylor (2013).

Following Csiszar (1975) duality approach, one can easily show that:

$$\hat{\psi}_t = \frac{1}{T\left(1 + \hat{\boldsymbol{\lambda}}(\theta)' \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\hat{\phi}} \mathbf{R}_t^e\right)} \quad \forall t = 1..T, \tag{30}$$

where $\hat{\phi}$ and $\hat{\boldsymbol{\lambda}} \equiv \hat{\boldsymbol{\lambda}}(\hat{\phi}) \in \mathbb{R}^n$ are the solution to the dual optimisation problem:

$$\hat{\phi} = \arg\min_{\phi\in\mathbb{R}} \; -\sum_{t=1}^{T} \ln\left(1 + \hat{\boldsymbol{\lambda}}(\phi)' \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right) \tag{31}$$

$$\hat{\boldsymbol{\lambda}}(\phi) = \arg\min_{\boldsymbol{\lambda}\in\mathbb{R}^n} \; -\sum_{t=1}^{T} \ln\left(1 + \boldsymbol{\lambda}(\phi)' \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right) \tag{32}$$

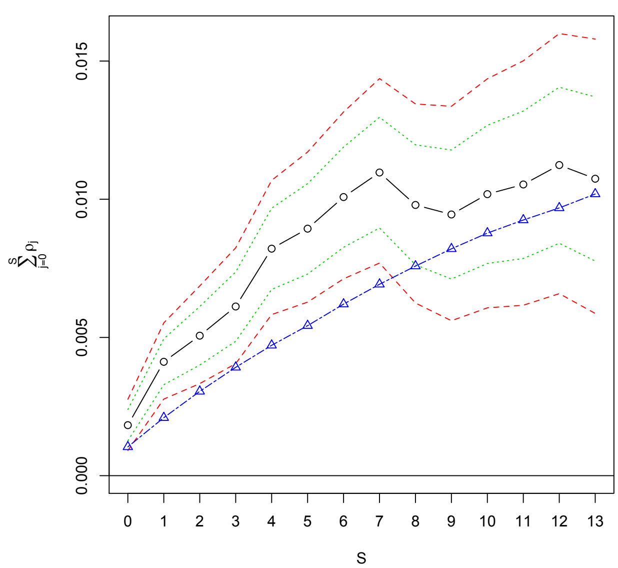
The dual problem is usually solved via the combination of internal and external loops (Kita-

[20]We assume absolute continuity of both $P$ and $\Psi$.

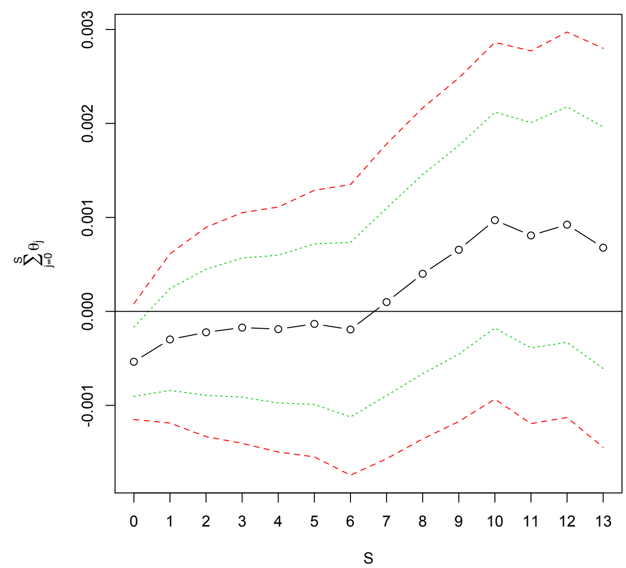
mura (2001)): first, for each $\phi$ find the optimal values of the Langrange multipliers $\lambda$, as in equation (32); then minimize the value of the dual objective function w.r.t. $\phi(\hat{\lambda})$, following equation (31).

Empirical likelihood estimator is known not only for its nonparametric likelihood interpretation, but also for its convenient asymptotic representation and properties. It belongs to the family of Generalised Empirical Likelihood estimators (Newey and Smith (2004)), with other notable members including the Exponentially Tilted Estimator (ET, Kitamura and Stutzer (1997)) and the Continuously Updated GMM (CU-GMM, Hansen, Heaton, and Yaron (1996)). While the whole family enjoys the same asymptotic distribution of the parameter estimates, achieves the semiparametric efficiency bound of Chamberlain (1987), and shares the standard battery of tests for moment equalities (e.g. $J$-test), the empirical likelihood estimator is also higher-order efficient (Newey and Smith (2004), Anatolyev (2005)).

## A.5 Additional Figures




**(a)** cumulated consumption response to common factor, $f_t$, shocks




**(b)** cumulated consumption response to e to bond factor, $g_t$, shocks

**Figure A1:** Slow consumption adjustment response to the latent factors $f_t$ and $g_t$ shocks.

Posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions. Estimation based on the two-factor model in equations (5) and (6). Triangles denote Bansal and Yaron (2004) implied values.

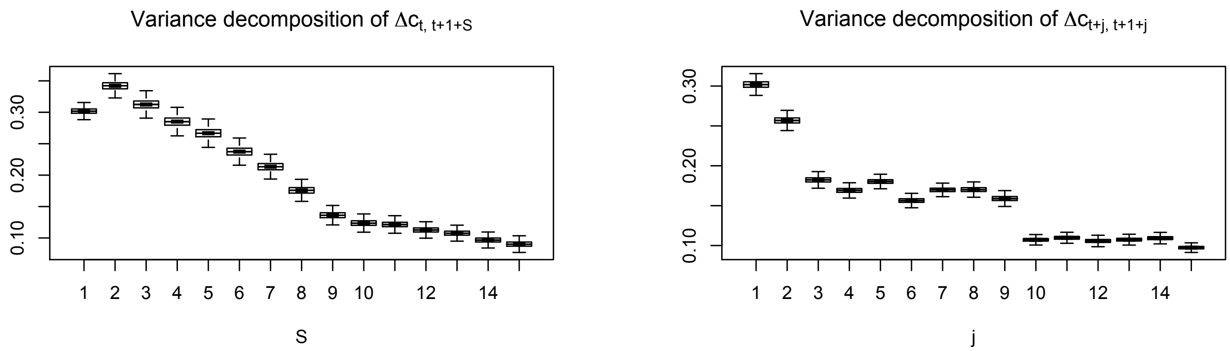



**Figure A2:** Variance of consumption growth explained by the MA components $f$ and $g$.

Box-plots (posterior 95% coverage area) of the percentage of time series variances of consumption growth explained by the MA components $f$ and $g$. Left panel: cumulated consumption growth $\Delta c_{t,t+1+S}$. Right panel: one period consumption growth $\Delta c_{t+j,t+1+j}$.

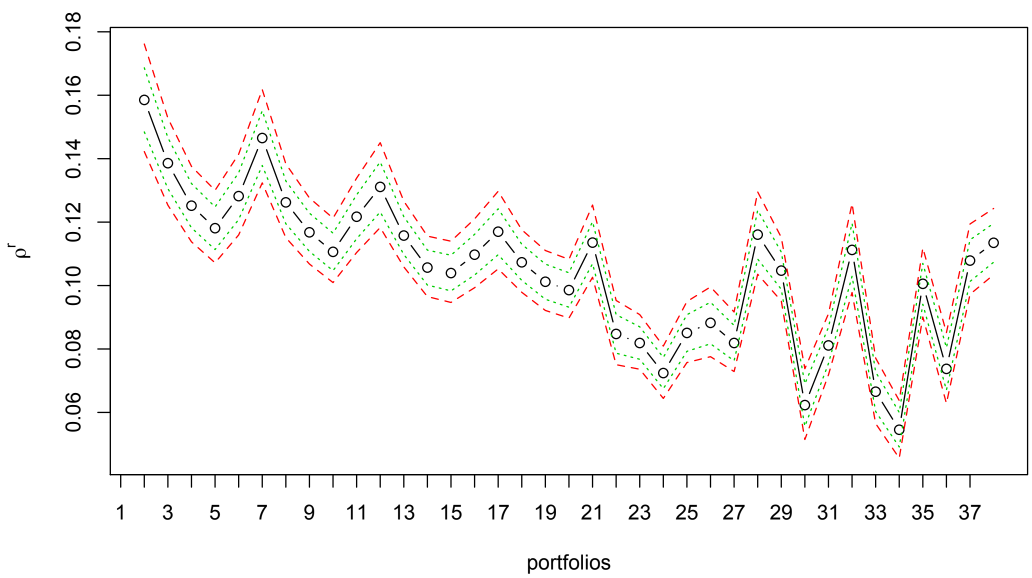



**Figure A3:** Common factor loadings ($\boldsymbol{\rho}^r$) of the stock portfolios in the two-factor model.

The graph presents posterior means of the stocks factor loadings on $f_t$ (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions in the two latent factors model. Ordering of portfolios: 25 Fama and French (1992) size and book-to-market sorted portfolios (e.g. portfolio 2 is the smallest decile of size and the second smaller decile of book-to-market ratio), and 12 industry portfolios.

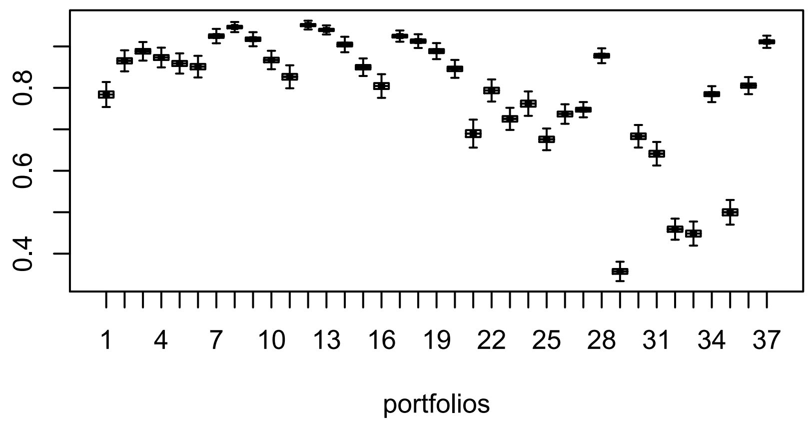



**Figure A4:** Share of stock portfolios' return variance explained by the $f$ component in the two-factor model.

Box-plots (posterior 95% coverage area) of the percentage of time series variances of individual stock portfolio returns explained by the $f$ component in the two-factor model. Ordering of portfolios: 25 Fama and French (1992) size and book-to-market sorted portfolios (e.g. portfolio 2 is the smallest decile of size and the second smaller decile of book-to-market ratio), and 12 industry portfolios.

## A.6 Additional Tables

**Table A1:** Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

| | Empirical Likelihood | | | | | Generalised Method of Moments | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Horizon S (Quarters) | $R^2_{adj}(\%)$ (1) | $\alpha_b$ (2) | $\alpha_s$ (3) | $\phi$ (4) | $J$-test (5) | $R^2_{adj}(\%)$ (6) | $\alpha_b$ (7) | $\alpha_s$ (8) | $\phi$ (9) | $J$-test (10) |
| *Panel A: 9 Bonds and Fama-French 6 portfolios* | | | | | | | | | | |
| 0 | -6 | 0.0001 | 0.0162 | -74 | 59.7497 | 71 | 0.0003 | 0.0137 | 33 | 38.3181 |
| | | (0.0002) | (0.0045) | (21.2) | [0.0003] | | (0.0003) | (0.0069) | (42.1) | [0.0001] |
| 10 | 54 | 0.0004 | 0.0105 | 22 | 24.2148 | 29 | 0.0006 | 0.0132 | 28 | 17.3421 |
| | | (0.0003) | (0.0046) | (3.9) | [0.6184] | | (0.0004) | (0.0046) | (6.9) | [0.1372] |
| 11 | 51 | 0.0005 | 0.0099 | 24 | 24.2189 | 44 | 0.0008 | 0.0131 | 30 | 17.6300 |
| | | (0.0003) | (0.0047) | (3.7) | [0.6181] | | (0.0003) | (0.0048) | (8.3) | [0.1274] |
| 12 | 52 | 0.0005 | 0.0093 | 22 | 22.1532 | 53 | 0.0009 | 0.0136 | 32 | 18.6997 |
| | | (0.0003) | (0.0049) | (3.5) | [0.7295] | | (0.0003) | (0.0050) | (9.0) | [0.0960] |
| *Panel B: 9 Bonds and Fama-French 25 portfolios* | | | | | | | | | | |
| 0 | 50 | -0.0006 | 0.0130 | 60 | 62.3266 | 61 | 0.0011 | 0.0125 | 50 | 226.2077 |
| | | (0.0002) | (0.0045) | (21.2) | [0.0007] | | (0.0001) | (0.0038) | (15.3) | [0.0000] |
| 10 | 72 | -0.0002 | 0.0104 | 19 | 23.1802 | 26 | 0.0019 | 0.0063 | 37 | 44.4437 |
| | | (0.0003) | (0.0038) | (3.9) | [0.8425] | | (0.0003) | (0.0018) | (5.9) | [0.0558] |
| 11 | 79 | -0.0002 | 0.0096 | 18 | 20.8589 | 56 | 0.0020 | 0.0052 | 39 | 33.7601 |
| | | (0.0002) | (0.0039) | (3.9) | [0.9156] | | (0.0003) | (0.0020) | (6.5) | [0.3355] |
| 12 | 78 | -0.0001 | 0.0096 | 17 | 20.4496 | 64 | 0.0018 | 0.0065 | 42 | 28.8556 |
| | | (0.0002) | (0.0040) | (3.7) | [0.9257] | | (0.0002) | (0.0015) | (7.2) | [0.5768] |
| *Panel C: 9 Bonds, Fama-French 6, and Industry 12 portfolios* | | | | | | | | | | |
| 0 | 64 | 0.0000 | 0.0119 | -14 | 59.4323 | -33 | 0.0006 | 0.0239 | 31 | 124.6547 |
| | | (0.0002) | (0.0041) | (22.3) | [0.0001] | | (0.0001) | (0.0027) | (18.4) | [0.0000] |
| 10 | 72 | 0.0003 | 0.0131 | 14 | 21.9269 | -77 | 0.0016 | 0.0140 | 32 | 44.9201 |
| | | (0.0003) | (0.0039) | (4.3) | [0.5836] | | (0.0003) | (0.0024) | (6.3) | [0.0060] |
| 11 | 70 | 0.0004 | 0.0119 | 11 | 24.8752 | -53 | 0.0018 | 0.0140 | 34 | 37.2377 |
| | | (0.0002) | (0.0040) | (3.9) | [0.4126] | | (0.0003) | (0.0023) | (7.0) | [0.0414] |
| 12 | 72 | 0.0004 | 0.0115 | 10 | 24.4976 | 2 | 0.0019 | 0.0107 | 38 | 29.5539 |
| | | (0.0002) | (0.0041) | (3.7) | [0.4335] | | (0.0003) | (0.0024) | (7.7) | [0.2000] |
| *Panel D: 9 Bonds, Fama-French 25, and Industry 12 portfolios* | | | | | | | | | | |
| 0 | 54 | 0.0005 | 0.0124 | 23 | 78.2258 | 36 | 0.0007 | 0.0146 | 58 | 269.4971 |
| | | (0.0002) | (0.0036) | (16.7) | [0.0008] | | (0.0002) | (0.0027) | (13.4) | [0.0000] |
| 10 | 61 | -0.0002 | 0.0114 | 6 | 55.7091 | -29 | 0.0018 | 0.0093 | 36 | 71.4739 |
| | | (0.0003) | (0.0034) | (2.5) | [0.0926] | | (0.0003) | (0.0013) | (4.7) | [0.0041] |
| 11 | 62 | -0.0002 | 0.0112 | 6 | 53.6016 | 8 | 0.0020 | 0.0090 | 37 | 60.1299 |
| | | (0.0002) | (0.0034) | (2.4) | [0.1289] | | (0.0003) | (0.0013) | (4.8) | [0.0430] |
| 12 | 62 | -0.0002 | 0.0111 | 6 | 51.8898 | 25 | 0.0019 | 0.0082 | 42 | 47.2360 |
| | | (0.0002) | (0.0034) | (2.2) | [0.1659] | | (0.0003) | (0.0012) | (5.4) | [0.3036] |

The table reports the pricing of excess returns of stocks and bonds, allowing for separate asset class-specific intercepts. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.

**Table A2:** Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

| | Empirical Likelihood | | | | Generalised Method of Moments | | | |
|---|---|---|---|---|---|---|---|---|
| Horizon S (Quarters) | $R^2_{adj}(\%)$ (1) | $\alpha$ (2) | $\phi$ (3) | $ELR$-test (4) | $R^2_{adj}(\%)$ (5) | $\alpha$ (6) | $\phi$ (7) | $J$-test (8) |
| *Panel A: B Bonds and Fama-French 6 portfolios* | | | | | | | | |
| 0 | -30 | 0.0002 | -16 | 30.1955 | 73 | 0.0007 | 73 | 35.0646 |
| | | (0.0002) | (25.0) | [0.0044] | | (0.0003) | (27.1) | [0.0008] |
| 10 | 94 | 0.0008 | 23 | 11.5946 | 91 | 0.0009 | 29 | 24.9738 |
| | | (0.0003) | (6.0) | [0.5611] | | (0.0004) | (6.8) | [0.0233] |
| 11 | 95 | 0.0006 | 24 | 10.4758 | 94 | 0.0011 | 32 | 24.4029 |
| | | (0.0003) | (6.7) | [0.6546] | | (0.0003) | (8.5) | [0.0276] |
| 12 | 92 | 0.0005 | 23 | 11.1154 | 96 | 0.0012 | 34 | 25.2110 |
| | | (0.0003) | (6.5) | [0.6011] | | (0.0003) | (9.3) | [0.0217] |
| *Panel A: B Bonds and Fama-French 25 portfolios* | | | | | | | | |
| 0 | 54 | -0.0004 | 52 | 78.6597 | 60 | 0.0018 | 61 | 321.3738 |
| | | (0.0002) | (18.2) | [0.0000] | | (0.0001) | (15.3) | [0.0000] |
| 10 | 74 | -0.0001 | 19 | 68.5008 | 38 | 0.0025 | 38 | 48.0606 |
| | | (0.0002) | (3.7) | [0.0002] | | (0.0003) | (5.7) | [0.0340] |
| 11 | 76 | 0.0000 | 20 | 67.9188 | 62 | 0.0024 | 40 | 35.1659 |
| | | (0.0002) | (3.7) | [0.0002] | | (0.0003) | (6.0) | [0.3205] |
| 12 | 70 | 0.0000 | 18 | 71.0791 | 67 | 0.0029 | 44 | 30.5687 |
| | | (0.0002) | (3.4) | [0.0001] | | (0.0002) | (7.4) | [0.5390] |
| *Panel C: 9 Bonds, Fama-French 6, and Industry 12 portfolios* | | | | | | | | |
| 0 | -6 | 0.0001 | -6 | 63.2328 | 61 | 0.0017 | 55 | 273.0204 |
| | | (0.0002) | (21.9) | [0.0002] | | (0.0002) | (15.2) | [0.0000] |
| 10 | 56 | 0.0009 | 14 | 56.6896 | -24 | 0.0037 | 35 | 51.9830 |
| | | (0.0003) | (4.0) | [0.0003] | | (0.0003) | (6.4) | [0.0012] |
| 11 | 51 | 0.0009 | 12 | 58.4329 | -9 | 0.0042 | 37 | 38.4378 |
| | | (0.0002) | (3.7) | [0.0002] | | (0.0003) | (6.9) | [0.0419] |
| 12 | 52 | 0.0009 | 12 | 58.0225 | 10 | 0.0039 | 41 | 29.1776 |
| | | (0.0002) | (3.6) | [0.0002] | | (0.0003) | (6.5) | [0.2566] |
| *Panel D: 9 Bonds, Fama-French 25, and Industry 12 portfolios* | | | | | | | | |
| 0 | 26 | -0.0003 | 22 | 146.685 | 54 | 0.0016 | 69 | 356.9325 |
| | | (0.0002) | (15.2) | [0.0000] | | (0.0002) | (13.5) | [0.0000] |
| 10 | 38 | -0.0002 | 8 | 141.4802 | -25 | 0.0039 | 39 | 77.4115 |
| | | (0.0002) | (2.5) | [0.0000] | | (0.0003) | (4.8) | [0.0014] |
| 11 | 38 | -0.0002 | 8 | 140.6384 | 16 | 0.0041 | 39 | 66.0979 |
| | | (0.0002) | (2.3) | [0.0000] | | (0.0003) | (4.9) | [0.0172] |
| 12 | 37 | -0.0002 | 8 | 140.8904 | 29 | 0.0041 | 43 | 51.9741 |
| | | (0.0002) | (2.2) | [0.0000] | | (0.0003) | (5.6) | [0.1912] |

The table reports the pricing of excess returns of stocks and bonds, allowing for a common intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.

**Table A3:** Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

| | Empirical Likelihood | | | | Generalised Method of Moments | | | |
|---|---|---|---|---|---|---|---|---|
| Horizon S (Quarters) | $R^2_{adj}(\%)$ (1) | $\alpha$ (2) | $\phi$ (3) | $ELR$-test (4) | $R^2_{adj}(\%)$ (5) | $\alpha$ (6) | $\phi$ (7) | $J$-test (8) |
| 0 | -30 | 0.0002 | -16 | 30.1955 | 73 | 0.0007 | 73 | 35.0646 |
| | | (0.0002) | (25.0059) | [0.0044] | | (0.0003) | (27.1) | [0.0008] |
| 1 | 50 | 0.0005 | 55 | 19.8352 | 64 | 0.0006 | 50 | 26.7987 |
| | | (0.0003) | (16.5936) | [0.0994] | | (0.0003) | (16.4) | [0.0133] |
| 2 | 3 | 0.0008 | 50 | 15.9515 | 39 | 0.0008 | 45 | 20.5230 |
| | | (0.0004) | (11.4430) | [0.2517] | | (0.0004) | (11.3) | [0.0829] |
| 3 | 27 | 0.0007 | 45 | 14.3198 | 55 | 0.0007 | 40 | 20.5313 |
| | | (0.0004) | (9.3960) | [0.3517] | | (0.0004) | (9.4) | [0.0827] |
| 4 | -33 | 0.0004 | 40 | 12.6842 | 16 | 0.0004 | 36 | 18.8278 |
| | | (0.0003) | (7.8412) | [0.4725] | | (0.0003) | (7.7) | [0.1285] |
| 5 | 58 | 0.0004 | 29 | 11.8102 | 42 | 0.0005 | 31 | 19.6120 |
| | | (0.0003) | (6.5887) | [0.5433] | | (0.0003) | (6.7) | [0.1053] |
| 6 | 67 | 0.0004 | 27 | 12.0794 | 53 | 0.0005 | 29 | 20.0162 |
| | | (0.0003) | (6.0256) | [0.5211] | | (0.0003) | (6.2) | [ 0.0948] |
| 7 | 61 | 0.0002 | 26 | 11.9012 | 43 | 0.0004 | 28 | 22.5791 |
| | | (0.0003) | (5.8619) | [0.5358] | | (0.0003) | (6.0) | [0.0470] |
| 8 | 89 | 0.0003 | 25 | 12.3113 | 74 | 0.0006 | 29 | 23.9049 |
| | | (0.0003) | (5.8866) | [0.5023] | | (0.0003) | (6.3) | [0.0320] |
| 9 | 95 | 0.0003 | 25 | 13.0312 | 92 | 0.0009 | 29 | 24.9160 |
| | | (0.0003) | (5.9862) | [0.4454] | | (0.0003) | (6.4) | [0.0237] |
| 10 | 94 | 0.0008 | 23 | 11.5946 | 91 | 0.0009 | 29 | 24.9738 |
| | | (0.0003) | (5.9595) | [0.5611] | | (0.0004) | (6.8) | [0.0233] |
| 11 | 95 | 0.0006 | 24 | 10.4758 | 94 | 0.0011 | 32 | 24.4029 |
| | | (0.0003) | (6.7275) | [0.6546] | | (0.0003) | (8.5) | [0.0276] |
| 12 | 92 | 0.0005 | 23 | 11.1154 | 96 | 0.0012 | 34 | 25.2110 |
| | | (0.0003) | (6.5436) | [0.6011] | | (0.0003) | (9.3) | [0.0217] |
| 13 | 86 | 0.0004 | 22 | 11.8978 | 96 | 0.0012 | 35 | 26.5862 |
| | | (0.0003) | (6.3313) | [0.5360] | | (0.0003) | (9.6) | [0.0142] |
| 14 | 85 | 0.0004 | 23 | 11.7044 | 97 | 0.0013 | 42 | 18.5716 |
| | | (0.0003) | (6.5983) | [0.5520] | | (0.0005) | (13.2) | [0.1370] |
| 15 | 79 | 0.0005 | 21 | 13.4734 | 96 | 0.0021 | 43 | 32.4073 |
| | | (0.0003) | (6.1575) | [0.4120] | | (0.0004) | (12.7) | [0.0021] |

The table reports the pricing of 9 excess bond holding returns and 6 Fama-French portfolios, sorted on size and book-to-market. We report the results for various values of the horizon parameters S and allow for a common intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.