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Multi-Attribute Decision Making (MADM) and Associated Assessment Techniques in Support of Army Studies and Analyses

Code of Best Practices (COBP)

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14. ABSTRACT
This document is intended to assist study directors and analysts in recognizing situations suited to multi-attribute decision making and how to best apply the process as a tool for analysis to aid decision makers.

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Content

- Purpose
- Background
- MADM Basics
- MADM Definitions and Conventions
- Multi-attribute Value Analysis with the Illustrative Problem
- Qualitative Modeling & Formulating Hierarchical Structures
- Measurement Scales & Quantification of Value Functions
- Useful Assessment Techniques for Determining Weights & Priorities
- Group Assessment Considerations
- The Analytic Hierarchy Process (AHP) and Its Contributions
- Cautions and Summary

Content

This document addresses the conduct of Multi-Attribute Decision Making (MADM) and associated assessment techniques in support of military studies and analyses. The COBP begins by acknowledging recent background developments that have led to the development of this document. This COBP then covers basic MADM definitions and conventions used throughout the document before illustrating the general processes and methodologies that support the conduct of MADM analyses.

In general, MADM concentrates on decision making situations where there are multiple competing objectives that require the consideration of tradeoffs among the objectives and their attributes; attributes reflect the varying degrees of attainment associated with the objectives.

Throughout MADM, there are certain guiding principles or tenets that, when adhered to, facilitate completion of structured, defensible studies and analyses. This COBP addresses these tenets and then follows with discussions of assessment techniques that support MADM analyses.

Purpose

To provide guidelines and techniques, in the form of a code of best practice (COBP) for the employment of Multi-Attribute Decision Making (MADM) and Associated Assessment Techniques in the conduct of Army studies and analyses.

Primary References

- **Keeney, R.L. and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. New York: John Wiley & Sons, 1976.**
- **Keeney, R.L. *Value-Focused Thinking: A Path to Creative Decisionmaking*. Cambridge, Massachusetts: Harvard University Press, 1992.**
- **Kirkwood, C.W. *Strategic Decision Making: Multi-Objective Decision Analysis with Spreadsheets*. Belmont, CA: Duxbury Press, 1997.**
- **Saaty, T.L., *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*. New York: McGraw-Hill, 1980.**

TRADOC and Army personnel who have led or conducted MADM analyses provided practical input for this COBP

Purpose

The purpose of this COBP is to provide guidelines for conducting MADM analysis and its associated assessment techniques. The guidelines describe things that must be done, things that should be done, and things that must not be done when conducting MADM analysis. The intended audience for the COBP is TRADOC personnel who conduct MADM analysis and who are new to, or re-visiting, the component concepts of MADM analysis.

A large portion of the source material for developing this COBP came from the primary references listed in the previous chart. These sources provided the basic concepts underlying this COBP. In addition, several secondary references were instrumental in developing this COBP. These will be specified throughout this document when discussing their relevance. A complete list of all references is appended at the back of this document.

In addition, many of the practical aspects associated with this COBP came from inputs provided by TRADOC and Army personnel who have conducted MADM analyses. Their execution of MADM analysis provided several lessons learned and have been incorporated into this COBP.

Background: Why the Focus on MADM?

- **Multi-attribute decision making's (MADM's) general usefulness in addressing selection and evaluation problems.**
- **Quick reaction studies & efforts in support of senior decision makers.**
- **Joint Capabilities Integration and Development Systems (JCIDS) Analysis & Requirements.**
  - **Evaluation & prioritization of functions in conjunction with Functional Area Analysis (FAA).**
  - **Evaluation & prioritization of needs in conjunction with Functional Needs Analysis (FNA).**
  - **Evaluation & prioritization of solutions in conjunction with Functional Solutions Analysis (FSA).**

For JCIDS applications, this MADM COBP should be used in conjunction with: TRADOC Analysis Center, *Joint Capabilities Integration and Development System (JCIDS) Analysis Code of Best Practice (COBP)*, TRAC-TD-05-012, Fort Leavenworth, Kansas, June 2005.

Background: Why the Focus on MADM?

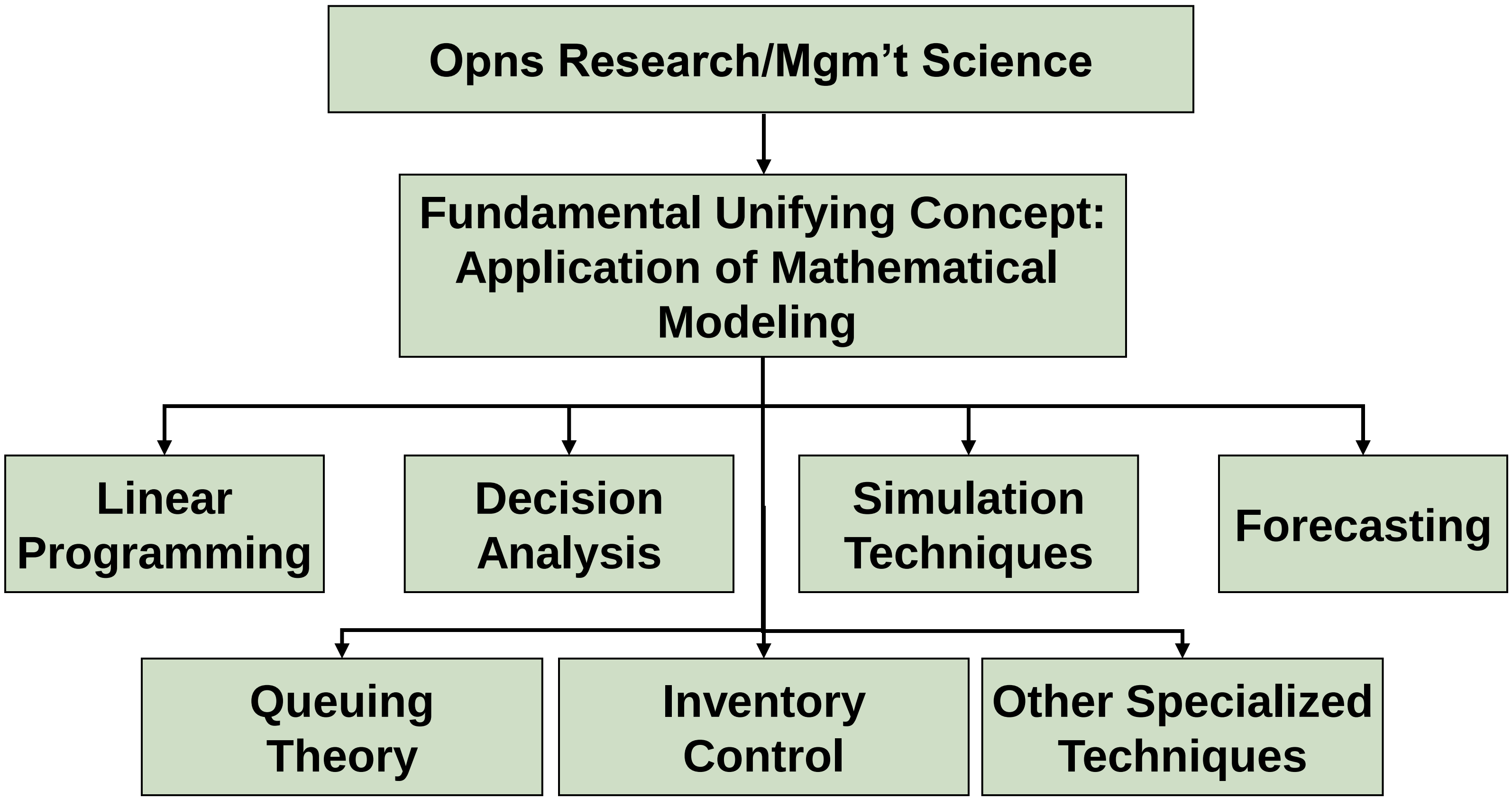
In recent years, more emphasis has been focused on providing analysis to support decisions extending beyond strict lethality-based systems to multifaceted systems that are critical in the conduct of modern warfare (e.g., C3I systems, systems incorporating both lethal & non-lethal components, logistical support systems, etc.). Thus, some of the traditional study and analysis techniques, which focused primarily on lethality capabilities, now need to be supplemented or replaced with broader analytical techniques that address both the tangible and intangible characteristics associated with modern military systems. MADM techniques provide an alternative to traditional military modeling techniques to deliver this broader analytical perspective and general usefulness.

Additionally, decision makers are continually seeking prompt analytically based underpinnings to aid them in making important decisions. MADM techniques can be implemented more rapidly than many of the traditional military study and analysis techniques to allow for more timely analytical support.

And in particular, MADM techniques have been found useful in conducting studies and analyses in support of the Joint Capabilities Integration and Development System (JCIDS), as prescribed by Chairman Joint Chiefs of Staff Instruction (CJCSI 3170.01E, 11 May 2005) and Chairman Joint Chiefs of Staff Manual (CJCSM 3170.01 B, 11 May 2005). In JCIDS analysis, the Functional Area Analysis (FAA) is the first analytical step and the step in which operational tasks, conditions, and standards are identified and prioritized. Subsequently, the Functional Needs Analysis (FNA) identifies shortfalls, which are known as capability gaps, when inabilities to perform to standard or across conditions are projected while operating in the context of current or programmed capabilities. During the Functional Solutions Analysis (FSA), proposed DOTMLPF actions to mitigate capability gaps are assessed and relative priorities determined. In each of the foregoing JCIDS steps, MADM techniques are appropriate to assist in performing the required evaluations and prioritizations. A separate TRAC COBP exists for conducting JCIDS analysis, and that should be used in conjunction with this COBP when it comes to applying MADM procedures during the JCIDS process.

Decision Analysis

From an analytic perspective, decision analysis is one of the primary branches of operations research and management science. MADM is a component branch of this broader field of decision analysis.



Decision Analysis

MADM techniques and the soon to be addressed assessment techniques are a branch of decision analysis (DA), which in turn is a field of operations research or management science as shown. In general, all the fields of operations research or management science are based on the application of mathematical modeling as the unifying, fundamental concept. In addition to DA, other operations research and management science fields include: linear programming, simulation, forecasting, queuing, inventory control, and many other specialized techniques for classes of problems that frequently occur.

In DA, different structured approaches are taken to represent and analyze various decision making situations depending on their common characteristics. Within this COBP, we are specifically focusing on MADM, which is a particular branch of decision analysis.

COBP Definitions

- **Decision. An act of judgment made among available choices.**
- **Decision Maker (DM). A person or governing body making the judgment.**
- **Objective(s). What the DM hopes to achieve as a result of the decision.**
- **Attribute(s). Performance parameters, components, factors, characteristics, and properties such that:**
  - **They provide a means of evaluating the attainment level of an objective.**
  - **Each alternative can be characterized by a number of attributes.**
- **Alternatives. Courses of action that might be taken.**
- **Value(s). Things important to the DM and especially those that are relevant to a decision.**
- **Goal(s). Setting specific degrees of attainment for objectives.**

COBP Definitions

For this COBP, the decision analytic definitions on the previous chart will apply. All of these terms are fairly standard except that sometimes in the decision analytic literature "objective" and "goal" are used interchangeability and result in confusion.

As defined on the previous chart, "increasing one's wealth" is an objective while "achieving a wealth of 1 million dollars (i.e., becoming a millionaire)" is a goal.

For a more complete description of the lexicon of decision making, the following website can be consulted: http://faculty.fuqua.duke.edu/daweb/lexicon.htm.

MADM Basics

Multi-attribute Decision Making (MADM) situations possess four common characteristics:

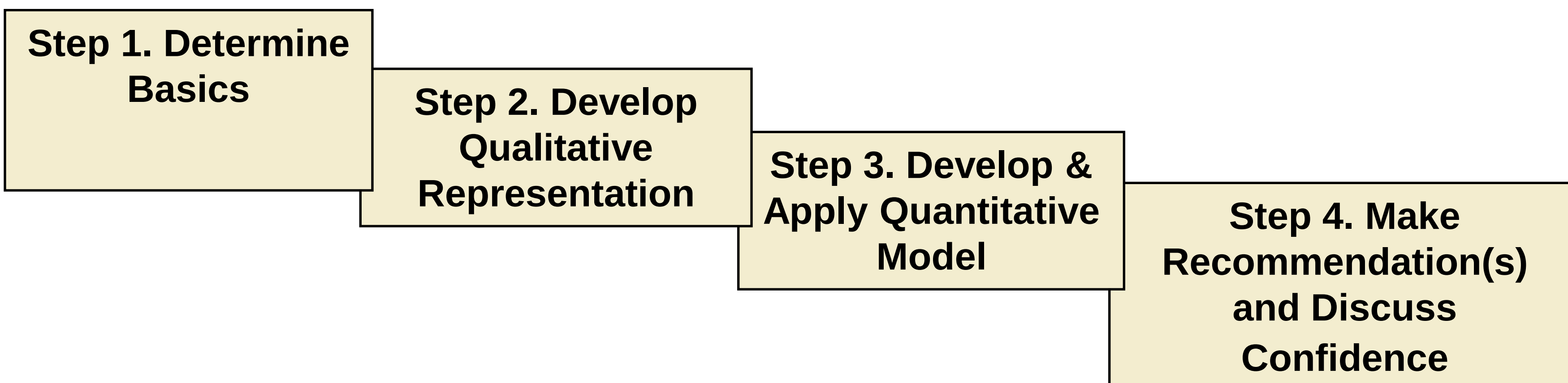
1. **There are multiple objectives and attributes.**
2. **Objectives & attributes conflict with each other to some extent and tradeoffs apply (i.e., increasing the level of achievement for one objective or attribute will often result in a decrease in the level of achievement for one or more of the remaining objectives and attribute).**
3. **Units of measurement are not the same across all attributes.**
4. **Purpose of analysis is to evaluate all alternatives and/or select best alternative.**

MADM Basics

As stated on the prior chart, situations appropriate for MADM analysis exhibit four characteristics. First, there are several objectives and attributes of interest when it comes to making a decision. Secondly, these objectives and attributes conflict with one another so that tradeoffs among the objectives and their levels of attainment become part of the decision making process. Thirdly, the decision making process is complicated because the attributes are not measured on a common scale. This means that some form of scale transformation is required in order to combine attribute scores into an overall value that reflects an alternative's worth or, in other words, "degree of goodness". Lastly, MADM situations are aimed at either selecting the single best alternative from a group of specified alternatives or evaluating the entire group of alternatives for their relative degree of worth based on the decision criteria (i.e., the objectives, their attributes, and the tradeoffs deemed appropriate).

General Steps for Addressing MADM Situations

- **Determine the basics of the decision making situation (Decision Makers (DMs), Subject Matter Experts (SMEs), stakeholders and fundamental objectives).**
- **Develop and construct a qualitative value model fitting the given situation.**
- **Develop and apply an appropriate quantitative model.**
- **Make analytically based recommendation(s) and discuss confidence concerning resulting recommendation(s).**



General Steps for Addressing MADM Situations

To initially get started in addressing a MADM situation, the analyst or analytical team needs to think hard about the basics underlying the decision. The team must identify the decision maker(s), various stakeholders, subject matter experts, and the fundamental objectives driving the decision process. The ultimate worth of a decision analysis often reflects the quality put forth during this initial step; if a decision analysis falters in determining the proper perspective or relevant principals and participants, a sub-standard decision analysis is almost assured.

The second general step in conducting a MADM analysis is to develop a qualitative model and understanding conforming to the decision situation. In this step, the decision situation basics are expanded so that the fundamental objectives are partitioned into supporting objectives as need be, and eventually into attributes, which are used to measure the attainment on the applicable objectives. Additionally during this step, alternatives are developed and specified. Typically, the qualitative model is set forth in a visual, hierarchical representation from decision focus down through the levels of objectives, attributes, and eventually to alternatives.

The aspects of quantification occur only when the third general MADM step is undertaken. In this COBP, two of the more popular quantitative MADM techniques, multi-attribute value analysis and the Analytic Hierarchy Process, will be explained, compared, and contrasted. Both procedures use well documented quantitative techniques to ultimately derive numeric evaluations for the alternatives which, when implemented properly, correspond to the decision maker's perceived overall value associated with the alternatives.

The final step of every MADM analysis should conclude with recommendations in the form of relative priorities or rankings for the alternatives along with a sensitivity analysis reflecting the confidence that should be placed in the priorities or rankings supporting the recommendations.

Emphasis on Two MADM Techniques

- **Multi-attribute value analysis.**
  - **Will specifically address problems of value where uncertainty is not considered a relevant factor in the evaluation of alternatives (i.e., this COBP deals with multi-attribute decision analysis problems under certainty, where no elements are consider subject to chance).**
  - **Will further restrict ourselves to the study of the compensatory model, as implemented via "additive value functions."**

- **Analytic Hierarchy Process.**
  - **To extend our perspective some, this COBP will address the Analytic Hierarchy Process (AHP) and its components as promoted by T.L. Saaty as an alternative MADM technique.**
  - **This method is touted as an another generalized MADM process particularly useful for planning, priority setting, and resource allocation problems.**

Emphasis on Two MADM Techniques

As indicated on the prior couple of charts, this COBP will focus on multi-attribute value analysis and the Analytic Hierarchy Process as two procedures for conducting MADM analyses. Actually, both procedures are quite similar during the first two general MADM steps and primarily differ with respect to the third (i.e., quantification) step and the subsequent fourth (i.e., sensitivity analysis) step.

Multi-attribute value analysis is the more traditional and standard of the two techniques. Its theory and practice are derived from Keeney & Raiffa's book entitled Decisions with Multiple Objectives: Preferences and Value Tradeoffs. In this book, the authors discuss preference and tradeoff problems under both certain and uncertain conditions. In general, both the certain and uncertain procedures are categorized into the general class of problems termed Multi-Attribute Utility Theory (MAUT) problems. However, the procedure described in this COBP is more appropriately termed multi-attribute value analysis since uncertainty is not a characteristic of the applications discussed, and the term utility is primarily associated with uncertainty. Additionally, this COBP will only deal with multi-attribute value analysis problems where an additive, linear mathematical model underlies the process for making tradeoffs among the decision criteria. In such a model, a decrease on one decision criterion may be counteracted (i.e., compensated for) by an increase on one or more of the remaining decision criteria. Decreases and increases are accounted for through subtraction and addition respectively.

The Analytic Hierarchy Process is a popular MADM technique, but it is less traditional and more controversial than multi-attribute value analysis. This process was first set forth by T. L. Saaty in a 1977 monograph and then followed up with a book published in 1980. Since then, hundreds of articles and applications have been published regarding this technique's strengths, weaknesses, and usefulness. Although AHP shares many characteristics with multi-attribute value analysis, it differs in the final step of quantifying the alternatives, and the nature of this quantification makes the procedure controversial. This COBP will focus on specifying and illustrating the usefulness of two of the AHP's component procedures while subsequently explaining the controversy that makes the overall AHP procedure suspect according to the traditionalists.

“The Illustrative Problem” Introduction (1 / 2)

- The Rapid Equipping Force (REF) is an organization that takes its operational guidance from the G-3 and reports directly to the Vice Chief of Staff of the Army. It has a broad mission to rapidly increase mission capability while reducing risk to Soldiers and others. The REF accomplishes this mission in three ways:
  - EQUIP operational commanders with off-the-shelf (government or commercial) solutions or near-term developmental items that can be researched, developed and acquired quickly.
  - INSERT future force technology solutions that our engaged and deploying forces require. It does this by developing, testing, and evaluating key technologies and systems under operational conditions.
  - ASSESS capabilities and advise Army stakeholders of findings that will enable our forces to rapidly confront an adaptive enemy.

“The Illustrative Problem” Introduction (2 / 2)

- **Consider an organization whose mission is to help warfighters and Soldiers address urgent needs on the battlefield that require a quicker response than the normal fielding process or for which there are no currently available means for acquisition.**
- **This organization’s in-theater personnel collect warfighters' battlefield needs that have not been addressed. Based upon both operational priority and the potential for the problem to be solved rapidly, they work to solve these technical problems and bring a solution to theater.**
- **As part of their mission, they examine currently available commercial and governmental off-the-shelf items to meet operational requirements. Given the rapid nature of their mission, MADM techniques are particularly useful.**

"The Illustrative Problem"

Request for a solution:

- **Problem Statement: Multiple key lines of communication (LOC) in the area of operation are covered by tall reeds. These reeds can get upwards of 10 feet tall and allow threat cover and concealment from friendly forces. Attacks are consistently worse on routes that have heavy reed cover.**
- **Justification: The unit requests thirty brushcutters to eliminate reeds along key LOC. A brush-cutting device can prevent threat from planting devices by eliminating their concealment and egress routes.**
- **Key Required Characteristics:**
  - **Able to remove with stocks up to 16" in diameter.**
  - **Operated from the roadside by dismounted personnel.**
  - **Has to be able to reach downward into canals.**
  - **Impact on relations with host nation minimal.**

“The Illustrative Problem” (continued)

- To help focus the discussion of MADM procedures, this COBP will employ the selection of a solution to the problem presented to the REF. While this problem is relatively simple from a military perspective, it is of a size and magnitude sufficient enough to demonstrate MADM concepts without encountering the numerical burden associated with larger problems featuring many objectives, attributes, and alternatives. It also demonstrates one key aspect of the MADM technique, the generally quick turn nature of analysis and yet retaining the capability to provide sound recommendations. In this respect, the illustrative problem can be construed as a “textbook-type” of problem to convey concepts.
- As this COBP proceeds, this illustrative problem will be used to discuss the principles underlying a MADM analysis and for the demonstration of multi-attribute value analysis and AHP procedures. Once these principles and procedures are set forth in this COBP, the aim is that these principles and procedures will be explained well enough to guide the reader through larger and more complex situations where major military and defense alternatives are evaluated.

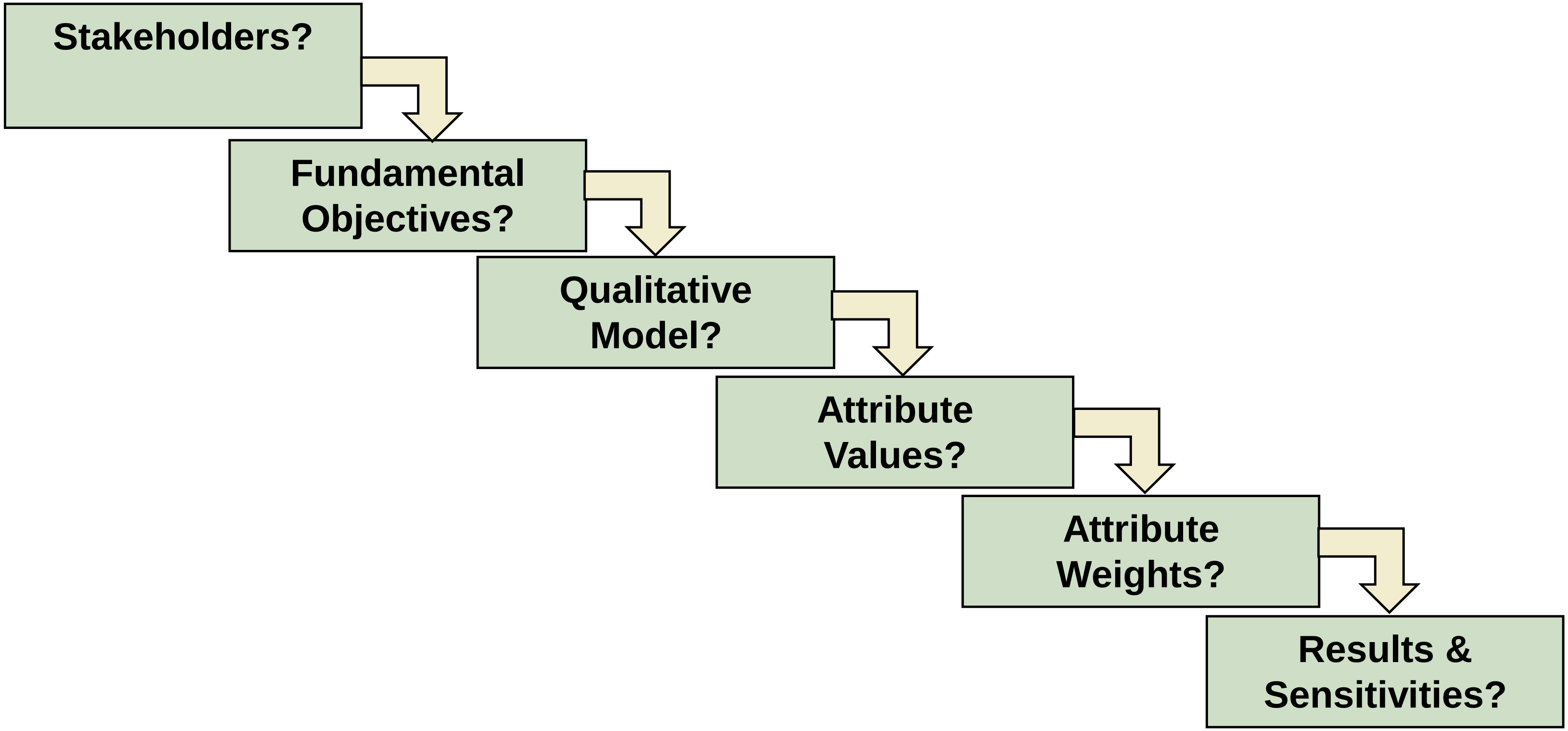
! Although this example is based upon an actual requirement received by the REF, the analysis conducted as part of this example is not based in any way upon the actual analysis conducted by the REF or the solution implemented.

Multi-attribute Value Analysis: Basic Steps & Process

- **Step 1. Identify stakeholders.**
  - Decision Makers (DM's), Subject Matter Experts (SME's), & others affected by the decision to be made.
- **Step 2. Identify fundamental objectives.**
  - Use existing documentation, leadership interviews, brainstorming, etc.
  - Time is usually the driving constraint in this important step.
- **Step 3. Develop the qualitative model.**
  - Write down and structure the fundamental objectives & supporting attributes
  - Develop graphical representation for communication purposes.
- **Step 4. Develop quantitative value model.**
  - Start with identifying measures for attribute & value functions that represent returns to scale of the measures.
  - Assess value functions (usually done with SMEs as DMs usually only care about results & why functions have shapes they do).

Multi-attribute Value Analysis: Basic Steps & Process (continued)

- **Step 5. Determine weighting factors for attributes.**
  - **Various methods possible & we will consider several of these.**
- **Step 6. Generate the results & conduct sensitivity analysis.**



Multi-attribute Value Analysis: Basis Steps & Process

In concert with the general MADM processes described previously, six steps have been detailed for conducting multi-attribute value analysis on the prior two charts. As this COBP continues, it will become apparent that the first three steps as listed on the first chart remain constant regardless of the quantitative technique applied in carrying out steps four through six. In sequence, steps four and five are concerned with generating numerical attribute values and attribute weights in accord with the qualitative model identified in step three. In the final step, results are synthesized from the attribute values and weights found in steps four and five into summary quantities, and these quantities are converted into recommendation(s) and subjected to sensitivity analysis.

This COBP will now demonstrate the steps of multi-attribute value analysis by applying them to the illustrative problem of answering the REF request for determining the best reed mitigation solution.

Identifying Stakeholders

- **Determine the decision makers, subject matter experts and other possible stakeholders in your decision.**
  - **REF: decision maker**
  - **Supported unit commander: major stakeholder**
  - **Supported unit soldiers: stakeholders**
  - **Other potential units: stakeholders**
  - **Combat veterans: subject matter experts**
  - **Others?**

Output from Step 1

! Necessary that all important perspectives identified!

Identifying Stakeholders

In the illustrative problem, certainly the REF (who will be providing the solution) and the supported commander and his soldiers are primary stakeholders in the purchase decision. Hopefully, combat veterans can qualify as subject matter experts in the selection of a solution for the specified problem. Can you think of any other stakeholders pertaining to this decision situation?

The intent underlying the identification of stakeholders is to determine who should provide inputs and expertise during the subsequent steps of the decision analysis. We must also make sure the decision analysis will include all perspectives necessary to arrive at acceptable decision recommendations.

In community or public interest decision problems, there are often competing stakeholders consisting of powerful constituents and groups that represent them. For example, when trying to formulate recommendations concerning drilling for oil on public land, both the oil producers as well as the environmentalists are major stakeholders. Naturally, such groups often hold opposing viewpoints, and this is one of the challenging aspects of executing comprehensive decision analysis. The decision analysis must acknowledge the objectives of all major stakeholders and incorporate reasonable tradeoffs among these objectives if the evaluation of decision alternatives is expected to be respected among the stakeholders.

As such, the identification of stakeholders and their roles is the first step in structuring a decision analysis and leads into the second step, the identification of fundamental objectives.

Identifying Fundamental Objectives

- **Following discussions with the requesting unit about important desires & requirements; conversations with potential previous combat veterans; etc., you determine you are ready to specify your fundamental objectives.**
- **After some preliminary attempts & restatements, you arrive at three fundamental objectives:**
  - **Mitigate the reed problem with a practical solution.**
  - **Minimize the risk of negative impact on relations with the host nation.**
  - **Select a solution which is easy for a few soldiers to operate.**

Output from Step 2

- **Note that well stated objectives start with verb & outline a desired action the DM(s) would like to achieve.**

Identifying Fundamental Objectives

After some thought and discussions with the other stakeholders and subject matter experts, you arrive at three fundamental objectives for the illustrative problem: functionality, operability, and impact on relations. In this illustrative situation, the three fundamental objectives all seem natural and straightforward. However, in instances where different viewpoints are held and greater complexity underlies the decision situation, the determination of fundamental objectives can become a difficult task. In particular, the development of a set of clear, comprehensive, and non-duplicate fundamental objectives is the aim of this step, and it usually requires a fair amount of thought and eventual refinement. As in the first step, any serious shortcomings in this step promulgate themselves throughout the rest of the decision analysis.

Note that when stating objectives, they start with a verb and indicate something that the DM would like to achieve. In this respect there is a direction associated with the objectives (i.e., either more is better or less is better).

After the fundamental objectives are specified, the next step is to further capture the decision situation in a more complete representation. This is termed the qualitative model, and it consists of supporting levels of objectives, pertinent attributes, and ultimately, the alternatives.

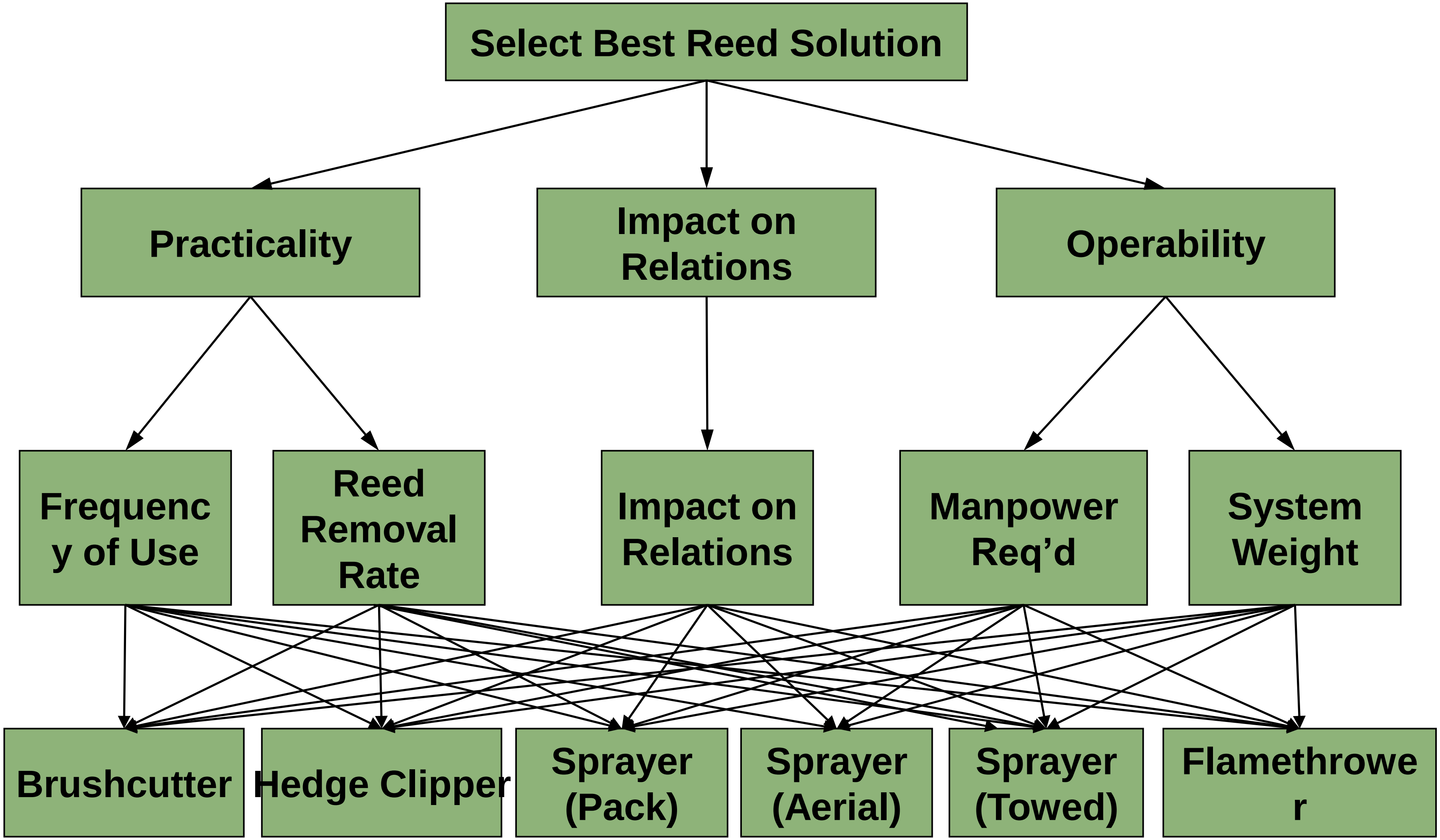
Developing the Qualitative Model

- **Once you are self-assured that your fundamental objectives reflect the DM's true desires and values, it is time to refine those, and subsequent, thoughts into the qualitative model.**
  - **Add level(s) of more specific fundamental objectives which further refine fundamental objectives.**
  - **Add attributes which allow for evaluation & measurement of lowest level of fundamental objectives specified.**
- **For "illustrative problem", we insert no more specific objectives & determine the following attributes are applicable:**
  - **Frequency of use and reed removal rate are determined to be attributes relating to "practicality".**
  - **Host nation relation impact, itself, is an intangible attribute related to "risk of negative impact" & we know it when we see it.**
  - **Operating manpower and weight are deemed to be attributes of "operability".**

Developing the Qualitative Model (Continued)

- So far nothing has been dependent on the alternatives, and that is the way it should be.
  - Development of the objectives & attributes breakdown occurs totally independent of the alternatives available.
- In the “illustrative problem”, we now turn to the community of experts which proceeds to help us identify six alternatives:
  - Brushcutter
  - Hedge Clipper
  - Backpackable Chemical Sprayer
  - Towed Chemical Sprayer
  - Aerial Chemical Sprayer
  - Backpackable Flamethrower
- As a device for communicating among the stakeholders & interested parties, we summarize our qualitative model in a hierarchical manner as shown on the next chart.

Developing the Qualitative Model (continued)



Output from Step 3 of Multi-Attribute Value Analysis Process

Developing the Qualitative Model

Building off the fundamental objectives from the second step, sub-levels of more specific objectives are added to help clarify the initial, fundamental objectives. After a reasonable degree of clarity is reached, attributes are sought which will help us measure the degree of attainment with respect to the objectives. As refinements are made and this process proceeds, the qualitative model evolves in this third step. The aim is to eventually arrive at a qualitative model that represents the apparent relationships among objectives and attributes and expresses them in clear verbal form; the model could also be summarized in hierarchical form. Ideally, the determination of the objectives and attributes occurs without any regard for the alternative courses of action, but the qualitative modeling process ultimately concludes by identifying and appending the alternatives below the attributes used for evaluation. Typically, the qualitative model is summarized via a hierarchical structure that can be depicted graphically.

The prior chart depicts such a hierarchical structure and qualitative model for our illustrative problem of "choosing the best reed mitigation solution." In this qualitative model, three fundamental objectives are foremost: make a practical choice that performs well, avoids negative relations with host nation, and meets operability constraints. Specific attributes relating to the objectives are frequency of use and reed removal rate with respect to the practicality, impact on relations as an attribute and an objective itself, and required manpower and system weight for the operability objective. Subsequently, the five properties under consideration are at the lowest level of the hierarchy.

During the fourth step, the quantitative procedure commences via application of the compensatory additive value model. In essence, the compensatory additive value model provides for trade-offs among alternatives by viewing the composite value associated with each alternative as the sum of weighted attribute values determined independently as described on the next two charts.

Develop Quantitative Model: Compensatory Additive Value Model

- **In compensatory models, overachievement on one criterion can be offset by underachievement on another criterion (i.e., can tradeoff value between attributes).**
- **In additive model, the values associated with the attributes can be added together to determine an overall value when appropriately weighted.**
- **The total value for alternative j equals the weight for attribute 1 times value on attribute 1, plus the weight for attribute 2 times value on attribute 2, plus and so on**

$$V_J = w_1 v_1 + w_2 v_2 + \ldots + w_n v_n = \Sigma_k w_k v_{jk} .$$

- **However, this equation requires a common scale for values based on the attribute's measured level of attainment.**

Compensatory Additive Value Model (continued)

- **Not always applicable for all decision environments; sometimes minimum thresholds values apply.**
  - **In JCIDS, minimum threshold requirements often set for Key Performance Parameters (KPP).**
  - **Can screen alternatives based on KPP & then apply compensatory model (i.e., apply concept of identifying satisticing solutions, which exceed all of the aspiration levels of each attribute).**
- **Research by Nobel Laureate Henry Simon shows decisions makers often merely seeking “satisficing” solution, while analysts focus on seeking the best solution.**

Compensatory Additive Value Model

The MADM techniques illustrated in this COBP are of the compensatory type; such techniques can trade off between favorable and unfavorable performance on attributes to arrive at the same value score for an alternative. In other words, over-achievement on one attribute can be counter balanced by under-achievement on another attribute.

In the mathematical model, the total value for alternative j (V_J) is equal to the sum of the importance weights (w_k) times the corresponding values (v_k) across all attributes.

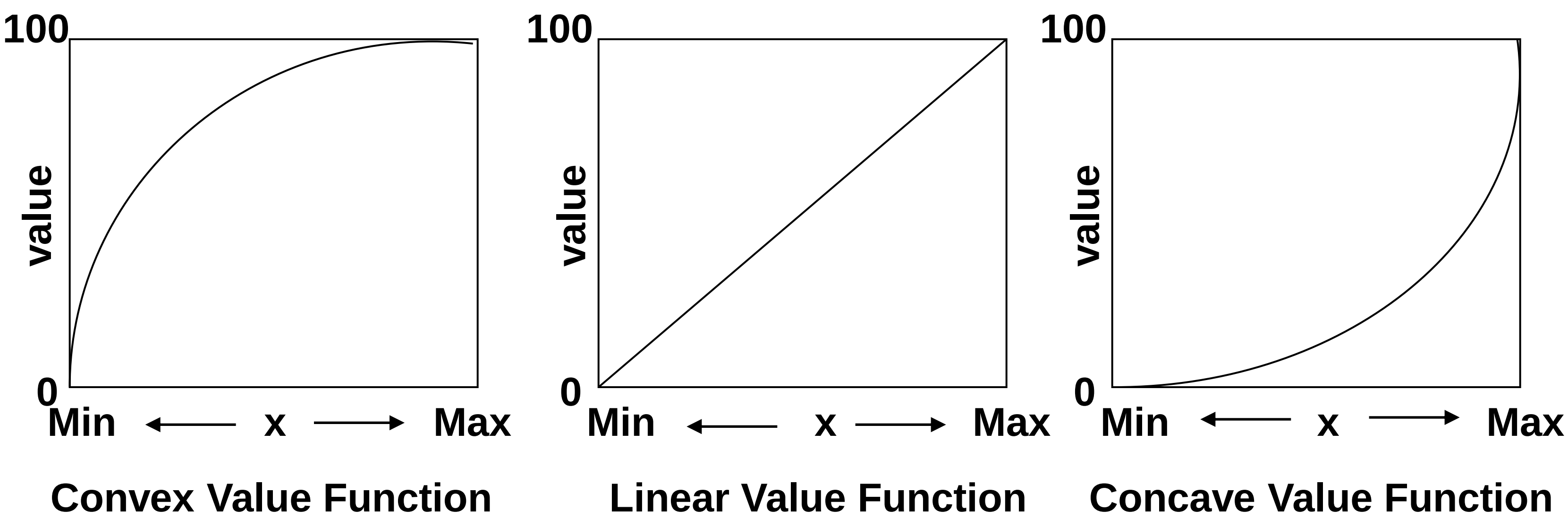
Thus, to apply this model, eventually estimates must be derived for both the attribute importance weights as well as the single attribute values.

In JCIDS analysis, often there are minimum threshold capabilities that the key performance parameters (KPP) must achieve. Thus, screening the alternatives based on KPP thresholds and then applying a compensatory model to select the "best" of the "acceptable" solutions makes sense. However, in some JCIDS analyses, like during the Armed Reconnaissance Helicopter (ARH) and Attack Helicopter Fleet (AHF) Analysis of Alternatives (AoAs), the compensatory model was used throughout, and the presentation of results made clear all the shortfalls in meeting the KPP thresholds.

As noted by Herbert Simon in his Nobel work, decision makers are often just concerned with achieving "satisficing" (i.e., acceptable) solutions rather than determining the "best" (i.e., optimal) solution. Various non-compensatory or non-additive techniques for identifying satisficing solutions have been proposed and can be researched and applied. However, this COBP will continue to focus on the compensatory additive value model which underlies the primary MADM techniques used to differentiate value among alternatives.

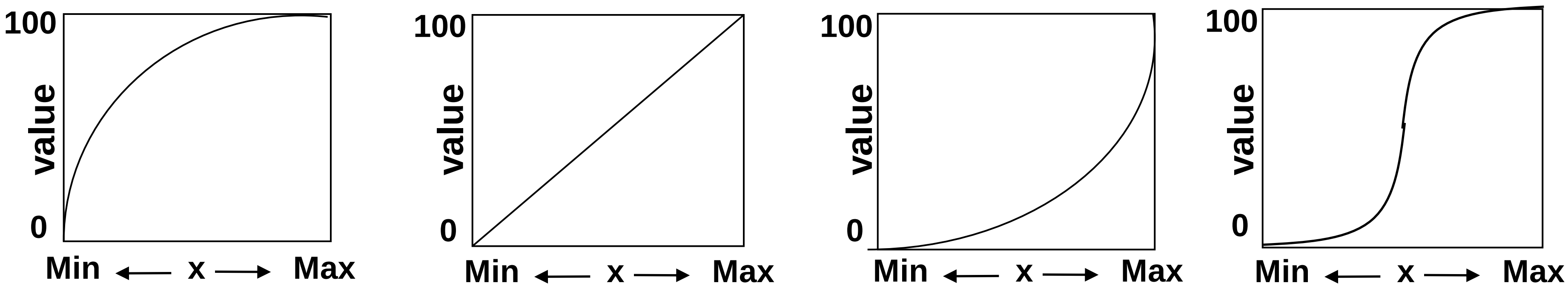
Single Attribute Value Functions

- For each attribute, there is a function that relates the measured attribute level of attainment to value and it may be displayed graphically.

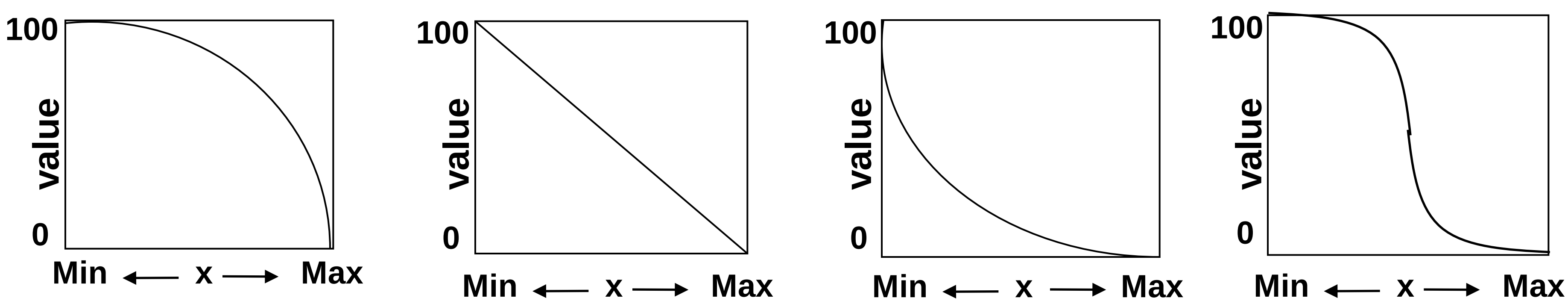


Single Attribute Value Functions

- For each attribute, there is a function that relates the measured attribute level of attainment to value and it may be displayed graphically.
- Examples below show representative functions for attributes for which more is better (top figures) and less is better (bottom figures).



Convex Value Function (decreasing returns to scale)	**Linear Value Function** (equal returns to scale)	**Concave Value Function** (decreasing returns to scale)	**S-Curve** (variable returns to scale)



Single Attribute Value Functions

As noted previously, in compensatory additive value models, there is a requirement to transform the attribute measurements to a common value scale. The concept for making this transformation for each attribute is that of single attribute value functions. In these functional relationships, each unique score of x_1 maps into a corresponding value, v_1, on the value scale (i.e., $v_1 = f(x_1)$).

As illustrated, the single attribute value functions may be curvilinear (convex or concave) or linear as shown by the middle graph. Note that the x-scale for each attribute ranges from the minimally realistic score for x to the maximally realistic score for x.

In the graphs depicted, the values range from 0 to 100 which could be revised to any common value scale; for instance, from 0 to 1, or from 0 to 10.

To estimate the single attribute value functions, interaction with the subject matter experts (SMEs) and/or decision makers (DMs) (or designated proxy DM(s)) is required to gauge the relationship shape and correspondence between attribute attainment and corresponding value assignment.

The Model with Single Attribute Value Functions

- **From the prior equation for the compensatory additive value model, the model's equation can now be rewritten in this alternative form:**

$$V_J = w_1 f_1(x_1) + w_2 f_2(x_2) + \ldots + w_n f_n(x_n) = \Sigma_k \, w_k f_k(x_{kj})$$

where the single attribute functions are indicated.

- **In order to correctly apply the compensatory additive value model, a couple of conditions should exist or be highly approximated:**
  - **The contribution of one attribute to total value of an alternative must not depend on values in other attributes (a trait termed <u>preferential independence</u>)**
  - **The specification of the differences in values within an attribute do not depend on values in other attributes (a trait termed <u>difference independence</u>)**

The Model with Single Attribute Value Functions

Just to clarify, the mathematical model could be written in this alternative form where the single attribute value functions are explicitly included.

Although the concept of single attribute value functions have been discussed and it has been noted that they must be evaluated using information from the SMEs and/or DM (or designated proxy DM(s)), the importance weights for the attributes have not been discussed. These weighting factors must also be estimated from inputs provided by the DM or the designated proxy DM(s). Later in this COBP, methods for obtaining estimates of these weights will be demonstrated.

Before getting into the specifics of both single attribute function and weight estimation, it must be pointed out that a couple of conditions (preferential independence and difference independence) are also necessary for the single attribute value formulation to be correctly applied.

Preferential independence exists if the DM's preference specification for one attribute does not depend on fixed values of other attributes. If $X_1,\ldots,X_n$ represent decision attributes, then attribute X_1 is said to be independent of the remaining attributes if the DM's preference specification at different levels of X_1, holding $X_2,\ldots,X_n$ constant, does not depend on where we hold $X_2,\ldots,X_n$ constant. For example, if X_1 is removal rate and the other attributes are from the REF example, then a higher removal rate is still preferred to any lower removal rate for any fixed combination of the other attributes (required frequency of use, impact on host nation, operating manpower, and weight). However, if we had a model including attributes of thickness of reeds cleared and removal rate, the attributes would be preferentially dependent if the value the DM associated with the removal rate depended upon the thickness of the reeds that the alternative could clear.

The additive model also assumes that a DM can specify differences in value within an attribute, which goes a step further than preferential independence. Difference independence requires that the specification of the differences in values within the attribute do not change given

The Model with Single Attribute Value Functions (continued)

fixed levels of outcomes in other attributes. For example, the difference in perceived value between a removal rate of 40 ft^2 per hour versus 1000 ft^2 per hour does not change for any fixed combination on the other attributes (frequency of use, impact on host nation, operating manpower, and weight). However, if some other model combined the attributes of removal rate and reed thickness, hopefully you can see how a difference in perceived value between a removal rate of 40 ft^2 per hour versus 100 ft^2 per hour could depend on reed thickness.

Obviously, if either of the preferential or difference independence conditions are severely violated, the results of using the compensatory additive value model will be distorted and the validity of the technique questioned.

Two Possible Error Sources

- Error can arise from two different sources: modeling error and elicitation error.
- Modeling Error – Due to applying the model with simplifying assumptions (violating the strict conditions of preference independence & difference independence somewhat) & accepting simplified value function representations.
- Elicitation Error – Arises when measures obtained do not accurately reflect DM's true preferences. The more complicated the questions put to the DM, the greater the chance for elicitation error (e.g., elicitation techniques involving hypothetical alternatives).
- As analysts, of course, our aims are to minimize both types of errors to the degree possible in our applications.

Two Possible Error Sources

Beyond the independence conditions just discussed, two major types of error, modeling and elicitation, can occur when attempting to apply the additive value model.

Modeling error deals with violating the mathematical assumptions underlying the theoretical constructs or using simplified mathematical representations for some of the more complex theoretical components.

Elicitation error on the other hand deals with not accurately determining the DM's true preferences. This can be caused by the confusion brought on by the mode of questioning used with the DM or the use of proxy DMs that are not in sync with the DM's true preferences.

Of course, as analysts we want to minimize all errors, but we must recognize the possibility of these two which often work against one another to some extent so that some type of balance must be reached regarding their effects. For if a complex mathematical representation is employed, then in turn, complex questioning of the DMs/SMEs or their proxies usually results. Many DMs/SMEs or their proxies will be overcome by the numerous questions needed for mathematical rigor and begin responding without sufficient thought; the result is elicitation error. Conversely, the mode of elicitation can become more straightforward if less rigorous mathematical representations are assumed to underlie value estimation and aggregation, but such simplifying assumptions may result in substantial modeling error.

With this basic understanding of the quantitative modeling process, the next few charts demonstrate quantification in regard to the reed mitigation problem.

Alternatives by Attributes Matrix

	Frequency of Use	Removal Rate	Negative Impact on Relations	Operating Manpower	Weight
	/month	ft^2/hr	rating (1-5)	soldiers	lbs
Brushcutter	5	40	1	1	18.3
Hedge Clipper	4	70	1	1	14.7
Backpack Spray	2	200	3	1	42
Towed Spray	1	600	3	2	238
Aerial Spray	1	1000	5	4	1000*
Flame-thrower	2	150	4	1	44

* This option also requires an aircraft which is implicitly captured in the large value here.

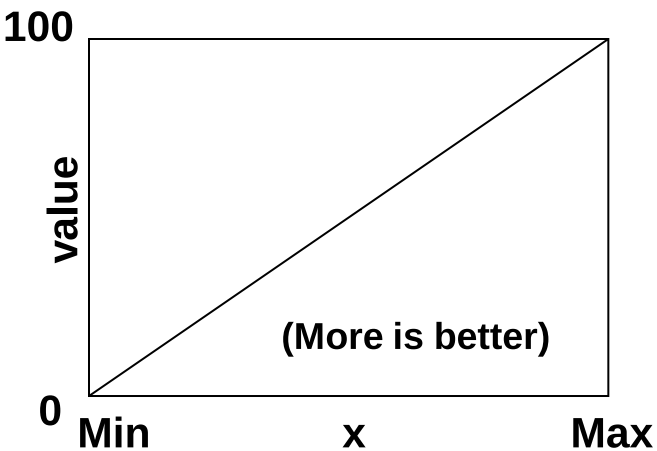
Alternatives by Attributes Matrix

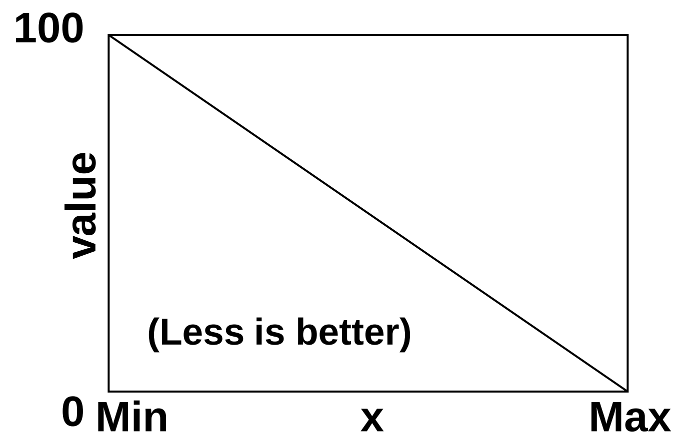
To commence the quantification of the single attributes, an alternatives by attributes matrix is usually developed initially and populated with attribute scores. This is a basic measurement step in which attributes are quantified on naturally occurring or constructed scales that make sense. Naturally occurring scales are ones that possess a common interpretation by everyone (e.g., dollars for business decisions), while constructed scales are ones developed for a particular decision problem to measure the degree of attainment of an objective (e.g., the risk of negative relations rating on a scale of 1-to-5). Additionally, such scales may be distinguished as either direct or proxy scales. Direct scales directly measure the degree of attainment of an objective, while proxy scales reflect the degree of attainment of its associated objective but does not directly measure the objective. For instance, the use of Gross National Product (GNP) as a proxy measure for an objective of increasing a country's economic well-being would qualify as such a proxy scale.

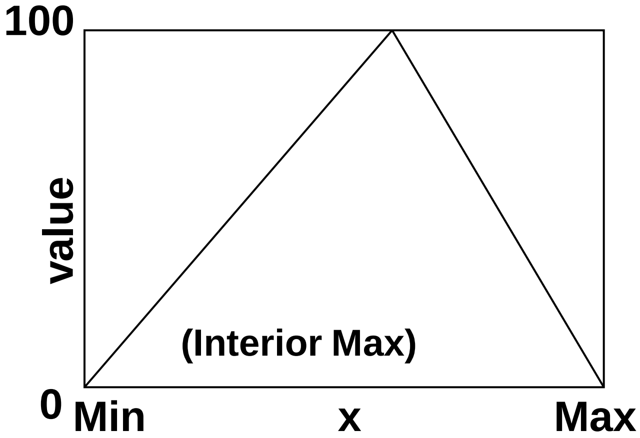
After the alternatives by attributes matrix is set forth, any dominated alternatives are eliminated from further consideration. For an alternative to be dominated, each attribute's score must be inferior, or at least indifferent, for the corresponding scores on at least one of the other alternatives. In the illustrative example, the brushcutter is dominated by the hedge clipper because it has an inferior usage frequency, lower removal rate, and weighs more. We have left the brushcutter in the comparison for purposes of showing another option to the task, but eliminating dominated alternatives screens the alternatives to the minimum number that must be considered in the subsequent analytical steps.

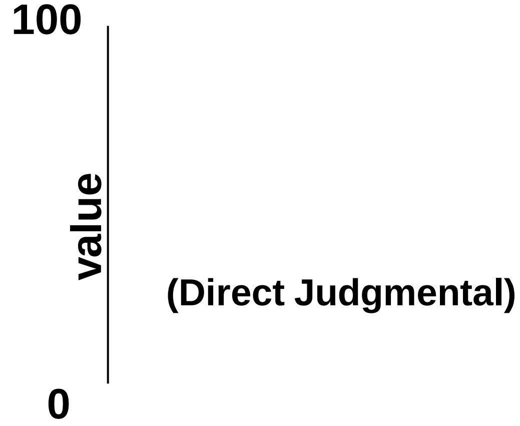
Develop Single Attribute Values

Four approximate forms for single attribute value functions are generally useable (more is better, less is better, internal maximum, or direct judgmental).









Develop Single Attribute Values

Ward Edwards and F. Hutton Barron (1994), in their work based on the simple multi-attribute rating technique (SMART), instituted the simplifying concept that straight line single attribute value approximations are sufficient to substitute for possible curvilinear value functions in most cases. They did this because the elicitation of detailed value functions can be tedious and demanding, and they argued that this approximation rarely affected the decision outcomes unless the ratio of maximum slope to minimum slope on the underlying curvilinear value function exceeds a ratio of 2:1. Thus, Edwards and Barron argued for the acceptance of some modeling error in order to reduce the possibility of elicitation error. Edwards and Barron’s four approximations are shown (more is better, less is better, internal maximum, & direct judgment) as basic forms, and they also noted that an internal minimum might also be a possibility, but its occurrence should be rare.

During the remainder of the illustrative problem, the straight line single attribute functional case will be utilized. However, if one is unwilling to accept the straight line approximation or faces an application where the curvilinear ratio straight exceeds 2:1, Kirkwood (1997, pp. 62-68) discusses the estimation and use of piece-wise linear and exponential value functions.

Developing Values from Alternatives by Attributes Matrix

For each attribute determine minimal and maximal plausible endpoints across all relevant alternatives.

	Frequency of Use	Removal Rate	Impact on Relations	Operating Manpower	Weight
	/month	ft^2/hr	rating (1-5)	soldiers	lbs
Minimal	1	0	1	1	0
Maximal	5	1000	5	4	1000

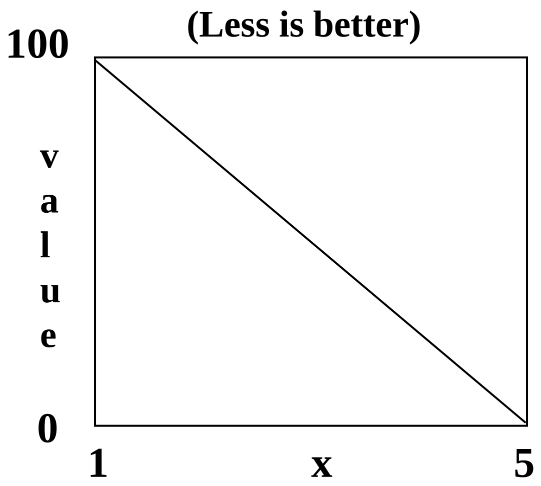
Convert scores in Alternatives by Attributes matrix to values using appropriate single dimension form, attribute scores, & plausible endpoints.

Developing Values from Alternatives by Attributes Matrix

Employing this concept of linear single dimension value functions, initially the maximum and minimum plausible attribute scores are determined across the relevant alternatives. For our REF data, the determination of plausible scores is shown on the previous chart.

Thereafter, the conversion of attribute scores to attribute values occurs using the appropriate linear single dimension utility form. The next chart provides a specific example of this process where the removal rate attribute from the illustrative problem is converted.

Value Example: Frequency of Use



Value(Brushcutter) = 100 from graph

Value(Aerial Spray/Towed Spray) = 0 using graph

- In general, $\text{Value}_J = 100*(\text{Max} - \text{Freq}_J)/(\text{Max} - \text{Min})$
- Value(Hedge Clipper) = 100*(5 – Freq(Hedge Clipper))/(5 – 1) =

  = 100*(5 – 4)/(5 – 1) = 25
- Value(Towed and Aerial Spray) = 100*(5 – 1)/(5 – 1) = 0

! In some cases it may be preferred to use some specific value for the maximum or minimum rather than the alternative ranges. For instance, there may be a maximum or minimum specified in the study tasker or anticipated alternative not initially specified with values outside the current ranges.

Value Example: Removal Rate

In considering removal rate attribute, this is a case of more is better. The plausible endpoints for removal rate have been determined to be 0 and 1000 and are represented on the x-scale. The corresponding single attribute values ranging from 0 to 100 are those values expressed in the y-dimension which are mapped from the x-dimension. The corresponding single attribute values for specific x-values can be found graphically or through the equation.

With respect to the removal rate attribute, the single attribute values are: 4 for the brushcutter, 7 for the hedge clipper, 20 for the backpack spray, 60 for the towed spray, 100 for the aerial spray, and 15 for the flamethrower.

Values For Choosing Best Reed Removal Solution

Results when completing transformation to single attribute values.

	Frequency of Use	Removal Rate	Impact on Relations	Operating Manpower	Weight
Brushcutter	0	4	100	100	81
Hedge Clipper	25	7	100	100	85
Backpack Spray	75	20	50	100	58
Towed Spray	100	60	50	50	0
Aerial Spray	100	100	0	0	0
Flame-thrower	75	15	25	100	56

Output from Step 4 of Multi-Attribute Value Analysis Process

Values For Choosing Best Reed Removal Solution

The prior chart displays the values of the alternative properties with respect to all five of the attributes considered in the illustrative problem. This completes the quantification step of the multi-attribute value analysis process where the alternatives by attribute scores matrix is converted to the alternative values by attributes matrix.

The next step in the multi-attribute value analysis process is to determine the weighting factors to be associated with the individual attributes. A procedure for determining these weights is set forth in the next four charts.

Commence Weighting Process: Determine Ranking Relationships

With respect to choosing the best reed mitigation solution, what ranks should be assigned to the following attributes? (1 – highest; 5 – lowest)

Attribute	Rank
Frequency of Use	2
Removal Rate	1
Impact on Relations	3
Operating Manpower	4
Weight	5

Weight Assessment (Estimation)

Start by ordering according to ranks & assigning lowest ranking attribute a relative value of 10.

Attribute	Rank	Relative Value
Removal Rate	1	
Frequency of Use	2	
Impact on Relations	3	
Operating Manpower	4	
Weight	5	10

Weight Assessment (Continued)

Proceed by asking how important is the rank 4 attribute relative to the rank 5 attribute: Twice as important? Three times as important? Assume answer turns out to be 1.6 times as important.

Repeat for rank 3 attribute relative to rank 5 attribute, rank 2 attribute relative to rank 5 attribute, & rank 1 attribute relative to rank 5 attribute? Assume result to be 2.2, 2.5, & 3.1 times as important.

Assign relative values based on multiplicative relation of results.

Attribute	Rank	Value
Removal Rate	1	31
Frequency of Use	2	25
Impact on Relations	3	22
Operating Manpower	4	16
Weight	5	10

Weight Assessment (Estimation): Normalization Step

Normalize by summing & then dividing each relative value by the sum.

Attribute	Rank	Rel. Value	Norm. Value
Removal Rate	1	31	.298
Frequency of Use	2	25	.240
Impact on Relations	3	22	.212
Operating Manpower	4	16	.154
Weight	5	10	.096
Sum		*104*	*1.000*

Output from Step 5 of Multi-Attribute Value Analysis Process

Weight Assessment (Estimation)

During this illustrative weighting step of the multi-attribute value analysis process, the weighting of attributes commences by identifying the perceived ranking relationships. In our illustrative reed mitigation problem, these rankings were determined from the DM or his/her representatives.

After rating the attributes for importance, the attributes are rearranged into into rank order, and then a base value is assigned to the lowest ranked element. Ten works well, although one could just as easily use 1 or 100.

Thereafter, the DM or his proxy representative is sequentially asked to provide his relative assessment of each attribute's value relative to the base item (lowest ranked item) and its base value. As shown, the respondent has provided data indicating estimates of 16, 22, 25, and 31. Take note of the multiplicative characteristic we are attempting to get the respondent to provide during this assessment. The deliberate aim is to get the respondent to provide the weights in a ratio preserving (i.e., multiplicative relational) manner.

After obtaining the relative weight judgments from the respondent, the estimates are normalized so that the relative ratio weights sum to 1.000. These represent the weighting factors desired from the weighting step of the multi-attribute value analysis process.

The next step is to synthesize the attribute values and attribute weighting factors into an overall estimate of value for each of the alternatives.

Multiply Attribute Weights Times Values for Multi-Attribute Value Analysis Results

V_{BC}	= .240(0)	+ .298(4)	+ .212(100)	+ .154(100)	+ .096(81)	= 45.5
V_{HC}	= .240(25)	+ .298(7)	+ .212(100)	+ .154(100)	+ .096(85)	= 52.8
V_{BS}	= .240(75)	+ .298(20)	+ .212(50)	+ .154(100)	+ .096(58)	= 55.5
V_{TS}	= .240(100)	+ .298(60)	+ .212(50)	+ .154(50)	+ .096(0)	= 60.2
V_{AS}	= .240(100)	+ .298(100)	+ .212(0)	+ .154(0)	+ .096(0)	= 53.8
V_{FT}	= .240(75)	+ .298(15)	+ .212(25)	+ .154(100)	+ .096(56)	= 48.6

Output from Step 6 of Multi-Attribute Value Analysis Process

In the final step, the attribute weights and values are combined to determine the overall values associated with the alternative properties.

- The more overall value the better.

Multiply Attribute Weights Times Values for Multi-Attribute Value Analysis Results

The calculations indicate that the towed sprayer is deemed the most favorable solution with 60.2 value points and would be selected if one alternative was to be purchased.

On the other hand, if this was an evaluation or prioritization effort, the towed sprayer would be followed by the backpack sprayer in second with 55.5 value points, the aerial sprayer in third with 53.8 value points, the hedge clipper in fourth with 52.8 value points , the flamethrower in fifth with 48.6 value points, and the brushcutter in sixth with only 45.5 value points.

Conduct Sensitivity Analysis

- **Examine how variation in values changes the results and recommendations.**
  - **For example, decrease all values associated with the towed chemical sprayer by 10% and see if it is still top ranking alternative.**
- **Examine how variation in weights changes the results and recommendations.**
  - **For example, assign equal weights to all attributes and see if this changes the ranking of alternatives.**

This completes the “best reed mitigation solution” example of how multi-attribute value analysis can be executed in basic decision situation.

Now COBP will expand on multi-attribute value analysis concepts and techniques so that process can be executed during more diverse decision situations and analytical circumstances.

Conduct Sensitivity Analysis

In concluding a multi-attribute value analysis, the sensitivity of the outcomes to variation among valuing and weighting estimates derived during the analysis process should be explored. The aim of the sensitivity analysis is to provide the DM and other interest parties a sense of how robust the outcomes and conclusions are with respect to value and weighting variations. The previous chart indicates a couple of examples of possible sensitivities that can be explored if automated software is not being utilized. An advantage of using an automated MADM computer package, such as Logical Decisions, etc., is that they nearly always offer options for an automated sensitivity analysis.

This completes the illustrative example of multi-attribute value analysis through the application of a six step process: (1) identifying stakeholders, (2) identifying fundamental objectives, (3) developing the qualitative model, (4) developing the quantitative value model, (5) determining weighting factors for attributes, and (6) generating results and conducting sensitivity analysis. Thus, via the illustrative example the reader should now have a basic idea of how a multi-attribute value analysis proceeds and develops results. However the basics of this COBP now need to be extended to provide a more comprehensive view of qualitative modeling as well as possible assessment techniques for quantifying attribute values and determining weighting factors via prioritization techniques. Starting on the next chart, qualitative modeling and the formulation of hierarchical structures will commence.

Qualitative Modeling & Formulating Objectives Hierarchies

- Objectives hierarchies are the backbone of most MADM techniques as well as other many other analytical procedures.
- Objectives hierarchies start with a fundamental objective or central focus element followed by subsequent levels of more specific fundamental objectives, attributes/criteria, and alternatives.
- Although several sources offer guidelines and principles for constructing objectives hierarchies, no hard and fast prescriptive procedures exist for hierarchy development (i.e., hierarchy development is an art not a science).
- Best achieved by understanding guidelines and principles as well as studying examples.

Qualitative Modeling & Formulating Objectives Hierarchies

Objectives hierarchies are a fundamental component of most MADM procedures, and this section is devoted to helping the reader understand how to approach the development of objectives hierarchies as well as provide a few examples.

Normally, objectives hierarchies start with some central focus and thereafter branch out into multiple levels consisting of objectives, attributes, and alternatives. Although many MADM sources suggest principles and guidelines for the development of objectives, attributes, and alternatives as well as their eventual depiction in hierarchical form, no explicit procedures and rules exist for hierarchy formulation as almost every application is unique in some respect. Thus, hierarchy development takes on more of the characteristics of an "art" rather than a "science".

Despite the lack of explicit rules, this section will set forth some of the general guidelines and principles for constructing objectives hierarchies, while also setting forth a few examples used in previous military applications.

Characteristics of Good Objectives Hierarchies

- **Objectives and attributes span all issues of concern to decision maker(s).**
- **Attributes are reflective of the degree to which each objective can be met.**
- **Included alternatives should reflect only courses of action the DM can reasonably implement and the alternatives should be characterized through effective description.**
- **The analytical decomposition from upper level elements to lower level elements flows in a logical manner.**
- **Hierarchy should not contain overlapping measures.**
- **Hierarchy should include the minimum number of elements necessary.**

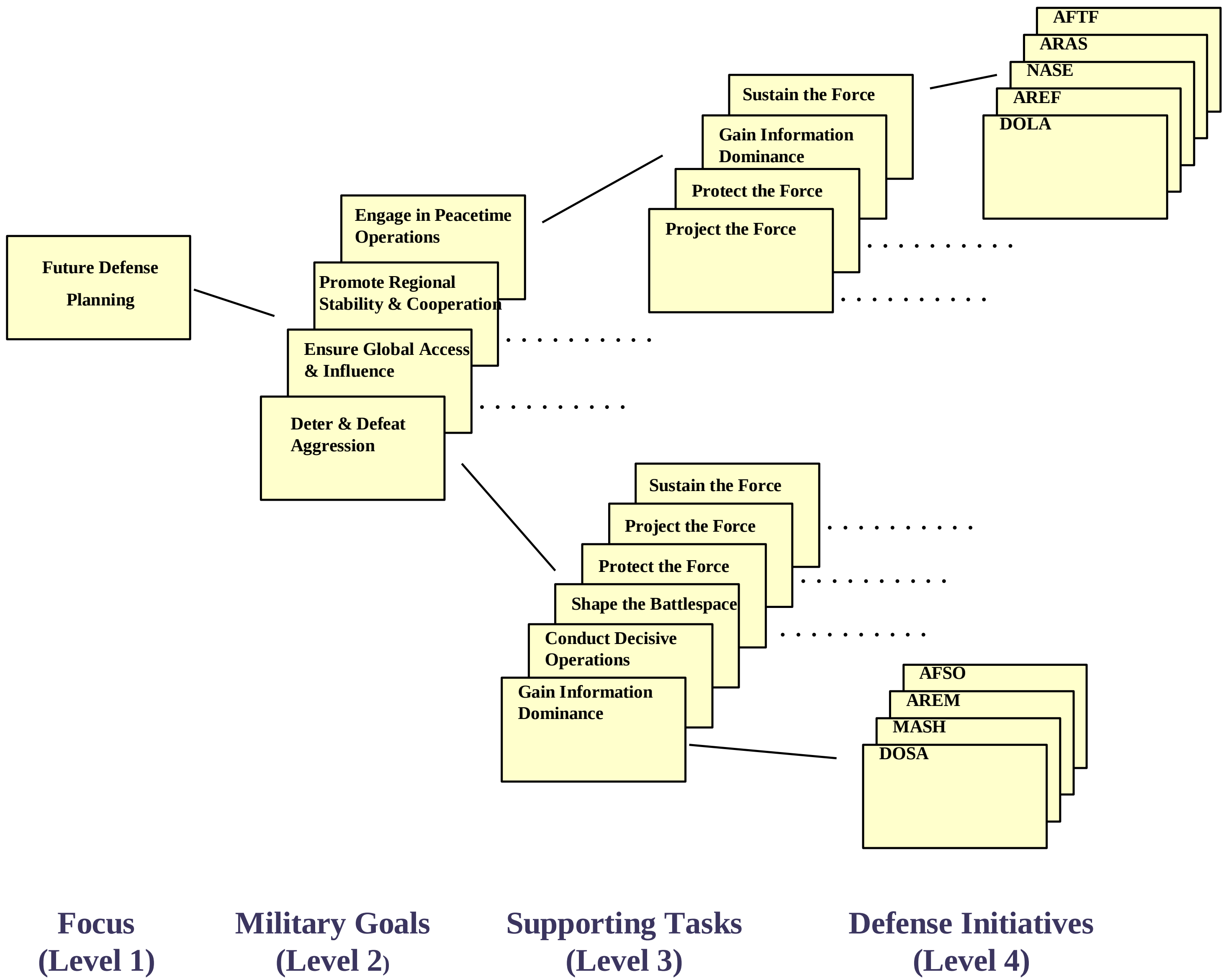
Characteristics of Good Objectives Hierarchies

Listed here are several major principles and guidelines that provide for the construction of effective hierarchies. These stated principles and guidelines are derived both from literature on decision making as well as from practical experience.

For more thorough discussions dealing with the nuances of constructing effective hierarchies, one can consult the following references: Keeney (1992), Keeney and Raiffa (1976), Olson (1996), Saaty (1977, 1980, and 1982), or just about any other comprehensive book dealing with decision making. In particular, Keeney's 1992 Value-Focused Thinking book is an excellent reference source, and probably the foremost one, on dealing with the qualitative aspects of constructing objectives hierarchies. During his discussion on building objectives hierarchies, Keeney (Table 3.2, p. 82) explicitly sets forth nine desirable properties guiding the development of fundamental objectives. His guiding properties include deriving fundamental objectives that are (1) essential, (2) controllable, (3) complete, (4) measurable, (5) operational, (6) decomposable, (7) non-redundant, (8) concise, and (9) understandable. As can be seen, the previously stated major principles and guidelines embody Keeney's desired properties and, vice versa.

For illustration, a couple of military hierarchies will now be considered.

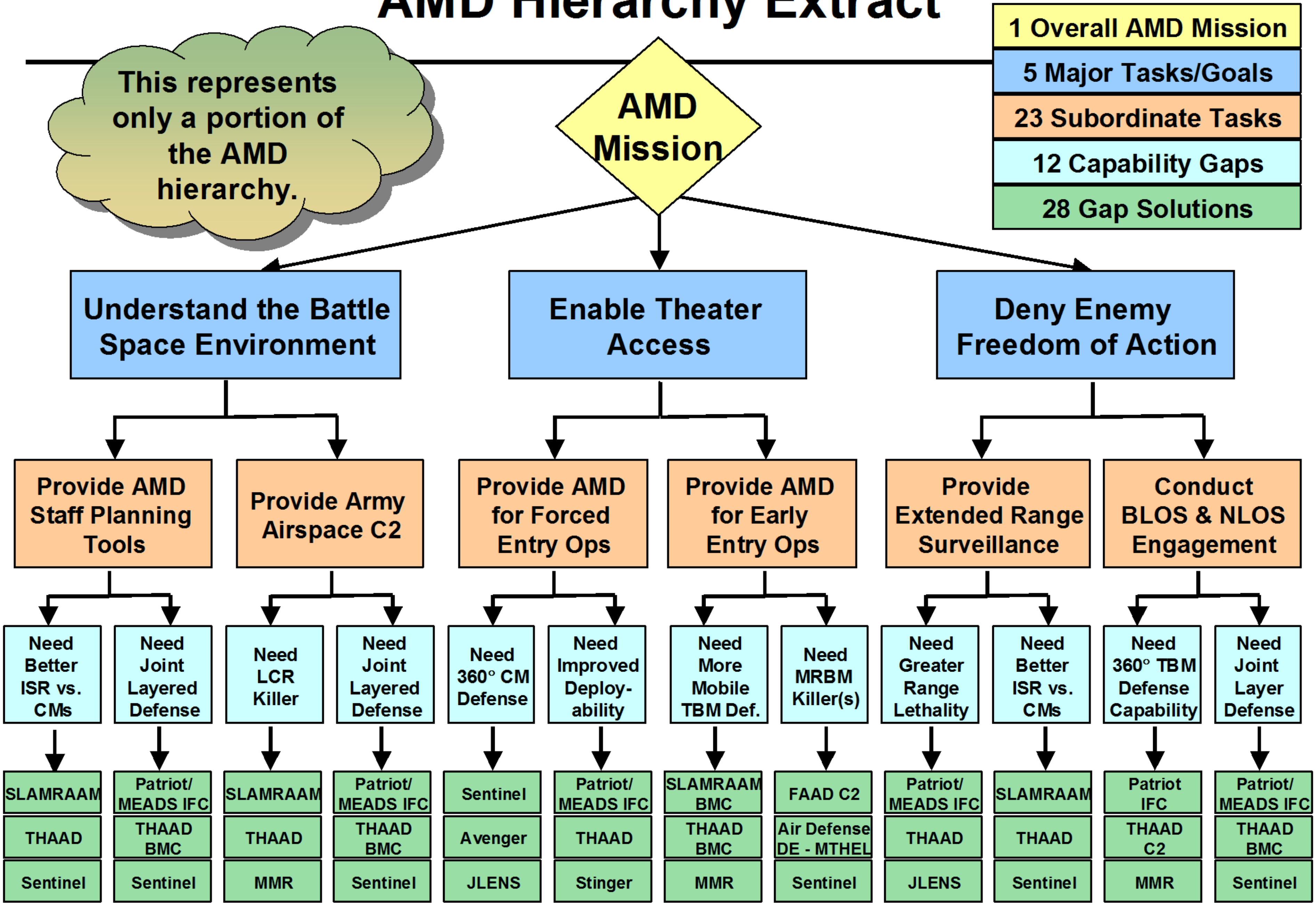
Example Military Hierarchy



Example Military Hierarchy

The prior chart depicts a hierarchical structure that formed the basis for some Army investment work done several years back and was inspired by the national military strategy in place at that time. The central focus for the hierarchy was on Future Defense Planning. Four fundamental military objectives comprised the fundamental objectives level: (1) Deter and Defeat Aggression, (2) Ensure Global Access and Influence, (3) Promote Regional Stability and Cooperation, and (4) Engage in Peacetime Operations. Subsequently, six primary force attributes were identified as characteristics that supported the four fundamental objectives. These attributes pertained to: (1) gaining information dominance, (2) conducting decisive operations, (3) shaping the battlespace, (4) protecting the force, (5) projecting the force as well as (6) sustaining the force. Note that as shown all six force attributes pertain to Detering & Defeating Aggression, while only four of the force attributes supported Engaging in Peacetime Operations (i.e., Conducting Decisive Operations and Shaping the Battlespace do not apply). The next lower hierarchical level contained the defense initiative alternatives, which were coded via four-character identifiers. These identifiers represented the proposed modernization actions at the time, which were designed to enhance the force attribute areas. In the DoD’s Programming, Planning, Budgeting, and Execution System (PPBES), these defense initiatives are known as MDEPs (Management Decision Packages) and are the alternatives considered to receive funding.

AMD Hierarchy Extract



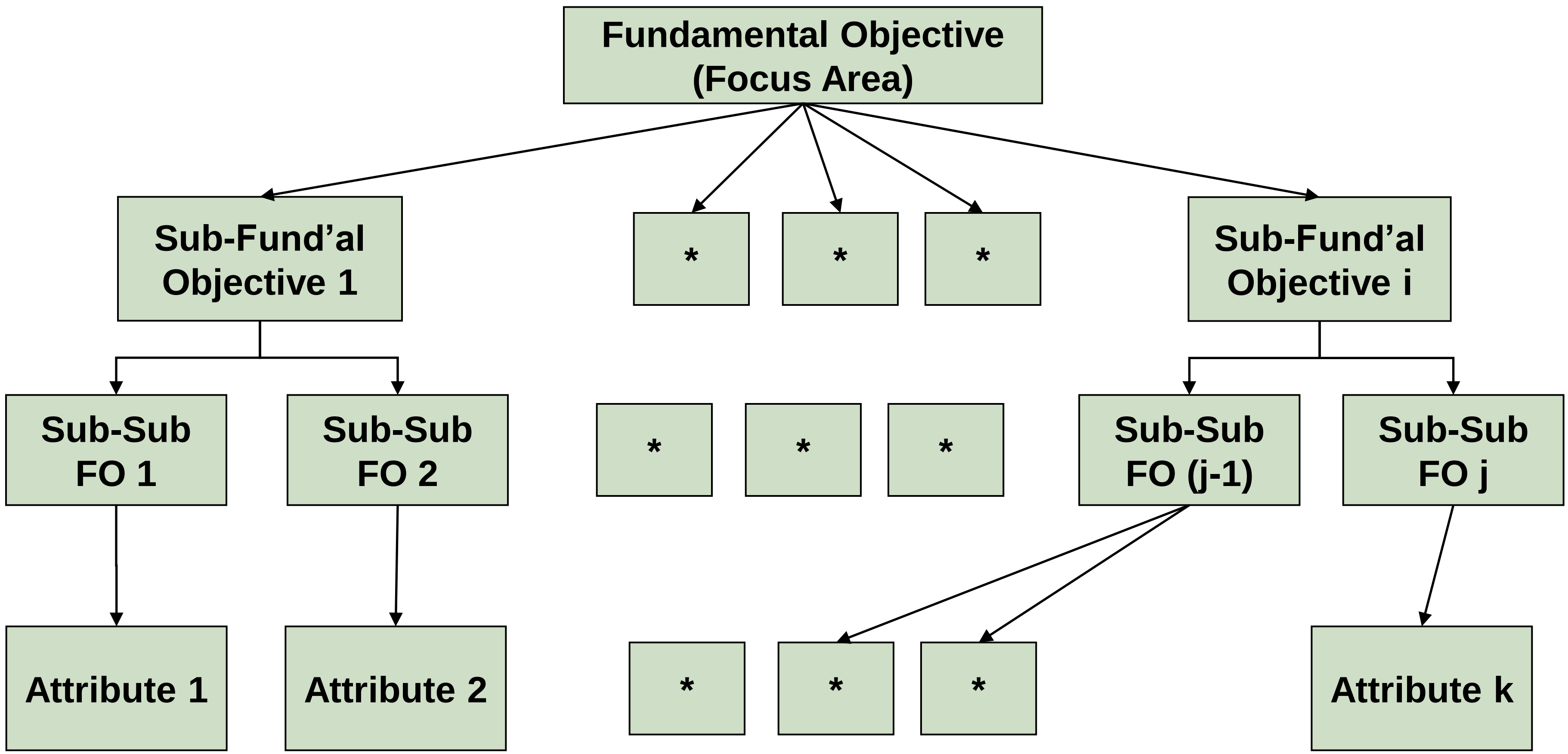
Air and Missile Defense Hierarchy Extract

The prior figure indicates a hierarchy used in conjunction with the Air and Missile Defense End-to-End Analysis. Note the size of the full hierarchy – it is quite a bit larger than the typical textbook example.

Some authors, in fact, will argue that hierarchies with these dimensions are too large to be manageable. Their contention is levels with more than 7-9 elements exceed the capabilities of the normal human mind, and they cite psychological research to support this point. It is definitely agreed that the smaller one can keep the hierarchy the better, but the practical necessities associated with the military type of problems faced often force applications that stretch the limits beyond the ideal ones.

In general, the following chart provides a useful as a starting framework for objectives hierarchy development.

Basic Objectives and Attributes Hierarchical Framework



Qualitative model is foundation for subsequent analysis.
Must devote enough time and effort to get it "right".

Basic Objectives and Attributes Hierarchical Framework

Via the previous discussion and depiction of the general framework underlying objectives hierarchies, this COBP has aimed to enhance the qualitative modeling process. Although there are guidelines and principles for constructing hierarchies and examples to work from, there are no hard and fast rules covering all situations. Because of the ambiguity associated with the development of adequate hierarchies, this COBP contends that often <u>not</u> enough time and effort is allocated to this fundamental step, and this lack of sufficient work can cause numerous difficulties later in the analytical process. This COBP believes the analyst just cannot think too much about this fundamental step because it is the foundation on which the rest of the analysis will be built. Of course, clearly defining the hierarchical terms utilized helps the builder understand what is being set forth in the hierarchy as well as can later help the eventual users understand the builders intent.

Attribute Scaling and Quantification

- **“On the Theory of Scales of Measurement”**
  - **S. S. Stevens (1946), Science, pp 677-680**
- **Four Scales**
  - **Nominal**
  - **Ordinal**
  - **Interval**
  - **Ratio**

Attribute Scaling and Quantification

During the illustrative problem, the quantification of single attribute value functions was discussed. To further that discussion, this COBP section will focus on some of the theory surrounding scaling and quantification in general; in particular, this section will focus on its applicability to attribute scaling and quantification.

S.S. Stevens' notable work concerning measurement scales will provide the start point. In this work, S.S. Stevens is credited with first describing the four fundamental measurement scales that possess distinctive characteristics regarding permissible mathematical operations and applicable statistics. As listed, these fundamental measurement scales are known as the nominal, ordinal, interval, and ratio scales. The following four charts denote the characteristics associated with each as well as indicate some examples of each.

Nominal Scale

- **Assignment of numerals as identifiers for individual elements or classes of elements.**
  - **numerals on basketball jerseys (1-to-1 assignment)**
  - **numerals indicating marital status (e.g., 1- unmarried, 2- married, 3 - widowed, etc.)**
- **For classes of elements can count the number of elements & describe most frequent element as mode.**
- **Qualitative measurement**
- **No basis for deriving single attribute value functions based on nominal data.**

Ordinal Scale

- Arises from the operation of rank ordering.
- Classic examples are hardness of minerals, quality of leather, bond risk categorization, etc.
- Likert scales of "very poor", "poor", "average", "good", "very good".
- In addition to counting operations, can compute median and percentiles (non-parametric statistics appropriate).
- ! Still technically qualitative measurement although sometimes "stretched" to treat as quantitative measurement (weak measurement combined with strong statistics through application of parametric statistical procedures).
- Ordinal data can form the basis for constructing single attribute value functions.

Interval Scale

- Equivalent to equally spaced number line with a stationary (but arbitrarily chosen) zero point.
  - Fahrenheit & Centigrade temperature scales.
  - One scale can be converted to other by linear transformation $x' = ax + b$.
  - most psychological measurement processes aim for this level of measurement (e.g., SAT and ACT scores).
- Parametric statistical procedures appropriate (e.g., mean, standard deviation, product-moment correlation).
- Quantitative measurement.
- Interval data can form the basis for constructing single attribute value functions.

Ratio Scale

- **Usually encountered in physical measurements & physics.**
- **True absolute zero point implied, even though the zero value may never be produced (e.g., absolute temperature).**
  - **Fundamental measures such as length, weight, electrical resistance, dollars (currencies).**
  - **Derived measures such as density, force, elasticity, debt ratio.**
- **Comparison of ratios meaningful (e.g., coefficient of variation).**
- **Quantitative scales where multiplicative relationships are applicable.**
- **Ratio data can form the basis for constructing single dimension value functions.**

Nominal, Ordinal, Interval, & Ratio Scaling

As shown, the nominal scale is the most basic of the measurement scales. Numerals are simply employed for identification and classification of objects or outcomes. Since nominal scaling only results in identification and classification, there is no basis for estimating singular attribute values functions with regard to this level of measurement.

The second most basic scale of measurement is ordinal. Ordinal scales arise when objects or outcomes can be rank ordered. In addition, ordinal or ranking data provides a sufficient basis for the development of single attribute value functions.

The interval scale is a step up from the nominal and ordinal scales. It features equal intervals and the concept of a number line, but the zero point is arbitrary. In addition to some typical example scales such as degrees Fahrenheit and degrees Centigrade, large amounts of psychological and educational measurement are associated with this level of measurement. As with ordinal data, single attribute value functions can be based on interval level measurements.

The remaining scale is the ratio scale. Ratio scales are based on the scale possessing an absolute or theoretical zero point, and this zero point ultimately provides for multiplicative relationships and interpretations among the objects or outcomes. Naturally, single attribute value functions can be based on this foremost level of measurement scaling.

The next chart provides a summary of the basic operations, mathematical relationships, and permissible statistics that accompany each level of measurement.

Comparison of Measurement Scales

Scale	Basic Operations	Math Relation	Permissible Statistics
Nominal	Determination of Equality	1-1 Substitution	Number of cases Mode
Ordinal	Determination of greater or less	Monotonic > or < function	Median Percentiles
Interval	Determination of equality of intervals or differences	Linear relationship x’ = ax + b	Mean Standard Deviation Product-moment correlation
Ratio	Determination of equality of ratios	x’ = ax	Coeff. of variation (std dev/mean)

Comparison of Measurement Scales

This chart depicts a comparison of the scale characteristics. It is important to note that all the mathematical characteristics and statistics associated with the nominal, ordinal, and interval levels can be applied to ratio data as well. In a similar manner, all the nominal and ordinal characteristics apply at the interval level, and nominal characteristics apply at the ordinal level.

One of the major differences between the so called hard and soft sciences are the normal levels of measurement that underlie these fields of study. Often in the soft sciences, the aim is to be able to measure entities on an interval scale, while in the hard sciences, ratio-type data is the standard for entities.

Previously, the concept of multi-attribute value analysis was illustrated via a process of transforming single attribute scores on to a common value scale. As such, the aim for the value scale employed in multi-attribute value analysis is an interval one in terms of its intended level of measurement.

In decision making situations, often the decision makers, as well as their analysts, possess attribute/criteria data originating from different measurement levels. Just how to combine such data from the various measurement levels is a fundamental issue underlying many decision science approaches and also a reason so many alternative analytical techniques are available and touted. Some of these techniques are built on strong assumptions for combining data, while others are built on weak assumptions.

The next collection of charts will illustrated some procedures for transforming ordinal, interval, and ratio attribute scores into single attribute values. This discussion will commence with the transformation of ordinal Likert-type data which is commonly found used in surveys and rating scales. The next chart provides an example of Likert-type data collection drawn from a study on Task Force Modularity, and the subsequent chart illustrates the transformation from Likert-type data to single attribute values.

Collecting Likert Type Data

Task Force Modularity Questionnaire

As a portion of the Task Force Modularity effort, five proposed Brigade UAs have been designed to replace the current BCT organization. Detailed descriptions of the five proposed Brigade UAs have been provided separately as courses of action (COA) 1, 2, 3a, 3b, and 5. After studying the 5 alternative organizations you are asked to respond to the following questions with a checkmark or "X" based on the following scale:

EW: Extremely Worse
HW: Highly Worse
MW: Moderately Worse
SW: Slightly Worse
E: Equal
SB: Slightly Better
MB: Moderately Better
HB: Highly Better
EB: Extremely Better

General Issue 1: Is the proposed UA enabled to function directly for a joint task force (JTF) headquarters (HQ) better than the current Brigade Combat Team (BCT) within a joint, multinational and interagency framework?

1-1. With respect to the current BCT, is COA1 better enabled to work for a JTF HQ?

_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____
EW		**HW**		**MW**		**SW**		**Equal**		**SB**		**MB**		**HB**		**EB**

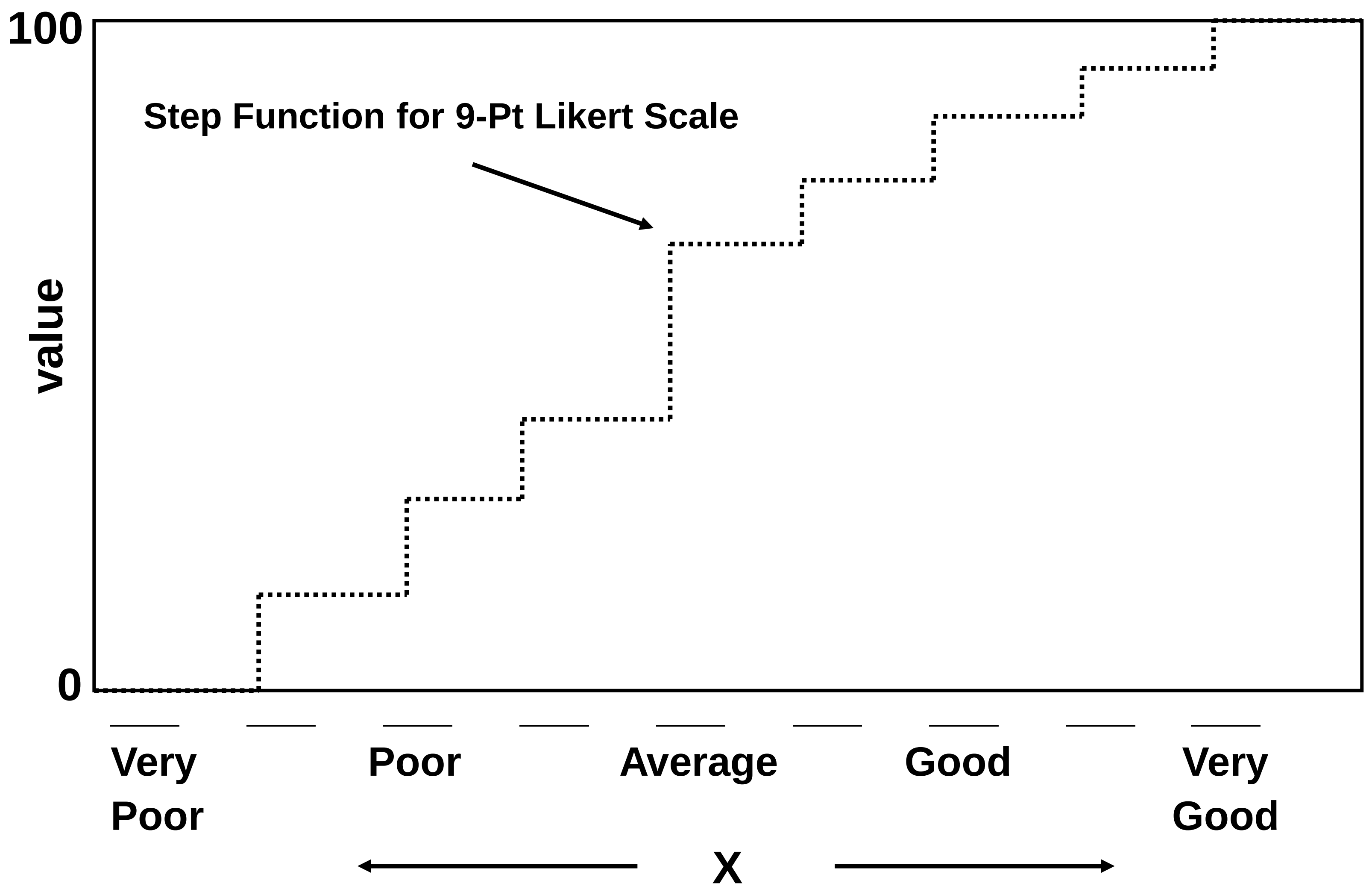
1-2. With respect to the current BCT, is COA2 better enabled to work for a JTF HQ?

_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____
EW		**HW**		**MW**		**SW**		**Equal**		**SB**		**MB**		**HB**		**EB**

[As before for COA's 3a, 3b, and 5 & remaining issues.]

Ordinal Single Attribute Value Function

Likert Type Data



Ordinal Single Attribute Value Function
Likert Type Data

The Task Force Modularity Questionnaire example illustrates the actual mode of Likert-type questioning that was imposed while collecting ordinal type data regarding the comparability of five unit of action (UA) brigades with the current brigade combat team (BCT) on a 17-point scale ranging from "extremely worse" to "extremely better". The subsequent chart depicts the ultimate single attribute function conversion between the Likert-type data and corresponding values. In this case, the chart demonstrates the conversion between a 9-point scale and the corresponding value scale using a stepwise function that is the standard functional form when converting from Likert categories to value assessments.

To support such conversions, a process for assigning values corresponding to the various Likert categories must be employed. This process generally relies on soliciting such information from subject matter experts (SMEs) independently from collecting the Likert data. For example, a separate panel of SMEs is asked how much value is each succeeding category is worth, and the average SME value assessments are used in conjunction with normalization to define the stepwise single attribute function.

Basic Transformations for Interval & Ratio Data

Linear Scale Transformation (transformed proportionally so that relative order of magnitude remains the same for data & values).

$v_{ij} = x_{ij} / x_{max}$ **where** x_{max} = **maximum** x_{ij} **(benefit attributes)**

$v_{ij} = 1 - (x_{ij} / x_{max})$ **where** x_{max} = **maximum** x_{ij} **(cost attributes)**

Vector Normalization Transformation (transformed where all attributes have same unit length value vector).

$v_{ij} = x_{ij} / \sqrt{\Sigma x_{ij}^2}$ **(**$1 - v_{ij}$ **for cost attributes)**

Non-proportional Transformation (transformed so that values vary from 0 to 1 for each attribute, but does not lead to proportional change as in linear case).

$v_{ij} = (x_{ij} - x_{jmin}) / (x_{jmax} - x_{jmin})$ **(benefit attributes)**

$v_{ij} = (x_{jmax} - x_{ij}) / (x_{jmax} - x_{jmin})$ **(cost attributes)**

Transformation Example (Benefit Attribute)

	Removal Rate ft^2/hr	Linear Transform	Vector Norm	Non-proportional Transform
Brushcutter	40	0.040	0.033	0.000
Hedge Clipper	70	0.070	0.059	0.031
Backpack Sprayer	200	0.200	0.167	0.167
Towed Sprayer	600	0.600	0.502	0.583
Aerial Sprayer	1000	1.000	0.837	1.000
Flamethrower	150	0.150	0.125	0.115

No definitive right or wrong choice when picking one of these transformations.

Basic Transformations for Interval & Ratio Data

When dealing with interval and ratio level data, the three identified standard transformations (linear scale, vector normalization, and non-proportional) as derived from Hwang and Yoon's (1981) book are often employed during MADM procedures to convert attribute scores onto a common value scale.

An advantage of the linear scale transformation is that all scores are transformed in a linear (i.e., proportional) manner such that the order of magnitude remains equal. However, the resulting attribute values do not necessary span the same ranges since the ultimate ranges depend on the maximum and minimum plausible endpoints defined for each attribute.

The vector normalization of attribute scores yields dimensionless units that facilitate inter-attribute comparisons. On the other hand, due to the non-linearity of this transformation, a straight forward comparison is still difficult because the minimum and maximum scaled values are not necessarily equal, and the ranges are not necessarily equal for each attribute.

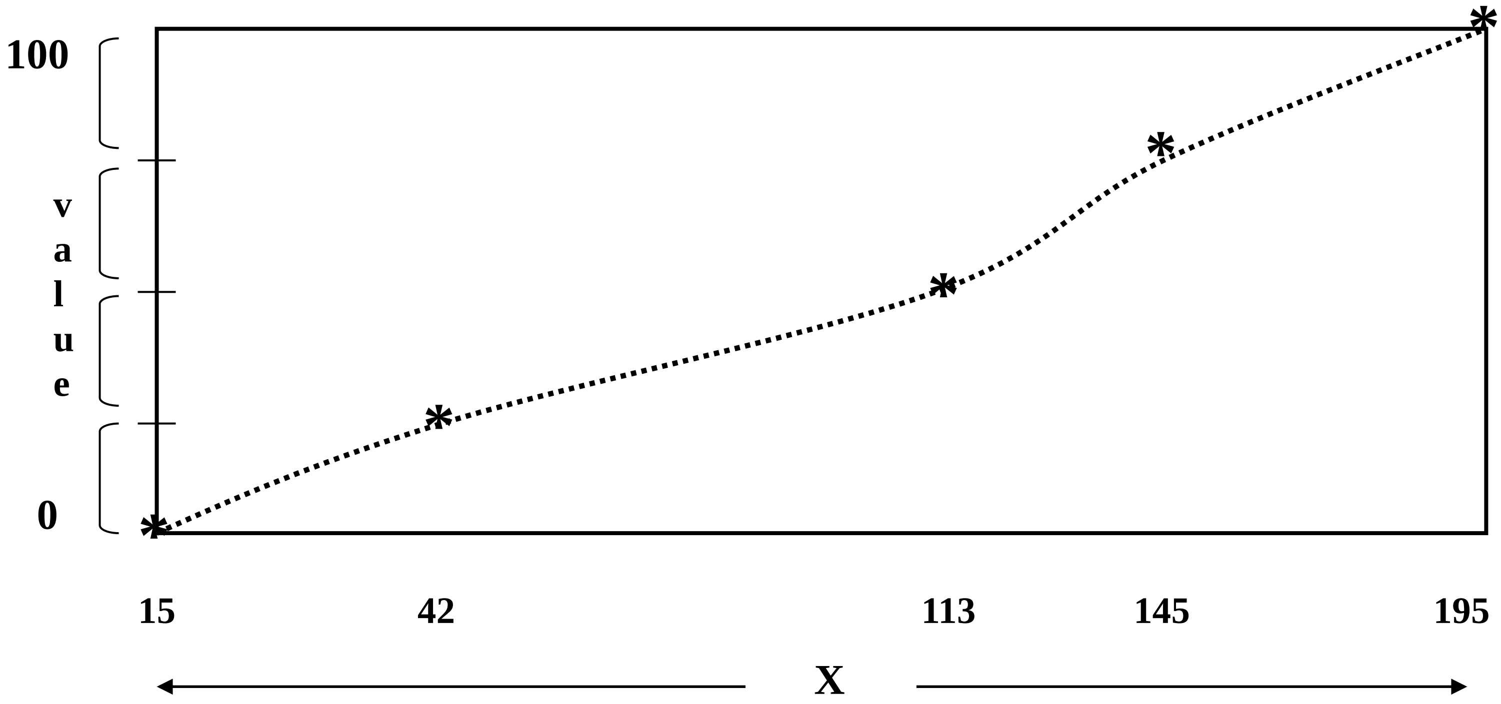
The non-proportional transformation does yield a scale of measurement that varies precisely from zero to one for each attribute and can be viewed as an advantage by imposing equivalent ranges. However, this transformation does not result in proportional change in the scaled values matching the proportional differences among attribute scores.

Hence, there is really no definitive right and wrong choice when picking one of these transformations for use during a MADM analysis. Generally, all these transformations will result in the same ordering of alternatives; however, the apparent magnitudes among their differences may be either embellished or diminished depending on the transformation chosen. Thus, this is another factor that can be examined by sensitivity analysis -- in this case, based on the change of transformation employed.

Value Incremental Approach for Building Value Functions

- **Assume we are using a value scale ranging for 0 to 100.**
- **Assume plausible scores for attribute range from 15 (lowest) to 195 (highest).**
  - **We know value function starts at coordinates (15,0) and ends at (195,100).**
- **Partition the value scale into 4 equal intervals: 0-25, 25-50, 50-75, 75-100.**
- **Ask SME, when the value increases from 0 to 25, where relative to 15 is the x-score? (Presume a response of 42.) Ask SME, when the value then increases from 25 to 50, where relative to 42 is the x-score? (Presume a response of 113.) Ask SME, when the value then increases from 50 to 75, where relative to 113 is the x-score? (Presume a response of 145.)**
- **Plot identified data points on graph: (15,0), (42,25), (113,50), (145,75), & (195,100).**
- **Draw smooth curve thru these points to generate value function.**

Value Incremental Approach (continued)



- Can then interpolate other values using graph
- Alternatively one could use curve-fitting program to generate best equation (plot) corresponding to data points

Value Incremental Approach for Building Value Functions

As a final attribute score to value conversion procedure, the value incremental approach to building single attribute value functions can be implemented as set forth in the prior two charts when needed or desired. The concept for building the single dimension attribute function is to divide the value scale into several equal intervals and determine the change required in attribute scores to match the corresponding change in the successive value increments from SMEs. Thus, the process is to determine the change needed in the x-dimension (i.e., attribute-dimension) to bring about the incremental changes specified in the y-dimension (i.e., the value-dimension). Thereafter, the corresponding ordered pairs determined through the aforementioned process are plotted on a graph. Curve-fitting software or free-hand drawings can then be employed to generate the best fitting equations or graphs that represent the single attribute functions.

Reliability & Measurement

- **Consistency among measurements brought about by accuracy and error free measurement.**
- **Ratio of true variance in a set of measurements to total variance in a set of measurements (true variance/(true variance + error variance)).**
  - **Theoretical concept as true score variance always unknown but estimated through a variety of reliability measures (e.g., ANOVA & correlation estimation procedures).**
- **Two specific types of reliability of interest.**
  - **Inter-rater reliability – agreement (i.e., consistency) among raters or judges (think average correlation among raters).**
  - **Within-person reliability – agreement (i.e., consistency) among judgments made by an individual.**

Reliability & Measurement

As a result of the scale characteristics just considered, the soft sciences are also highly focused on achieving reliability and validity, as well.

Reliability centers on achieving consistency in the measurement process whether it is over a time period, over parallel forms of a test designed to measure the same trait, between judges or raters when evaluating the same trait, or within personal perceptions provided by the same individual when responding in identical situations.

In theory, reliability is the ratio of true score variance divided by the total score variance (i.e., error score variance + true score variance). Reliability coefficients derived from test data are estimates for this theoretical ratio.

As indicated in our further decision analytic work, the interest will be particularly on inter-rater consistency as well as within-person consistency.

Validity & Measurement

- **A measurement process is valid if it measures what it purports to measure.**
  - **Process may be valid in some circumstances & same process might be invalid under other circumstances.**
- **Measurement process cannot be valid unless it is reliable (however, reliability in measurement process does not guarantee validity).**

Validity & Measurement

Measurement validity means that a measurement instrument or process is actually measuring what it intends to measure.

For the hard sciences this is pretty straight forward; however, when one is dealing with soft science measures, it is not so straight forward. For instance, are the SAT and ACT tests valid for all cultures and socio-economic classes? As you are probably quick to realize, such validity questions can and do spark quite emotional responses when there is a lot riding on the outcomes. The fact of the matter is that a measurement instrument or process may be valid some circumstances while it may not be valid at all in others. In other words, a measurement instrument or process may exhibit predictive validity in some instances, while it may exhibit little or no predictive validity in others.

The main COBP point here, however, is the last bullet. A measurement instrument or process cannot be valid unless it is reliable, for inconsistency in the measurement instrument or process obviously limits potential validity. This, in turn, means that as analysts using SME & PMJ data, we must aim for achieving as much reliability as possible if we want our work to be deemed valid.

Useful Assessment Techniques for Determining Weights and Priorities

- **Following techniques can be helpful in multi-attribute value analysis weighting and prioritization efforts.**
  - **Ranking & Sorting.**
  - **Direct Assessments.**
  - **Swing Weighting via Swing Weight Matrix.**
  - **Rank-Ordered Centroid (ROC) Assessments.**

Useful Assessment Techniques for Determining Weights and Priorities

Various methods have been proposed for determining appropriate attribute weights in the application of the compensatory additive value model. Among these are the four procedures listed which can be utilized during attribute weighting in multi-attribute valuing analysis efforts as well as often in support of independent prioritization efforts. This COBP section will now focus on explaining these four procedures so that the reader may consider them for attribute weighting during future multi-attribute value analyses and elsewhere where the independent prioritization of elements is an objective.

The direct assessment technique for weighting attributes was already explained and used during the illustrative application of multi-attribute value analysis regarding the choice of reed mitigation solution, but it is included again so that this section is comprehensive.

The subsequent explanation will begin by considering an example from the basic process of ranking. In this case, the ranking procedure will be illustrated using a set of four military objectives.

Ranking of Elements

With respect to future defense planning, what ranks should be assigned to the following military objectives? (1 – highest; 4 – lowest)

Objective	**Rank**
Promote Regional Stability & Cooperation	_____
Ensure Global Access & Influence	_____
Deter & Defeat Aggression	_____
Engage in Peacetime Operations	_____

Ranking of Elements: Notional Result

With respect to future defense planning, what ranks should be assigned to the following military objectives? (1 – highest; 4 – lowest)

Objective	Rank
Promote Regional Stability & Cooperation	3
Ensure Global Access & Influence	2
Deter & Defeat Aggression	1
Engage in Peacetime Operations	4

Ranking of Elements

When it comes to ranking, this COBP believes it is best to just ask the rater a simple, direct ranking question such as the one illustrated. Note the first clause of the ranking question reminds the respondent of the frame of reference for his or her responses, and the parenthetical expression alleviates any question about whether to rank in ascending or descending order. In other words, do not assume the respondent will automatically grasp the frame of reference and his or her task.

In general, when working with respondents, be as explicit in the questioning and instructions as possible; you may be surprised by the number of times respondents do not answer using the anticipated format or, obviously, do not consider the question carefully.

To complete the illustration, the ranking question along with notional responses have been included. It is evident that this assessment procedure is pretty straightforward.

Sequential Sorting

- When there are large number of elements to rank (e.g, more than about 15 – 20 elements), place element statements on cards that can be sorted.
- Have raters sort cards into 5 ordinal piles of "very high," "moderately high," "average," "moderately low," and "very low" on characteristic being ranked.
- Subsequently, sort cards in each pile into another ordinal set of five piles (or into priority order if number of elements allow).
- Repeat until eventually elements are ranked in full priority order.
- Thereafter, have raters mark rankings with tied rankings if allowed.

Sequential Sorting

There are times when respondents need to rank a considerable number of items. Here, sequential sorting can be employed.

In this procedure, statements reflecting the elements to be ranked are placed on cards (3X5, 4X6, or other). Subsequently, the cards are ranked via sequential sorting as described on the prior chart. Using this procedure, eventually the respondent places the sorted cards (i.e., ranked) into 1-to-n order. After having the respondent document his or her order and return the cards, his or her priority judgments and achieved ranking on a large number of items has been captured.

Depending on study considerations, tied ranks may or may not be allowed (strict ranking versus ranking with ties).

This type of sequential sorting technique was used during the aviation analysis of alternatives (AoAs) when 84 performance attributes had to be weighted with respect to the armed reconnaissance helicopter and 68 performance attributes were weighted with respect to the attack helicopter.

The next few charts illustrate the direct assessment technique again.

Direct Assessment (Estimation)

Start by ordering according to ranks & assigning lowest ranking objective a relative value of 10.

Objective	Rank	Relative Value
Deter & Defeat Aggression	1	
Ensure Global Access & Influence	2	
Promote Regional Stability & Cooperation	3	
Engage in Peacetime Operations	4	10

Direct Assessment (Estimation): Notional Result

- **Proceed by asking how important is rank 3 objective is relative to rank 4 objective: Twice as important? Three times as important? Assume answer turns out to be 2.2 times as important.**
- **Repeat for rank 2 objective relative to rank 4 objective & rank 1 objective relative to rank 4 objective? Assume result to be 4.1 & 5.7 times as important.**
- **Assign relative values based on multiplicative relation of results.**

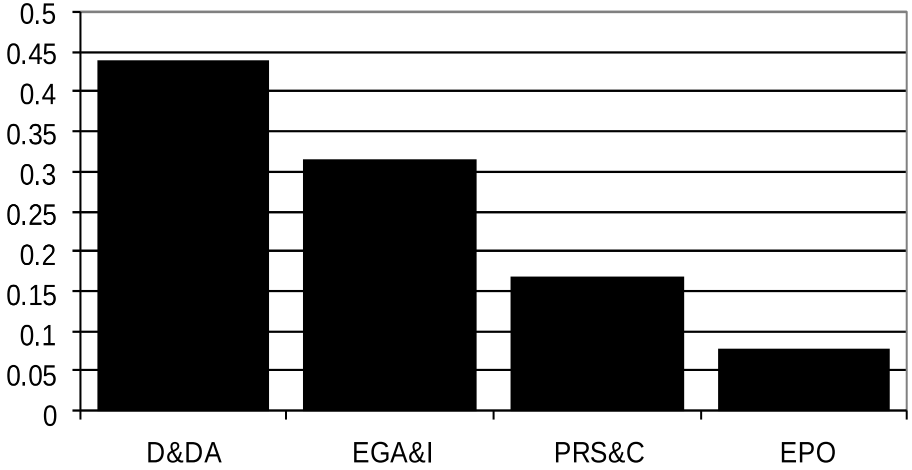
Objective	Rank	Value
Deter & Defeat Aggression	1	57
Ensure Global Access & Influence	2	41
Promote Regional Stability & Cooperation	3	22
Engage in Peacetime Operations	4	10

Direct Assessment (Estimation): Normalization Step

Normalize by summing & then dividing each relative value by the sum.

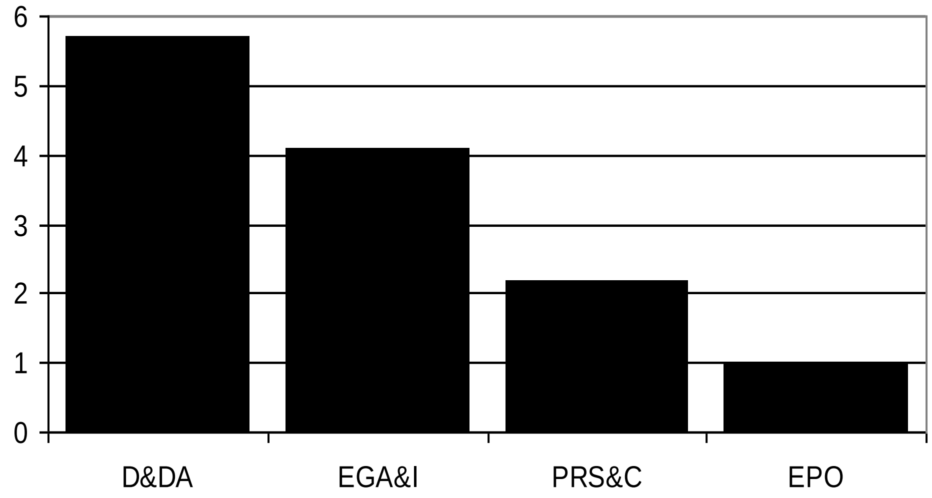
Objective	Rank	Rel. Value	Norm. Value
Deter & Defeat Aggression	1	57	.438
Ensure Global Access & Influence	2	41	.315
Promote Regional Stability & Coop	3	22	.169
Engage in Peacetime Operations	4	10	.077
Sum		*130*	*1.000*

Possible Graphic Depictions



Normalize so that weights sum to 1.000

Normalize so that lowest ranking weight is assigned a value of 1.000



Direct Assessment

The direct assessment weighting procedure is another assessment procedure that is often useful. This procedure seems to work best when the analyst can work side-by-side with the respondent and lead him or her through the procedure. In other words, it works best when there are only 1-3 respondents to work with at a time.

As a first task, the respondent ranks the elements, and the elements are then rearranged into rank order. A base value is assigned to the lowest ranked element. Ten (10) works good, although one could just as easily use 1 or 100. Next, the respondent is sequentially asked to provide his relative assessment of each element's value relative to the base item (lowest ranked item) and its base value.

As shown, the respondent has provided data indicated values of 22, 41, and 57. Note that the aim is to get the respondent to provide the multiplicative characteristic. This aim is deliberate as the objective is to get the respondent to provide as data on a relative ratio scale.

The term "relative ratio" is used because the effort is to explicitly try to generate data with with ratio scale qualities even though there is no zero point identified. This COBP contends that this type of data lies somewhere between the measurement levels Stevens identified as interval and ratio. It is multiplicative data with an arbitrary zero point.

After obtaining the relative value judgments from the respondent, the data is normalized so that the relative ratio weights sum to 1.000. The standard is usually to normalize to 1.000 so that weights across raters/judges can be compared.

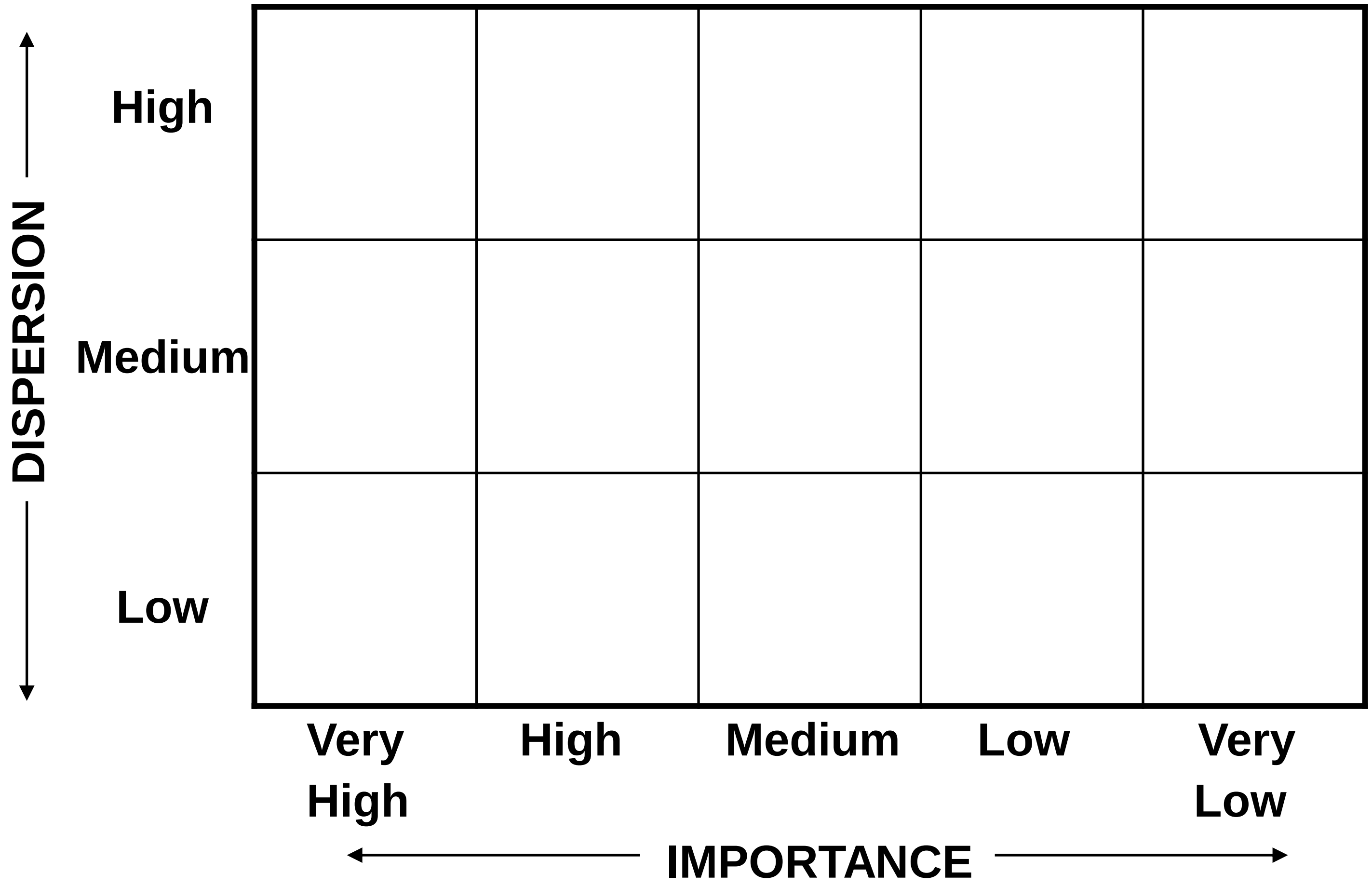
Graphically, these data can be depicted with the normalized weights as proportions (upper chart) or based on the multiplicative relationship (lower chart) via the convenient base value to normalize against (1.00 in the depicted graph).

The next set of charts demonstrate the swing weighting procedure via the swing weight matrix approach.

Swing Weighting via Swing Weight Matrix

- **Concept: both importance & dispersion among elements being rated should be considered when arriving at overall weights.**
  - **For example, in pricing of autos, if alternatives vary from price of $15,000 thru $35,000 versus varying from $15,000 thru $18,000, the weight assigned to pricing may change.**
- **Swing weight matrix approach attempts to take both importance & dispersion into consideration when determining weights.**
- **Swing weight matrix steps.**
  - **Define the importance and dispersion dimensions.**
  - **Place elements to be weighted in the matrix.**
  - **Assess the swing weights.**
  - **Calculate the overall weights.**

Define Importance & Dispersion Dimensions



Place Elements to be Weighted Into Matrix

DISPERSION					
High	*Deter & Defeat Aggression*				
Medium		*Ensure Global Assess & Influence*			
Low		*Promote Regional Stability & Cooperation*	*Engage in Peace Opns*		
	Very High	High	Medium	Low	Very Low
	IMPORTANCE				

Seek & Get Concurrence with Placement

Assess the Swing Weights

DISPERSION \ IMPORTANCE	Very High	High	Medium	Low	Very Low
High	*Deter & Defeat Aggression* 100	85	75	65	50
Medium	80	*Ensure Global Assess & Influence* 65	50	35	20
Low	60	*Promote Regional Stability & Cooperation* 45	*Engage in Peace Opns* 30	15	1

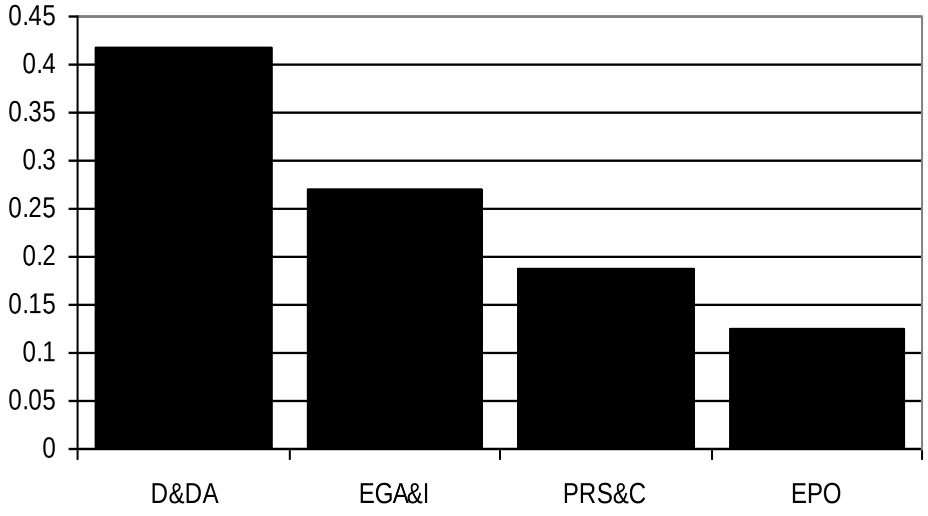
Seek & Get Concurrence with Swing Weights

Swing Weight (Estimation): Normalization Step

Normalize by summing & then dividing each relative value by the sum.

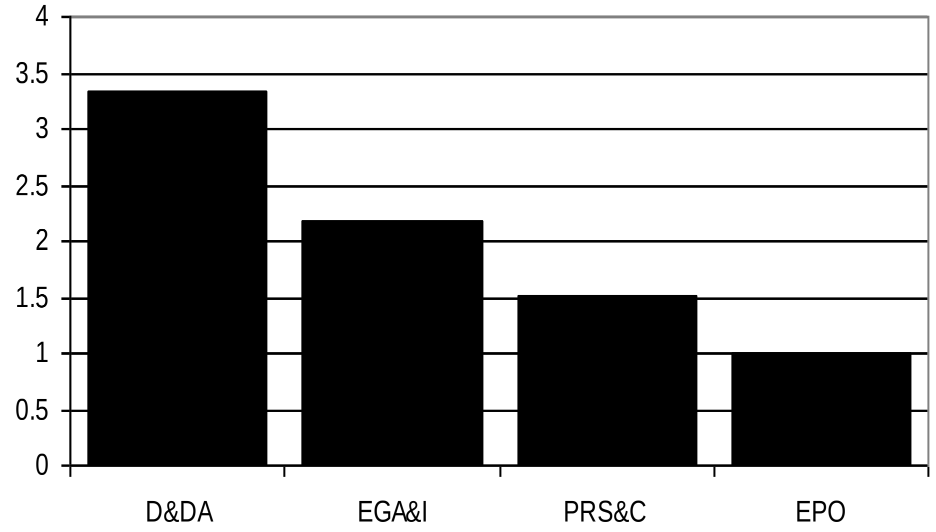
Objective	Swing Weight	Norm. Value
Deter & Defeat Aggression	100	.417
Ensure Global Access & Influence	65	.271
Promote Regional Stability & Cooperation	45	.188
Engage in Peacetime Operations	30	.125
Sum	*240*	*1.001*

Possible Graphic Depictions



Normalize so that weights sum to 1.000

Normalize so that lowest ranking weight is assigned a value of 1.000



Swing Weighting via Swing Weight Matrix

During the process of weighting attributes, it has been noted that the range among the plausible attribute values is a consideration as well as the inherent importance of the attribute. For example, one can see how the weight assigned to the price attribute could be lower if only autos in the price range from $15,000 to $18,000 were considered versus when autos vary in price from $15,000 to $35,000.

The swing weight matrix approach attempts to incorporate the dispersion among the attribute scores into the weighting process and can be implemented in the four steps as shown.

The first explanation chart shows how the swing weight matrix is constructed. On the x-axis, ordinal categories of importance are specified, while on the y-axis, ordinal categories of dispersion are specified. The size of the matrix is variable and can be adjusted to the number of elements to be weighted. In the example, only four objectives are weighted, so a large matrix is not required. For an actual application, see Ewing et al (2006) article where a 6- by 3-matrix is used when they had 40 attributes to weight in their Army Base Realignment and Closure (BRAC) 2005 Military Value Analysis.

During the next step of the swing weighting process, the elements are placed into the matrix cells where they are perceived to belong. As you can see, all four objectives are deemed to be of medium importance or above. When it comes to dispersion, military freedom of action is limited in peacetime operations and promoting regional stability and, hence, these objectives are assigned as having low dispersion. Military freedom of action is highest when considering the latitude in “Deter & Defeating Aggression” actions and, thus, deemed to possess high dispersion. Ensuring Global Access & Influence is assigned to the medium dispersion category as military freedom of action is greater than that found in EPO & PRS&C operations but less than for D&DA operations. Note that this layout makes an excellent visual for seeking & obtaining concurrence with the analysis team’s judgments or for receiving guidance to modify the placement of individual elements.

Swing Weighting via Swing Weight Matrix (continued)

In the subsequent step of the swing weighting process, values are added to the cells to indicate a relative scaling with respect to the two dimensions, importance and dispersion. Note that in this example, two orders of magnitude (i.e., values from 1-to-100) were allowed in the separation between the “low-low” cell and the “high-high” cell. As before this visual can be shared with the DM (or the DM body) when addressing concurrence or revision issues.

After obtaining the relative value judgments from an agreed upon swing weight matrix, the values are normalized so that the relative ratio weights sum to 1.000.

Graphically, the normalized weights can be depicted as proportions (upper chart) or shown based on the multiplicative relationship (lower chart) using a convenient base value to normalize against (1.00 in the depicted graph).

Next an approximation weighting technique know as the “Rank Order Centroid” will be illustrated.

Rank Ordered Centroid Assessment (Estimation)

- **Based on work of Edwards and Barron (1994) as described as portion of the Simple Multi-Attribute Rating Technique Exploiting Ranks (SMARTER), their version of procedures to address basic multi-attribute value analysis situations.**
- **Commence again with the rankings.**

Military Objective	Rank
Deter & Defeat Aggression	1
Ensure Global Access & Influence	2
Promote Regional Stability & Cooperation	3
Engage in Peacetime Operations	4

Conversion of Ranks to Relative Values

Rank order centroid (ROC) weights

$$w_i(\mathrm{ROC}) = (1/n) \sum_{j=i}^{n} (1/j), \quad i = 1, \ldots, n$$

Relative values for military objectives generated using ROC procedure

$$v(\mathrm{D\&DA}) = 1/4(1/1 + 1/2 + 1/3 + 1/4) = .5208$$

$$v(\mathrm{EGA\&I}) = 1/4(1/2 + 1/3 + 1/4) = .2708$$

$$v(\mathrm{PRS\&C}) = 1/4(1/3 + 1/4) = .1458$$

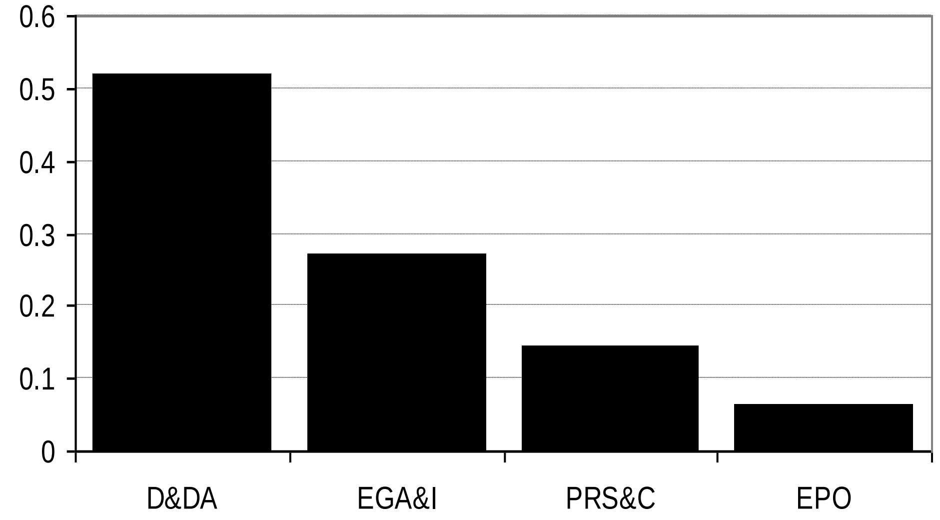
$$v(\mathrm{EPO}) = 1/4(1/4) = .0625$$

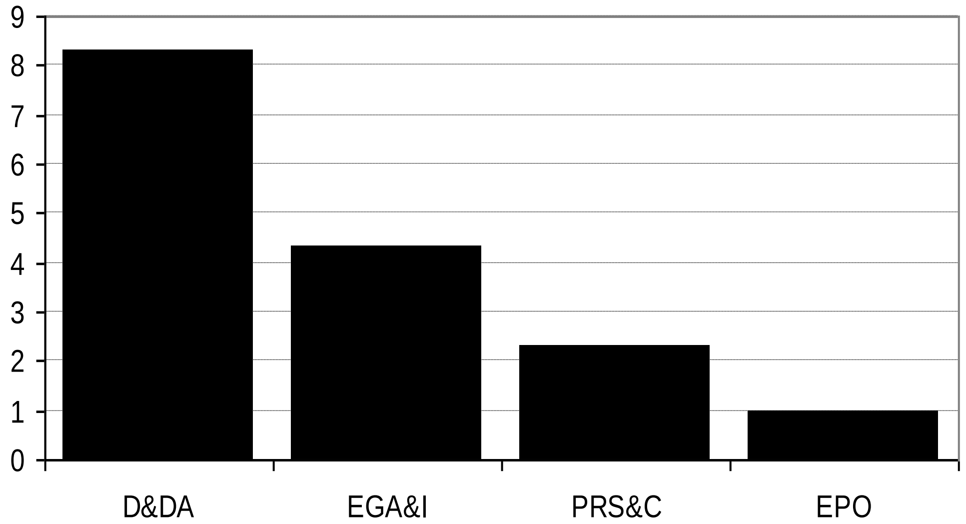
Excel Method

	Rank 1 Element	Rank 2 Element	Rank 3 Element	Rank 4 Element
	1.0000	0.0000	0.0000	0.0000
	0.5000	0.5000	0.0000	0.0000
	0.3333	0.3333	0.3333	0.0000
	0.2500	0.2500	0.2500	0.2500
Sum	2.0833	1.0833	0.5833	0.2500
# of elements	4	4	4	4
ROC Weight	***0.5208***	***0.2708***	***0.1458***	***0.0625***

Possible Graphic Depictions

Normalize so that weights sum to 1.000





Normalize so that lowest ranking weight is assigned a value of 1.000

Rank Ordered Centroid Assessment (Estimation)

Now for some magic! The Rank Ordered Centroid (ROC) method sets forth a procedure for converting ordinal data into relative ratio data. Since this procedure produces relative ratio results from ordinal rankings, it definitely must be considered an estimation technique, but it is often useful in practical applications.

On the preceding charts, the formula for ROC estimation is shown in general form, and the calculations are illustrated as applied when considering the ranks associated with the four military objectives. These ranks are consistent with the notional ranks used to illustrate the direct assessment technique as well as the swing weight matrix technique.

In essence, the formulators of this procedure justify this procedure mathematically and statistically as being the estimation procedure which results in the minimum loss of fidelity as ordinal (i.e., ranked) information is transformed into relative ratio information.

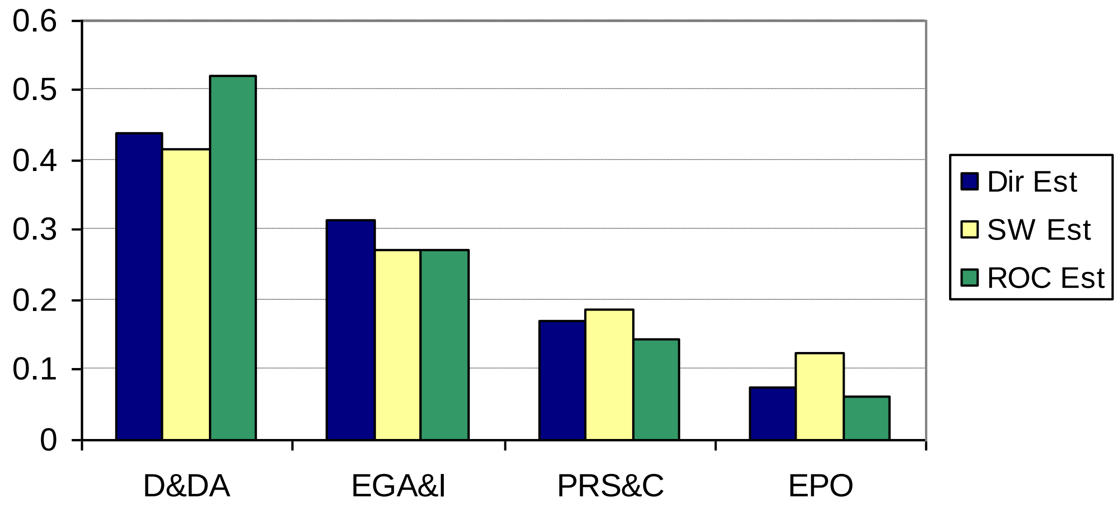
Instead of using the direct assessment procedure or swing weight matrix approach, ROC estimation can be used to compute the attribute weights which are then multiplied times the values obtained from the single dimension value functions.

As a practical matter, one of the preceding charts demonstrates how ROC estimates can be calculated quickly in Excel. With a little examination, it can be seen how this is just a convenient way to apply the formula for computing ROC weights.

As before, the graphical depiction options are once again shown to highlight the two potential ways to portray the results to DMs or decision making bodies.

Comparison of Methods

	Direct	Swing Weight	ROC
PRS&C	.169	.188	.146
EGA&I	.315	.271	.271
D&DA	.438	.417	.521
EPO	.077	.125	.063
Total	*.999*	*1.001*	*1.001*



As long as ranks are constant, estimation results demonstrate some consistency for these methods in this case.

Comparison of Methods

At this point, three assessment techniques for weighting attributes or determining priorities have been applied to the same notional problem of deriving weights for the same four military objectives. These procedures were the direct weighting procedure, the swing weight matrix approach, and the rank-order centroid method.

The prior chart demonstrates that the results from these procedures were fairly consistent when applied to the military objectives situation, as long as the ranks remained identical.

Thus, this COBP concludes that with respect to weighting attributes or prioritizing entities:

- The ROC procedure minimizes the burden on raters/judges/DMs to provide input (i.e., just ranks), while generating relatively good estimates for the relative values.
- The swing weight matrix procedure with its visually implemented stepwise approach is the best for seeking concurrence from the raters/judges/DMs, and ordinarily produces the most refined estimates for it takes both the importance and dispersion aspects into account.
- The direct assessment procedure is available and can be applied when circumstances allow. It probably produces estimates better than the ROC ones but inferior to the swing weight matrix estimation ones.

In the next section of this COBP, some group assessment topics are highlighted and discussed.

Group Assessment Considerations

- **Picking experts.**
- **Summarizing group rankings.**
- **Delphi technique.**

Group Assessment Considerations

This section of the COBP focuses on group assessment considerations. The first topic discussed will offer a few principles for choosing respondents to provide subject matter expertise (SME) and professional military judgment (PMJ). Thereafter, procedures for summarizing group rankings will be discussed along with procedures for turning the summary ranks into rank ordered centroid (ROC) weights. Lastly, the focus will shift to the implementation of the Delphi process. The Delphi process aims at enhancing the reliability achieved from expert judgment through repeated iterations of collecting individual assessments, assembling group summarization data, and then providing this data as feedback as a prelude to starting a new iteration of the process. This process terminates when the variation from iteration to iteration stabilizes.

Picking Experts

- Seek PMJ & SME based on execution experience & direct knowledge of topic; not general education or military rank.
- Quite possible to use different SMEs at different levels of hierarchy.
- Quality, not quantity, is most important factor .
  - N does not need to be 30 or more as in statistics.
  - A few knowledgeable SMEs trumps several marginal SMEs every time.
- For reporting, document qualifications of SMEs that point toward the validity of their responses.

Picking Experts

As pointed out, the best option to obtain good professional military judgment (PMJ) and subject matter expert (SME) inputs is to seek out individuals with direct experience and knowledge related to the endeavor being studied. While general education and military rank may impress, they do not necessarily translate into specific expertise related to a particular decision analysis.

Also, different experts may be appropriate at different levels of the hierarchy. For example, general officers may be most appropriate when considering the importance of military objectives during both JCIDS and other military studies, while colonels and lieutenant colonels may understand the ramification of the gaps best in JCIDS analyses. As a specific example, in the Armed Reconnaissance Helicopter (ARH) and Attack Helicopter Fleet (AHF) Analyses of Alternatives (AoAs), Kiowa Warrior and Apache pilots were used to evaluate the alternatives, while captains through colonels from the Aviation Center supplied judgments at the statement of required capabilities (SORC) and gap levels of the qualitative model.

With respect to achieving reliability and validity, the quality of respondents is more important than quantity of respondents. This principle is sometimes compromised through the belief that sample sizes of thirty or more are required for statistical reasons. However, when seeking expert judgment, it is far better to limit participation to true experts than to include marginal experts who may induce increased random variation into the process because of their limited understanding.

As a portion of every report or briefing which utilizes PMJ or SME inputs, the qualifications of the respondents should be summarized and discussed so that the recipients of the analytical work can form an opinion on the quality of inputs received.

Summarizing Rankings

Median is appropriate & available in Excel.

	SME 1	SME 2	SME 3	SME 4	SME 5	SME 6	Median	Summary Rank
Obj A	4	4	3	4	3	3	3.5	**3-4**
Obj B	2	3	2	1	2	4	2.0	**2**
Obj C	7	6	7	7	6	7	7.0	**7**
Obj D	1	1	1	2	1	1	1.0	**1**
Obj E	3	2	4	3	4	5	3.5	**3-4**
Obj F	5	5	6	5	5	2	5.0	**5**
Obj G	6	7	5	6	7	6	6.0	**6**

Summarizing Rankings

This sample Excel spreadsheet shows that the median is the appropriate statistic for summarizing group rankings across a set of SME ranks. There is an Excel function, MEDIAN, that will perform the desired calculations. Summary ranks for the entire data set may then be assigned based on the median values with ties always being possible – like depicted here with a tie between third and fourth places.

In particular, the following chart demonstrates how to calculate ROC weights for data where there are tied ranks.

ROC Weights with Tied Ranks

Computation of ROC weights with tied rankings via Excel.

Rank	1	2	3-4	3-4	5	6	7
	Obj D	Obj B	Obj A	Obj E	Obj F	Obj G	Obj C
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
3-4	0.2500	0.2500	0.2500	0.2500	0.0000	0.0000	0.0000
3-4	0.2500	0.2500	0.2500	0.2500	0.0000	0.0000	0.0000
5	0.2000	0.2000	0.2000	0.2000	0.2000	0.0000	0.0000
6	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.0000
7	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
Sum	2.5095	1.5095	1.0095	1.0095	0.5095	0.3095	0.1429
# Ranked	7	7	7	7	7	7	7
ROC	*0.3585*	*0.2156*	*0.1442*	*0.1442*	*0.0728*	*0.0442*	*0.0204*

ROC Weights with Tied Ranks

For completeness, this chart indicates how to use Excel when it comes to generating ROC values from data containing tied ranks. Note the placement of the reciprocal of the lowest tied element in the matrix where the tied entries occur. Thereafter the standard procedures for calculating ROC weights from the EXCEL matrix containing the reciprocal data is followed.

The next set of charts describe and provide an example of using the Delphi approach to refine expert judgments.

Delphi Process

- **E. S. Quade, Analysis for Public Decisions, 2nd Edition, 1982.**
- **The Delphi process refers to a repeated process of eliciting and refining the judgments and/or opinions of a group of experts based on their individual inputs.**
- **Intent is to improve the quality of the group judgmental approach in arriving at estimates or forecasts by subjecting the views of individual participants to each others' thoughts and criticisms in ways that avoid unstructured face-to-face confrontations and the associated drawbacks that accompany such interactions.**
- **Under the guidance of study director or his designee, response information generated from assessment techniques is fed back to SMEs so that they may revise their previous judgments.**
  - **redundant & irrelevant material can be eliminated.**
  - **identities concealed.**

Delphi Process

The Delphi process is another tool the analyst can employ in the attempt to build better reliability among respondents. The procedure outlined here is taken E.S. Quade's Analysis for Public Decisions (1982). The Delphi process aims at refining a group's judgment process though a step wise process starting with: (1) promoting group interaction and discussion, (2) collecting individual judgments anonymously, (3) distilling these via a third-party, impartial process into group trends, (4) providing feedback to all group members regarding trends, and (5) thereafter repeating the prior discussion, collection, distillation, and feedback steps until the data trends indicate the group refinement process has stabilized. In the end, this process may result in a single common viewpoint or multiple generalized viewpoints. The key, however, is to synthesize the initial individual judgments into refined group judgments and, thereby, interject greater consistency and, hence, reliability into the resulting outcomes.

Delphi Example

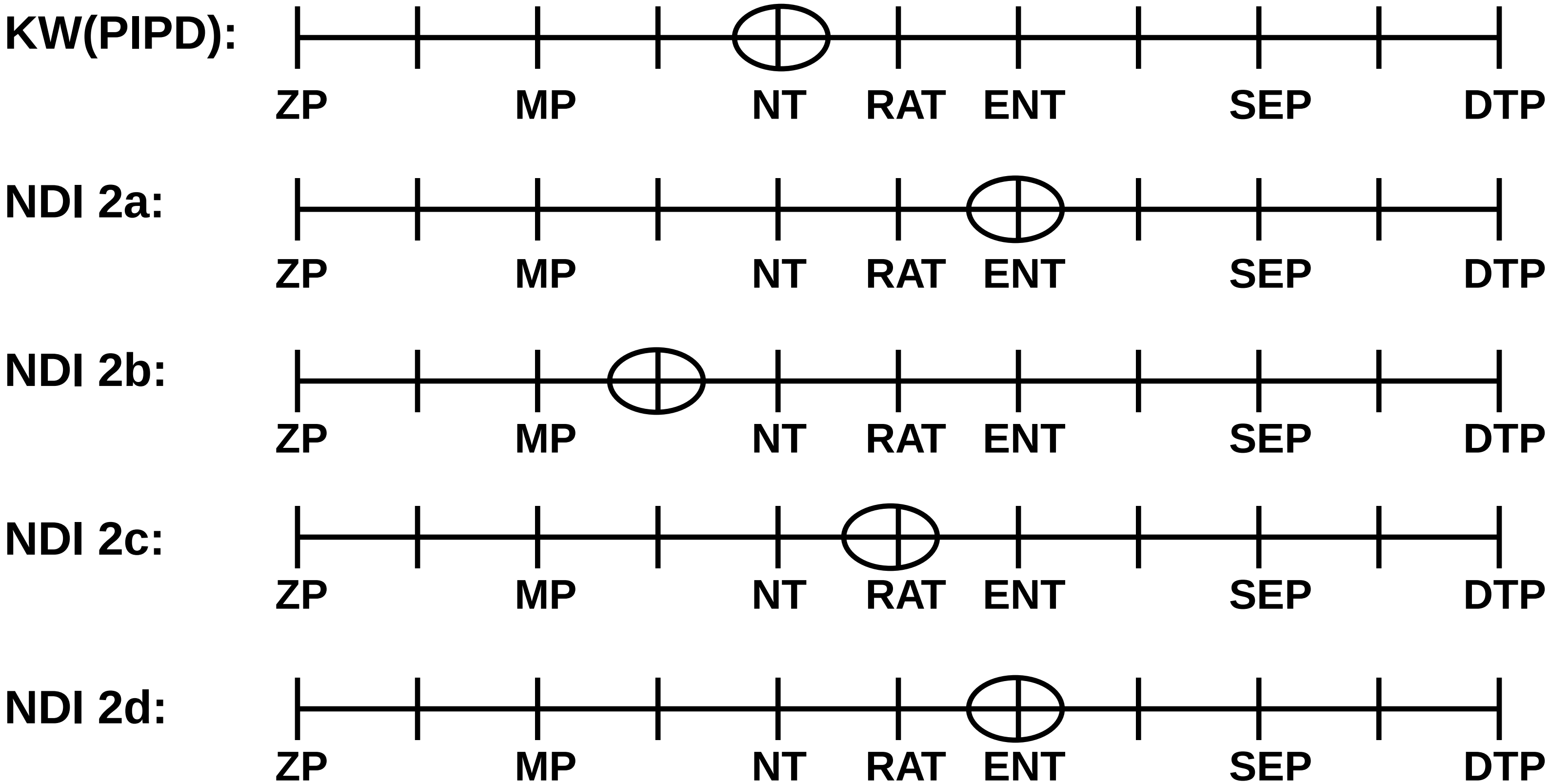
During ARH AoA posed the questions such as the following to pilots:

Performance Attribute 1: Cruise Airspeed (as defined in task, conditions, and standards reference handout)

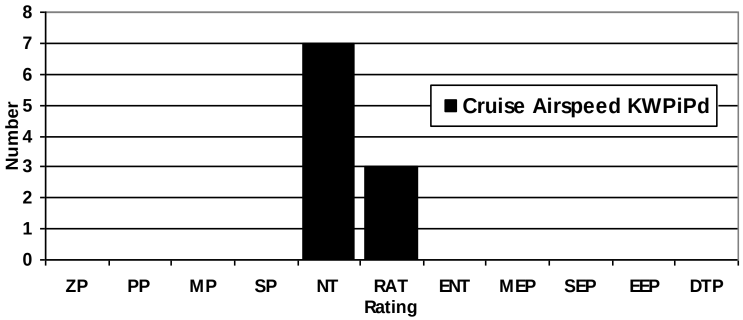
With regard to this needed performance attribute, how well do you think each ARH alternative addresses the threshold requirement?

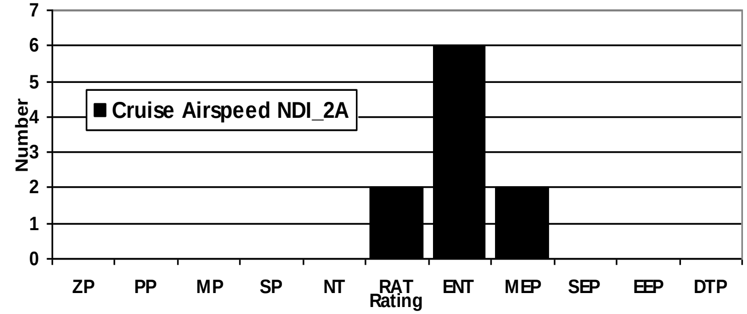
- **ZP – Zero or almost zero capability to meet performance threshold (0 to 20% capability)**
- **MP – Moderate capability to meet performance (40 to 60% capability)**
- **NT – Near performance threshold capability (80 to 99% capability)**
- **RAT – Right at performance threshold capability (100% capability)**
- **ENT – Excess performance capability near threshold, (101% to 120% capability)**
- **SEP – Significant excess performance capability (140-160% capability)**
- **DTP – Double or near double threshold performance capability (180% to 200% capability or beyond)**

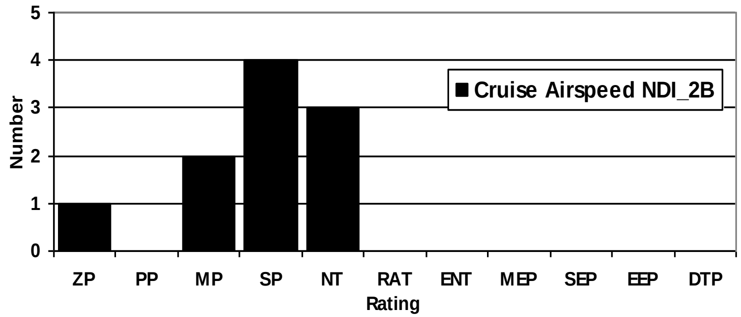
Delphi Example (continued)

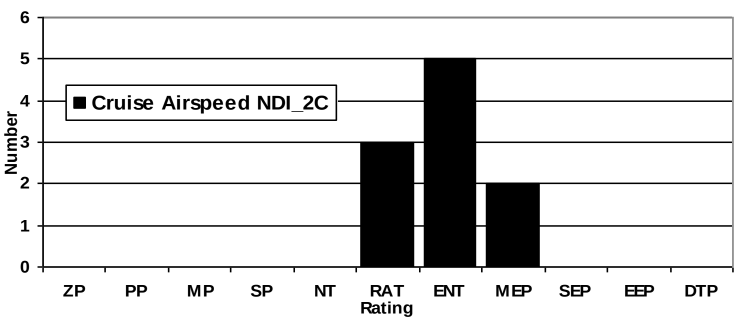


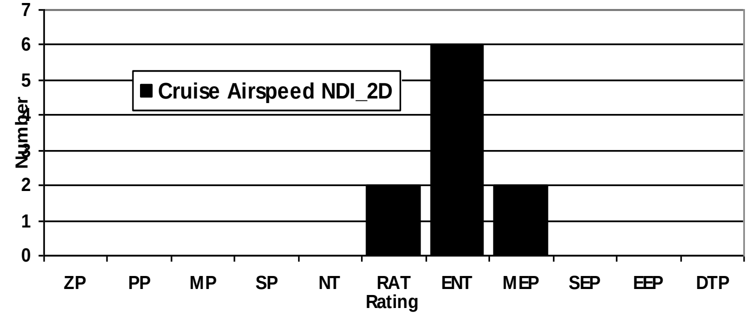
Feedback Display











Delphi Example (Concluded)

- **Based on visual display & discussion took second measurement using same question format.**
- **Only second measurement counted in computation of final results.**

Delphi intended to increase reliability and, thus, validity of results.

Delphi Example

As an example, questions like the "Cruise Airspeed" one were posed to the Kiowa Warrior (KW) pilots who acted as respondents during the armed reconnaissance helicopter (ARH) AoA. This type of questioning was performed across all 84 ARH performance attributes where the current KW pilots acted as experts in evaluating the five ARH aircraft alternatives. The KW(Pip'd) alternative refers to product improved Kiowa Warrior (OH-58D) and the other alternatives represent four classes of non-developmental items (i.e., non-developmental commercial helicopters that could be militarized in this case).

During the evaluation procedure, each of the ten pilots individually circled their projected performance ratings for each alternative using a Likert-type scale where the scale descriptors ranged from "Zero Performance" through "Double Threshold Performance". Thereafter, the subsequent chart depicts the type of notional bar charts that were generated for each of the 84 performance attributes like "Cruise Airspeed" after the first set of pilot judgments were collected. Once the bar charts were reviewed and discussed in a group setting, a second set of individual judgments were obtained from the same set of pilots using the same questioning format. As indicated, only the second set of assessments were used in developing the final results. As such, this example shows how the Delphi process was used in order to increase reliability and, thereby, the validity of results.

This COBP section regarding group assessment considerations focused on highlighting some principles for picking experts, generating summary rankings and corresponding ROC weights, as well as implementing the Delphi process. The next section commences the discussion of the Analytic Hierarchy Process (AHP).

The Analytic Hierarchy Process (AHP)

- **Thomas L. Saaty (1977, 1980, 1982)**
- **Touted as generalized process for:**
  - **Planning**
  - **Priority Setting**
  - **Resource Allocation**
- **Overall, a questionable technique, but components of the quantitative technique can prove quite useful.**
  - **Pair-wise comparison technique for determining weights & priorities.**
  - **Technique for computing composite values between successive levels of the objectives hierarchy.**

The Analytic Hierarchy Process (AHP)

In this section of the COBP, the Analytic Hierarchy Process (AHP), a popular and widely used MADM technique, will be discussed as another potential MADM technique. AHP was developed by Thomas Saaty in the mid-to-late 1970s.

Among AHP's three primary uses, one is priority setting which makes it a logical procedure to consider for JCIDS analysis where prioritization is required within the FAA, FNA, and FSA steps. The other two primary uses relate to generalized planning and resource allocation which are also the topics of many military studies and decision analyses.

In general, a couple of the components taken from the theory underlying AHP can prove very useful in MADM analyses; however, the overall process is considered somewhat questionable by the traditional decision analytic community. The purpose of this section is to highlight and demonstrate the components that are useful while also illustrating the perceived shortcomings associated with the overall technique.

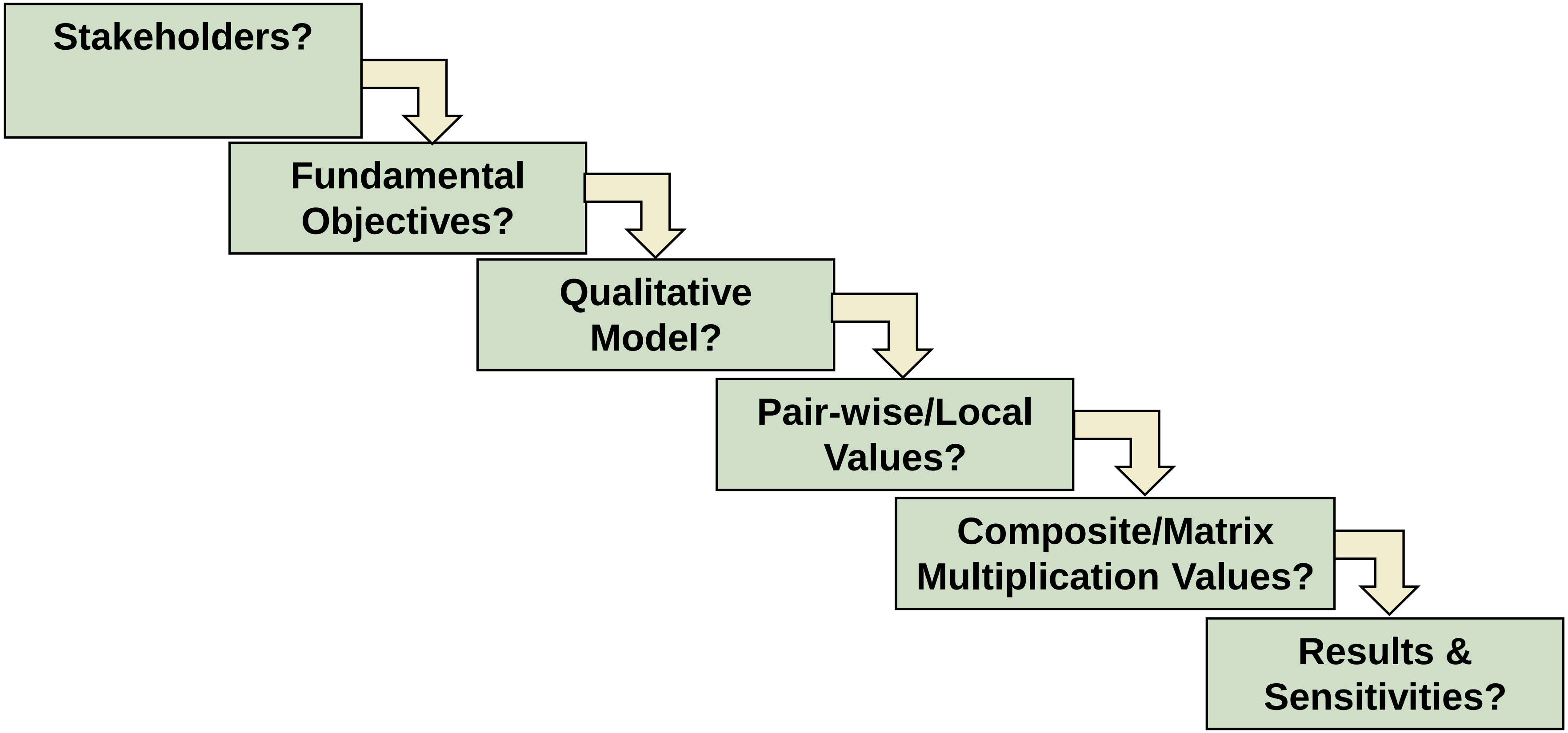
Based on the next couple of charts, the discussion of AHP starts by comparing its stepwise procedure with the stepwise procedure of multi-attribute value analysis.

AHP Analysis: Basic Steps & Process

- **Step 1. Identify stakeholders.**
  - **Decision Makers (DM's), Subject Matter Experts (SME's), & others affected by the decision to be made.**
- **Step 2. Identify fundamental objectives.**
  - **Use existing documentation, leadership interviews, brainstorming, etc.**
  - **Time is usually the driving constraint in this important step.**
- **Step 3. Develop the qualitative model.**
  - **Write down the fundamental objectives & supporting attributes.**
  - **Develop graphical representation for communication purposes.**
- **Step 4. Develop quantitative value model.**
  - **Performing comparative judgments through pair-wise comparisons.**
  - **Determine local values by analyzing pair-wise comparison results.**

AHP Analysis: Basic Steps & Process (continued)

- **Step 5. Synthesize local values from one level of hierarchy to next level (calculate composite values for the successive levels of the hierarchy).**
  - **Employ the method of matrix multiplication.**
- **Step 6. Generate the results & conduct sensitivity analysis.**



AHP Analysis: Basic Steps & Process

As can be seen, the first three steps of identifying stakeholders, identifying fundamental objectives, and developing the qualitative model for implementing an AHP analysis are the same as those for implementing a multi-attribute value analysis. However, the fourth and fifth steps, which implement the quantitative aspects of AHP, are quite different from those employed in multi-attribute value analysis. As this section of the COBP proceeds, these fourth and fifth AHP steps will be examined in detail. The sixth step remains essentially the same for both AHP and multi-attribute value analysis where results are derived and sensitivity analysis conducted.

The next chart commences the illustration of the pair-wise comparison process referenced in the fourth basic step of AHP.

Pair-wise Comparison Technique

- Consider again the four fundamental objectives from the "Future Defense Planning" objectives hierarchy.
- Objectives and their abbreviations.
  - Promote Regional Stability & Cooperation (PRS&C)
  - Ensure Global Access & Influence (EGA&I)
  - Deter & Defeat Aggression (D&DA)
  - Engage in Peacetime Operations (EPO)
- Start by formulating four military objectives against one another in pair-wise format.
  - 6 pairs (i.e., ${}_4C_2 = 6$)
  - In general, ${}_nC_2 = n! / ((n-2)!*2!)$

Pair-wise Comparison Technique

The illustration starts by considering the four fundamental objectives associated with the "Future Defense Planning" hierarchy and then determines how many pair-wise comparison items will be needed to conduct a pair-wise examination among these objectives.

Like the direct assessment technique, the pair-wise comparison technique also attempts to develop relative ratio weighting among the objectives, but it is more involved than the direct assessment technique. Saaty, and his followers, would argue that the pair-wise comparison technique is superior to the direct assessment technique because it partitions the respondents' overall task into many paired comparisons which are evaluated one at a time and, thereby, reduces the intellectual complexity associated with achieving the prioritization and derivation of relative weights.

With the four objectives, there are six unique paired comparisons to be made (i.e., mathematically, the combination of four things taken two at a time, or in general, the combination of n-things taken two at a time).

Henceforth, the following abbreviations will be used for the objectives:

- Promote Regional Stability & Cooperation (PRS&C)
- Ensure Global Access & Influence (EGA&I)
- Deter & Defeat Aggression (D&DA)
- Engage in Peacetime Operations (EPO)

The next chart demonstrates one questioning format for collecting pair-wise data from respondents with notional outputs shown on the following one.

Comparative Judgment Question

With respect to future defense planning, which military objective in each of the following pairs possesses greater value and what is the intensity associated with the more important element?

			Equal		Moderate		Strong		Very Strong		Extreme
1.	**PRS&C**	**EGA&I**	___	___	___	___	___	___	___	___	___
2.	**PRS&C**	**D&DA**	___	___	___	___	___	___	___	___	___
3.	**PRS&C**	**EPO**	___	___	___	___	___	___	___	___	___
4.	**EGA&I**	**D&DA**	___	___	___	___	___	___	___	___	___
5.	**EGA&I**	**EPO**	___	___	___	___	___	___	___	___	___
6.	**D&DA**	**EPO**	___	___	___	___	___	___	___	___	___

Comparative Judgment Question Notational Result

With respect to future defense planning, which military objective in each of the following pairs possesses greater value and what is the intensity associated with the more important element?

			Equal		Moderate		Strong		Very Strong		Extreme
1.	$\boxed{\textbf{PRS\&C}}$	**EGA&I**			X						
2.	**PRS&C**	$\boxed{\textbf{D\&DA}}$			X						
3.	$\boxed{\textbf{PRS\&C}}$	**EPO**				X					
4.	**EGA&I**	$\boxed{\textbf{D\&DA}}$						X			
5.	$\boxed{\textbf{EGA\&I}}$	**EPO**					X				
6.	$\boxed{\textbf{D\&DA}}$	**EPO**								X	

Comparative Judgment Question

Again, note the first clause of question reminds the respondent to be cognizant of the frame of reference.

Previously in the instructions section, the respondent would have been told that his task is to "circle" or "box" the more important objective and then use a "X" to mark the intensity associated with the greater importance. Note that there is an option to rate the objectives as having "equal" importance relative to the frame of reference and, in this case, the respondent would be instructed to indicate neither choice and place the "X" in the equals column.

Pair-wise Comparison Intensity Scale

Definition	Intensity of Importance	Explanation
Equal Importance	1	Two elements contribute equally to the objective.
Moderate Importance	3	Experience & judgment moderately favor one element over the other.
Strong Importance	5	Experience & judgment strongly favor one element over the other.
Very Strong Importance	7	An element is very strongly favored over the other.
Extreme Importance	9	There is an extreme degree favoritism of one element over the other.
	2,4,6,8	When compromise is needed.
	Reciprocals of above non-zero numbers.	If the elements are reversed in preference.

Pair-wise Comparison Intensity Scale

The definition of the pair-wise comparison scale and its components would also be a part of the directions for the respondent. In essence, the pair-wise comparison intensity scale is a seventeen point scale which runs from one-ninth through nine with a value of one (1.00) indicating equal importance and the middle of the scale. The values of two through nine indicate the preference of Quality A over Quality B with the reciprocals indicating just the opposite: the preference of Quality B over Quality A.

As shown on the next chart, the questionnaire-type data is transformed into a matrix format to aid in the analysis of the pair-wise comparison data.

Pair-wise Comparison Matrix from Example Respondent

	PRS&C	EGA&I	D&DA	EPO
PRS&C	1/1	3/1	1/3	4/1
EGA&I	1/3	1/1	1/6	5/1
D&DA	3/1	6/1	1/1	8/1
EPO	1/4	1/5	1/8	1/1

Each comparative judgment viewed as ratio of relative values (V_i / V_j)

Pair-wise Comparison Matrix from Example Respondent

Here, the question format has been translated to the matrix format with scale values indicating the scaled relative ratio relationships. Note the diagonal elements are 1/1 since each element is equal to itself. If the lower left-hand portion of the matrix were included, it would consist of the reciprocals of the upper right-hand portion of the matrix.

As noted at the bottom of the prior chart, the matrix entries are deemed to represent the ratio of the preference for one element when compared to the other.

System of Equations

$V_{PRS\&C} / V_{EGA\&I} = 3/1$ or $V_{PRS\&C} = 3 * V_{EGA\&I}$

$V_{PRS\&C} / V_{D\&DA} = 1/3$ or $V_{PRS\&C} = 1/3 * V_{D\&DA}$

$V_{PRS\&C} / V_{EPO} = 4/1$ or $V_{PRS\&C} = 4 * V_{EPO}$

$V_{EGA\&I} / V_{D\&DA} = 1/6$ or $V_{EGA\&I} = 1/6 * V_{D\&DA}$

$V_{EGA\&I} / V_{EPO} = 5/1$ or $V_{EGA\&I} = 5 * V_{EPO}$

$V_{D\&DA} / V_{EPO} = 8/1$ or $V_{D\&DA} = 8 * V_{EPO}$

and

$V_{PRS\&C} + V_{EGA\&I} + V_{D\&DA} + V_{EPO} = 1$

(i.e., 7 equations in 4 unknowns; several possible methods of solution: right eigenvalues, least squares, LP minimization of deviations, basic approximation method, etc.)

System of Equations

Based on an element-by-element formulation, a system of equations is derived. It consists of the six pair-wise comparison equations as well as a seventh equation which is added so that the relative values are normalized to sum to 1.00. The system of equations ultimately becomes the seven equations in four unknowns shown.

As indicated, there are several ways to derive estimates of the unknown values from such a system of equations. Saaty asserts that the only correct way to solve is to derive the right-hand eigenvalues for graph theoretical reasons, but other well known techniques exist. A statistician would probably minimize the least squares, a linear programming (LP) expert would probably minimize the absolute deviations using a LP formulation, and any analyst simply interested in reasonably good estimates might use the basic approximation method that can be implemented via Excel. This method is illustrated on the next chart.

Basic Approximation Method

Determine column sum for elements

	PRS&C	EGA&I	D&DA	EPO
PRS&C	1.000	3.000	0.333	4.000
EGA&I	0.333	1.000	0.167	5.000
D&DA	3.000	6.000	1.000	8.000
EPO	0.250	0.200	0.125	1.000
Sum	*4.583*	*10.200*	*1.625*	*18.000*

Normalize columns & determine weight as average row value

	PRS&C	EGA&I	D&DA	EPO	**Weight**
PRS&C	0.2182	0.2941	0.2051	0.2222	**0.2349**
EGA&I	0.0727	0.0980	0.1026	0.2778	**0.1378**
D&DA	0.6545	0.5882	0.6154	0.4444	**0.5757**
EPO	0.0545	0.0196	0.0769	0.0556	**0.0517**

Basic Approximation Method

The basic approximation steps are as follows:

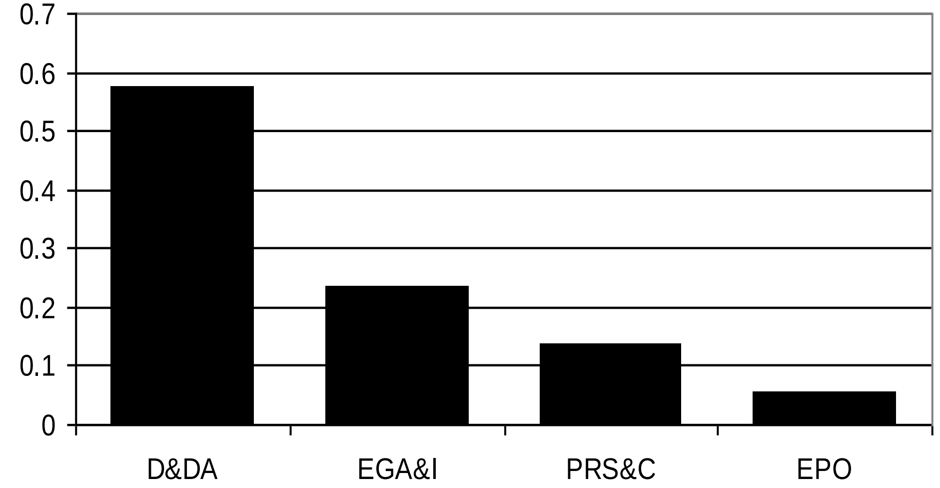
- **Place the entire pair-wise comparison matrix (with reciprocals) into Excel (in decimal format).**
- **Determine the column sums (can use the Excel Σ tool for this) based on the complete pair-wise comparison matrix.**
- **Normalize the pair-wise comparison entries based on the respective column sums (totals).**
- **Calculate the average mean value of the respective row entries to arrive at the basic weight approximation – the Excel function (f_x) named AVERAGE(number 1, number 2, …) can perform this task quickly.**

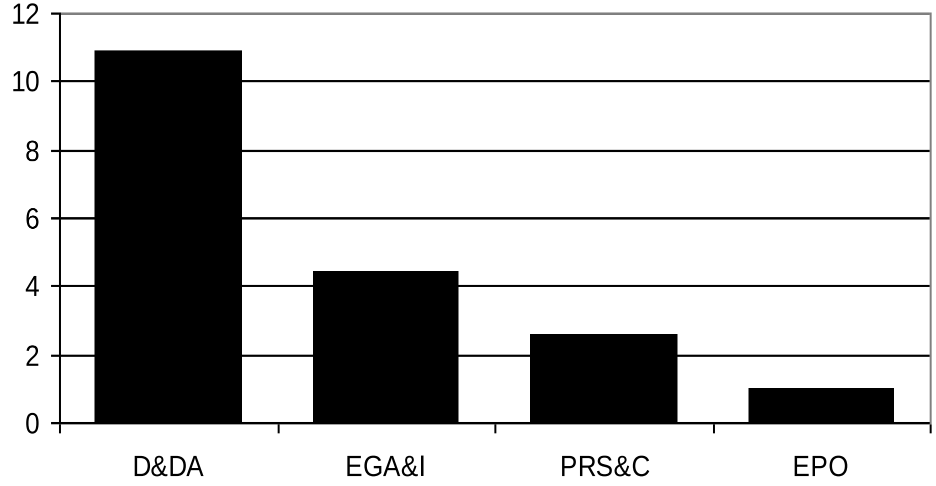
For our example, the estimated objective weights are as follows:

v(PRS&C) = .2349,
v(EGA&I) = .1378,
v(D&DA) = .5757, and
v(EPO) = .0517.

Possible Graphic Depictions

Normalize so that weights sum to 1.000





Normalize so that lowest ranking weight is assigned a value of 1.000

Possible Graphic Depictions

As before with the other weighting techniques, the same two chart options are available for depicting analytical results of the pair-wise comparison process.

At this point, the same objective weights have been estimated using the direct estimation technique, swing weighting via the swing weight matrix, the ROC method, and now the pair-wise comparison technique. Thus, the pair-wise comparison taken from AHP could be added to the earlier COBP section on "Useful Assessment Techniques for Determining Weights and Priorities" as yet another useful technique.

In addition, the pair-wise comparison technique possesses another advantage. From the pair-wise data for a respondent, a consistency index can be computed assessing that respondent's intra-personal consistency as shown on the next three charts. Based on the earlier measurement section of this COBP, this internal consistency estimate qualifies as a within person reliability measure (although Saaty doesn't ever explicitly state this).

Consistency Ratio

- **According to Saaty, consistency ratio (CR) should be less than 10% (< .10) or judgments deemed somewhat random & should perhaps be revised.**
- **Start CR calculation by computing Weighted Sum Vector.**

	PRS&C	EGA&I	D&DA	EPO
PRS&C	1.000	3.000	0.333	4.000
EGA&I	0.333	1.000	0.167	5.000
D&DA	3.000	6.000	1.000	8.000
EPO	0.250	0.200	0.125	1.000

*

Weight
0.2349
0.1378
0.5757
0.0517

=

Weighted Sum Vector
1.0468
0.5703
2.5203
0.2099

Consistence Ratio (continued)

From the data on the previous page, divide weighted sum elements by corresponding objective weights.

1.0468 / .2349 = 4.4560
.5703 / .1378 = 4.1394
2.5203 / .5757 = 4.3782
.2099 / .0517 = 4.0632

Average

4.2592

- **Derive Consistency Index:**

CI = (Average - # elements)/(# elements – 1)

CI = (4.2592 – 4)/(4 – 1) = .0864

- **Compare with Random Index (RI) for various size matrices.**

Size	1	2	3	4	5	6	7	8	9	10	11	12	13	14
RI	.00	.00	.58	.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.60

- **Consistency Ratio = Consistency Index / Random Index = .0864/.90 = .0960**
- **Just less than .1000 so deemed consistent enough with inconsistency of 9.60%.**

Consistency Ratio

As indicated, Saaty states the consistency ratio (CR) should be less than 10% for the pair-wise judgments to be considered acceptable. If greater than 10%, a redo may be indicated (or if the respondent is inconsistent over a number of pair-wise comparison matrices, it might be concluded that he is not really familiar with the topic or be an actual SME and, thus, be eliminated from the exercise).

Calculation of the CR starts by computing the weighted sum vector via matrix multiplication of the pair-wise comparison matrix by the vector of estimated objective weights. The Excel function MULT(array1,array2) can be used.

After dividing the respective weighted sums by estimated objective weights, the mean of the four computed values is found. In this case, the mean is 4.2592. In mathematical language (and Saaty's procedures), this is an estimate of the principal eigenvalue ($\lambda_{max} = 4.2592$). If the pair-wise matrix were completely consistent, this value would be 4.000 and consistent with the number of objectives being compared.

Since there is some inconsistency, we subtract 4 from the average (λ_{max}) and divide by the number of objectives minus 1 to arrive at what Saaty terms the consistency index (CI): .0864.

In order to see what CIs would result from putting random responses into the pair-wise comparison matrices, Saaty performed a simulation where this was done numerous times (n=500 in latest simulation and n=100 in earlier simulations). Saaty termed the average result to be the "random index", RI, and listed these averages in a table format based on matrix size . RI results are shown for matrix sizes 1 through 14.

The CR is then computed as CI/RI. In the example case, this calculate to 0.0960 as a proportion or 9.60 as a percent. Since the CR is less than 10%, or .1000 as a proportion, the judgments from our pair-wise matrix illustration are deemed to possess enough consistency to be acceptable.

Summarizing Paired Comparisons

- **Apply geometric mean to individual responses (i.e., multiply responses together & take n-th root of product).**
  - **GEOMEAN function in Excel.**

	SME 1	SME 2	SME 3	SME 4	SME 5	SME 6	SME 7	Geometric Mean
Item 1	3.0000	1.0000	2.0000	4.0000	3.0000	3.0000	4.0000	*2.6273*
Item 2	0.3333	0.2000	0.2500	1.0000	0.5000	0.3333	0.2500	*0.3538*
Item 3	4.0000	3.0000	4.0000	5.0000	5.0000	4.0000	3.0000	*3.9269*
Item 4	0.1667	0.1429	0.1111	0.2000	0.2500	0.2500	0.2000	*0.1820*
Item 5	5.0000	4.0000	3.0000	4.0000	3.0000	4.0000	5.0000	*3.9269*
Item 6	8.0000	6.0000	6.0000	7.0000	5.0000	6.0000	5.0000	*6.0665*

Proceed by solving 7 equations in 4 unknowns as before using geometric means (e.g., basic technique, right eigenvalues, least squares, etc.)

Summarizing Paired Comparisons

With respect to paired comparison data from multiple respondents, the appropriate technique is to average the individual responses across respondents using a geometric mean and then use one of the solution techniques (basic approximation, principal eigenvector, least squares etc.) to solve the system of equations built from the geometric means.

Why the geometric mean? Think of the extreme case where one respondent indicates attribute A is 9 times as important as B, while another respondent indicates just the reverse, where attribute A is 1/9 as important as B. These responses should counter balance one another and take on an "equal" importance rating (i.e., a value of 1 on the intensity scale defined by Saaty). If you multiply 9 times 1/9 your result is 1 and when you take the square root of 1 it equals 1, the desired result. Likewise, in the general case, if you multiply all the individual responses together and take the n-th root of the product, the desired result is obtained.

Contrast this with taking the arithmetic mean of 9 and 1/9, which yields 4.556 [(9+1/9)/2 = 4.556] - obviously not the desired value of 1.00 since it over estimates the desired average result of one based on the extreme score of 9.

Inter-rater Reliability

- **Think average correlation among raters.**
- **For previous paired comparison example can build inter-correlation matrix among using CORREL function of Excel.**

	SME 1	SME 2	SME 3	SME 4	SME 5	SME 6	SME 7
SME 1	1	0.974	0.968	0.955	0.875	0.984	0.900
SME 2		1	0.948	0.889	0.821	0.939	0.822
SME 3			1	0.973	0.945	0.977	0.815
SME 4				1	0.964	0.984	0.881
SME 5					1	0.942	0.806
SME 6						1	0.920
SME 7							1

- **Average inter-correlation equals .918.**
- **Could use analysis of variance (ANOVA) techniques as well to compute reliability among SMEs.**

Inter-rater Reliability

Now consider the inter-rater reliability for the six items and seven respondents on the previous chart. If the CORREL (correlation) function of Excel is used to derived the product moment correlations between the vectors of item responses, the inter-correlation matrix can be built between the respondents as shown. The mean of all the off-diagonal elements is .918; this is the average inter-correlation between any two randomly picked respondents. Average inter-correlations ranging from .85 to 1.00 are very good, and average inter-correlations ranging from .70 to .85 are more typical and generally acceptable. If the average inter-correlation value is much below .70 there really is not much consistency (i.e., reliability) among respondents and, hence, the validity is limited as was discussed earlier in this COBP's measurement section.

Based on the statistical principles associated with correlation and regression analysis, the square of the correlation coefficient, $r^{2,}$ reflects the percentage of the variance explained by the relationship between two random variables. Hence, if the inter-correlation coefficient falls below .70, less than half the average variance between any two randomly chosen response vectors is being explained.

This completes the explanation of the pair-wise comparison technique. As demonstrated, the pair-wise comparison technique is an alterative analytical technique that can be extracted from Saaty's AHP and used independently for the prioritization or weighting of a set entities. The availability of the consistency index, CI, is an important advantage associated with this technique along with the ability to derive average pair-wise comparison response vectors from the pair-wise data provided by multiple respondents.

Saaty Verification Looks

- Wealth of nations through their world influence.
- Distance between cities by frequent travelers.
- Illumination intensity and the inverse square law.

Good agreement found in all three cases.

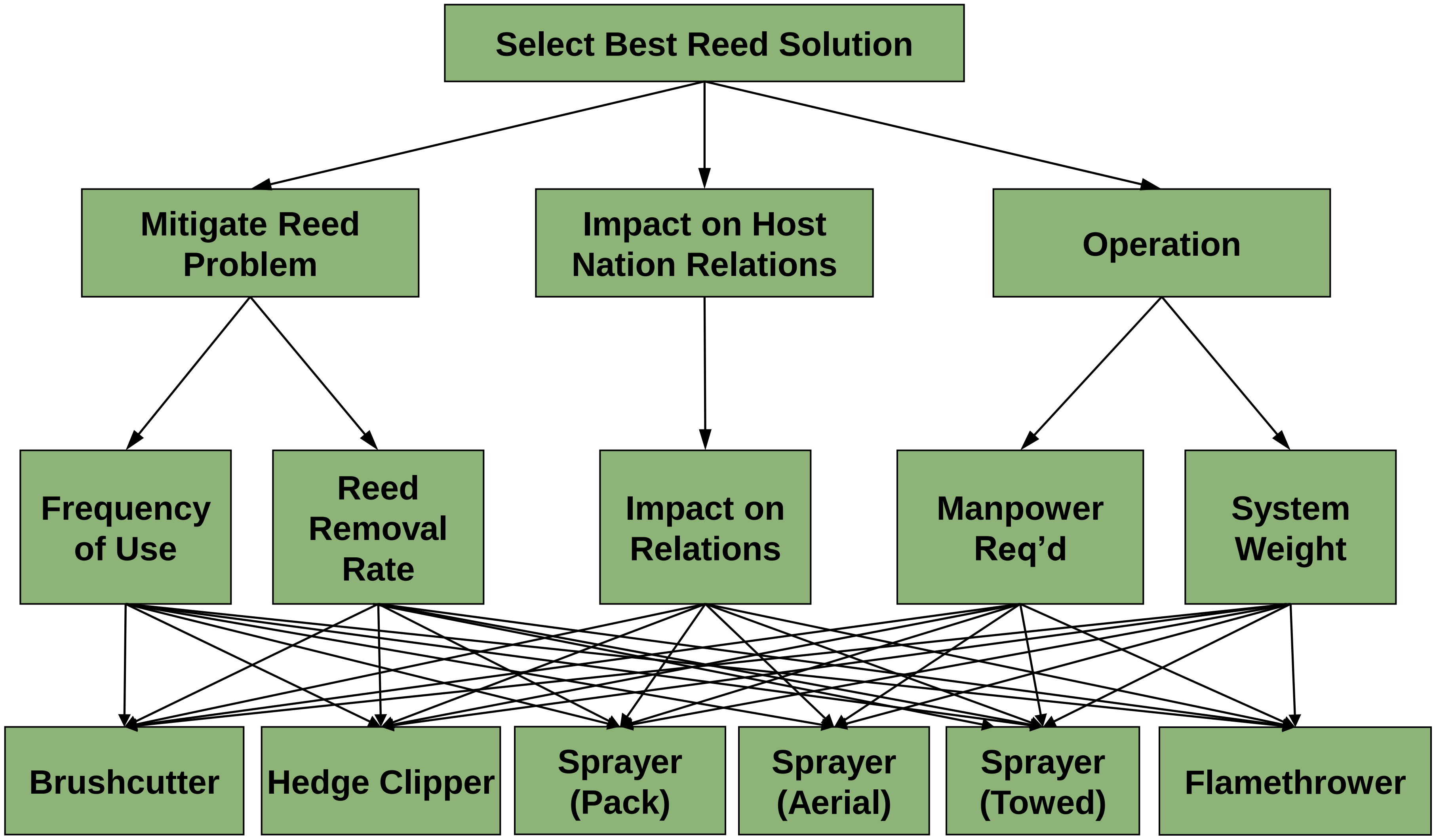
Saaty Verification Looks

At this point, a skeptical reader might question whether this pair-wise comparison process really works and should be applied. To address this question, Saaty, in his 1980 book, described three verification examinations that he undertook where the estimates derived via pair-wise judgments made by experts could be compared with actual known values. As indicated, the topics listed formed the basis for the examinations. The results of all three experiments resulted in estimates that were highly consistent with the actual known values, and it should be noted that his third experiment involved a known non-linear relationship from the physical sciences.

Any critical reader still realizes that these results were generated in situations where the known values could be looked up or computed from known formulas and that a leap of faith is still required to accept that that the pair-wise comparison procedure works well when known values do not exist and psychological factors come into play. However, based on Saaty's presented evidence, this COBP concludes that the pair-wise comparison approach probably works pretty well in general, but there are some scaling concerns if the rated elements differ by more than an order of magnitude since Saaty's comparison scale limits the rater from indicating one element is more than nine-times as important as another.

This COBP will now turn its attention back to the "selection of reed mitigation solution" and the computation of hierarchical values at successive levels of the hierarchy.

Refresher: Choose The Best Reed Removal Solution



Output from Step 3 of AHP and Multi-Attribute Value Analysis Process

Refresher: Choose The Best Reed Removal Solution

As before, this is the hierarchical structure developed based on the REF request for a reed removal solution.

To review, there are three fundamental objectives: practical, relations, and operable. Specific attributes relating to the objectives are frequency and rate with respect to the practical objective, relations as an attribute as well as the objective itself, and manpower and weight are attributes relating to the operable objective. Finally, at the lowest level of the hierarchy are the six alternatives under consideration.

Just as this example was used to illustrate multi-attribute value analysis, it will now be used to demonstrate the procedures of AHP. The next chart shows the AHP commencing with collecting and analyzing pair-wise comparison data at the fundamental objective level.

Solicit Comparative Judgment among 3-Elements at First Level

- With respect to ***choosing a reed removal solution***, which ***objective*** in each of the following pairs possesses greater value and what is the intensity associated with the more important goal?

			Equal		Moderate		Strong		Very Strong		Extreme
1.	Practical	Relations	___	X	___	___	___	___	___	___	___
2.	Practical	Operability	___	___	X	___	___	___	___	___	___
3.	Relations	Operability	___	X	___	___	___	___	___	___	___

	Practical	Relations	Operable
Practical	1	2	3
Relations	0.5	1	0.5
Operable	0.333333	2	1
Sum	1.833333	5	4.5

	Practical	Relations	Operable	
Practical	0.545	0.400	0.667	**0.537**
Relations	0.273	0.200	0.111	**0.195**
Operable	0.182	0.400	0.222	**0.268**
Sum	1	1	1	

Solicit Comparative Judgment among 3-Elements at First Level

As illustrated in the prior pair-wise comparison assessment section, the initial step is to determine the relative values for the fundamental objectives using paired comparisons and the basic approximation method. This chart illustrates the development of the relative values as 0.537, 0.195, and 0.268 for the practical, relations and operable objectives, respectively.

The next set of charts demonstrate how the pair-wise comparison process and determination of relative values is completed for the second level of the hierarchy with respect to the first level containing the fundamental objectives.

Solicit Comparative Judgment for Elements at Second Level

With respect to the *practical* objective, which attribute in the following pair possesses greater value and what is the intensity associated with the more important factor?

	Equal		Moderate		Strong		Very Strong		Extreme	
Frequency	X	_	___	___	___	___	___	___	___	Rate

With respect to the *operability* objective, which attribute in the following pair possesses greater value and what is the intensity associated with the more important factor?

	Equal		Moderate		Strong		Very Strong		Extreme	
Manpower	_	X	___	___	___	___	___	___	___	Weight

Develop Attributes by Objectives Matrix

	Frequency	Rate	
Frequency	1.000	1.000	
Rate	1.000	1.000	
Sum	2	2	
	Frequency	Rate	
Frequency	0.500	0.500	**0.500**
Rate	0.500	0.500	**0.500**
	1	1	

	Impact	
Impact	1	
Sum	1	
	Impact	
Impact	1.000	**1.000**
Sum	1	

	Manpower	Weight	
Manpower	1.000	2.000	
Weight	0.500	1.000	
Sum	1.5	3	
	Manpower	Weight	
Manpower	0.667	0.667	**0.667**
Weight	0.333	0.333	**0.333**
Sum	1	1	

		Objectives		
		Practical	Relations	Operable
	Freq	**0.500**	0	0
	Rate	**0.500**	0	0
Attributes	Impact	0	**1**	0
	Manpower	0	0	**0.66666667**
	Weight	0	0	**0.33333333**

Develop Attributes by Objectives Matrix

Next, the paired comparisons of the attributes with respect to the objectives are considered. No paired comparison with respect to the appearance objective is required since appearance only has one element, itself, at the attribute level.

Using the basic approximation technique, the attributes by objectives matrix is assembled which indicates the appropriate relationships and their strengths. As can be seen, an attribute value is deemed to have zero value if it is not related to an objective.

The next chart shows how the composite AHP relative values are found at the second hierarchical level using matrix multiplication.

Composite 2nd Level Values Determined by Matrix Multiplication

$$
\begin{array}{l}
\\
\text{Frequency} \\
\text{Rate} \\
\text{Impact} \\
\text{Manpower} \\
\text{Weight}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \text{Practical} & \text{Relations} & \text{Operable} \end{array} \\
\begin{pmatrix}
.500 & .000 & .000 \\
.500 & .000 & .000 \\
.000 & 1.000 & .000 \\
.000 & .000 & .667 \\
.000 & .000 & .333
\end{pmatrix}
\end{array}
*
\begin{pmatrix} .537 \\ .195 \\ .268 \end{pmatrix}
\begin{array}{l} \text{Practical} \\ \text{Relations} \\ \text{Operable} \end{array}
$$

$$
= \begin{pmatrix} .133 \\ .267 \\ .200 \\ .200 \\ .200 \end{pmatrix}
\begin{array}{l} \text{Frequency} \\ \text{Rate} \\ \text{Impact} \\ \text{Manpower} \\ \text{Weight} \end{array}
$$

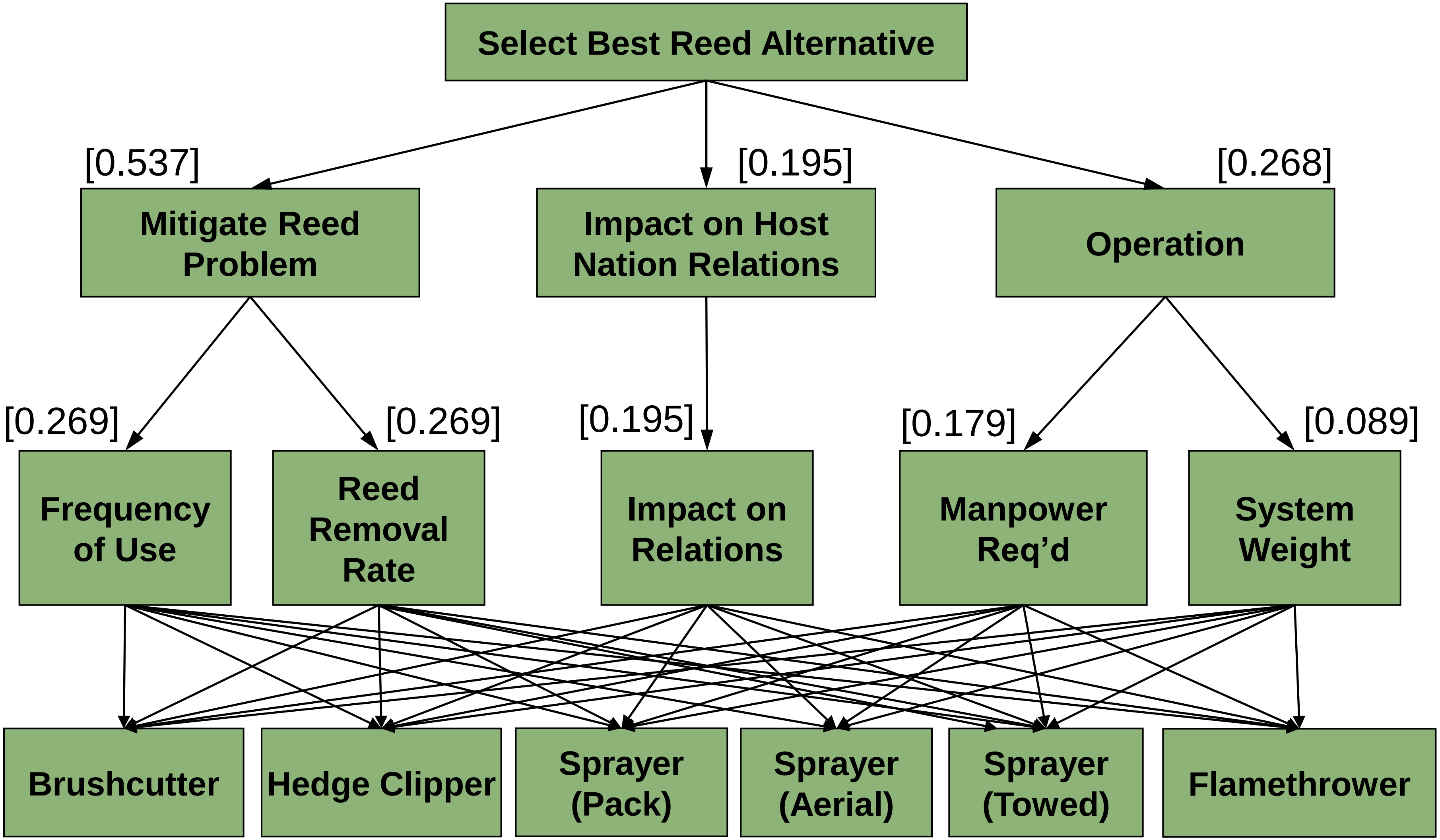
Composite 2nd Level Values Determined by Matrix Multiplication

To derive the composite values for the second hierarchical level, an EXCEL matrix multiplication, MMULT, between the attributes by objectives matrix (array 1) values and the vector of fundamental objective values (array 2) is performed. Composite values are determined to be: .269 for frequency importance, .269 for rate importance, .195 for relations importance, .179 for manpower importance, and .893 for weight importance.

Note in EXCEL that the entire resulting array cells must be highlighted, and then by hitting "Control-Shift-Enter" concurrently, the full range of matrix multiplication results are computed and displayed in the spreadsheet.

The next chart summarizes the results down to the attribute level and, thereafter, the following five charts display notional pair-wise comparison results for the alternatives which are evaluated via pair-wise comparisons with respect to the five attributes.

Housing Selection: Weights at 1st & 2nd Hierarchical Levels



Output for fundamental objectives and attributes of this AHP example

Comparative Judgment Question with respect to Frequency

With respect to the *frequency* attribute, which *alternative* in each of the following pairs possesses greater value and what is the intensity associated with the more important alternative?

			Equal		Moderate		Strong		Very Strong		Extreme
1	Brushcutter	Hedge Clipper	X								
2	Brushcutter	Backpack Spray		X							
3	Brushcutter	Towed Spray					X				
4	Brushcutter	Aerial Spray				X					
5	Brushcutter	Flamethrower		X							
6	Hedge Clipper	Backpack Spray		X							
7	Hedge Clipper	Towed Spray				X					
8	Hedge Clipper	Aerial Spray			X						
9	Hedge Clipper	Flamethrower		X							
10	Backpack Spray	Towed Spray		X							
11	Backpack Spray	Aerial Spray		X							
12	Backpack Spray	Flamethrower	X								
13	Towed Spray	Aerial Spray	X								
14	Towed Spray	Flamethrower		X							
15	Aerial Spray	Flamethrower		X							

Comparative Judgment Question with respect to Rate

With respect to the *rate* attribute, which *alternative* in each of the following pairs possesses greater value and what is the intensity associated with the more important alternative?

			Equal		Moderate		Strong		Very Strong		Extreme
1	Brushcutter	Hedge Clipper		X							
2	Brushcutter	Backpack Spray					X				
3	Brushcutter	Towed Spray								X	
4	Brushcutter	Aerial Spray									X
5	Brushcutter	Flamethrower			X						
6	Hedge Clipper	Backpack Spray			X						
7	Hedge Clipper	Towed Spray								X	
8	Hedge Clipper	Aerial Spray									X
9	Hedge Clipper	Flamethrower		X							
10	Backpack Spray	Towed Spray				X					
11	Backpack Spray	Aerial Spray		X							
12	Backpack Spray	Flamethrower		X							
13	Towed Spray	Aerial Spray		X							
14	Towed Spray	Flamethrower				X					
15	Aerial Spray	Flamethrower							X		

Comparative Judgment Question with respect to Relations

With respect to the *relations* attribute, which *alternative* in each of the following pairs possesses greater value and what is the intensity associated with the more important alternative?

			Equal		Moderate		Strong		Very Strong		Extreme
1	Brushcutter	Hedge Clipper	X								
2	Brushcutter	Backpack Spray		X							
3	Brushcutter	Towed Spray			X						
4	Brushcutter	Aerial Spray								X	
5	Brushcutter	Flamethrower					X				
6	Hedge Clipper	Backpack Spray		X							
7	Hedge Clipper	Towed Spray			X						
8	Hedge Clipper	Aerial Spray							X		
9	Hedge Clipper	Flamethrower				X					
10	Backpack Spray	Towed Spray		X							
11	Backpack Spray	Aerial Spray					X				
12	Backpack Spray	Flamethrower			X						
13	Towed Spray	Aerial Spray				X					
14	Towed Spray	Flamethrower		X							
15	Aerial Spray	Flamethrower				X					

Comparative Judgment Question with respect to Manpower

With respect to the *manpower* attribute, which *alternative* in each of the following pairs possesses greater value and what is the intensity associated with the more important alternative?

			Equal		Moderate		Strong		Very Strong		Extreme
1	Brushcutter	Hedge Clipper		X							
2	Brushcutter	Backpack Spray	X								
3	Brushcutter	Towed Spray		X							
4	Brushcutter	Aerial Spray							X		
5	Brushcutter	Flamethrower		X							
6	Hedge Clipper	Backpack Spray		X							
7	Hedge Clipper	Towed Spray		X							
8	Hedge Clipper	Aerial Spray						X			
9	Hedge Clipper	Flamethrower		X							
10	Backpack Spray	Towed Spray		X							
11	Backpack Spray	Aerial Spray			X						
12	Backpack Spray	Flamethrower		X							
13	Towed Spray	Aerial Spray		X							
14	Towed Spray	Flamethrower		X							
15	Aerial Spray	Flamethrower				X					

Comparative Judgment Question with respect to Weight

With respect to the *weight* attribute, which *alternative* in each of the following pairs possesses greater value and what is the intensity associated with the more important alternative?

			Equal		Moderate		Strong		Very Strong		Extreme
1	Brushcutter	Hedge Clipper	X								
2	Brushcutter	Backpack Spray			X						
3	Brushcutter	Towed Spray				X					
4	Brushcutter	Aerial Spray									X
5	Brushcutter	Flamethrower				X					
6	Hedge Clipper	Backpack Spray				X					
7	Hedge Clipper	Towed Spray				X					
8	Hedge Clipper	Aerial Spray									X
9	Hedge Clipper	Flamethrower			X						
10	Backpack Spray	Towed Spray					X				
11	Backpack Spray	Aerial Spray								X	
12	Backpack Spray	Flamethrower	X								
13	Towed Spray	Aerial Spray					X				
14	Towed Spray	Flamethrower						X			
15	Aerial Spray	Flamethrower									X

Comparative Judgment Results

Using the basic approximation technique, relative values are produced for each of the alternative relative to each of the attributes. This process assumes that "real data" on the alternatives is not available (which might be true of a future system or concept of operations).

For the frequency attribute, the respective weights for the brushcutter, hedge clipper, backpack sprayer, towed sprayer, aerial sprayer, and flamethrower are 0.070, 0.076, 0.145, 0.301, 0.277, and 0.131.

For the rate attribute, the respective weights for the brushcutter, hedge clipper, backpack sprayer, towed sprayer, aerial sprayer, and flamethrower are 0.031, 0.045, 0.142, 0.316, 0.401, and 0.065.

For the relations attribute, the respective weights for the brushcutter, hedge clipper, backpack sprayer, towed sprayer, aerial sprayer, and flamethrower are 0.310, 0.294, 0.175, 0.110, 0.032, and 0.079.

For the manpower attribute, the respective weights for the brushcutter, hedge clipper, backpack sprayer, towed sprayer, aerial sprayer, and flamethrower are 0.263, 0.231, 0.154, 0.094, 0.042, and 0.216.

For the weight attribute, the respective weights for the brushcutter, hedge clipper, backpack sprayer, towed sprayer, aerial sprayer, and flamethrower are 0.298, 0.298, 0.140, 0.061, 0.021, and 0.182.

Composite Alternative Valuing

	Frequency	Rate	Impact	Manpower	Weight
Brushcutter	**0.070**	**0.031**	**0.310**	**0.263**	**0.298**
Hedge Clipper	**0.076**	**0.045**	**0.294**	**0.231**	**0.298**
Backpack Spray	**0.145**	**0.142**	**0.175**	**0.154**	**0.140**
Towed Spray	**0.301**	**0.316**	**0.110**	**0.094**	**0.061**
Aerial Spray	**0.277**	**0.401**	**0.032**	**0.042**	**0.021**
Flame-thrower	**0.131**	**0.065**	**0.079**	**0.216**	**0.182**

*

Composite Result at Level 2	
0.269	Frequency
0.269	Rate
0.195	Impact
0.179	Manpower
0.089	Weight

=

Composite Result at Level 3	
0.161	Brushcutter
0.158	Hedge Clipper
0.151	Backpack Spray
0.209	Towed Spray
0.198	Aerial Spray

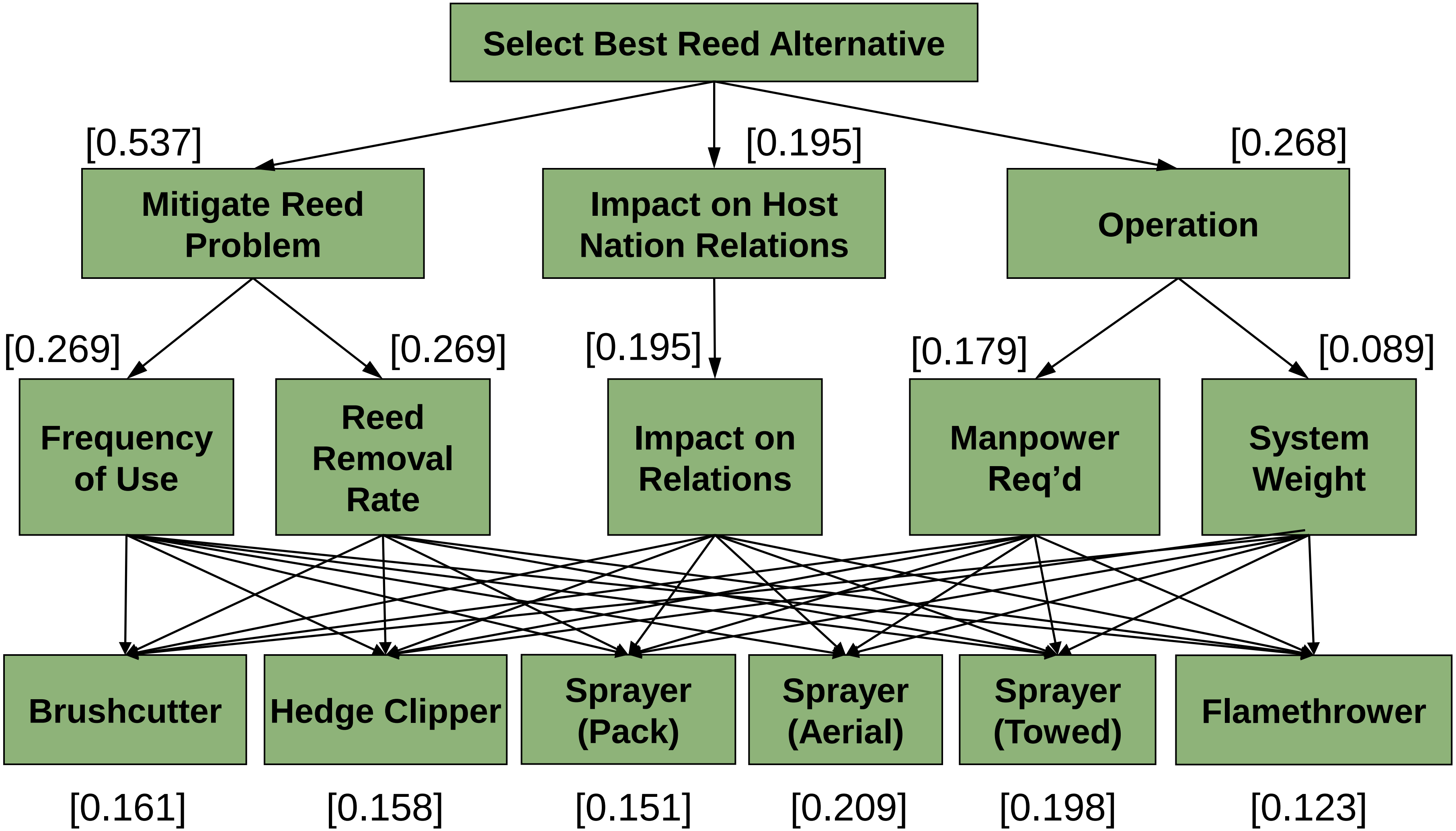
Composite Alternative Valuing

As before, from the paired comparison data, the alternative values relative to the attributes are computed using the basic approximation method, and then these values are organized into a properties by attributes matrix.

Subsequently, the composite alternative values vector is computed through matrix multiplication of the alternatives by attributes array by the vector of composite attribute values generated earlier. This results in a composite alternative values of: 0.161 for the brushcutter, 0.158 for hedge clipper, 0.151 for the backpack sprayer, 0.209 for the towed sprayer, 0.198 for the aerial sprayer, and 0.123 for the flamethrower. Again, note that these are properly considered relative values that sum to 1.000.

In AHP, Saaty promotes the illustrated matrix multiplication technique all the way down through the alternatives level. This actually results in alternatives of a MADM problem being evaluated using a relative ratio scale.

Reed Mitigation Selection: Weights at All Hierarchical Levels



Output for objectives, attributes, and alternatives of this AHP example

Reed Mitigation Selection: AHP Weights at All Hierarchical Levels

Visually, the preceding chart shows the complete AHP valuing process, which results in relative alternative values that sum to 1.000 at every level. And, in fact, these composite relative values might be used to imply the towed sprayer is favored over the flamerthrower by a ratio of about 1.7 (.209/.123 = 1.70).

Recall that back in the multi-attribute value analysis procedure, the alternatives were valued using single attribute value functions independently rather than being scored on a relative basis and then their values were multiplied by the weights associated with the attributes, which resulted in an overall value score on an interval scale.

Is this difference in procedures a cause for concern? Quite vocally, the traditional decision analytic community has responded that the AHP is flawed due to an argument that has been termed the "identical alternative conundrum".

See the following references for arguments and counter arguments: Dyer, J.S., "Remarks on the Analytic Hierarchy Process." <u>Management Science</u>, Vol. 36, No. 3, March 1990, pp. 249-258; Saaty, T.L., "An Exposition of the AHP in Reply to the Paper 'Remarks on the Analytic Hierarchy Process'." <u>Management Science</u>, Vol. 36, No. 3, March 1990, pp. 259-268; Harker, P.T. & L.G. Vargas, "Reply to 'Remarks on the Analytic Hierarchy Process' by J.S. Dyer." <u>Management Science</u>, Vol. 36, No. 3, March 1990, pp. 269-273; and Dyer, J.S., "A Clarification of 'Remarks on the Analytic Hierarchy Process.'" <u>Management Science</u>, Vol. 36, No. 3, March 1990, pp. 274-275.

Identical Alternative Conundrum

- **Consider the AHP & multi-attribute value analysis techniques if an identical alternative is added to the set of alternatives considered in a problem.**
  - **Under multi-attribute value analysis the new alternative receives the same value score as the alternative it is identical to and the other alternatives receive the same scores; even though now there are two tied alternative scores, no actual change in original rank ordering has occurred.**
  - **Under AHP final level composite weights for all alternatives must adjust since they must sum to 1.000 and, in fact, rank ordering of alternatives may change.**

Identical Alternative Conundrum (continued)

- **Two camps of decision theorists.**
  - **Those who think AHP fundamentally unsound (Dyer, Smith, von Winterfeldt).**
  - **Those who see usefulness in AHP (Saaty, Harker, Vargas, Foreman).**
- **Argument for AHP seems to lie in situation where scarce resources are at issue; idea is that if many alternatives possess same characteristics, those with scarce characteristics may be entitled to greater value.**
- **A good read for those wanting to understand this conundrum is: E. H. Forman, "Relative vs Absolute Worth," Mathematical Modeling, Vol. 9, No. 3-5, pp 195-202, 1987.**
- **Except when scarcity is major consideration, multi-attribute value analysis is preferred.**

Identical Alternative Conundrum

As indicated, the identical alternative conundrum has divided the decision science community into two camps -- one that thinks AHP is fundamentally unsound and those that see value in AHP.

The essential differences are discussed in Forman's "Relative versus Absolute Worth" paper. In this article, he argues that "economic value is dependent on scarcity and that scarcity by its very nature is a relative property." His premise is that relative worth is usually more appropriate in priority setting situations. He discusses the relative worth versus absolute worth approaches in terms of valuing basketball players and that, sometimes, counterintuitive results make sense.

Despite the arguments of Forman and other AHP supporters, this COBP generally supports the concept that if an identical alternative is added to an analysis, the outcome values assigned to all alternatives should be consistent with the values placed on the original alternatives. Thus, this COBP contends for almost all military analyses, worth as determined by multi-attribute value analysis is usually preferable to AHP and its relative worth concept. Only when scarcity is a significant factor of the analysis should a complete AHP analysis be considered.

AHP Conclusions

- Overall the AHP procedure is a questionable technique in its entirety.

- But AHP components can often times be useful as components in MADM analyses.
  - Pair-wise comparison technique for determining weights & priorities.
  - Technique for computing composite values between successive levels of the objectives hierarchy.

AHP Conclusions

Note that the multi-attribute value analysis and AHP procedures are pretty similar except at the lowest level of the objectives hierarchy where single attribute value functions are used in multi-attribute value analysis to evaluate alternatives versus as relative values determined via paired comparisons in the AHP procedure. As this COBP has shown, it is the relative valuing of the alternatives at the lowest level of the objectives hierarchy that makes the overall AHP questionable.

On the other hand, two of the components associated with AHP often can prove useful during the conduct of MADM analyses. The pair-wise comparison technique as devised by Saaty, along with its consistency index to measure within-person reliability, works well as a weighting procedure and could be used interchangeably with the direct estimation, swing weighing via the swing weight matrix, or the ROC weighting techniques as discussed for attribute weighting in multi-attribute value analysis. In fact, as AHP supporters have pointed out, the complexity placed on respondents may actually be reduced by using the pair-wise process, where only two characteristics are considered at a time versus processes where the multiple characteristics are considered concurrently.

Additionally, Saaty's AHP concept for using the local weights derived from the pair-wise data and then applying matrix multiplication to compute composite weights at successive hierarchical levels appears both appealing and legitimate. It provides a logical way to derive weights down and throughout the objectives hierarchy until all the attributes in a MADM analysis receive weights.

Thus, this COBP views the two components of AHP to be legitimate and often useful to incorporate in MADM analyses where attributes are to be weighted or prioritization is desired at multiple levels of the objectives hierarchy.

⚠ Cautions ⚠

- Think though the entire MADM analysis process before taking action; there are a number of relationships and data/participant requirements that require coordinated planning to ensure a smooth analysis.
- Be clear on the scope of the MADM analysis; work with the DM and his/her representatives and stakeholders to ensure analysis meets their needs.
- Do not conduct MADM analysis with the intention of justifying a preconceived alternative as the most favorable outcome.
- Qualitative modeling is a vital step since it provides the foundation for the quantitative analysis; take time to do it right.
- Use fundamental objectives as the basis for identifying attributes; do not define attributes based on a particular alternative's specifications.
- Make sure SMEs (and any DMs providing input) have appropriate expertise and competencies.
- Aim to instill reliability and validity during the decision analysis process.

Cautions

Throughout this COBP, key potential pitfalls are preceded with caution symbols. These represent those concepts in which the decision analytic team must take particular care and are summarized below.

It should be evident that each step in a MADM analysis builds on the previous step. To ensure a comprehensive analysis using appropriate techniques and participants, plan the the whole process before starting to take action. Don't begin a decision analysis without knowing the analytical route that will be undertaken and the study's desired destination.

MADM analyses can be challenging because the analytical team must conduct numerous tasks in response to a study's perceived scope. Seek clarity on the scope of the analysis expected by the DM and his/her representatives as well as any other major stakeholders. If expectations become problematic, immediately begin discussions with the principals to refine the expectations on a mutual basis.

Be sure to avoid efforts where MADM analysis are aimed at justifying a preconceived, favorite alternative. As such, be prepared to deal with those that are convinced there is a favorite alternative that the decision analysis should support.

The qualitative modeling must be well done. Take care to perform this step in an exceptional manner or the analysis may have to revisit this step or, ultimately, suffer from validity concerns as a result.

Make sure the analysis is objectives-based by dictating the attributes examined rather than being alternatives-based, where the alternatives drive the attributes that are considered in the analysis.

MADM analyses and supporting assessment techniques depend on inputs from SMEs and DMs. Make sure the SMEs and DMs have the right competencies in terms of experience and expertise to provide the informed judgments sought.

Remember that reliability in the form of consistency among responses is a necessary condition for validity. Hence, take all actions possible to enhance the reliability received from SMEs and other participants as they proceed to make their judgments and assessments.

Summary

- Section 1: Getting started
  - Purpose
  - Background
  - MADM basics
  - MADM definitions and conventions
  - Multi-attribute value analysis analysis with the illustrative problem
- Section 2: Qualitative modeling & formulating hierarchies
- Section 3: Measurement scales & quantification of value functions
- Section 4: Useful assessment techniques for determining weights and priorities
- Section 5: Group assessment considerations
- Section 6: The Analytic Hierarchy Process (AHP) and Its contributions
- Section 7: Cautions and Summary

Summary
(1 of 3)

This COBP has focused on the conduct of MADM and associated assessment techniques in support of military studies and analyses. After discussing the recent developments leading to the development of this document and basic MADM definitions and conventions, four general steps for addressing MADM situations were stated: (1) determine the basics of the decision making situation (DMs, SMEs, stakeholders, and fundamental objectives); (2) develop and construct a qualitative value model fitting the given situation; (3) develop and apply an appropriate quantitative model; and finally, (4) make analytically based recommendation(s) and discuss confidence concerning resulting recommendation(s).

After stating that this COBP would emphasize two MADM techniques, multi-attribute value analysis and the Analytic Hierarchy Process, attention turned to illustrating the basic steps and procedures for conducting a multi-attribute value analysis. A problem of "choosing the best reed mitigation solution" was used to demonstrate the multi-attribute value analysis process. Six steps were discussed and illustrated: (1) identifying stakeholders, (2) identifying fundamental objectives, (3) developing the qualitative model, (4) developing the quantitative value model, (5) determining the weighting factors for attributes and (6) generating results and conducting sensitivity analysis. While no prescriptive procedures are available to accomplish steps 1-3, this COBP attempted to emphasize that these steps should not be shortchanged in terms of time and effort, as the foundations of a decision analysis nearly always affect the adequacy of the entire analysis.

Subsequently, this COBP began focusing on explaining and illustrating the quantitative aspects of conducting a multi-attribute value analysis. The discussion commenced by pointing out the concept underlying multi-attribute value analysis is based on a compensatory additive value model, where favorable and unfavorable performance on attributes can be traded off among attributes. From there, the discussion evolved into one of determining single attribute values based on the "choose the best reed mitigation solution" example, so that the individual single attribute scores were converted to corresponding single attribute values on a common scale.

Summary
(2 of 3)

Thereafter, a process for allocating relative weights to the attributes was set forth for the example problem. The process of synthesizing the attribute values and attribute weighting factors into an overall estimates of value for each of the alternatives concluded this portion of the COBP along with a discussion of the need for sensitivity analysis. The overall aim of this section of the COBP was to discuss and illustrate a basic application of multi-attribute value analysis before further expanding on multi-attribute value analysis concepts in the subsequent sections.

Despite acknowledging that there are no prescriptive procedures for formulating objectives hierarchies and qualitative modeling in general, the subsequent COBP section attempted to relate some characteristics of good hierarchies, provide a couple of military examples, as well as set forth a basic framework for objectives and attributes hierarchies in support of multi-attribute value analysis steps 1-3.

The subsequent COBP section dealt with the principles of attribute scaling and the quantification associated with attribute valuing during multi-attribute value analysis step 4. Initially, the focus was on developing an understanding of nominal, ordinal, interval, and ratio scales. Thereafter, procedures and techniques for transforming ordinal, interval, and ratio scores into attribute values were discussed and illustrated. This section ended with a discussion of the need to instill measurement reliability and validity into the value measurement process.

Useful assessment techniques for determining attribute weights in support of multi-attribute value analysis step 5, as well as in support of individual prioritization efforts, were discussed in the next COBP section. After discussing ranking and sorting procedures, three separate assessment techniques were set forth and compared: direct assessment, swing weighting via a swing weight matrix, and the rank-ordered centroid procedure.

Next, this COBP addressed some issues that surround the use of groups during any assessment processes. Initially, this section discussed some principles for picking experts to provide inputs during studies and analyses. Thereafter, use of the Excel median function was described as a procedure for summarizing group rankings and, in the case of tied rankings, the computation of ROC weights was illustrated. Finally in this section the Delphi process was

Summary
(3 of 3)

described as a procedure for molding group viewpoints and as a tool for increasing reliability within group outputs.

Next, this COBP discussed the AHP as an alternative for conducting MADM-type analyses. After noting steps 1-3 of the AHP were the same as those for a multi-attribute value analysis, the differing quantitative components of AHP were discussed and illustrated: the pair-wise comparison procedure for determining local priorities among a set of elements, the composite valuing across multiple and successive levels of the objectives hierarchy via matrix multiplication in order to place weights on attributes, and the extension of this matrix multiplication concept to determine the relative values of the alternatives at the lowest hierarchical level. Thereafter, the perceived strengths and weakness with AHP and its components were discussed. After noting the merit associated with the pair-wise comparison process and composite valuing at successive levels of the objectives hierarchy, the identical alternative conundrum argument was presented concerning the prioritization and valuation of decision alternatives. This is AHP’s major fault from the viewpoint of traditional decision analysts.

In the last section, several overall caution areas were highlighted. Foremost among these was the need to think through the entire MADM analytic process before taking action. Other cautionary notes included developing a clear scope for the MADM analysis, avoiding any tendencies or desires to use MADM analysis to justify a preconceived or favored alternative, recognize qualitative modeling as a vital step and afford it the attention it deserves, deriving decision attributes from objectives and not from the alternatives, ensuring SMEs have appropriate expertise and competencies, and aiming for reliability and validity during the decision analysis process.

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