Generalizations
Part 1: Three-Way ANOVA

STAT 705: Regression and Analysis of Variance

Introduction

- There can be any number of factors in a designed experiment
- We have looked at one-factor and two-factor experiments
- We now turn our attention to three-factor experiments
- We still have an ANOVA table, F tests, t tests, main effects and interactions
- Look at the interactions first

Multiple Factor Studies

- Studies involving several factors are often focused on identifying how the factors work together to affect the outcome, as opposed to simply comparing the means of the combinations.
- Questions that are typically asked with ANOVA are:
  - What is the most important factor?
  - Can any factor be ignored?
  - Do any of the factors interact?

A Three Factor Example

- A study was done to determine factors that may affect the efficiency of a solar water heater.
- The factors are
  - the capacity of the water heater
  - the flow rate of the water through the system
  - the length of exposure of the solar collector to direct sunlight
- The experiment has two levels (denoted ‘low’ and ‘high’) for each of the factors, abbreviated as
  - Cap (low, high), Flo (low, high) and Exp (low, high)
- The response is Efficiency, denoted Eff

Solar Water Heater Data

Capacity	Flow	Exposure	Efficiency
high	high	high	41.6
high	high	high	41.3
high	high	low	39.9
high	high	low	39.7
high	low	high	51.9
high	low	high	52.4
high	low	low	43.0
high	low	low	44.9
low	high	high	39.2
low	high	high	38.4
low	high	low	37.5
low	high	low	35.0
low	low	high	50.2
low	low	high	51.3
low	low	low	41.3
low	low	low	43.5

There are two replicates for each combination of factors

Three-way Means

- There are two replicates for each combination of factors
- Average the two replicates to get the three-way means

Three-way Means			
Capacity	**Flow**	**Exposure**	**Mean**
high	high	high	41.45
high	high	low	39.80
high	low	high	52.15
high	low	low	43.95
low	high	high	38.80
low	high	low	36.25
low	low	high	50.75
low	low	low	42.40

Two-way Means

- These are the means for each of the combinations of the levels of two factors when averaged over the levels of the third factor
- Two-way means for Flow by Exposure are shown below

Three-way Means			
Capacity	**Flow**	**Exposure**	**Mean**
high	high	high	41.45
high	high	low	39.80
high	low	high	52.15
high	low	low	43.95
low	high	high	38.80
low	high	low	36.25
low	low	high	50.75
low	low	low	42.40

Two-way Means		
Flow	**Exposure**	**Mean**
high	high	40.125
high	low	38.025
low	high	51.450
low	low	43.175

For example . . .
for high Flow and high Exposure,
the two-way mean is
(41.45 + 38.80) / 2 = 40.125

One-way Means

- These are the means for the levels of one factor averaged over the levels of the other two
- One-way means for Cap are shown below

Three-way Means			
Capacity	**Flow**	**Exposure**	**Mean**
high	high	high	41.45
high	high	low	39.80
high	low	high	52.15
high	low	low	43.95
low	high	high	38.80
low	high	low	36.25
low	low	high	50.75
low	low	low	42.40

One-way Means	
Capacity	**Mean**
high	44.3375
low	42.0500

For example . . .
for high Capacity, the one-way mean is
(41.45 + 39.80 + 52.15 + 43.95) / 4
= 44.3375

Generic 3-Way ANOVA Table

- Denote the factors by A, B, C
- The ANOVA table contains sums of squares, degrees of freedom, mean squares, F-statistics and p-values for
  - A, B, C main effects
  - Two-way interactions: A*B, A*C, and B*C
  - Three interaction: A*B*C
  - Error

ANOVA for Solar Water Heater

- The terms with p-values 0.05 or less are the main effects of Cap, Flo and Exp, and the two-way interaction Flo*Exp.

Source	DF	SS	MS	F	p-value
cap	1	20.93	20.93	19.78	0.0021
flo	1	271.43	271.43	256.52	<.0001
cap*flo	1	2.64	2.64	2.50	0.1528
exp	1	107.64	107.64	101.73	<.0001
cap*exp	1	0.28	0.28	0.26	0.6236
flo*exp	1	38.13	38.13	36.04	0.0003
cap*flo*exp	1	0.14	0.14	0.13	0.7249
Error	8	8.47	1.06		
Total	**15**	**449.65**			

Interpreting the ANOVA Table

- Use the significant terms in the ANOVA table to direct your attention to the most important means to look at
  - If the 3-way interaction is significant, look at the 3-way means
  - If the 3-way interaction is not significant, look at the 2-way means for any factors that have significant 2-way interactions
  - If a factor is not involved in a significant 3-way or 2-way interaction but has a significant main effect, then look at the one-way means involving this factor

Example

- The water heater ANOVA table shows that Cap is not involved in any significant interactions and it has a significant main effect.
- The main effect means (below) show that the high capacity water heaters have greater efficiency.
- Because there is no interaction involving Capacity, we can say that high capacity water heaters have greater efficiency regardless of the setting for flow rate or length of exposure to direct sunlight.

One-way Means	
Capacity	**Mean**
high	44.3375
low	42.0500

Example, continued

- The factors Flo and Exp have a significant two-way interaction.
- We disregard their main effect means and go to the two-way means involving these factors.
- We see that low flow rate has higher efficiency regardless of the exposure, but the largest efficiency occurs when low flow rate is combined with a high exposure.

Two Way Means for Flo*Exp

Flo	Exp	Mean Eff
high	high	40.125
high	low	38.025
low	high	51.450
low	low	43.175

Example, continued

- We still have not determined which means are statistically significantly different from the others. A SAS analysis will show this.
- Regardless, we have able to figure out the essential features of the data by looking at the ANOVA table and the means that the ANOVA table tells us to look at.
- In summary
  - A large capacity heater is better.
  - A low flow rate is advantageous especially when combined with a solar collector that is exposed for a longer time to direct sunlight.

What You Should Know

- Be able to interpret a 3-way ANOVA table to decide which means (one-way, two-way, or three-way) are the appropriate ones to examine
- Be able to summarize your findings

The complete SAS code for the water heater data is available on the course website, along with its annotated output.