From Winston's Book

2 During each 4-hour period, the Smalltown police force requires the following number of on-duty police officers: 12 midnight to 4 A.M.—8; 4 to 8 A.M.—7; 8 A.M. to 12 noon—6; 12 noon to 4 P.M.—6; 4 to 8 P.M.—5; 8 P.M. to 12 midnight—4. Each police officer works two consecutive 4-hour shifts. Formulate an LP that can be used to minimize the number of police officers needed to meet Smalltown's daily requirements.

Group B

4 A company has nine projects under consideration. The NPV added by each project and the capital required by each project during the next two years is given in Table 9. All figures are in millions. For example, Project 1 will add $14 million in NPV and require expenditures of $12 million during year 1 and $3 million during year 2. Fifty million is available for projects during year 1 and $20 million is available during year 2. Assuming we may undertake a fraction of each project, how can we maximize NPV?

TABLE 9

Year 1 Outflow	12	54	6	6	30	6	48	36	18
Year 2 Outflow	3	7	6	2	35	6	4	3	3
NPV	14	17	17	15	40	12	14	10	12

8 You are a CFA (chartered financial analyst). Madonna has come to you because she needs help paying off her credit card bills. She owes the amounts on her credit cards shown in Table 43. Madonna is willing to allocate up to $5,000 per month to pay off these credit cards. All cards must be paid off within 36 months. Madonna's goal is to minimize the total of all her payments. To solve this problem, you must understand how interest on a loan works. To illustrate, suppose Madonna pays $5,000 on Saks during month 1. Then her Saks balance at the beginning of month 2 is

$$20{,}000 - (5{,}000 - .005(20{,}000))$$

This follows because during month 1 Madonna incurs .005(20,000) in interest charges on her Saks card. Help Madonna solve her problems!

TABLE 43

Saks Fifth Avenue	20,000	.5
Bloomingdale's	50,000	1
Macys	40,000	1.5

2 A company faces the following demands during the next three periods: period 1, 20 units; period 2, 10 units; period 3, 15 units. The unit production cost during each period is as follows: period 1—$13; period 2—$14; period 3—$15. A holding cost of $2 per unit is assessed against each period's ending inventory. At the beginning of period 1, the company has 5 units on hand.

In reality, not all goods produced during a month can be used to meet the current month's demand. To model this fact, we assume that only one half of the goods produced during a period can be used to meet the current period's demands. Formulate an LP to minimize the cost of meeting the demand for the next three periods. (*Hint:* Constraints such as $i_1 = x_1 + 5 - 20$ are certainly needed. Unlike our

2 O.J. Juice Company sells bags of oranges and cartons of orange juice. O.J. grades oranges on a scale of 1 (poor) to 10 (excellent). O.J. now has on hand 100,000 lb of grade 9 oranges and 120,000 lb of grade 6 oranges. The average quality of oranges sold in bags must be at least 7, and the average quality of the oranges used to produce orange juice must be at least 8. Each pound of oranges that is used for juice yields a revenue of $1.50 and incurs a variable cost (consisting of labor costs, variable overhead costs, inventory costs, and so on) of $1.05. Each pound of oranges sold in bags yields a revenue of 50¢ and incurs a variable cost of 20¢. Formulate an LP to help O.J. maximize profit.

Group A

1 Sunco Oil has three different processes that can be used to manufacture various types of gasoline. Each process involves blending oils in the company's catalytic cracker. Running process 1 for an hour costs $5 and requires 2 barrels of crude oil 1 and 3 barrels of crude oil 2. The output from running process 1 for an hour is 2 barrels of gas 1 and 1 barrel of gas 2. Running process 2 for an hour costs $4 and requires 1 barrel of crude 1 and 3 barrels of crude 2. The output from running process 2 for an hour is 3 barrels of gas 2. Running process 3 for an hour costs $1 and requires 2 barrels of crude 2 and 3 barrels of gas 2. The output from running process 3 for an hour is 2 barrels of gas 3. Each week, 200 barrels of crude 1, at $2/barrel, and 300 barrels of crude 2, at $3/barrel, may be purchased. All gas produced can be sold at the following per-barrel prices: gas 1, $9; gas 2, $10; gas 3, $24. Formulate an LP whose solution will maximize revenues less costs. Assume that only 100 hours of time on the catalytic cracker are available each week.

19† Brady Corporation produces cabinets. Each week, it requires 90,000 cu ft of processed lumber. The company may obtain lumber in two ways. First, it may purchase lumber from an outside supplier and then dry it in the supplier's kiln. Second, it may chop down logs on its own land, cut them into lumber at its sawmill, and finally dry the lumber in its own kiln. Brady can purchase grade 1 or grade 2 lumber. Grade 1 lumber costs $3 per cu ft and when dried yields 0.7 cu ft of useful lumber. Grade 2 lumber costs $7 per cubic foot and when dried yields 0.9 cu ft of useful lumber. It costs the company $3 to chop down a log. After being cut and dried, a log yields 0.8 cu ft of lumber. Brady incurs costs of $4 per cu ft of lumber dried. It costs $2.50 per cu ft of logs sent through the sawmill. Each week, the sawmill can process up to 35,000 cu ft of lumber. Each week, up to 40,000 cu ft of grade 1 lumber and up to 60,000 cu ft of grade 2 lumber can be purchased. Each week, 40 hours of time are available for drying lumber. The time it takes to dry 1 cu ft of grade 1 lumber, grade 2 lumber, or logs is as follows: grade 1—2 seconds; grade 2—0.8 second; log—1.3 seconds. Formulate an LP to help Brady minimize the weekly cost of meeting the demand for processed lumber.