Multiple Regression

Part 6: Influence and Outliers

STAT 705: Regression and Analysis of Variance

KANSAS STATE UNIVERSITY

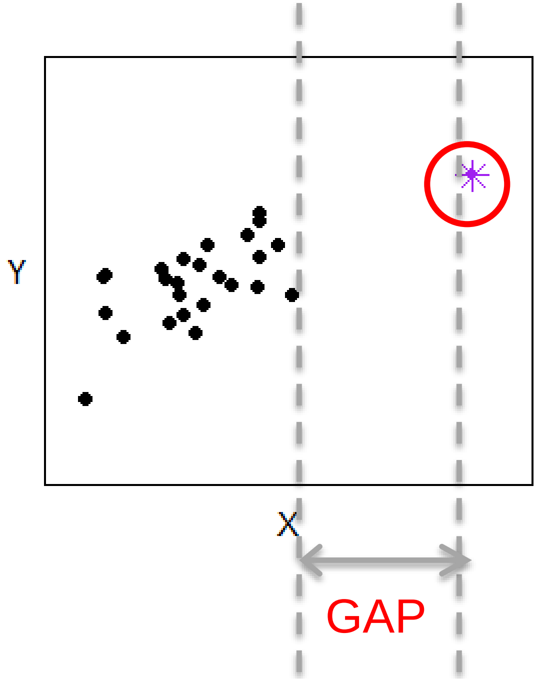
Extreme Observations

- An ‘observation’ is one row in the data set
    - Includes the measured Y and all the X’s for the subject
    - We call this a ‘data point’
- Ways in which an observation can be extreme
    - The combination of X’s may be unusual
    - The value of Y, given the X’s, may be unusual
    - Both the X’s and the Y may be unusual
- Classify rows according to how they affect the model
    - Leverage, Outlier, Influence

Leverage

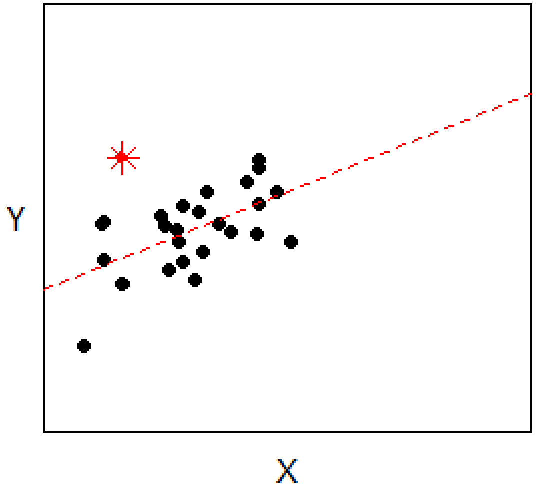
- A data point has high leverage if its combination of values for the X's is unusual in relation to all the other rows in the data

- If there is only one X variable, points with high leverage appear separated (to the right or the left) of the other points.
- The Y value is not used in calculating leverage
- The point may or may not seem to follow the least squares line



Outliers

- Are observations that have unusually large or small Y values, in relation to the values of the X's that are recorded for subject
- Measured by the residuals
  - Large positive or large negative
  - Ordinary residuals may have large variance (estimated by MSE)
  - 'Large' residuals are large *relative to the MSE*



Studentized Residuals

- By assumption, ε_i ~ N(0,σ^2)
- Error variance is estimated by MSE
- ‘Standardize’ the residuals
  - Subtract the mean and divide by standard deviation
  - These are not independent, so distribution is not known
- ‘Studentize’ the residuals
  - Divide the standardized residuals by square root of (1 - leverage)
  - These follow an approximate t distribution

Identifying Outliers

From normal distribution theory,

- Approximately 95% of observations should fall within 2 std. dev. of the mean
  - Observations that fall outside 2 std. dev. are potential outliers
- Approximately 99.7% of observations should fall with 3 std. dev. of the mean
  - Observations that fall outside 3 std. dev. are extreme outliers
- Studentized residuals have mean 0 and std. dev. 1
  - Potential outliers have studentized residual > 2 or < -2
  - Extreme outliers have studentized residual > 3 or < -3

Influence

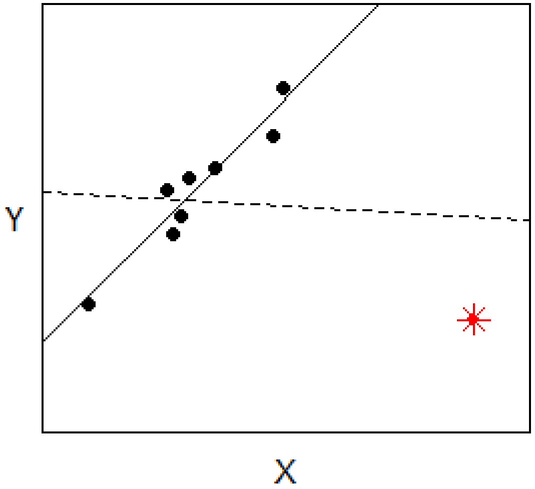
- The influence of a point is a measure of how much the fitted model would change if the point was removed from the data set
- The influence can be measured as change in
  - the fitted values
  - estimates for the individual coefficients
- A point can be influential because
  - it has high leverage
  - it is an outlier
  - both high leverage and outlier

Visualize Influence

The point marked with a red star is influential

If we exclude this point

- $Y = 1.17 + 1.27X$
- $R^2 = 91.3\%$
- RMSE = 0.92



If we include this point

- $Y = 14.36 - 0.07X$
- $R^2 = 0.7\%$
- RMSE = 3.55

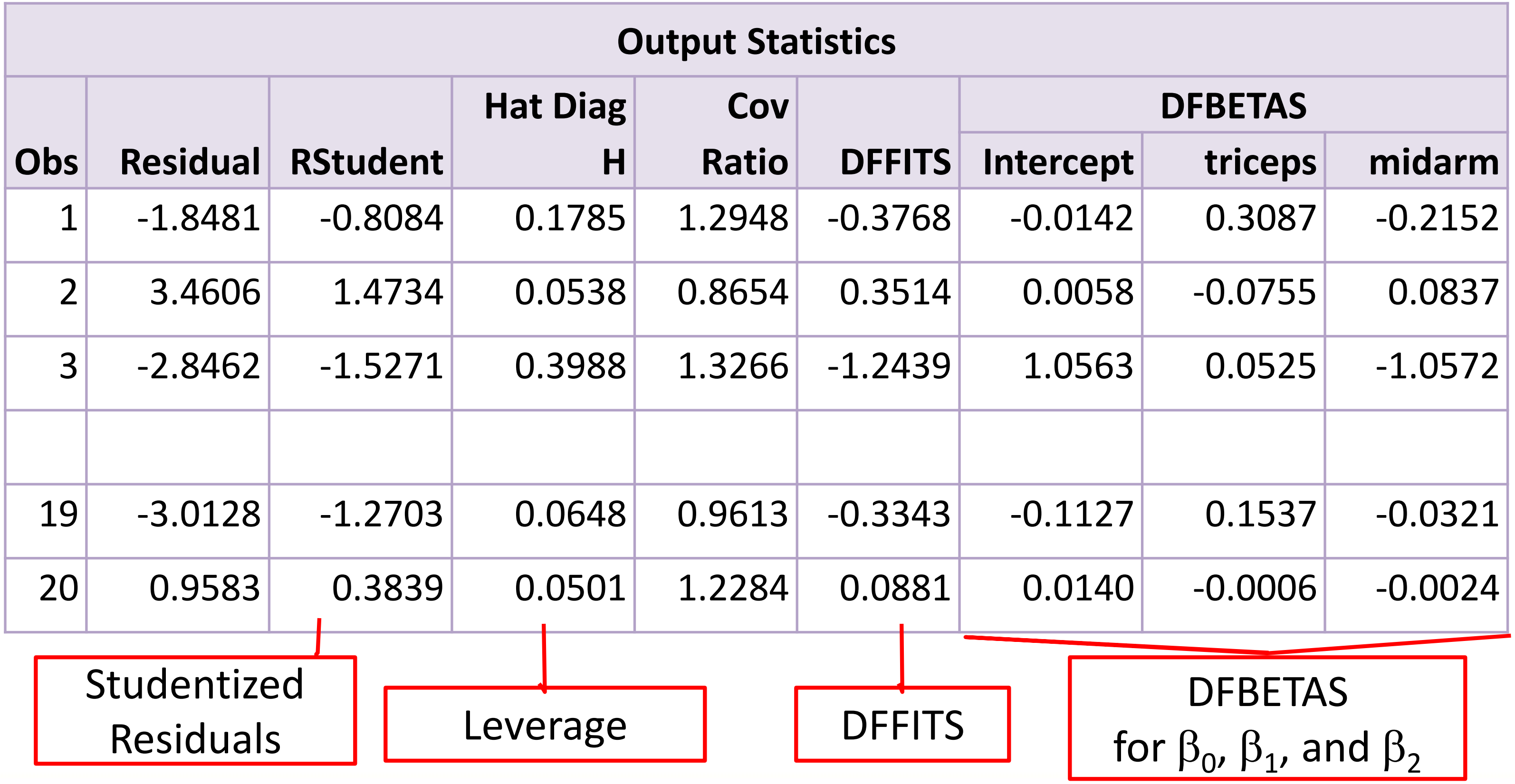
Identifying Influential Points

- Approach: “Leave one out”
  - Omit a single observation, re-fit the model and evaluate how inference changes
- Criteria for evaluating change in inference:
  - DFFITS - Influence on Single Fitted Values
  - Cook’s distance - Influence on all Fitted Values
  - DFBETAS – Influence on Regression Coefficients
- All of these can be generated in SAS

```
proc reg data=fat;
    model bodyfat = triceps midarm /influence press;
run;
```

Influence: SAS Output

This is the body fat data. The program is ‘Influence.BodyFat.sas’
This table is generated by the ‘influence’ option on the model statement.



Output Statistics

			Hat Diag	Cov		DFBETAS		
Obs	Residual	RStudent	H	Ratio	DFFITS	Intercept	triceps	midarm
1	-1.8481	-0.8084	0.1785	1.2948	-0.3768	-0.0142	0.3087	-0.2152
2	3.4606	1.4734	0.0538	0.8654	0.3514	0.0058	-0.0755	0.0837
3	-2.8462	-1.5271	0.3988	1.3266	-1.2439	1.0563	0.0525	-1.0572
19	-3.0128	-1.2703	0.0648	0.9613	-0.3343	-0.1127	0.1537	-0.0321
20	0.9583	0.3839	0.0501	1.2284	0.0881	0.0140	-0.0006	-0.0024

Studentized Residuals

Leverage

DFFITS

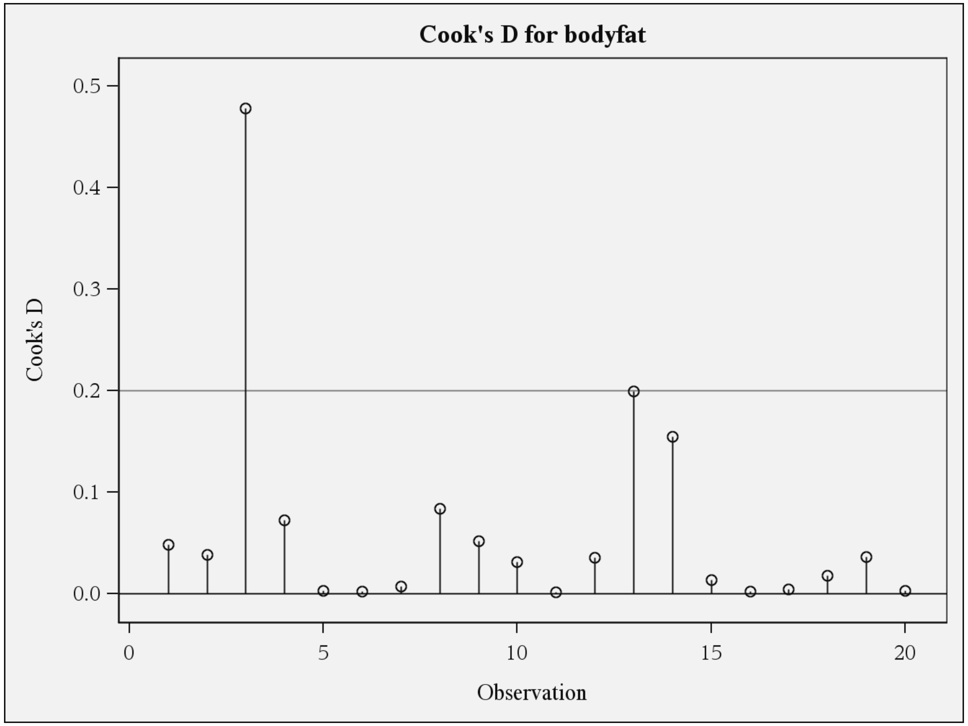
DFBETAS for β_0, β_1, and β_2

Evaluate the Output

- Studentized residuals
  - Values > 3 or < -3
  - Unusual Y value for the observed X’s
- Leverage
  - Values $> \frac{2}{n} \cdot (\#\,\text{parameters})$
  - Unusual combination of X values
- DFBETAS
  - Values $> \frac{2}{\sqrt{n}}$
  - Observation influences the specific parameter estimate

Cook’s D

- Results for Cook’s distance are shown in a graph
- Taller points above the horizontal line are more influential
- Before analyzing these data, we should make sure there are no errors in observation 3



What You Should Know

- How to generate influence measures in SAS
- Identify influential observations

Notes:

- These techniques simply identify unusual points in the data.
- We do NOT automatically remove influential points.
- First make sure there are no errors in the data.
- Then determine if there are any unusual conditions under which the observation was collected (that might explain why this point is different).
- Ultimately, it is the experience and knowledge of the researcher that dictates whether to keep or remove a data point.