4.5. 3.

z	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
1	-2	1	-1	0	0	0	0	
0	3	1	1	1	0	0	60	20
0	1	-1	2	0	1	0	10	10* Enter x_1 in row 2
0	1	1	-1	0	0	1	20	20

1	0	-1	3	0	2	0	20	
0	0	4	-5	1	-3	0	30	15/2
0	1	-1	2	0	1	0	10	None
0	0	2	-3	0	-1	1	10	5* Enter x_2 in row 3

z	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
1	0	0	3/2	0	3/2	1/2	25	
0	0	0	1	1	-1	-2	10	
0	1	0	1/2	0	1/2	1/2	15	
0	0	1	-3/2	0	-1/2	1/2	5	

This is an optimal tableau with optimal solution z = 25, s_1 = 10, x_1 = 15, x_2 = 5, s_2 = s_3 = 0.

4.5.5. Initial Tableau

z	X1	X2	S1	S2	S3	RHS
1	-1	-1	0	0	0	0
0	4	1	1	0	0	100
0	1	1	0	1	0	80
0	1	0	0	0	1	40

We could choose to enter either X1 or X2 into the basis. We arbitrarily choose to enter X2 into basis. Row 2 is the pivot row yielding the following (optimal) tableau.

z	X1	X2	S1	S2	S3	RHS
1	0	0	0	1	0	80
0	3	0	1	-1	0	20
0	1	1	0	1	0	80
0	1	0	0	0	1	40

Optimal solution is z = 80, x1=0, x2=80, s1=s2=0, s3=40.

6.

z	X1	X2	X3	S1	S2	S3	RHS
1	-1	-1	-1	0	0	0	0
0	1	2	2	1	0	0	20
0	2	1	2	0	1	0	20
0	2	2	1	0	0	1	20

We arbitrarily choose X1 to enter basis. Then we arbitrarily choose X1 to enter the basis in ROW 3. The resulting tableau follows:

Z	X1	X2	X3	S1	S2	S3	RHS
1	0	0	-.5	0	0	.5	10
0	0	1	1.5	1	0	-.5	10
0	0	-1	1	0	1	-1	0
0	1	1	.5	0	0	.5	10

Now X3 enters basis in Row 2. The resulting tableau follows:

Z	X1	X2	X3	S1	S2	S3	RHS
1	0	-.5	0	0	.5	0	10
0	0	2.5	0	1	-1.5	1	10
0	0	-1	1	0	1	-1	0
0	1	1.5	0	0	-.5	1	10

X2 enters basis in ROW 1 yielding the following optimal tableau:

Z	X1	X2	X3	S1	S2	S3	RHS
1	0	0	0	.2	.2	.2	12
0	0	1	0	.4	-.6	.4	4
0	0	0	1	.4	.4	-.6	4
0	1	0	0	-.6	.4	.4	4

The optimal solution to the LP is Z=12, X1=X2=X3=4.

4.6.1

1.

z	x_1	x_2	s_1	s_2	s_3	RHS	Ratio	
1	-4	1	0	0	0	0		
0	2	1	1	0	0	8	8	
0	0	1	0	1	0	5	5*	Enter x_2 in row 2
0	1	-1	0	0	1	4	None	

1	-4	0	0	-1	0	-5
0	2	0	1	-1	0	3
0	0	1	0	1	0	5
0	1	0	0	1	1	9

The current tableau is optimal because each variable has a non-positive coefficient in the current tableau. Thus the optimal solution to the LP is z = -5, s_1 = 3, x_2 = 5, s_3 = 9,

x_1 = s_2 = 0. Observe that the optimal objective function value for an LP can be negative.

4.7.2

2.

z	x_1	x_2	s_1	s_2	RHS	Ratio
1	3	-6	0	0	0	
0	5	7	1	0	35	5
0	-1	2	0	1	2	1* Enter x_2 in row 2

1	0	0	0	3	6
0	17/2	0	1	-7/2	28
0	-1/2	1	0	1/2	1

This is an optimal tableau with optimal solution z = 6, s_1 = 28, x_2 = 1, s_2 = x_1 = 0. Since the non-basic variable x_1 has a zero coefficient in Row 0 we can enter x_1 into the basis to obtain the alternative optimal solution z = 6, x_1 = 56/17, x_2 = 45/17. By averaging these two optimal solutions, a third optimal solution may be obtained. This yields the optimal solution z = 6, x_1 = 28/17, x_2 = 31/17.

4.8.1

1.

z	x_1	x_2	s_1	s_2	RHS	Ratio
1	0	-2	0	0	0	
0	1	-1	1	0	4	None
0	-1	1	0	1	1	1* Enter x_2 in row 2

z	x_1	x_2	s_1	s_2	RHS
1	-2	0	0	2	2
0	0	0	1	1	5
0	-1	1	0	1	1

Since x_1 has a negative coefficient in Row 0 and a non-positive coefficient in each constraint, we have an unbounded LP. From the final tableau we find that (holding $s_2 = 0$)

$z = 2 + 2x_1$
$s_1 = 5$
$x_2 = 1 + x_1$
$s_2 = 0$

Thus if $2 + 2x_1 = 10,000$ or $x_1 = 4,999$ we can find a point in the feasible region with $z = 10,000$.

Thus $z = 10,000$, $s_1 = 5$, $x_1 = 4,999$, $x_2 = 5,000$, $s_2 = 0$ is a point in the feasible region having $z>=10,000$.

4.12.3

3. Note that since $x_1 + x_2 = 3$, the constraint $x_1 + x_2>=3$ is automatically satisfied so we may omit this constraint from consideration. Then we wish to

max $z = 3x_1 + x_2 - Ma_2$
s.t. $2x_1 + x_2 + s_1 = 4$
$x_1 + x_2 + a_2 = 3$

Eliminating a_2 from $z - 3x_1 - x_2 + Ma_2$ we obtain
$z -(M + 3)x_1 - (M + 1)x_2 = -3M$. Proceeding with the simplex we obtain

z	x_1	x_2	s_1	a_2	RHS
1	-M-3	-M-1	0	0	-3M
0	2	1	1	0	4
0	1	1	0	1	3

1	0	(1-M)/2	(M + 3)/2	0	6-M
0	1	1/2	1/2	0	2
0	0	1/2	-1/2	1	1
z	x_1	x_2	s_1	a_2	RHS

1	0	0	2	M-1	5
0	1	0	1	-1	1
0	0	1	-1	2	2

This is an optimal tableau with optimal solution z = 5, x_1 = 1 x_2 = 2, s_1 = 0.