

3.  $x_1$  = Number of hours of Process 1 and  $x_2$  = Number of hours of Process 2. Then the appropriate LP is

$$\min z = 4x_1 + x_2$$

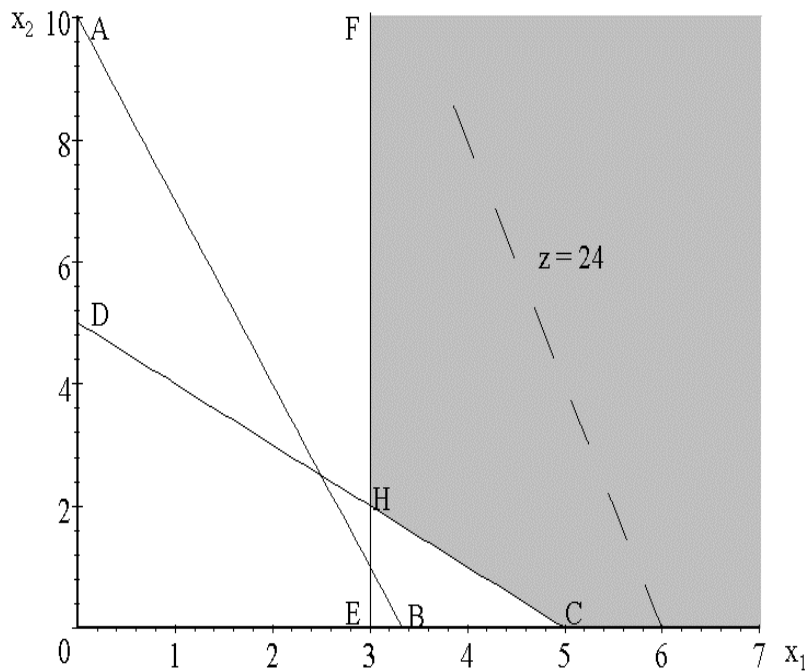
s.t.  $3x_1 + x_2 \geq 10$  (A constraint)

$x_1 + x_2 \geq 5$  (B constraint)

$x_1 \geq 3$  (C constraint)

$x_1, x_2 \geq 0$

AB is  $3x_1 + x_2 = 10$ . CD is  $x_1 + x_2 = 5$ . EF is  $x_1 = 3$ . The feasible region is shaded. Dotted line is isocost line  $4x_1 + x_2 = 24$ . Moving isocost line down to left we see that H (where B and C constraints intersect) is optimal. Thus optimal solution to LP is  $x_1 = 3$ ,  $x_2 = 2$ ,  $z = 4(3) + 2 = \$14$ .



5. Let  $x_1$  = desks produced,  $x_2$  = chairs produced. LP formulation is

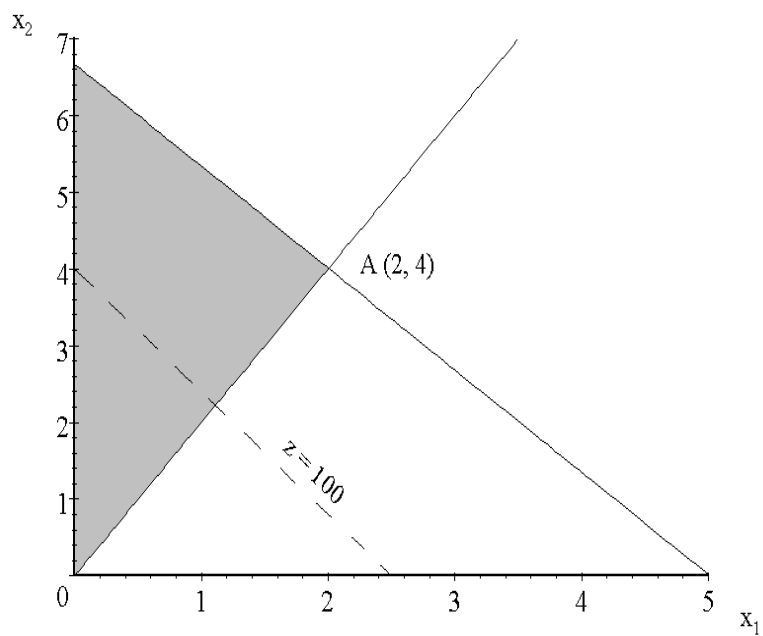
$$\max z = 40x_1 + 25x_2$$

s.t.  $-2x_1 + x_2 \geq 0$

$4x_1 + 3x_2 \leq 20$

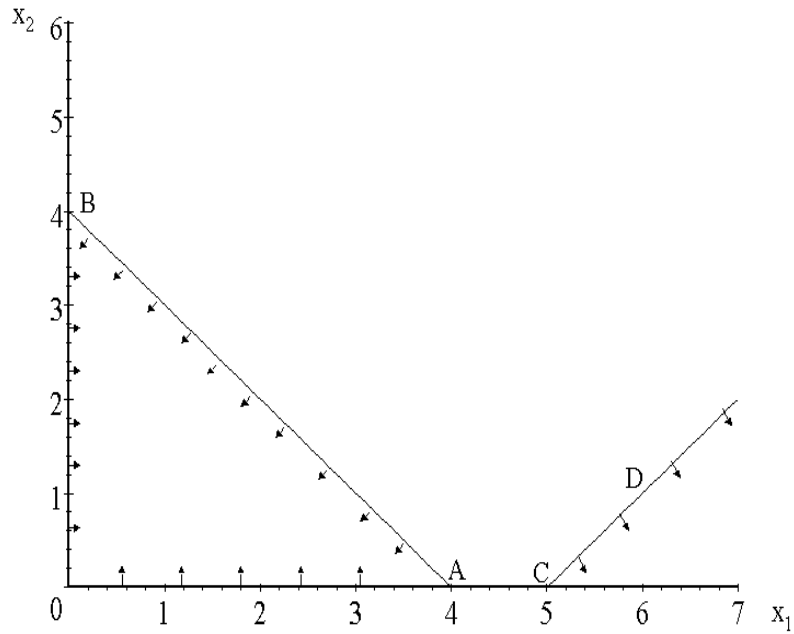
$x_1, x_2 \geq 0$

Graphically we find the optimal solution to be  $x_1 = 2$ ,  $x_2 = 4$  and  $z = 180$ .

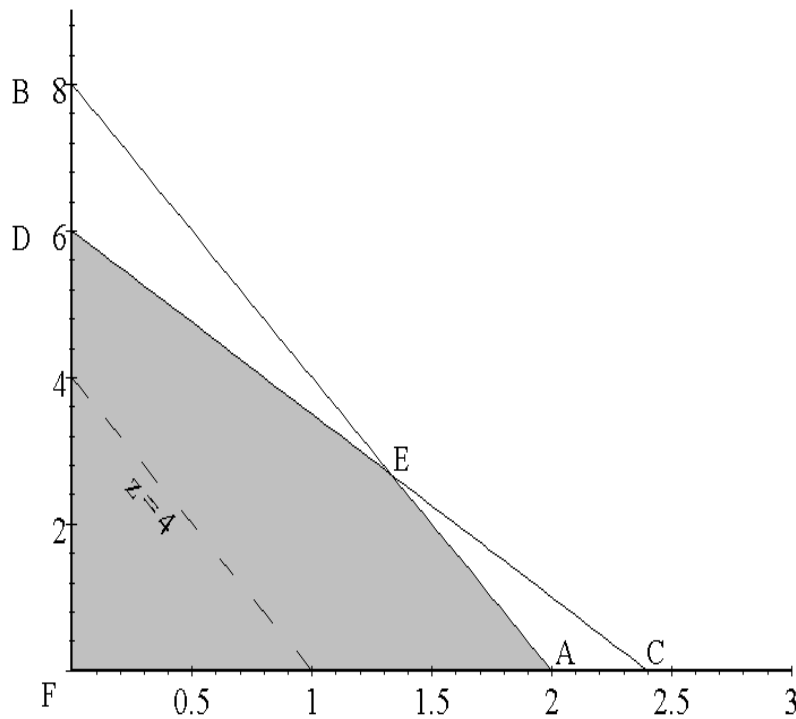


### Section 3.3

1. AB is  $x_1 + x_2 = 4$ . CD is  $x_1 - x_2 = 5$ . From graph we see that there is no feasible solution (Case 3).

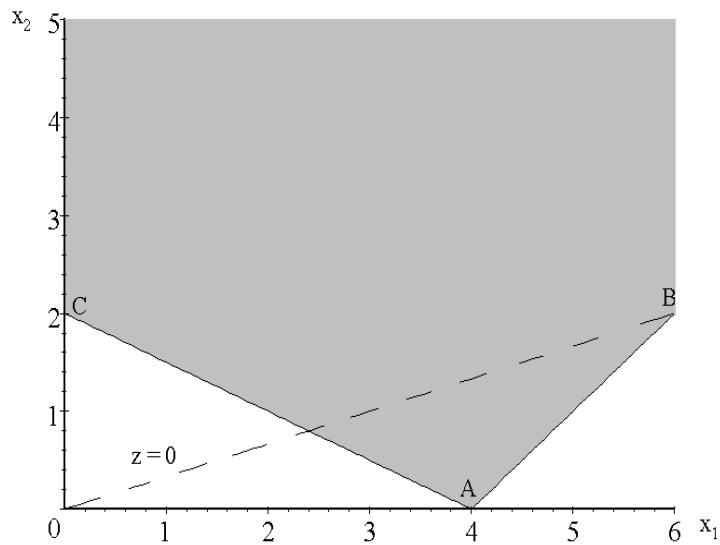


2. AB is  $8x_1 + 2x_2 = 16$ . CD is  $5x_1 + 2x_2 = 12$ . Dotted line



is  $z = 4x_1 + x_2 = 4$ . Feasible region is bounded by AEDF. Since isoprofit line is parallel to AE, entire line segment AE is optimal. Thus we have alternative or multiple optimal solutions.

3. AB is  $x_1 - x_2 = 4$ . AC is  $x_1 + 2x_2 = 4$ . Feasible region is bounded by AC and infinite line segment AB. Dotted line is isoprofit line  $z = 0$ . To increase  $z$  we move parallel to isoprofit line in an upward and 'leftward' direction. We will never entirely lose contact with the feasible region, so we have an unbounded LP (Case 4).



4. AB is  $2x_1 + x_2 = 6$ . CD is  $x_1 + 3x_2 = 9$ . The feasible region is bounded by AECF. Dotted line is  $3 = 3x_1 + x_2$ . Moving up and to right (and parallel to isoprofit line) we find that point A is optimal. A is where constraints  $2x_1 + x_2 \leq 6$  and  $x_2 \geq 0$  are binding.

Thus E has  $2x_1 + x_2 = 6$  and  $x_2 = 0$ . Optimal solution to the LP is  $x_1 = 3$ ,  $x_2 = 0$ ,  $z = 9$ .

