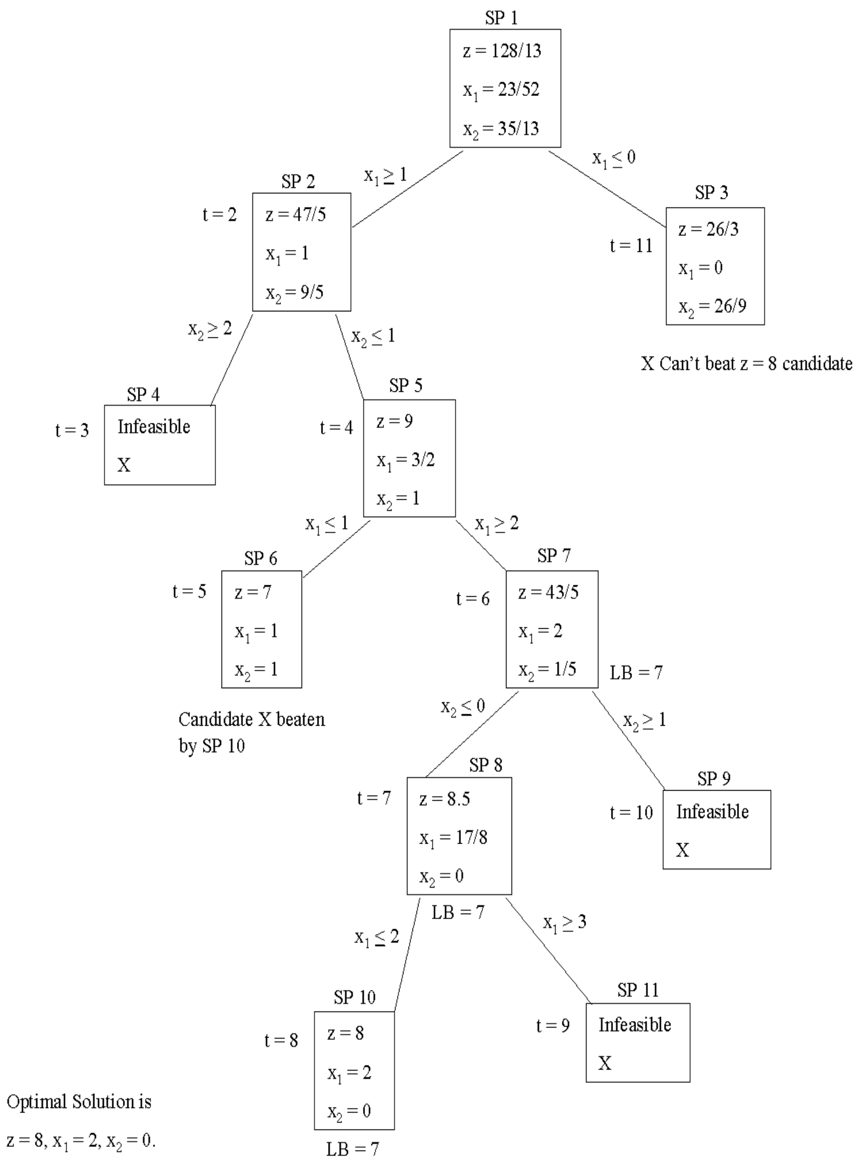
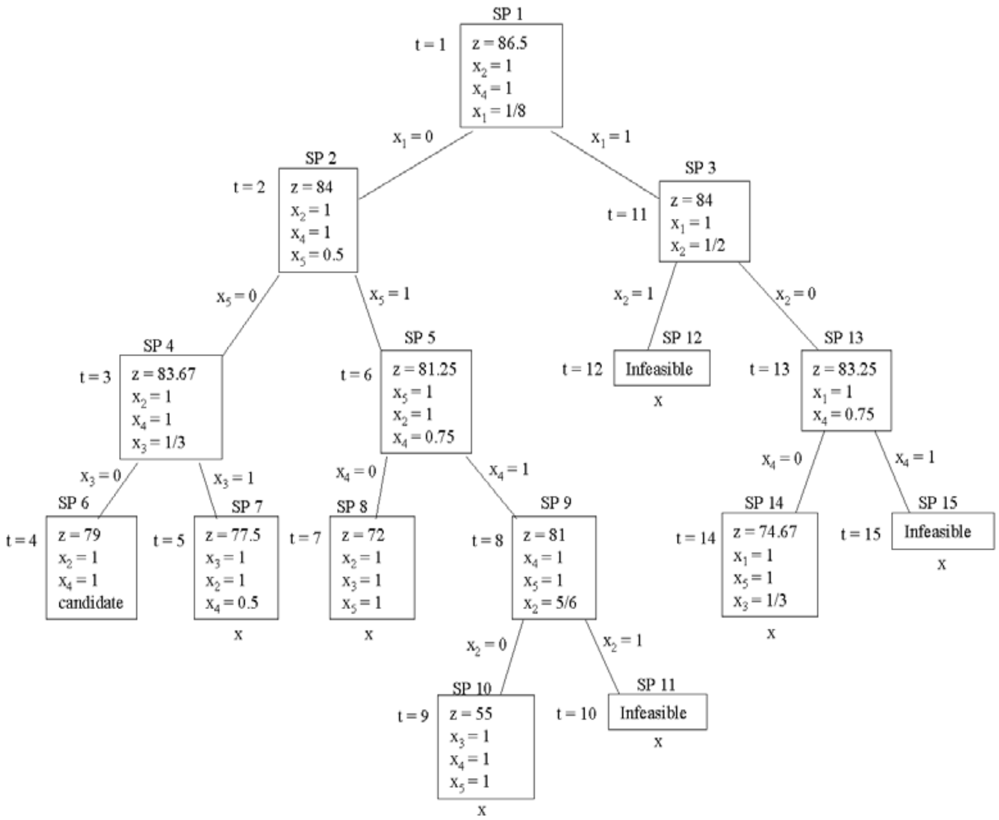
Integer Programming Homework

Do problems in Section 9.3 # 4
Do problems in Section 9.5 # 2, 3
Model problem in Section 9.2 # 16, 21, 31, 35

9.3.4.



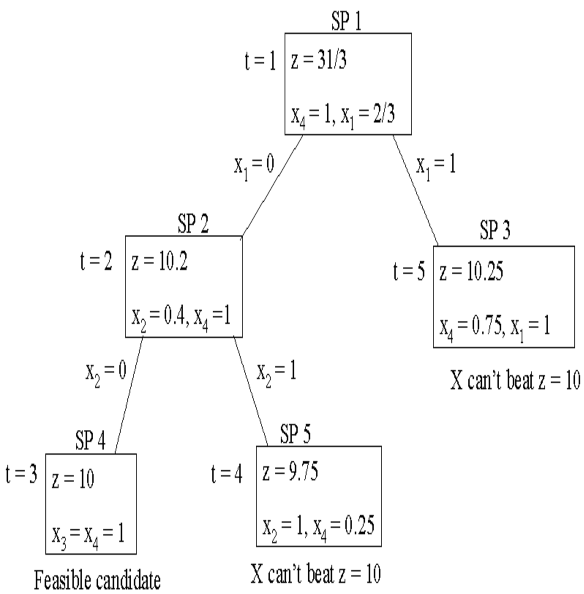
9.5 2



3. Letting $x_i = 1$ if item i is chosen and $x_i = 0$ otherwise yields the following knapsack problem:

$$\max z = 5x_1 + 8x_2 + 3x_3 + 7x_4$$
$$\text{st } 3x_1 + 5x_2 + 2x_3 + 4x_4 \le 6$$
$$x_i = 0 \text{ or } 1$$

We obtain the following tree (for each subproblem any omitted variable equals 0):



9.2.16

16. Let H = homes built and A = apartments built. Also let $y_1 = 1$ if marina is built, $y_1 = 0$ otherwise and $y_2 = 1$ if tennis court is built, $y_2 = 0$, otherwise. Then a correct IP formulation is

(objective function is in thousands of dollars of NPV)

max $z = 48A + 46H - 40A - 40H - 1200y_1 - 2800y_2$
st $A + H \le 10{,}000$, $y_1 + y_2 = 1$
$3A - H \le 30{,}000y_3$, $y_1 \le 30{,}000(1 - y_3)$
A, H non-negative integers and $y_i = 0$ or 1
We chose M = 30,000 because $3A - H \le 30{,}000$ will always hold.

9.2.21

21. Let xij = number of air conditioners (in thousands) produced in city i for region j (i = 1 is NY, j = 1 is East, etc.). Also let yj = 1 if factory is operated in city j, yj = 0 otherwise. Then the appropriate IP is(z is in thousands of dollars)
min z = 6000y1 + 5500y2 + 5800y3 +6200y4 + 206x11 + 225x12 + 230x13 +290x14 + 225x21 + 206x22 + 221x23 + 270x24 + 230x31 + 221x32 + 208x33 + 262x34 + 290x41 + 270x42 + 262x43 + 215x44
st x11 + x21 + x31 + x41≥100 (East)
x12 + x22 + x32 + x42≥150 (South)
x13 + x23 + x33 + x43≥110 (Midwest)
x14 + x24 + x34 + x44≥90 (West)
x11 + x12 + x13 + x14≤150y1 (NY)
x21 + x22 + x23 + x24≤150y2 (Atl)
x31 + x32 + x33 + x34≤150y3 (Chic)
x41 + x42 + x43 + x44≤150y4 (LA)
(Either x13≥50 or x23≥50)
50 - x13≤50y
50 - x23≤50(1 - y)
All xij integer; y, all yi = 0 or 1

9.2.31

31. Let $x_{ic} = 1$ if product i is assigned to compartment c, $x_{ic} = 0$ otherwise. e_i = shortage of product i (1 = super 2 = regular, 3 = unleaded), g_{ic} = gallons of product i in compartment c.

$$\min z = 10e_1 + 8e_2 + 6e_3$$

st
$$g_{11} + g_{21} + g_{31} \le 2700$$
$$g_{12} + g_{22} + g_{32} \le 2800$$
$$g_{13} + g_{23} + g_{33} \le 1100$$
$$g_{14} + g_{24} + g_{34} \le 1800$$
$$g_{15} + g_{25} + g_{35} \le 3400$$
$$g_{11} + g_{12} + g_{13} + g_{14} + g_{15} + e_1 - f_1 = 2900$$
$$g_{21} + g_{22} + g_{23} + g_{24} + g_{25} + e_2 - f_2 = 4000$$
$$g_{31} + g_{32} + g_{33} + g_{34} + g_{35} + e_3 - f_3 = 4900$$
$$g_{i1} \le 2700x_{i1},\ g_{i2} \le 2800x_{i2},\ g_{i3} \le 1100x_{i3},\ g_{i4} \le 1800x_{i4}$$
$$g_{i5} \le 3400x_{i5}\ (i = 1,2,3)$$
$$x_{1j} + x_{2j} + x_{3j} \le 1\ (j = 1, 2, 3, 4, 5)$$
All $x_{ij} = 0$ or 1 All other variables ≥ 0
$$e_1 \le 500, e_2 \le 500, e_3 \le 500$$

9.2.35

38. Let $X_{ij} = 1$ if size i box is used to meet demand for type i,i+1,...j boxes. Let $Y_i = 1$ if type i box is used at all and $Y_i = 0$ if type i box is not used. Answer is on LINDO printout

Section 9.2 Problem 38 Printout

```
MIN     13200 X11 + 9900 X12 + 16500 X13 + 23100 X14 + 6600 X15
      + 13200 X16 + 6600 X17 + 9000 X22 + 15000 X23 + 21000 X24 + 6000 X25
      + 12000 X26 + 6000 X27 + 13000 X33 + 18200 X34 + 5200 X35 + 10400 X36
      + 5200 X37 + 16800 X44 + 4800 X45 + 9600 X46 + 4800 X47 + 3800 X55
      + 7600 X56 + 3800 X57 + 7200 X66 + 3600 X67 + 3400 X77 + 1000 Y1
      + 1000 Y2 + 1000 Y3 + 1000 Y4 + 1000 Y5 + 1000 Y6 + 1000 Y7
 SUBJECT TO
        2)   X11 =    1
        3)   X12 + X22 =    1
        4)   X13 + X23 + X33 =    1
        5)   X14 + X24 + X34 + X44 =    1
        6)   X15 + X25 + X35 + X45 + X55 =    1
        7)   X16 + X26 + X36 + X46 + X56 + X66 =    1
        8)   X17 + X27 + X37 + X47 + X57 + X67 + X77 =    1
        9)   X11 - Y1 <=   0
       10)   X12 - Y1 <=   0
       11)   X13 - Y1 <=   0
       12)   X14 - Y1 <=   0
       13)   X15 - Y1 <=   0
       14)   X16 - Y1 <=   0
       15)   X17 - Y1 <=   0
       16)   X22 - Y2 <=   0
       17)   X23 - Y2 <=   0
       18)   X24 - Y2 <=   0
```

```
     19)   X25 - Y2 <=   0
     20)   X26 - Y2 <=   0
     21)   X27 - Y2 <=   0
     22)   X33 - Y3 <=   0
     23)   X34 - Y3 <=   0
     24)   X35 - Y3 <=   0
     25)   X36 - Y3 <=   0
     26)   X37 - Y3 <=   0
     27)   X44 - Y4 <=   0
     28)   X45 - Y4 <=   0
     29)   X46 - Y4 <=   0
     30)   X47 - Y4 <=   0
     31)   X55 - Y5 <=   0
     32)   X56 - Y5 <=   0
     33)   X57 - Y5 <=   0
     34)   X66 - Y6 <=   0
     35)   X67 - Y6 <=   0
     36)   X77 - Y7 <=   0
 END
```