Homework 6 Problems with solutions

**8 .1 a Show that x1+2x2+4x3≤4, x≥0 is an integral polytope. B. Show it is not TDI. C.**

**Convert it to a TDI.**

**Determine whether or not the matrices in 8.4 are TUM.**

**A**

**B**

**C**

**D**

**8.5 Show this matrix isn’t TUM but it has integer solutions if b is integer.**

**Solutions**

**8 .1 a Show that x1+2x2+4x3≤4, x≥0 is an integral polytope. B. Show it is not TDI. C.**

**Convert it to a TDI.**

8.1 A

From LP theory, at most one variable will be will be basic. Thus, the optimal solution will be solving either that x1=4, 2x2=4, 4x3=4, or s1=4. Clearly each such point will be integer.

8.1b

The dual of this problem with a cost function of (1,3,5) is

min 4y

s.t. y≥1

2y≥3

4y≥5

Clearly the answer is y=1.5, which isn’t integral.

8.1c

Show that 1/4x1+1/2x2+x3≤1, x≥0 is TDI. For any integers c1 c2 c3  the following polytope is clearly integral.

min 4y

s.t. 1/4y≥c1

1/2y≥c2

1y≥c3

**Determine whether or not the matrices in 8.4 are TUM.**

**Here they are. A**

**B**

**C**

**D**

To show the A is TUM, Let us examine any subset of rows Q, We will divide the rows as in class by alternating between Q1 and Q2.

If |Q|=2. there can be no 1 –1

1 1 matrix or variation of it, so all determinants are 0,1 or –1.

If |Q| =3, and there are three consecutive rows (this may allow for looping), then Q1 can equal all three rows and the theorem is satisfied. If they aren’t consecutive, then there exists 2 columns with with a –1 and a 1 and they can both be in Q1, the other row is in Q2, which satisfies the theorem. single 1.

If |Q|=4, then you can assume the first row is deleted. Then Q1 for the next two rows and Q2 for the final two rows satisfies the theorem.

If |Q|= 5, then letting all of the rows be in Q1 and the result is complete.

Thus the matrix is TUM.

To show B is TUM, again examine any set of Q rows.

If |Q|=2, then divide these into a Q1 and Q2. Clearly since there are at most 2 ones in each column, the theorem is satisfied.

If |Q|=3 and the first row is in Q, then select Q1 as the first row and the other two get the Q2 and this clearly satisfies the theorem. If the first row isn’t selected, then you can just alternate the Q1, Q2 and Q1 and it is also true.

If |Q|=4, then if the top row isn’t included, then you can alternate the Q1, Q2, Q1 and Q2 and it is true. If the top row is in Q, then two rows will have exactly 2 1s in them. Both of these rows are in Q2 and the other two rows are in Q1.

If |Q|=5, then the alternating Q’s work.

Thus the matrix is TUM.

C Taking rows 2,3,5 and columns 2,3,5 we get

0 1 1

1 1 0

1 0 1

This has a determinant of –2 so it isn’t TUM

4th matrix look at row 2 and 4 column 2 and 3 and you get

1 1

1 -1 which has a determinat or –2 so it isn’t TUM.

**8.5 Show this matrix isn’t TUM but it has integer solutions if b is integer.**

The matrix isn’t TUM since it has the square submatrix

1 1

-1 1 which has a determinant of 2.

However solving

 Always has

x1=b3,

x2=b2+b3

x3=b1-b2-b3-b3= b1-b2-2b3.

Since the b’s are integers, the solution will always be integer. Thus, there exists matrices that have integer extreme points that are not TUM, which is why we study balanced and totally balanced matrices.