Generalizations
Part 2: Randomization and Blocking

STAT 705: Regression and Analysis of Variance

Properties of a Good Experiment

- There are two properties that we would like all experiments to have, regardless of the area of study.
  1. We would like bias to be small so that the experiment does not unfairly favor one treatment over another.
  2. We would like the random variation to be small so that the effects of the treatments can be more clearly seen.
- Randomization and blocking are statistical tools that can be used to reduce bias and reduce the adverse effects of random variability in experimentation.

Terminology of Experiments

- Experimentation is the process of applying treatments to experimental units and measuring the responses.
- The statistical design of an experiment is the plan that determines which experimental units go with which treatments.
- Randomization and blocking are essential elements in devising the plan for data collection.

An Agronomy Example

- We will work through this example to illustrate the two most common statistical designs.
- Suppose we wish to compare how four types of fertilizer affect the yields of wheat. We have a field with 12 plots (numbered 1 through 12) to use as our experimental units, as shown in the diagram.

1	**3**	**5**	**7**	**9**	**11**
2	**4**	**6**	**8**	**10**	**12**

A Completely Randomized Design

- In a CRD, experimental units are assigned randomly to the treatments.
- For our example, think of putting 12 numbers in a hat and drawing them out one by one. The first three would be assigned to treatment 1, the next three to treatment 2, etc.
- Here is one possible random assignment:

Treatment	Plots
1	11, 6, 4
2	9, 2, 3
3	5, 12, 8
4	10, 1, 7

Schematic of CRD

- This is the resulting plan that shows which treatments are applied to which plots.
- The agronomist would fertilize the plots according to this plan.
- Note that there are three plots (experimental units) that receive each treatment, so this design is balanced.

1 Trt = 4	3 Trt = 2	5 Trt = 3	7 Trt = 4	9 Trt = 2	11 Trt = 1
2 Trt = 2	4 Trt = 1	6 Trt = 1	8 Trt = 3	10 Trt = 4	12 Trt = 3

Why Randomize?

- We randomize to guard against biases that could occur because of unknown or uncontrollable differences among experimental units.
- For instance, if a fungus were dormant in the soil of several adjacent plots but this was not known at the start of the study, a systematic application of the treatments to the plots could results in all of one treatment being applied to the "bad" plots. Randomization helps avoid this as illustrated in Example 1 on the next two slides.

Example 1

- Suppose there is a hidden “bad” spot in one part the field as shown by the shaded area in the diagram below. If the agronomist assigned to treatments systematically to the plots (as shown below) treatments 1 and 3 would be adversely affected by the bad plots, resulting in negative biases for these treatments in comparison to the other two.

1 Trt = 1	3 Trt = 1	5 Trt = 1	7 Trt = 2	9 Trt = 2	11 Trt = 2
2 Trt = 3	4 Trt = 3	6 Trt = 3	8 Trt = 4	10 Trt = 4	12 Trt = 4

Example 1, continued

Even with the completely random design (shown below) any treatment that includes the bad spot would be adversely affected. However, the adverse effects would tend to randomly spread out among the treatments, so relative comparisons among the treatments would not be affected as much as they would be with the systematic design.

1 Trt = 4	3 Trt = 2	5 Trt = 3	7 Trt = 4	9 Trt = 2	11 Trt = 1
2 Trt = 2	4 Trt = 1	6 Trt = 1	8 Trt = 3	10 Trt = 4	12 Trt = 3

Homogeneous Experimental Units

- The CRD works best if the experimental units are homogeneous, that is, if they are as alike as possible.
- If this is not the case, differences among experimental units can cause the experimental error to be large, which make it difficult to obtain statistically significant differences among treatments.
- Unfortunately, in many situations such as field trials or experiments dealing with people or animals, variability among experimental units is unavoidable.
- This leads us to consider blocking.

Blocking

- Suppose that it is known that there are differences among the plots in our field experiment.
- For example, suppose that as we move from west to east across the field the plots tend to have increased moisture due to the lay of the land.
  - Even though the plots are not homogeneous as we would want for a CRD, we can still get precise comparisons using a device called blocking.
- A block is a group of experimental units that have the same characteristics.
  - Units within a block are homogeneous.
  - Units in different blocks may differ.

A Picture of Blocking in a Field

- This is a picture of plots that have been grouped together in blocks of 4, moving west to east across the field.
- Blocks are represented by different shading.
- Since we are assuming moisture varies from west to east, plots within a block should have similar moisture content.

West

1	3	5	7	9	11
2	4	6	8	10	12

East

Randomized Complete Block Design

- Abbreviated RCB
- Each block has the same number of experimental units as there are treatments.
- All treatments appear once in each block.
- Within each block, experimental units are randomly assigned to treatments.
- One such random assignment is shown below.

1 Trt = 2	3 Trt = 1	5 Trt = 4	7 Trt = 3	9 Trt = 4	11 Trt = 1
2 Trt = 3	4 Trt = 4	6 Trt = 1	8 Trt = 2	10 Trt = 3	12 Trt = 2

How Does an RCB Design Work?

- The experimental units within blocks are homogeneous, which gives us small within-block random variability.
- By using ANOVA in the right way, only the variability within blocks will contribute to the MSE (more on this in the next lesson).
- This allows us to get precise comparisons among treatments even though the experimental units as a whole have a lot of variability among them.

Other Uses of Blocking

- Blocking applies to more than just the physical characteristics of experimental units. It also applies to experimental conditions that change over time or place. We block in order to create experimental conditions within each block as consistent as possible.
- Example:
  - If we are baking bread using 3 recipes and we have two ovens for doing the baking, we may block on ovens. We do all three recipes with one oven (block one) and all three recipes with the other oven (block 2). If the experiment must be done over several days, we can block on days, doing all three recipes each day.

Factorial Treatments

- Both the CRD and the RCB can have factorial treatments. We form the treatments from the combinations of the factors, then assign the experimental units to the treatments according to either the CRD plan or the RCB plan.
- Suppose the fertilizer treatments in our agronomy experiment consist of combinations of Nitrogen (0, 10) and Phosphorus (0, 5). We form the 4 combinations as Trt 1 = (0N,0P), Trt 2 = (0N,5P), Trt 3 = (10N,0P), and Trt 4 = (10N,5P). Then we can use either or CRD or RCB plan for four treatments as the plan to determine which combinations of N and P are applied to which plots.

ANOVA

- The CRD is analyzed using the ANOVA tools we've already developed, i.e., one-way, two-way, three-way ANOVA, with contrasts and multiple comparisons as appropriate.
- The RCB can also be analyzed using the ANOVA tools we've developed. We simply consider Block as one of the factors.
  - There are additional issues involving blocks that we will deal with in the next lesson, but in essence we have the tools we need to analyze the RCB.

Other Experimental Designs

- There are many other experimental designs, including
  - Latin squares
  - incomplete block designs
  - split-plot designs
  - strip-plot designs
  - repeated measures designs
- These are discussed in an experimental design course (at K-State, this is STAT 720).
- All of these designs are used often in research at K-State.

Additive Block Effects

- In the analysis of the randomized complete block design (RCB) we will assume that the effect of the blocking factor is additive.
- In other words, we assume that, in going from one block to another the responses (on average) will either increase the same amount or decrease the same amount as a result of the changing block conditions, regardless of the effects of the treatments.
- Another way to say this is that there is no interaction between the blocks and the treatments.

Agronomy Study, Re-visited

- Recall the agronomy example from the previous lesson
- Suppose that, as we move from west to east, the soil moisture of the field becomes more favorable for growing wheat. If this causes the yields to increase the same amount (on average) regardless of the effects of the treatments, then the blocking factor is additive.

West

1 **Trt = 2**	**3** **Trt = 1**	**5** **Trt = 3**	**7** **Trt = 4**	**9** **Trt = 4**	**11** **Trt = 1**
2 **Trt = 3**	**4** **Trt = 4**	**6** **Trt = 1**	**8** **Trt = 2**	**10** **Trt = 3**	**12** **Trt = 2**

East

The Model

- The mathematical model for an RCB design can be expressed symbolically as

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

where

Y_{ij} is the response for the i^{th} EU in the j^{th} block

μ is the overall mean response (over all EUs and all blocks)

α_i is the effect of the i^{th} treatment

β_j is the effect of the j^{th} block

ε_{ij} is the random error for the i^{th} EU in the j^{th} block

The Model

- In words, the mathematical model for an RCB design can be expressed as:

  The response is . . .

  an overall effect . . .

  plus an effect due to treatment . . .

  plus an effect due to block . . .

  plus some random error

- This is the additive model for ANOVA where one of the factors is Treatment and the other factor is Block.

Generic SAS code for RCB

- Suppose that “trt” denotes the treatment variable and “blk” denotes the blocking variable.
- The Proc GLM statements for the RCB are the same as that for two-way ANOVA, except that the interaction terms (trt*blk) are not included.

```
proc glm;
 class trt blk;
 model response = trt blk / ss3;
 lsmeans trt blk / stderr pdiff;
 run;
```

Degrees of Freedom for RCB

- Suppose that there are “t” treatments and “b” blocks.
- Because treatments appear once in each block, the total number of observations in an RCB is tb.
  - Total df = tb – 1
  - df treatments = # treatments – 1 = t – 1
  - df blocks = # blocks – 1 = b – 1
  - df error = Total df – df treatments – df blocks

    = (tb – 1) – (t – 1) – (b – 1)

    = (t – 1)(b – 1)

The MSE in an RCB

- For every ANOVA, we must have a measure of experimental error, that is, we must have an MSE.
- Because we assume that the RCB has only one observation for each block by treatment combination, we cannot use the observations within blocks to compute MSE.
- However, because there is no interaction between blocks and treatments, the MS for block by treatment interaction is affected only by random error. Thus we may use the block by treatment mean square as the MSE in an RDB design.

Example

- Suppose that the observations in our agronomy example turned out to be as shown below.

	1 Trt = 2 39.4	3 Trt = 1 40.5	5 Trt = 3 43.0	7 Trt = 4 42.0	9 Trt = 4 46.1	11 Trt = 1 48.3	
West	2 Trt = 3 38.3	4 Trt = 4 38.1	6 Trt = 1 45.4	8 Trt = 2 44.1	10 Trt = 3 46.2	12 Trt = 2 47.0	East

Example: Data

- Here is the same data in a table
- Notice how much the responses vary in going from west to east for each treatment.
  - Treatment 1: 48.3 - 40.5 = 7.8
  - Treatment 2: 47.0 - 39.4 = 7.6
  - Treatment 3: 46.2 - 38.3 = 7.9
  - Treatment 4: 46.1 - 38.1 = 8.0

	Blocks		
	West	**Middle**	**East**
Trt 1	40.5	45.4	48.3
Trt 2	39.4	44.1	47.0
Trt 3	38.3	43.0	46.2
Trt 4	38.1	42.0	46.1

- The difference is about the same for each treatment, so an additive model (i.e. no block by treatment interaction) seems appropriate

Example: Ignore Blocking

- Suppose we ignore the blocking factor and do a one-way ANOVA on the data.
- Because of the large variability of the observations within treatments, it appears that there is not a significant difference between the treatments (p = 0.8442)

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	12.65	4.22	0.27	0.8442
Error	8	124.09	15.51		
Corrected Total	11	136.74			

Example: Incorporate Blocking

- When we account for the blocking factor and do an analysis of the blocked design, then we see that treatment is highly significant (p = 0.0002)

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Treatment	3	12.65	4.22	46.13	0.0002
Block	2	123.55	61.77	675.93	<.0001
Error	6	0.55	0.091		
Corrected Total	11	136.74			

Example: Compare Results

- Why is there such a big difference in the results?
- When blocks are ignored, the SS for block goes into SS Error.
  - This makes SS Error larger
  - SS Error goes into the denominator of the F statistic
  - This makes the F statistic smaller
  - F statistics are significant when they are larger (not smaller)
  - Larger SS Error makes it more difficult to find significant differences in the treatments
- For the agronomy example, SS Block is 123.55, which is almost all of the SS Error (124.09) in the one-way analysis
- The RCB analysis extracts this from SS Error, leaving us with the “correct” SS Error of 0.54 and a “large” F statistic of 46.13.

Factorial Treatments for RCB

- Suppose there are two factors A and B that comprise the factorial treatment combinations.

  For example, the four fertilizer treatments in our agronomy example might consist of combinations of Nitrogen (0, 10) and Phosphorus (0, 5).

- We may assume that the factors A and B interact with each other, so that an A*B term may be included in the model.
- However, there would be no interactions between the block and A or the blocks and B.

Generic SAS Code for RCB with Factorial Treatments

- Suppose the treatments consists of the factorial combinations of A and B, and the blocking variable is denoted 'blk'.
- The SAS GLM statements would look like this

```
proc glm;
 class A B blk;
 model response = A B A*B blk / ss3;
 lsmeans A B A*B blk / stderr pdiff;
 run;
```

- This could also be written

```
proc glm;
 class A B blk;
 model response = A|B blk / ss3;
 lsmeans A|B blk / stderr pdiff;
 run;
```

Other Cases of Blocking

- Incomplete Blocks

  It is possible to have fewer experimental units per block than there are treatments. For instance, there may be 4 treatments but only 3 experimental units per block. I recommend consulting your friendly statistician if you plan to use incomplete blocks in a study.

- Multiple Experimental Units per Block

  There may be more experimental units than there are treatments per block. For instance, there may be 4 treatments and 8 experimental units per block. Generally speaking, you can use the same GLM statements that you would use with an RCB.

What You Should Know

- Understand the difference between Completely Randomized and Randomized Complete Block experimental designs
- Recognize when blocking should be used in an experiment
- Understand how the randomization (assigning experimental units to treatments) is fundamentally different for CRD vs. RCB designs
- Understand how blocks and treatments are incorporated into the linear model and be able to
  - identify the degrees of freedom for blocks, treatments and error
  - write the SAS code
  - interpret the SAS output
  - write a report of the results