Example: Orange Juice Sweetness

STAT 705, Simple Linear Regression, Part 6

The quality of the orange juice produced by a manufacturer is constantly monitored. There are numerous sensory and chemical components that combine to make the best tasting orange juice. For example, one manufacturer has developed a quantitative index of the "sweetness" of orange juice. (The higher the index, the sweeter the juice.) Is there a relationship between the sweetness index and a chemical measure such as the amount of water-soluble pectin (in parts per million) in the orange juice? Data collected on these two variables for 24 production runs at a juice manufacturing plant are shown in the table. Suppose a manufacturer wants to use simple linear regression to predict the sweetness from the amount of pectin.

Run	SweetIndex	Pectin
1	5.2	220
2	5.5	227
3	6.0	259
4	5.9	210
5	5.8	224
6	6.0	215
7	5.8	231
8	5.6	268
9	5.6	239
10	5.9	212
11	5.4	410
12	5.6	256
13	5.8	306
14	5.5	259
15	5.3	284
16	5.3	383
17	5.7	271
18	5.5	264
19	5.7	227
20	5.3	263
21	5.9	232
22	5.8	220
23	5.8	246
24	5.9	241

Questions

1. Specify the equation for the linear regression model, including all subscripts.
2. If we fit a regression model to these data, which variable is the response (Y) and which is the predictor (X)?
3. Does a linear model seem reasonable for these data? Why or why not?
4. Does it appear that any of the assumptions of simple linear regression are violated? Explain.
5. Specify the estimated regression equation.
6. For the ANOVA F test
    a. What are the null and alternative hypotheses?
    b. Identify the value of the test statistic.
    c. Identify the value of the critical value.
    d. Identify the value of the p-value.
    e. Interpret the results of this test.
7. Consider the t test for H0: $\beta_1 = 0$ vs. Ha: $\beta_1 \neq 0$.
    a. Identify the value of the test statistic.
    b. Identify the value of the critical value.
    c. Identify the value of the p-value.
    d. Interpret the results of this test.
8. Consider a single run of orange juice that has 250 parts per million pectin.
    a. Provide a point estimate for the sweetness index for this run of orange juice.
    b. Provide an interval estimate for the sweetness index of this run of orange juice.
9. Consider all runs of orange juice that have 250 parts per million pectin.
    a. Provide a point estimate for the mean sweetness index of these runs of orange juice.
    b. Provide an interval estimate for the mean sweetness index of these runs of orange juice.