Generalizations
Part 1: Three-Way ANOVA

STAT 705: Regression and Analysis of Variance

Introduction

- There can be any number of factors in a designed experiment

- We have looked at one-factor and two-factor experiments

- We now turn our attention to three-factor experiments

- We still have an ANOVA table, F tests, t tests, main effects and interactions

- Look at the interactions first

Multiple Factor Studies

- Studies involving several factors are often focused on identifying how the factors work together to affect the outcome, as opposed to simply comparing the means of the combinations.

- Questions that are typically asked with ANOVA are:
  - What is the most important factor?
  - Can any factor be ignored?
  - Do any of the factors interact?

A Three Factor Example

- A study was done to determine factors that may affect the efficiency of a solar water heater.

- The factors are

  - Cap = the capacity of the water heater (80 or 120 gallons)

  - Flo = the flow rate of the water through the system (high or low)

  - Exp = the amount of exposure to sunlight (4 or 6 kilowatt hours per square meter per month)

- The response is Efficiency, denoted Eff

Solar Water Heater Data

Capacity	Flow	Exposure	Efficiency
120	high	6	41.6
120	high	6	41.3
120	high	4	39.9
120	high	4	39.7
120	low	6	51.9
120	low	6	52.4
120	low	4	43.0
120	low	4	44.9
80	high	6	39.2
80	high	6	38.4
80	high	4	37.5
80	high	4	35.0
80	low	6	50.2
80	low	6	51.3
80	low	4	41.3
80	low	4	43.5

There are two replicates for each combination of factors

Three-way Means

- There are two replicates for each combination of factors
- Average the two replicates to get the three-way means

Three-way Means			
Capacity	**Flow**	**Exposure**	**Mean**
120	high	6	41.45
120	high	4	39.80
120	low	6	52.15
120	low	4	43.95
80	high	6	38.80
80	high	4	36.25
80	low	6	50.75
80	low	4	42.40

Two-way Means

- These are the means for each of the combinations of the levels of two factors when averaged over the levels of the third factor

- Two-way means for Flow by Exposure are shown below

Three-way Means			
Capacity	**Flow**	**Exposure**	**Mean**
120	high	6	41.45
120	high	4	39.80
120	low	6	52.15
120	low	4	43.95
80	high	6	38.80
80	high	4	36.25
80	low	6	50.75
80	low	4	42.40

Two-way Means		
Flow	**Exposure**	**Mean**
high	6	40.125
high	4	38.025
low	6	51.450
low	4	43.175

For example . . .
for high Flow and Exposure 6, the two-way mean is
(41.45 + 38.80) / 2 = 40.125

One-way Means

- These are the means for the levels of one factor averaged over the levels of the other two

- One-way means for Cap are shown below

Three-way Means			
Capacity	**Flow**	**Exposure**	**Mean**
120	high	6	41.45
120	high	4	39.80
120	low	6	52.15
120	low	4	43.95
80	high	6	38.80
80	high	4	36.25
80	low	6	50.75
80	low	4	42.40

One-way Means	
Capacity	**Mean**
120	44.3375
80	42.0500

For example . . .
for Capacity 120, the one-way mean is
(41.45 + 39.80 + 52.15 + 43.95) / 4
= 44.3375

Generic 3-Way ANOVA Table

- Denote the factors by A, B, C

- The ANOVA table contains sums of squares, degrees of freedom, mean squares, F-statistics and p-values for

  - A, B, C main effects

  - Two-way interactions: A*B, A*C, and B*C

  - Three-way interaction: A*B*C

  - Error

ANOVA for Solar Water Heater

- The terms with p-values 0.05 or less are the main effects of Cap, Flo and Exp, and the two-way interaction Flo*Exp.

Source	DF	SS	MS	F	p-value
Cap	1	20.93	20.93	19.78	0.0021
Flo	1	271.43	271.43	256.52	<.0001
Cap*Flo	1	2.64	2.64	2.50	0.1528
Exp	1	107.64	107.64	101.73	<.0001
Cap*Exp	1	0.28	0.28	0.26	0.6236
Flo*Exp	1	38.13	38.13	36.04	0.0003
Cap*Flo*Exp	1	0.14	0.14	0.13	0.7249
Error	8	8.47	1.06		
Total	**15**	**449.65**			

Interpreting the ANOVA Table

- Use the significant terms in the ANOVA table to direct your attention to the most important means to look at

  - If the 3-way interaction is significant, look at the 3-way means

  - If the 3-way interaction is not significant, look at the 2-way means for any factors that have significant 2-way interactions

  - If a factor is not involved in a significant 3-way or 2-way interaction but has a significant main effect, then look at the one-way means involving this factor

Example

- The water heater ANOVA table shows that Cap is not involved in any significant interactions and it has a significant main effect.

- The main effect means (below) show that the capacity 120 water heaters have greater efficiency.

- Because there is no interaction involving Capacity, we can say that high capacity water heaters have greater efficiency regardless of the setting for flow rate or length of exposure.

One-way Means	
Capacity	Mean
120	44.3375
80	42.0500

Example, continued

- The factors Flo and Exp have a significant two-way interaction.

- We disregard their main effect means and go to the two-way means involving these factors.

- We see that low flow rate has higher efficiency regardless of the exposure, but the largest efficiency occurs when low flow rate is combined with a high exposure.

Two Way Means for Flo*Exp

Flo	Exp	Mean Eff
high	4	38.025
high	6	40.125
low	4	43.175
low	6	51.450

Example, continued

- We still have not determined which means are statistically significantly different from the others. A SAS analysis will show this.

- Regardless, we have able to figure out the essential features of the data by looking at the ANOVA table and the means that the ANOVA table tells us to look at.

- In summary
  - A capacity 120 heater is better (more efficient) than capacity 80.
  - A low flow rate is advantageous especially when combined with a solar collector that is exposed to sunlight for a longer time.

What You Should Know

- Be able to interpret a 3-way ANOVA table to decide which means (one-way, two-way, or three-way) are the appropriate ones to examine

- Be able to summarize your findings

The complete SAS code for the water heater data is available on the course website, along with its annotated output.